Estimating reservoir parameters from seismic and electromagnetic data using stochastic rock-physics models and Markov chain Monte Carlo methods

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Summary

A Bayesian model is developed to estimate porosity, fluid saturation, and pore pressure in reservoirs using seismic and electromagnetic (EM) data. Within the Bayesian framework, unknown reservoir parameters at each pixel in space are considered as random variables and the co-located geophysical properties (seismic P- and S-wave velocity, density, and electrical conductivity), inverted from seismic and EM measurements, are considered as data. Rock-physics models are derived from borehole logs and are considered as random functions between the reservoir parameters and the geophysical properties. Using Markov chain Monte Carlo (MCMC) methods, many samples of each unknown variable are obtained from the Bayesian model, which subsequently are used to infer the unknown variable (reservoir parameter) as well as its uncertainty. A study, based on borehole data from a site in the Troll field, shows that the developed method is more effective for reservoir parameter estimation than traditional regression methods.

Introduction

Joint inversion of 2D or 3D seismic and EM data for reservoir parameter estimation is computationally expensive (Hoversten et al., 2005). One alternative to this inversion is a two-step process: (1) inverting seismic and EM data separately to produce seismic P- and S-wave velocity, density, and electrical conductivity, and (2) transforming the inverted data to reservoir properties using rock-physics models. Since the relationships between reservoir parameters and geophysical properties are non-unique and subject to uncertainty, traditional deterministic rock-physics models or regression methods are often ineffective and may lead to biases in reservoir parameter estimation. However, in recent years, efforts have been made to incorporate uncertainty into reservoir parameter estimation by using stochastic rock-physics models.

Avseth et al. (2001) developed an integrated method to map occurrence probabilities of different lithofacies and fluid properties from seismic amplitude variations with offset (AVO) data, for data collected from a North Sea site. They first defined seismic lithofacies and then used them as the link for tying fluid properties to seismic AVO data using statistical and rock-physics models. The success of the method relies heavily on the existence of seismic lithofacies and distinction in fluid properties among those facies. The method is site-specific and requires considerable geological, geophysical, and sedimentological information.

Bachrach et al. (2004) presented a method for quantitative estimation of reservoir parameters (porosity, water saturation, and effective stress) using seismic data. They considered the reservoir estimation given seismic data (seismic P- and S-wave velocity, density, or any function of the three variables) as a joint estimation problem within a Bayesian framework. The reservoir parameters in the Bayesian framework are considered as random variables, and the known geophysical attributes are considered as data. The rock-physics relations between the unknown reservoir parameters and the known geophysical data are used to define likelihood functions. The random variables, given data, likelihood functions, and other prior information together define a joint posterior probability distribution of all the unknown variables. The maximum a-posterior probability (MAP) and traditional Monte Carlo methods are used to find marginal posterior probability distribution from the joint posterior distribution function. Although the framework described by Bachrach et al. (2004) is general, the approaches for finding solutions are limited to a small number of unknown variables. As described, they can only deal with two unknown variables.

In this study, we generalize the method given by Bachrach et al. (2004) in the following ways: (1) we allow for simultaneous estimation of porosity, water saturation, gas or oil saturation, and pore pressure, (2) we incorporate EM as well as seismic data, and (3) we allow for the incorporation of various error distribution functions in the rock-physics models. The method can also incorporate spatial correlation and prior information of each unknown variable into the model. Most importantly, we use MCMC sampling methods to find solutions. This greatly enhances the generality of our developed methodology for reservoir parameter estimation.

