Event Generators and Perturbation Theory

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Event generators have proven very useful for understanding signals and backgrounds for large-Q^2 processes at colliders.

Two general classes, both based on perturbative QCD (pQCD):

- Fixed order matrix element: VECBOS, ...
- Parton shower: HERWIG, ISAJET, PYTHIA, ...

Will not discuss details of any of these.

Will try to answer two questions:

- Why does pQCD work for inclusive cross sections?
- Why does leading-order pQCD not work for event structure, and how do parton shower Monte Carlos fix this?

Preliminaries

QCD Lagrangian for massless quarks contains one dimensionless parameter g, so naively scale invariant. Structure at all scales.

But have ultraviolet divergences. Must introduce subtraction scale μ to define g.

Physics cannot depend on μ. Hence must satisfy renormalization group equation:

$$\mu \frac{d}{d\mu}\left(Z\Gamma\left(p, g, \mu\right)\right) = \left[\mu \frac{\partial}{\partial \mu} + \beta\left(g\right)\frac{\partial}{\partial g} + \gamma\right]\left(Z\Gamma\right) = 0$$

Running coupling $g(\mu^2)$ satisfies

$$\mu \frac{dg}{d\mu} = -g\left[b_0\left(\frac{\alpha_s}{4\pi}\right) + b_1\left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots\right]$$

$$b_0 = 11 - 2n_f/3$$

$$b_1 = 102 - 38n_f/3$$

The solution is

$$\frac{\alpha_s\left(\mu^2\right)}{4\pi} = \frac{1}{b_0 \ln\left(\mu^2/\Lambda^2\right)} - \frac{b_1 \ln\ln\left(\mu^2/\Lambda^2\right)}{b_0^3 \ln^2\left(\mu^2/\Lambda^2\right)}$$

Hence QCD is asymptotically free. For large Q^2 processes can hope to use perturbation theory with $\mu \sim Q$.

Conversely, α_s becomes strong at small μ. Quarks and gluons are confined.

Consider hadronic collision AB producing large-Q^2 process with initial partons (quarks or gluons).

- Interaction range is $\sim 1/Q$, so only one parton from each hadron participates.
- Interaction time is $\lesssim 1/Q$, so partons cannot rearrange themselves. Hence depend only on incoming hadron.

Hence expect (QCD-improved) parton model:

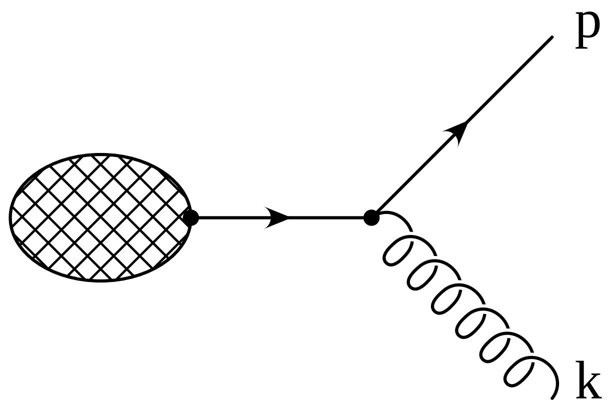
$$\sigma_{AB} = \sum_{ij} \int dx dx' \, \hat{\sigma}_{ij} \, f_{i/A}\left(x, Q^2\right) f_{j/B}\left(x', Q^2\right)$$

where $\hat{\sigma}$ is given by pQCD. Must show this is consistent with perturbation theory.

Infrared Singularities

pQCD would obviously produce series in α_s if Feynman diagrams were always finite.

Singularities arise from ultraviolet divergences and from internal propagators going on shell. This happens with massless on-shell external particles:



Two kinds of singularities:

Infrared: $p^2 = m^2$, $k \to 0 \Rightarrow (p+k)^2 = 0$.

Collinear: $p^2 = k^2 = 0$, $\theta_{pk} \to 0 \Rightarrow (p+k)^2 = 0$.

Want to show (to leading order in $1/Q^2$):

- Infrared singularities cancel.
- Collinear singularities factorize.

Physical origin of infrared singularity is radiation of infinite number of soft gluons from colored external particles. Just like QED case.

Probability of radiating zero soft gluons is

$$e^{-\bar{n}} \to 0$$

so all exclusive cross sections vanish.

