Summary

In summary, the development of some computational algorithms in two- and three-dimensions have been presented. Details of two computational codes, ARC2D and ARC3D have been presented. The basic algorithm used is the Beam and Warming implicit approximate factorization scheme or variants of that scheme such as the diagonalization. The codes employ improvements to enhance accuracy, (grid refinement, better boundary conditions, more versatile artificial dissipation model) and efficiency (diagonal algorithm, implicit treatment of artificial dissipation terms, variable time steps). Results for a wide variety of cases substantiate the accuracy and efficiency claims.

Future work is required to address improvement of boundary conditions, examining stability questions, eliminating cross term errors and more. We are also interested in developing new grid generation concepts both in terms of generation and grid quality. In three dimensions we see the area of zonal concepts as the newest horizon and envision substantial gains in solution capability as a result.

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Appendix

The flux Jacobian matrices of Eq. (5.4) have real eigenvalues and a complete set of eigenvectors. The similarity transforms are

$$\widehat{A} = T_\xi \Lambda_\xi T_\xi^{-1} \quad \text{and} \quad \widehat{B} = T_\eta \Lambda_\eta T_\eta^{-1} \tag{A.1}$$

where

$$\Lambda_\xi = \begin{bmatrix} U & & & \\ & U & & \\ & & U + a\sqrt{\xi_x^2 + \xi_y^2} & \\ & & & U - a\sqrt{\xi_x^2 + \xi_y^2} \end{bmatrix}, \tag{A.2a}$$

$$\Lambda_\eta = \begin{bmatrix} V & & & \\ & V & & \\ & & V + a\sqrt{\eta_x^2 + \eta_y^2} & \\ & & & V - a\sqrt{\eta_x^2 + \eta_y^2} \end{bmatrix}, \tag{A.2b}$$

with

$$T_\kappa = \begin{bmatrix} 1 & 0 & \alpha & \alpha \\ u & \widetilde{\kappa}_y \rho & \alpha(u + \widetilde{\kappa}_x a) & \alpha(u - \widetilde{\kappa}_x a) \\ v & -\widetilde{\kappa}_x \rho & \alpha(v + \widetilde{\kappa}_y a) & \alpha(v - \widetilde{\kappa}_y a) \\ \frac{\phi^2}{(\gamma-1)} & \rho(\widetilde{\kappa}_y u - \widetilde{\kappa}_x v) & \alpha\left[\frac{\phi^2 + a^2}{(\gamma-1)} + a\widetilde{\theta}\right] & \alpha\left[\frac{\phi^2 + a^2}{(\gamma-1)} - a\widetilde{\theta}\right] \end{bmatrix} \tag{A.3}$$

$$T_\kappa^{-1} = \begin{bmatrix} (1 - \phi^2/a^2) & (\gamma-1)u/a^2 & (\gamma-1)v/a^2 & -(\gamma-1)/a^2 \\ -(\widetilde{\kappa}_y u - \widetilde{\kappa}_x v)/\rho & \widetilde{\kappa}_y/\rho & -\widetilde{\kappa}_x/\rho & 0 \\ \beta(\phi^2 - a\widetilde{\theta}) & \beta[\widetilde{\kappa}_x a - (\gamma-1)u] & \beta[\widetilde{\kappa}_y a - (\gamma-1)v] & \beta(\gamma-1) \\ \beta(\phi^2 + a\widetilde{\theta}) & -\beta[\widetilde{\kappa}_x a + (\gamma-1)u] & -\beta[\widetilde{\kappa}_y a + (\gamma-1)v] & \beta(\gamma-1) \end{bmatrix} \tag{A.4}$$

and $\alpha = \rho/(\sqrt{2}a),\ \beta = 1/(\sqrt{2}\rho a),\ \widetilde{\theta} = \widetilde{\kappa}_x u + \widetilde{\kappa}_y v,$ and, forexample, $\widetilde{\kappa}_x = \kappa_x/\sqrt{\kappa_x^2 + \kappa_y^2}$.