Method

Bayesian Model

We develop a Bayesian model to estimate porosity (ϕ), water saturation (S_w), gas saturation (S_g), and pore pressure (P) at each pixel in space, given the inverted, co-located seismic P- and S-wave velocity (V_p and V_s), density (ρ), and natural logarithmic electrical conductivity (σ). We first transform seismic P- and S-wave velocity and density to bulk and shear modulus, using equations given by Mavko et al. (1998). Therefore, our actual input data are bulk modulus (k_b), shear modulus (k_s), density (ρ), and natural logarithmic electrical conductivity (σ). Based on Bayes' theorem (Stone, 1996), the joint posterior distribution of

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unknown variables at each pixel in space is given by the following formula:

$$f(\phi, S_w, S_g, P \mid k_b, k_s, \rho, \sigma) \propto f(k_b, k_s, \rho, \sigma \mid \phi, S_w, S_g, P) f(\phi, S_w, S_g, P). \qquad (1)$$

The first term on the right side of Equation (1) is referred to as the likelihood function, which is the connection between the given data and the unknown variables. The second term is referred to as the prior distribution function, which is a summary of all the information not included in the data. Equation (1) holds up to an unknown normalizing constant, which is not needed for MCMC sampling methods.

Likelihood Model

We can simplify the likelihood function in Equation (1), since bulk modulus, shear modulus, density, and electrical conductivity at each pixel are independent of one another. According to rock-physics theories (Mavko et al., 1998), bulk modulus and density are functions of porosity, fluid saturation, and pore pressure, whereas shear modulus is only a function of porosity and pore pressure. In addition, based on Archie's law, electrical conductivity is a function of porosity and water saturation. Consequently, we can write the likelihood function in Equation (1) as the product of several likelihood functions given as follows:

$$\begin{aligned} f(k_b, k_s, \rho, \sigma \mid \phi, S_w, S_g, P) = \\ f(k_b \mid \phi, S_w, S_g, P) f(k_s \mid \phi, P) \\ f(\rho \mid \phi, S_w, S_g, P) f(\sigma \mid \phi, S_w). \end{aligned} \qquad (2)$$

We develop each individual likelihood function in Equation (2) by fitting bulk modulus, shear modulus, density, and logarithmic electrical conductivity as functions of porosity, fluid saturation, and pore pressure. The fitted models can be written as follows:

$$\begin{aligned} k_b &= g_1(\phi, S_w, S_g, P) + \varepsilon_1 \\ ks &= g_2(\phi, P) + \varepsilon_2 \\ \rho &= g_3(\phi, S_w, S_g, P) + \varepsilon_3 \\ \sigma &= g_4(\phi, S_w) + \varepsilon_4. \end{aligned} \qquad (3)$$

In Equation (3), g_1, g_2, g_3, and g_4 represent the fitted functions of bulk modulus, shear modulus, density, and logarithmic conductivity, respectively, and ε_1, ε_2, ε_3, and ε_4 represent the residuals of the corresponding fitted functions. Each likelihood function is determined by the probability distribution function of its corresponding residuals. For example, if we assume that ε_4 has the Gaussian distribution with zero mean and standard deviation D, we can obtain the likelihood function of natural logarithmic electrical conductivity using the following:

$$f(\sigma \mid \phi, S_w) = \frac{1}{\sqrt{2\pi} D} \exp\left\{ -\frac{(\sigma - g_4(\phi, S_w))^2}{2D^2} \right\} \qquad (4)$$

Prior Model

We can simplify the prior distribution function in Equation (1) by assuming that porosity and pore pressure at each pixel in space are independent of each other, with both independent of water and gas saturation. The simplified prior distribution function is given by:

$$f(\phi, S_w, S_g, P) = f(\phi) f(P) f(S_w, S_g). \qquad (5)$$

We can use one of two approaches to determine the prior distribution functions of porosity and pore pressure. The first approach assumes that porosity and pore pressure at each pixel have uniform distributions over given ranges. The second approach assumes that porosity and pore pressure at each pixel are spatially correlated to their values at the adjacent pixels. Consequently, prior distribution functions of porosity and pore pressure at each pixel are the truncated Gaussian distribution, with mean and standard deviation determined by kriging methods.