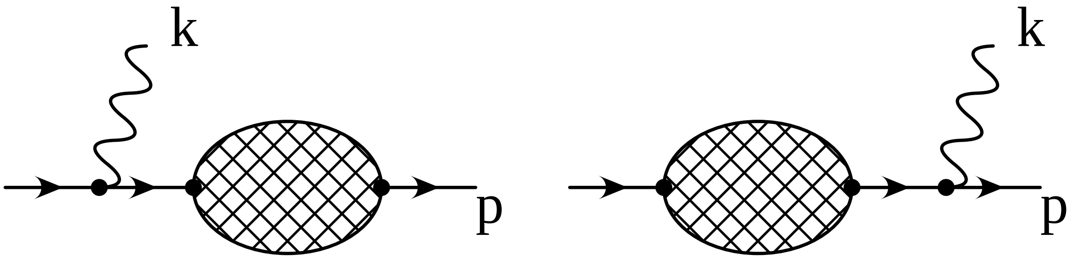
Color neutral hadrons cut off radiation, but still get expansion in

$$\alpha_s\left(Q^2\right) \ln Q^2/m_{\text{had}}^2 \sim \mathcal{O}\left(1\right)$$

Probability of radiating any number of soft gluons is one. Hence cross sections suitably averaged over soft radiation are finite [Kinoshita, Lee, Nauenberg]

Cancellation is between real and virtual graphs. Need to approximate emission amplitude and then do integrals.

Algebra involvs approximating matrix element and then doing integrals. First consider QED. [Weinberg]



Amplitude to radiate soft photon independent of spin, e.g.

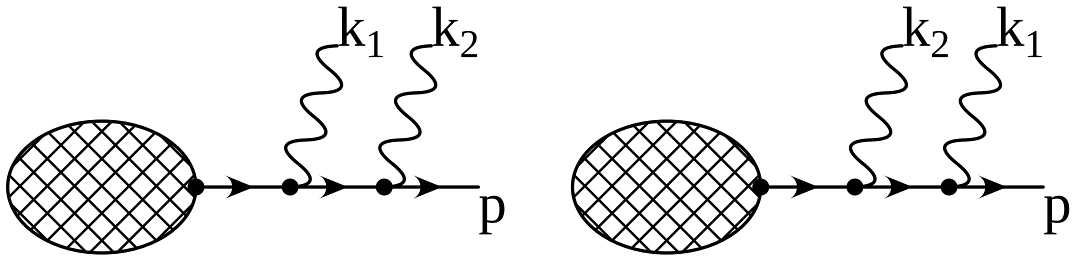
$$\bar{u}\left(p'\right)\gamma_\mu u\left(p\right) = \frac{\left(p+p'\right)_\mu}{2m}\bar{u}\left(p'\right)u\left(p\right) + \frac{i}{2m}\bar{u}\left(p'\right)\sigma_{\mu\nu}\left(p-p'\right)^\nu u\left(p\right) \rightarrow \frac{p_\mu}{m}$$

Inserting one photon gives

$$\mathcal{M}_1 \sim \mathcal{M}_0\left(-ie\right)\left(2p^\mu + \eta k^\mu\right)\frac{i}{\left[\left(p+\eta k\right)^2 - m^2 + i\epsilon\right]} \sim \mathcal{M}_0\frac{e\eta p^\mu}{\left[p\cdot k + i\eta\epsilon\right]},$$

where $\eta = \pm 1$ for final or initial line.

Two photons:



$$\mathcal{M}_2 \sim \mathcal{M}_0 \frac{e\eta p^{\mu_1}}{[p \cdot k_1 + i\eta\epsilon]} \frac{e\eta p^{\mu_2}}{[p \cdot (k_1 + k_2) + i\eta\epsilon]} + (1 \leftrightarrow 2)\,.$$

When this is summed over permutations the result is

$$\mathcal{M}_2 \sim \mathcal{M}_0 \frac{e\eta p^{\mu_1}}{[p \cdot k_1 + i\eta\epsilon]} \frac{e\eta p^{\mu_2}}{[p \cdot k_2 + i\eta\epsilon]}.$$

For N photons, get N factors:

$$\mathcal{M}_N \sim \mathcal{M}_0 \prod_{m,i} \frac{e_m \eta_m p_m^{\mu_i}}{[p_m \cdot k_i + i\eta_m\epsilon]}.$$

“Eikonal form.” Same form holds for gluons; non-Abelian graph cancels commutator of color matrices.