Relations exist between T_ξ and T_η of the form

$$\widehat{N} = T_\xi^{-1} T_\eta, \quad \widehat{N}^{-1} = T_\eta^{-1} T_\xi \tag{A.5}$$

where

$$\widehat{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & m_1 & -\mu m_2 & \mu m_2 \\ 0 & \mu m_2 & \mu^2(1+m_1) & \mu^2(1-m_1) \\ 0 & -\mu m_2 & \mu^2(1-m_1) & \mu^2(1+m_1) \end{bmatrix} \qquad (A.6a)$$

and

$$\widehat{N}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & m_1 & \mu m_2 & -\mu m_2 \\ 0 & -\mu m_2 & \mu^2(1+m_1) & \mu^2(1-m_1) \\ 0 & \mu m_2 & \mu^2(1-m_1) & \mu^2(1+m_1) \end{bmatrix} \qquad (A.6b)$$

with $m_1 = \left(\widetilde{\xi}_x\ \widetilde{\eta}_x + \widetilde{\xi}_y\widetilde{\eta}_y\right),\ m_2 = \left(\widetilde{\xi}_x\ \widetilde{\eta}_y - \widetilde{\xi}_y\widetilde{\eta}_x\right)$ and $\mu = 1/\sqrt{2}$.

It is interesting to note that the matrix $\widehat{N}$ is only a function of the metrics and not the flow variables.

In three dimensions the Jacobian matrices $\widehat{A}$ or $\widehat{B}$ or $\widehat{C}$ =

$$\begin{bmatrix} \kappa_t & \kappa_x & \kappa_y & \kappa_z & 0 \\ \kappa_x\phi^2 - u\theta & \kappa_t + \theta - \kappa_x(\gamma-2)u & \kappa_y u - \kappa_x(\gamma-1)v & \kappa_z u - \kappa_x(\gamma-1)w & \kappa_x(\gamma-1) \\ \kappa_y\phi^2 - v\theta & \kappa_x v - \kappa_y(\gamma-1)u & \kappa_t + \theta - \kappa_y(\gamma-2)v & \kappa_z v - \kappa_y(\gamma-1)w & \kappa_y(\gamma-1) \\ \kappa_z\phi^2 - w\theta & \kappa_x w - \kappa_z(\gamma-1)u & \kappa_y w - \kappa_z(\gamma-1)v & \kappa_t + \theta - \kappa_z(\gamma-2)w & \kappa_z(\gamma-1) \\ -\theta\left(\gamma e/\rho - 2\phi^2\right) & \kappa_x\left(\gamma e/\rho - \phi^2\right) - (\gamma-1)u\theta & \kappa_y\left(\gamma e\rho^{-1} - \phi^2\right) - (\gamma-1)v\theta & \kappa_z\left(\gamma e\rho^{-1} - \phi^2\right) - (\gamma-1)w\theta & \kappa_t + \gamma\theta \end{bmatrix} \qquad (A.7)$$

where

$$\theta = \kappa_x u + \kappa_y v + \kappa_z w$$

$$\phi^2 = (\gamma-1)(\frac{u^2+v^2+w^2}{2})$$

with $\kappa = \xi,$ or η or ζ for $\widehat{A}, \widehat{B},$ or, $\widehat{C}$ respectively.

The viscous flux Jacobian is

$$\widehat{M} = J^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ m_{21} & \alpha_1\partial_\zeta(\rho^{-1}) & \alpha_2\partial_\zeta(\rho^{-1}) & \alpha_3\partial_\zeta(\rho^{-1}) & 0 \\ m_{31} & \alpha_2\partial_\zeta(\rho^{-1}) & \alpha_4\partial_\zeta(\rho^{-1}) & \alpha_5\partial_\zeta(\rho^{-1}) & 0 \\ m_{41} & \alpha_3\partial_\zeta(\rho^{-1}) & \alpha_5\partial_\zeta(\rho^{-1}) & \alpha_6\partial_\zeta(\rho^{-1}) & 0 \\ m_{51} & m_{52} & m_{53} & m_{54} & \alpha_0\partial_\zeta(\rho^{-1}) \end{bmatrix} J \qquad (A.8a)$$