The prior distribution functions of water and gas saturation are determined jointly because they depend on each other at each pixel (since $S_w + S_g \le 1$). Let (a, b) and (c, d) be the ranges of water and gas saturation, respectively. We can thus assume that vector (S_w, S_g) is uniformly distributed on the intersection determined by the areas $a < S_w < b$, $c < S_g < d$, and $S_w + S_g \le 1$. Note that water and gas saturation may not be uniformly distributed over the ranges of (a, b) and (c, d).

Sampling Method

We use MCMC sampling methods, similar to the ones presented by Chen and Hoversten (2003) and Chen et al. (2004), to obtain many samples of each unknown variable from the joint posterior distribution function given in Equation (1). The method entails three major steps: (1) deriving a full conditional distribution function for each variable, (2) drawing samples sequentially from those conditional distributions, and (3) making inferences from those samples.

Synthetic Study

We demonstrated the effectiveness of our method for reservoir parameter estimation using a synthetic dataset, generated according to data collected from a borehole at the Troll site in the North Sea. We first fitted the borehole data

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with rock-physics models presented in Hoversten et al. (2003), and then generated synthetic bulk modulus, shear modulus, density, and electrical conductivity, using the rock-physics models (Table 1) and various reservoir logs. We added 10 percent relative Gaussian random noises to bulk modulus, shear modulus, and logarithmic conductivity, and 30 percent relative random noise to density data. We divided the synthetic data into two subsets: one is referred to as the training data set (60% of the data); the other is referred to as the testing dataset (40% of the data).

Table-1. Parameters of rock-physics models obtained by fitting borehole logs from a site in the Troll field, North Sea.

Parameter name	Fitted values
Grain shear modulus (Gpa)	22.5
Grain Poisson ratio	0.35
Grain density (g/cm^3)	2.567
Number of contacts/grain	13.5
Critical porosity	0.38
Oil API gravity	28.5
Gas gravity	0.59
Brine salinity (ppm/10^6)	0.07
Temperature (^{o}C)	65
Fluid conductivity (S/m)	1.2685
Porosity exponent	1.3091
Saturation exponent	0.1443
Gas correction	0.99

We first used regression methods to estimate reservoir parameters. We fitted porosity, water and gas saturation, and pressure as functions of bulk modulus, shear modulus, density, and electrical conductivity using the training data and stepwise deletion techniques (Stone, 1996). Next, we applied the fitted regression models to estimate reservoir parameters using the testing dataset. Finally, we compared the estimated results with their corresponding true values. Figure 1 shows the estimated mean values, true values, and 95 percent confidence intervals of porosity, water saturation, and gas saturation.

We also used our developed MCMC method to estimate reservoir parameters in the testing dataset. We first fitted bulk modulus, shear modulus, density, and natural logarithmic conductivity (see Equation 3) using rock-physics models given by Hoversten et al. (2003) and the training dataset. We analyzed the residuals of each fitting to get stochastic relationships between reservoir parameters and geophysical attributes. Using the method described in Section 2, we obtained posterior estimates of each unknown in the testing dataset. Figure 2 shows the means, true values, and 95 percent predictive intervals of porosity, water saturation, and gas saturation.

Conclusions

The MCMC method is clearly more effective than the traditional regression methods for reservoir parameter estimation, based on our synthetic study. First, the mean estimates of porosity and water and gas saturation obtained from the MCMC method closely follow the corresponding true values. Second, the 95 percent predictive intervals obtained from the MCMC method give more reasonable upper and lower bounds for the estimated values. However, the regression methods significantly underestimate the uncertainty in reservoir parameter estimation.

The MCMC method is theoretically more appealing than the traditional regression methods because the MCMC method combines physical connections among reservoir parameters and geophysical properties, and avoids the potential conflict among various regression functions. In addition, the MCMC method is more flexible. For example, we can incorporate the spatial correlation of reservoir parameters into the estimation, as well as lower and upper bounds of reservoir parameters obtained from other sources of information. We can also combine some direct measurements of reservoir parameters at certain locations into reservoir parameter estimation.