Now do the integrals. For virtual photons, tie soft lines together with propagators:

$$\mathcal{M}_{\text{vir.}} = \mathcal{M}_0 \sum_{nm} \frac{1}{2^N N!} \sum_{i=1}^{N} \int \frac{d^4 k_i}{(2\pi)^4} \frac{-i}{k_i^2 + i\epsilon} \times \frac{e_n e_m \eta_n \eta_m (p_n \cdot p_m)}{[p_n.k_i + i\eta_n \epsilon]\,[-p_m.k_i + i\eta_m \epsilon]}$$

Combinatorial factor $2^N N!$ comes from identical particles and permutations of ends.

Do sum to get exponential.

Do $k_{i,0}$ integral by picking up pole at

$$k_i^2 = k_{i,0}^2 - \vec{k}_i^2 = 0$$

Result is

$$\mathcal{M}_{\text{vir.}} \sim \mathcal{M}_0 \exp\left\{ \frac{-1}{2\,(2\pi)^3} \sum_{nm} \int_{\lambda}^{\Lambda} \frac{d^3 k}{2|k|} \times \frac{e_n e_m \eta_n \eta_m (p_n \cdot p_m)}{[p_n.k + i\eta_n \epsilon]\,[-p_m.k + i\eta_m \epsilon]} \right\}$$

where λ is infrared cutoff and Λ is cutoff for soft approximation.

Hence sum of soft virtual graphs gives

$$\mathcal{M}_{\text{vir.}} \sim \mathcal{M}_0 \left(\frac{\lambda}{\Lambda}\right)^{A/2}$$

$$A = \frac{-1}{8\pi^2} \sum_{n,m} \eta_n \eta_m e_n e_m \frac{1}{\beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right) > 0$$

$$\beta_{nm} = \left[1 - \frac{m_n^2 m_m^2}{(p_n \cdot p_m)^2}\right]^{1/2}$$

Physical interpertation: As cutoff $\lambda \to 0$, multiplicity of soft photons (gluons) diverges, so probability of zero emission vanishes.

Must be cancelled by real emission. Integrate eikonal amplitude over phase space:

$$\Gamma_{N,\,\text{real}} = \Gamma_0 \sum_{m,n} \int \prod_{i=1}^{N} \frac{d^3 k_i}{(2\pi)^3\, 2|k_i|} \frac{\eta_n \eta_m e_n e_m \,(p_n \cdot p_m)}{[p_n \cdot k_i]\,[p_m \cdot k_i]},$$

Angular integral gives A above, so

$$\Gamma_{N,\,\text{real}}(\omega_1, \ldots, \omega_N) = \Gamma_0 \frac{A^N}{N!} \int \frac{d\omega_1}{\omega_1} \cdots \frac{d\omega_N}{\omega_N}.$$

Integrate over smooth region $\sum \omega_i < E$:

$$\Gamma_{\text{real}}\left(\leq E\right) = \Gamma_0 \sum_N \frac{A^N}{N!} \int_\lambda^E \prod_{i=1}^N \frac{d\omega_i}{\omega_i} \vartheta\left(E - \sum_i \omega_i\right)$$

$$= \Gamma_0 \sum_N \frac{A^N}{N!} \int_\lambda^E \prod_{i=1}^N \frac{d\omega_i}{\omega_i} \int_{-\infty}^{\infty} d\sigma$$

$$\times \frac{\sin E\sigma}{\sigma} \exp\left(i\sigma \sum_i \omega_i\right)$$

Then

$$\Gamma_{\text{real}}\left(\leq E\right) = \Gamma_0 \left(\frac{E}{\lambda}\right)^A b\left(A\right)$$

$$b\left(x\right) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\sigma \frac{\sin\sigma}{\sigma} \exp\left[x \int_0^1 \frac{d\omega}{\omega}\left(e^{i\omega\sigma} - 1\right)\right]$$

Full result is

$$\Gamma\left(\leq E\right) = \Gamma_0 \left(\frac{E}{\Lambda}\right)^A b\left(A\right).$$

independent of cutoff λ.

Conclusion: Infrared singularities cancel between real and virtual graphs for *inclusive* cross sections in QED.

Same result holds for QCD to order $1/Q^2$, but general proof is complex. [Collins, Soper, Sterman; ...]

Exclusive cross sections are infrared divergent beyond leading order.

Typically, cross section for extra gluon diverges as cutoff λ vanishes. Virtual graphs give λ-dependent negative δ function.