where

$$
\begin{aligned}
m_{21} =& -\alpha_1\partial_\zeta(u/\rho) - \alpha_2\partial_\zeta(v/\rho) - \alpha_3\partial_\zeta(w/\rho) \\
m_{31} =& -\alpha_2\partial_\zeta(u/\rho) - \alpha_4\partial_\zeta(v/\rho) - \alpha_5\partial_\zeta(w/\rho) \\
m_{41} =& -\alpha_3\partial_\zeta(u/\rho) - \alpha_5\partial_\zeta(v/\rho) - \alpha_6\partial_\zeta(w/\rho) \\
m_{51} =& \alpha_0\partial_\zeta\left[-(e/\rho^2) + (u^2+v^2+w^2)/\rho\right] \\
& - \alpha_1\partial_\zeta(u^2/\rho) - \alpha_4\partial_\zeta(v^2/\rho) - \alpha_6\partial_\zeta(w^2/\rho) \\
& - 2\alpha_2\partial_\zeta(uv/\rho) - 2\alpha_3\partial_\zeta(uw/\rho) - 2\alpha_5\partial_\zeta(vw/\rho) \qquad (A.8b) \\
m_{52} =& -\alpha_0\partial_\zeta(u/\rho) - m_{21} \qquad m_{53} = -\alpha_0\partial_\zeta(v/\rho) - m_{31} \\
m_{54} =& -\alpha_0\partial_\zeta(w/\rho) - m_{41} \qquad m_{44} = \alpha_4\partial_\zeta(\rho^{-1}) \\
\alpha_0 =& \gamma\mu Pr^{-1}({\zeta_x}^2 + {\zeta_y}^2 + {\zeta_z}^2) \quad \alpha_1 = \mu[(4/3){\zeta_x}^2 + {\zeta_y}^2 + {\zeta_z}^2], \\
\alpha_2 =& (\mu/3)\zeta_x\zeta_y, \quad \alpha_3 = (\mu/3)\zeta_x\zeta_z, \quad \alpha_4 = \mu[{\zeta_x}^2 + (4/3){\zeta_y}^2 + {\zeta_z}^2], \\
\alpha_5 =& (\mu/3)\zeta_y\zeta_z, \qquad \alpha_6 = \mu[{\zeta_x}^2 + {\zeta_y}^2 + (4/3){\zeta_z}^2],
\end{aligned}
$$

The eigensystem decomposition of the three dimensional Jacobians have the form $\widehat{A} = T_\xi \Lambda_\xi T_\xi^{-1}$, $\widehat{B} = T_\eta \Lambda_\eta T_\eta^{-1}$, and $\widehat{C} = T_\zeta \Lambda_\zeta T_\zeta^{-1}$. The eigenvalues are

$$
\begin{aligned}
\lambda_1 &= \lambda_2 = \lambda_3 = \kappa_t + \kappa_x u + \kappa_y v + \kappa_z w \\
\lambda_4 &= \lambda_1 + \kappa a \quad \lambda_5 = \lambda_1 - \kappa a \\
\kappa &= \sqrt{\kappa_x^2 + \kappa_y^2 + \kappa_z^2}
\end{aligned} \qquad (A.9)
$$

The matrix T_κ, representing the left eigenvectors, is

$$
T_\kappa = \begin{bmatrix}
\widetilde{\kappa}_x & \widetilde{\kappa}_y & \widetilde{\kappa}_z & \alpha & \alpha \\
\widetilde{\kappa}_x u & \widetilde{\kappa}_y u - \widetilde{\kappa}_z\rho & \widetilde{\kappa}_z u + \widetilde{\kappa}_y\rho & \alpha(u + \widetilde{\kappa}_x a) & \alpha(u - \widetilde{\kappa}_x a) \\
\widetilde{\kappa}_x v + \widetilde{\kappa}_z\rho & \widetilde{\kappa}_y v & \widetilde{\kappa}_z v - \widetilde{\kappa}_x\rho & \alpha(v + \widetilde{\kappa}_y a) & \alpha(v - \widetilde{\kappa}_y a) \\
\widetilde{\kappa}_x w - \widetilde{\kappa}_y\rho & \widetilde{\kappa}_y w + \widetilde{\kappa}_x\rho & \widetilde{\kappa}_z w & \alpha(w + \widetilde{\kappa}_z a) & \alpha(w - \widetilde{\kappa}_z a) \\
\left[\widetilde{\kappa}_x\phi^2/(\gamma-1) + \rho(\widetilde{\kappa}_z v - \widetilde{\kappa}_y w)\right] & \left[\widetilde{\kappa}_y\phi^2/(\gamma-1) + \rho(\widetilde{\kappa}_x w - \widetilde{\kappa}_z u)\right] & \left[\widetilde{\kappa}_z\phi^2/(\gamma-1) + \rho(\widetilde{\kappa}_y u - \widetilde{\kappa}_x v)\right] & \alpha\left[(\phi^2+a^2)/(\gamma-1) + \widetilde{\theta}a\right] & \alpha\left[(\phi^2+a^2)/(\gamma-1) - \widetilde{\theta}a\right]
\end{bmatrix} \qquad (A.10)
$$