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References

Avseth, P., Mukerji, T., Jorstad, A., Mavko, G. and Veggeland, T., 2000, Seismic reservoir mapping from 3-D AVO in a North Sea turbidite system: Geophysics, **66**, 1157-1176.

Bachrach, R., and Dutta, N., 2004, Joint estimation of porosity and saturation and of effective stress and saturation for 3D and 4D seismic reservoir characterization using stochastic rock physics modeling and Bayesian inversion: 74th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

Chen, J., and Hoversten, M., 2003, Joint stochastic inversion of geophysical data for reservoir parameter estimation: 73th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

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Chen, J., Hoversten, M., Vasco, D. Rubin, Y., and Hou, Z., 2004, Joint stochastic inversion of seismic AVO and EM data for gas saturation estimation using a sampling-based stochastic model: 74th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

Chen, J., Hubbard, S., Rubin, Y., Murray, C., Roden, E., and Majer, 2004, Geochemical characterization using geophysical data and Markov chain Monte Carlo methods: a case study at the South Oyster Bacterial Transport Site in Virginia: Water Resources Research, **40**, W12412.

Hoversten, M., Cassassuce, F., Gasperikova, E., Newman, G., Chen, J., Rubin, Y., Hou, Z., and Vasco, D., 2005, Direct reservoir estimation using joint inversion of marine seismic AVA and CSEM data: Geophysics, submitted.

Hoversten, M., Gritto, R., Washbourne, J., and T. Daily, 2003, Pressure and fluid saturation prediction in a multi-component reservoir using combined seismic and electromagnetic imaging: Geophysics, **68**, 1580-1591.

Mavko, G., Mukerji, T., and Dvorkin, J., 1998, The rock physics handbook: Tools for seismic analysis in porous media: Cambridge University Press, New York.

Stone, C., 1996, A course in probability and statistics: Duxbury Press, New York.

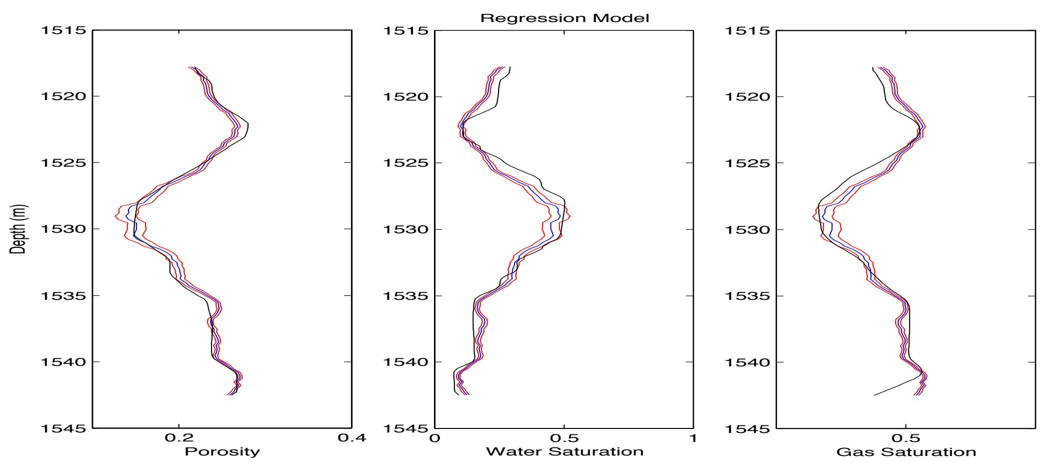

Figure 1. The estimated means (blue line), true values (black line), and 95 percent confidence intervals (red lines) of porosity, water saturation, and gas saturation for the testing dataset the using regression models. Note that the true values are out of bounds at many locations.