Can carry out cancellation analytically for particular inclusive cross sections.

In general, must keep finite infrared cutoffs. Produces large cancellations and numerical problems for generators beyond leading order in α_s.

Collinear Singularities

Collinear (or mass) singularities do not cancel, but they are *universal.*

Can be *consistently* absorbed into parton distributions.

Produce calculable Q^2 dependence described by Altarelli-Parisi equations.

Altarelli-Parisi derivation is deceptively simple. Consider deep inelastic scattering, and let

$$\tilde{f}_i\left(n, Q^2\right) = \int_0^1 dx\, x^{n-1} f_i\left(x, Q^2\right)$$

By the operator product expansion, the moments for the non-singlet distribution

$$f_{NS}\left(x, Q^2\right) = \sum_q \left[f_q\left(x, Q^2\right) - f_{\bar{q}}\left(x, Q^2\right)\right]$$

satisfy

$$\tilde{f}_{NS}\left(n, Q^2\right) = \tilde{f}NS\left(n, Q_0^2\right)\left[\frac{\alpha_s\left(Q_0^2\right)}{\alpha_s\left(Q^2\right)}\right]^{\gamma_{NS}/2\pi b_0}$$

But a product in Mellin transform space implies a convolution. Hence

$$\frac{df_{NS}\left(x,Q^2\right)}{d\ln Q^2} = \frac{\alpha_s\left(Q^2\right)}{2\pi}\int_x^1 \frac{dy}{y} f_{NS}\left(y,Q^2\right) \times \left[P\left(\frac{x}{y}\right) + c\delta\left(1-\frac{x}{y}\right)\right]$$

Determine $P(z)$ from singular part of 1-gluon emission. Determine c from conservation laws (or virtual graphs).

Can extend to non-singlet distributions.

Altarelli-Parisi derivation is simple and elegant, but hides what is really happening.

Solution corresponds to sum of ladder graphs; gives leading collinear singularities.

Each extra gluon $\Rightarrow$ factor in cross section. Hence QCD reduces (approximately) to classical Markov process, allowing parton shower algorithm.

Convenient to express deep inelastic scattering as discontinuity of forward photon-quark scattering amplitude $W^{\mu\nu}$:

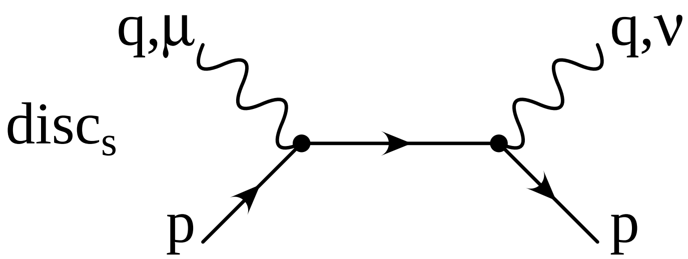
$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \left(p^\mu - \frac{\nu q^\mu}{q^2}\right)\left(p^\nu - \frac{\nu q^\nu}{q^2}\right) W_2$$

where W_1 and W_2 are functions of

$$\nu = p \cdot q, \quad q^2 = q \cdot q = -Q^2$$

Bjorken limit is $Q^2 \to \infty$ with $x = Q^2/2\nu$ fixed.

Lowest order graph:



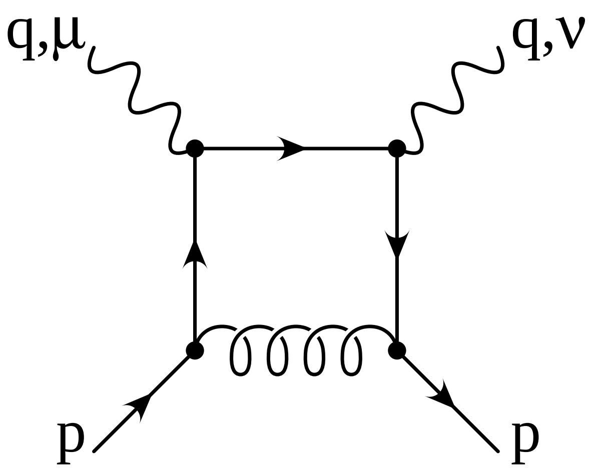
$$W^{\mu\nu} = \frac{1}{4} e^2 \delta\left((p+q)^2\right) \mathrm{Tr}\left[\not{p}\gamma^\mu\left(\not{p}+\not{q}\right)\gamma^\nu\right]$$
$$= e^2 \delta\left(2\nu - Q^2\right)\left[p^\mu (p+q)^\nu + p^\nu (p+q)^\mu - g^{\mu\nu} p\cdot q\right]$$