where

$$
\alpha = \frac{\rho}{\sqrt{2}a}, \quad \widetilde{\kappa}_x = \frac{\kappa_x}{\kappa}, \quad \widetilde{\kappa}_y = \frac{\kappa_y}{\kappa}, \quad \widetilde{\kappa}_z = \frac{\kappa_z}{\kappa}, \quad \widetilde{\theta} = \frac{\theta}{\kappa}
$$

The corresponding T_κ^{-1} is

$$T_\kappa^{-1} = \begin{bmatrix} \widetilde{\kappa}_x\left(1-\phi^2/a^2\right)-(\widetilde{\kappa}_z v-\widetilde{\kappa}_y w)/\rho & \widetilde{\kappa}_x(\gamma-1)u/a^2 \\ \widetilde{\kappa}_y\left(1-\phi^2/a^2\right)-(\widetilde{\kappa}_x w-\widetilde{\kappa}_z u)/\rho & \widetilde{\kappa}_y(\gamma-1)u/a^2-\widetilde{\kappa}_z/\rho \\ \widetilde{\kappa}_z\left(1-\phi^2/a^2\right)-(\widetilde{\kappa}_y u-\widetilde{\kappa}_x v)/\rho & \widetilde{\kappa}_z(\gamma-1)u/a^2+\widetilde{\kappa}_y/\rho \\ \beta(\phi^2-\widetilde{\theta}a) & -\beta[(\gamma-1)u-\widetilde{\kappa}_x a] \\ \beta(\phi^2+\widetilde{\theta}a) & -\beta[(\gamma-1)u+\widetilde{\kappa}_x a] \end{bmatrix}$$

$$\begin{bmatrix} \widetilde{\kappa}_x(\gamma-1)v/a^2+\widetilde{\kappa}_z/\rho & \widetilde{\kappa}_x(\gamma-1)w/a^2-\widetilde{\kappa}_y/\rho & -\widetilde{\kappa}_x(\gamma-1)/a^2 \\ \widetilde{\kappa}_y(\gamma-1)v/a^2 & \widetilde{\kappa}_y(\gamma-1)w/a^2+\widetilde{\kappa}_x/\rho & -\widetilde{\kappa}_y(\gamma-1)/a^2 \\ \widetilde{\kappa}_z(\gamma-1)v/a^2-\widetilde{\kappa}_x/\rho & \widetilde{\kappa}_z(\gamma-1)w/a^2 & -\widetilde{\kappa}_z(\gamma-1)/a^2 \\ -\beta[(\gamma-1)v-\widetilde{\kappa}_y a] & -\beta[(\gamma-1)w-\widetilde{\kappa}_z a] & \beta(\gamma-1) \\ -\beta[(\gamma-1)v+\widetilde{\kappa}_y a] & -\beta[(\gamma-1)w+\widetilde{\kappa}_z a] & \beta(\gamma-1) \end{bmatrix} \qquad (A.11)$$

where

$$\beta = \frac{1}{\sqrt{2}\rho a}$$

Thomas H. Pulliam
Mail Stop T27B-1, NASA Ames Research Center
Moffett Field, California, 94035
Phone : 415 - 604 - 6417
email : pulliam@nas.nasa.gov
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