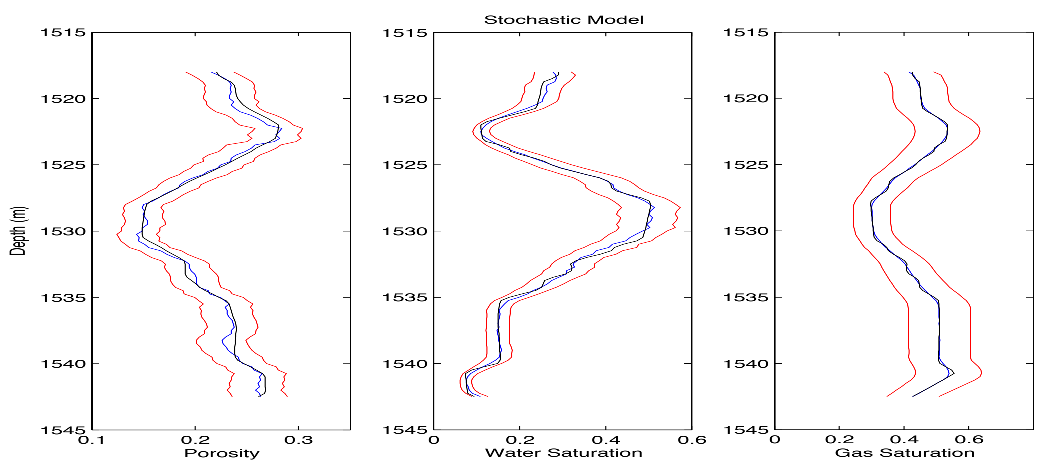

Figure 2. The estimated means (blue line), true values (black line), and 95 percent predictive intervals (red lines) of porosity, water saturation, and gas saturation for the testing dataset using the MCMC method.

EDITED REFERENCES

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REFERENCES

Avseth, P., T. Mukerji, A. Jorstad, G. Mavko, and T. Veggeland, 2000, Seismic reservoir mapping from 3-D AVO in a North Sea turbidite system: Geophysics, **66**, 1157-1176.

Bachrach, R., and N. Dutta, 2004, Joint estimation of porosity and saturation and of effective stress and saturation for 3D and 4D seismic reservoir characterization using stochastic rock physics modeling and Bayesian inversion: 74th Annual International Meeting, SEG, Expanded Abstracts, 1515-1518.

Chen, J., and M. Hoversten, 2003, Joint stochastic inversion of geophysical data for reservoir parameter estimation: 73rd Annual International Meeting, SEG, Expanded Abstracts, 726-729.

Chen, J., M. Hoversten, D. Vasco, Y. Rubin, and Z. Hou, 2004, Joint stochastic inversion of seismic AVO and EM data for gas saturation estimation using a sampling-based stochastic model: 74th Annual International Meeting, SEG, Expanded Abstracts, 236-239.

Chen, J., S. Hubbard, Y. Rubin, C. Murray, E. Roden, and Majer, 2004, Geochemical characterization using geophysical data and Markov chain Monte Carlo methods: a case study at the South Oyster Bacterial Transport Site in Virginia: Water Resources Research, **40**, W12412.

Hoversten, M., F. Cassassuce, E. Gasperikova, G. Newman, J. Chen, Y. Rubin, Z. Hou, and D. Vasco, 2005, Direct reservoir estimation using joint inversion of marine seismic AVA and CSEM data: Geophysics, submitted.

Hoversten, M., R. Gritto, J. Washbourne, and T. Daily, 2003, Pressure and fluid saturation prediction in a multicomponent reservoir using combined seismic and electromagnetic imaging: Geophysics, **68**, 1580-1591.

Mavko, G., T. Mukerji, and J. Dvorkin, 1998, The rock physics handbook: Tools for seismic analysis in porous media: Cambridge University Press.

Stone, C., 1996, A course in probability and statistics: Duxbury Press.