Compute $-g_{\mu\nu}W^{\mu\nu}$ and $p_\mu p_\nu W^{\mu\nu}$ for this and for W_i definition to find standard result:

$$\nu W_2 = 2xW_1$$
$$W_1 = \frac{1}{2}e^2\delta\left(1 - \frac{Q^2}{2\nu}\right)$$

Higher orders: Power counting suggests 1-particle exchange in amplitude (2-particle in $W^{\mu\nu}$) dominate.

Consider next [Dokshitser]



Convenient to work in momentum space with Sudakov variables:

$$k = \alpha q' + \beta p + k_T$$
$$q' = q + xp, \qquad q'^2 \approx p^2 \approx 0$$

To start, use Feynman gauge:

$$\mathcal{M}_{\mu\nu} = g^2\nu \int d\alpha d\beta d^2k_T \ \mathrm{Tr}\left[\not{p}\gamma^\sigma \not{k}\gamma_\nu \left(\not{k}+\not{q}\right)\gamma_\mu \not{k}\gamma_\tau\right] g^{\sigma\tau}$$
$$\times \frac{1}{\left[2\alpha\beta\nu + k_T^2 + i\epsilon\right]^2} \frac{1}{\left[2\left(\alpha+1\right)\left(\beta - x\right)\nu + k_T^2 + i\epsilon\right]}$$
$$\times \frac{1}{\left[-2\alpha\left(1-\beta\right)\nu + k_T^2 + i\epsilon\right]}$$

Do α contour integral, picking up pole at

$$\alpha = \frac{k_T^2}{2\left(1-\beta\right)\nu} + i\epsilon, \qquad 0 < \beta < 1$$

Contribution of order $\alpha_s \ln Q^2$ comes from integrating over

$$\mu^2 \ll k_T^2 \ll Q^2$$

so neglect $k_T^2 \ll \nu$ and $\alpha \ll 1$. Hence

$$\mathcal{M}_{\mu\nu} \approx i\pi g^2\nu \int_0^1 d\beta \int d^2k_T \ \mathrm{Tr}\left[\not{p}\gamma^\sigma \not{k}\gamma_\nu \left(\not{k}+\not{q}\right)\gamma_\mu \not{k}\gamma_\tau\right] g^{\sigma\tau}$$
$$\times \frac{\left(1-\beta\right)^2}{k_T^4} \frac{1}{\left[2\left(\beta - x\right)\nu + i\epsilon\right]} \frac{1}{\left[2\left(1-\beta\right)\nu\right]}$$

Next simplify trace. Note that

$$g_{\sigma\tau} = g^{\perp}_{\sigma\tau} + \frac{1}{\nu}\left(p_\sigma q'_\tau + p_\tau q'_\sigma\right)$$

where $g^{\perp}_{\sigma\tau}$ is the polarization for transverse gluons:

$$p^\sigma g^{\perp}_{\sigma\tau} = q^\sigma g^{\perp}_{\sigma\tau} = 0.$$

Hence

$$\begin{aligned}
\mathrm{Tr}\left[\ldots\right] &= \mathrm{Tr}\left[\not{p}\gamma^{\perp\sigma}\not{k}_T\gamma_\nu\left(\not{k}+\not{q}\right)\gamma_\mu\not{k}_T\gamma^{\perp}_\sigma\right] \\
&= \mathrm{Tr}\left[\gamma^{\perp\sigma}\not{k}_T\not{p}\gamma_\nu\left(\not{k}+\not{q}\right)\gamma_\mu\not{k}_T\gamma^{\perp}_\sigma\right] \\
&= 2k_T^2\,\mathrm{Tr}\left[\not{p}\gamma_\nu\left(\not{k}+\not{q}\right)\gamma_\mu\right] \\
&= -2k_T^2\nu g^{\perp}_{\mu\nu}
\end{aligned}$$

since $\gamma^{\perp\sigma}\gamma^{\perp}_\sigma = 2$.

Thus leading contribution from this graph is

$$\begin{aligned}
\mathcal{M}_{\mu\nu} &\approx 2i\pi g^2 g^{\perp}_{\mu\nu}\int_0^1 d\beta\int_{\mu^2}^{Q^2}\frac{d\left(-k_T^2\right)}{-k_T^2}\,\frac{1-\beta}{\beta-x+i\epsilon}, \\
&= 2i\pi g^2 g^{\perp}_{\mu\nu}\ln\left(\frac{Q^2}{\mu^2}\right)\int_0^1 d\beta\frac{1-\beta}{\beta-x+i\epsilon}
\end{aligned}$$

(Gasp.)

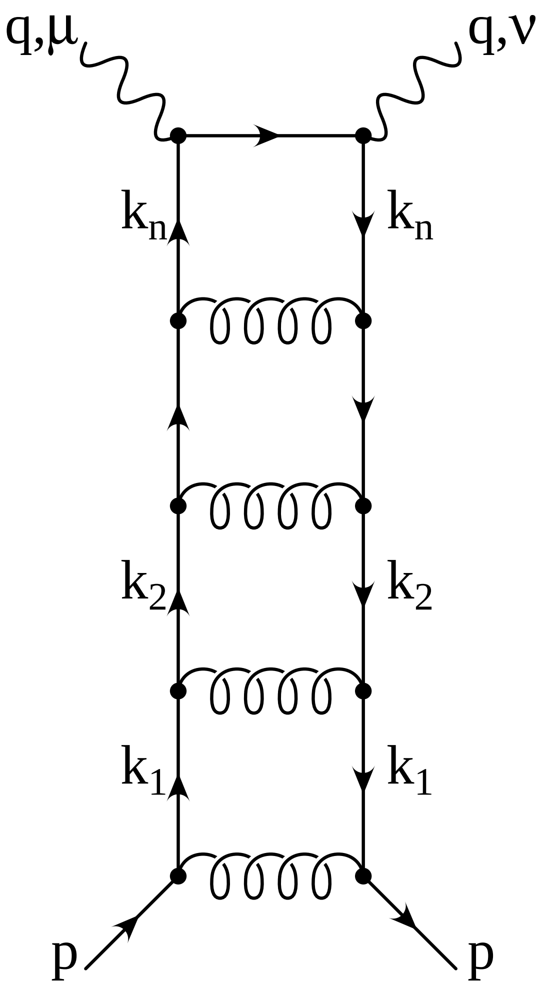
General analysis in Feynman gauge is hard because unphysical degrees of freedom propagate. Use instead axial gauge:

$$q'^{\mu} A_{\mu}(k) = 0,$$

where $q' = q + xp$. Propagator is

$$D_{\mu\nu}(k) = \frac{1}{k^2 + i\epsilon}\left[g_{\mu\nu} - \frac{k_\mu q'_\nu + q'_\mu k_\nu}{k \cdot q'}\right]$$

Also no ghosts.



Find in axial gauge:

- Ladder graph with N rungs give $(\alpha_s \ln Q^2)^N$ from the integration region:

$$\mu^2 \ll k_{1,T}^2 \ll k_{2,T}^2 \ll \ldots \ll k_{n,T}^2 \ll Q^2$$

- Crossed ladder graphs are subleading — fewer large k_T^2 intervals.
- Graphs with gluons between sides and rungs are subleading — extra k_T factors from vertex.
- Propagator and vertex corrections produce $\alpha_s(k_T^2)$, so each k_T integral gives factor of

$$\xi = \frac{1}{4\pi} \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \alpha_s\left(k^2\right) = \frac{1}{b_0} \ln\left(\frac{\alpha_s\left(\mu^2\right)}{\alpha_s\left(Q^2\right)}\right)$$

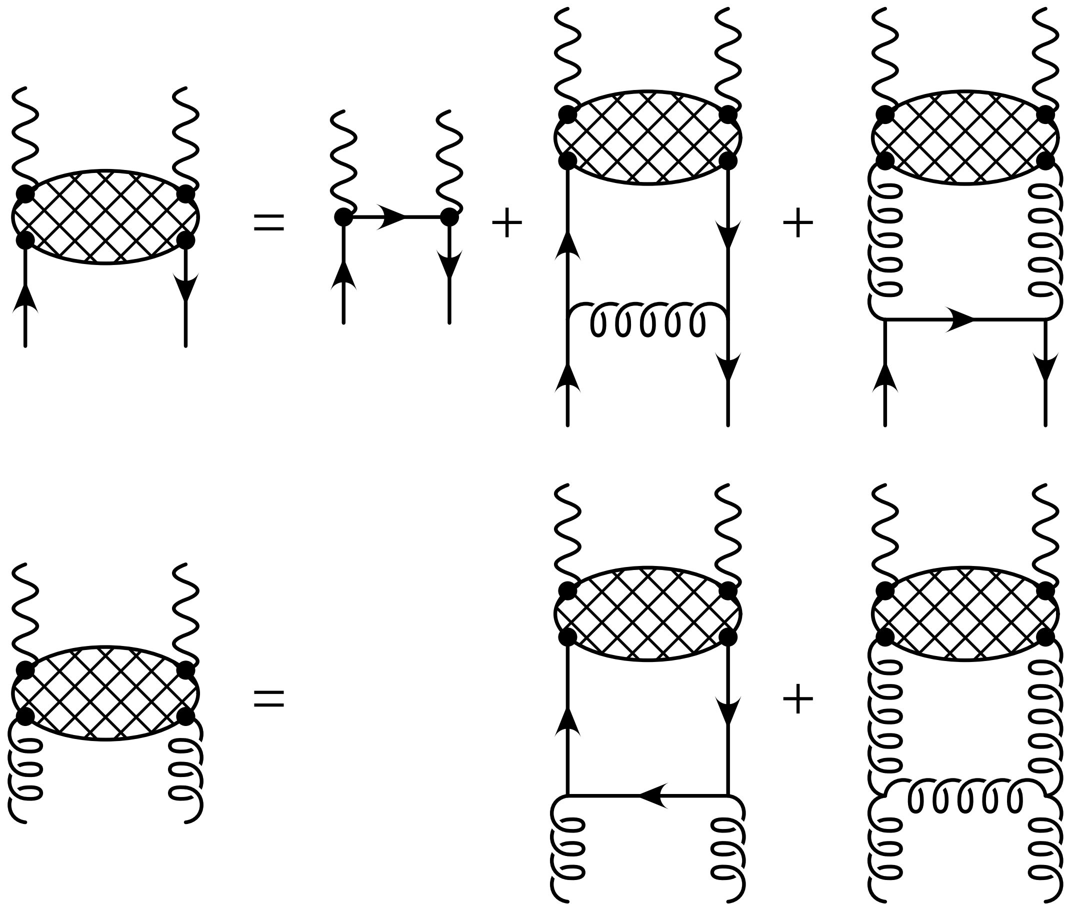
where $b_0 = 11 - 2/3 n_f$.

Conclusion: need only sum ladder graphs with dressed vertices. [Dokshitser]

Can derive integral equation for ladder graphs. Let $W_q^{\mu\nu}$ and $W_q^{\mu\nu}$ be sums with external quark and gluon lines.

$$W_q = -g_{\mu\nu} W_q^{\mu\nu}, \qquad W_g = -g_{\mu\nu} W_g^{\mu\nu}.$$

Then



Write loop integrals with Sudakov variables, do α integrals, and replace k_T integrals with ξ integrals. Result is

$$W_q(x,\xi) = e_q^2\delta(1-x) + 2\int_{\xi_0}^{\xi} d\xi' \int_0^1 dx'\, P_{qq}(x')\, W_q\left(\frac{x}{x'},\xi\right) + 2\int_{\xi_0}^{\xi} d\xi' \int_0^1 dx'\, P_{qg}(x')\, W_g\left(\frac{x}{x'},\xi\right)$$

$$W_g(x,\xi) = 2\int_{\xi_0}^{\xi} d\xi' \int_0^1 dx'\, P_{gq}(x') \sum_q W_q\left(\frac{x}{x'},\xi\right) + 2\int_{\xi_0}^{\xi} d\xi' \int_0^1 dx'\, P_{gg}(x')\, W_g\left(\frac{x}{x'},\xi\right)$$

Determine P_i's by calculating box graphs in axial gauge. For first term

$$\mathcal{M}_{\mu\nu} = \frac{g^2\nu c_F}{(2\pi)^4}\int d\alpha d\beta d^2k_T\, \mathrm{Tr}\left[\not{p}\gamma^\sigma \not{k}\gamma_\nu(\not{k}+\not{q})\gamma_\mu \not{k}\gamma_\tau\right] \times \left[g^{\sigma\tau} - \frac{(p-k)^\sigma q'^\tau + q'^\sigma (p-k)^\tau}{(p-k)\cdot q'}\right] \times \frac{1}{\left[2\alpha\beta\nu + k_T^2 + i\epsilon\right]^2}\frac{1}{\left[2(\alpha+1)(\beta-x)\nu + k_T^2 + i\epsilon\right]} \times \frac{1}{\left[-2\alpha(1-\beta)\nu + k_T^2 + i\epsilon\right]}$$

where $c_F = 4/3$ is the fundamental Casimir.

Do α integration and simplify trace plus propagators to

$$-2k_T^2 \nu g_{\mu\nu}^{\perp} \left(\frac{1+\beta^2}{1-\beta} \right).$$

After some algebra

$$\mathcal{M}_{\mu\nu} = \frac{4i\pi}{(2\pi)^4} c_F g_{\mu\nu}^{\perp} \int_0^1 d\beta \int_{\mu^2}^{Q^2} \frac{dk_T^2}{k_T^2} \bar{g}^2 \left(k_T^2\right) \left(\frac{1+\beta^2}{1-\beta} \right) \times \frac{1}{\beta - x + i\epsilon}$$

Taking the imaginary part and comparing with the integral equation finally gives

$$P_{qq}(\beta) = c_F \left(\frac{1+\beta^2}{1-\beta} \right)$$

Similarly, the other three box graphs give

$$P_{gq}(\beta) = P_{qq}(1-\beta)$$
$$P_{qg}(\beta) = \frac{1}{2} \left(\beta^2 + (1-\beta)^2 \right)$$
$$P_{gg}(\beta) = 2c_A \left(\frac{\beta}{1-\beta} + \frac{1-\beta}{\beta} + \beta(1-\beta) \right)$$

where $c_A = 3$ is the adjoint Casimir.

Have been sloppy about infrared singularities.

$P_{qq}(\beta)$ and $P_{gg}(\beta)$ have $1/(1-\beta)$ poles from soft gluons. Replace by distribution

$$\int d\beta\, \frac{1}{(1-\beta)_+} f(\beta) \equiv \int d\beta\, \frac{1}{(1-\beta)} \left[f(\beta) - f(1)\right]$$

and add $\delta(1-\beta)$ terms for virtual graphs:

$$P_{qq}(\beta) \Rightarrow c_F \left(\frac{1+\beta^2}{(1-\beta)_+}\right) + a_{qq}\delta(1-\beta)$$

$$P_{gg}(\beta) \Rightarrow 2c_A \left(\frac{\beta}{(1-\beta)_+} + \frac{1-\beta}{\beta} + \beta(1-\beta)\right) + a_{gg}\delta(1-\beta)$$

Momentum sum rule for all Q^2 requires

$$\int d\beta\, \left[P_{qq}(\beta)_+ + a_{qq}\delta(1-\beta) + P_{gq}(\beta)\right] = 0$$

$$\int d\beta\, \left[2n_f P_{qg}(\beta) + P_{gg}(\beta)_+ + a_{gg}\delta(1-\beta)\right] = 0$$

which determines δ function terms:

$$a_{qq} = \frac{3}{2}c_F, \qquad a_{gg} = \left(\frac{11}{6}c_A - \frac{1}{3}n_f\right)$$

QCD Branching

Can draw several implications from derivation of Altarelli-Parisi (A-P) equations.

(1) Collinear gluon radiation is important; A-P equations sum up terms of the form $[\alpha_s(Q^2)\ln Q^2]^n$.

(2) Each extra quark or gluon emission contributes to the cross section a factor

$$\sigma_{n+1} \sim \sigma_n \otimes \frac{\alpha_s\left(k^2\right)}{2\pi k^2} P_{ij}\left(z\right)$$

in the limit $k^2 \to 0$.

(3) Dominant region comes from strongly order $k^2 \sim k_T^2$. Hence radiations occur on all scales and are independent.

Thus, QCD is approximately equivalent to a classical Markov branching process.

This makes parton shower Monte Carlos possible.

Basic approximation:

- Start with some hard scattering process, generally to lowest non-trivial order
- Add additional QCD radiation using collinear singularities from A-P equations plus exact non-collinear kinematics.
- Stop at some cutoff and hadronize partons (and beam jets).

Approximation should be fairly good near collinear singularities, e.g., for low-p_T jets from initial state or for shape of jets.

Thus branching correctly describes dominant region for QCD radiation.

Approximation is poor for radiation of widely separated jets with p_T comparable to initial hard scattering. Need correct QCD matrix elements to describe these.