TECHNOLOGY LOCKS, CREATIVE DESTRUCTION AND NON-CONVERGENCE IN PRODUCTIVITY LEVELS

By

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ABSTRACT

This paper presents a simple solution to a new model that seeks to explain the distribution of plants across productivity levels within an industry, and empirically confirms some key predictions using the U.S. textile industry. In the model, plants are locked into a given productivity level, until they exit or retool. Convex costs of adjustment captures the fact that more productive plants expand faster. Provided there is technical change, productivity levels do not converge; the model achieves persistent dispersion in productivity levels within the context of a distortion free competitive equilibrium. The equilibrium, however, is rather turbulent; plants continually come on line with the cutting edge technology, gradually expand and finally exit or retool when they cease to recover their variable costs. The more productive plants create jobs, while the less productive destroy them. The model establishes a close link between productivity growth and dispersion in productivity levels; more rapid productivity growth leads to more widespread dispersion. This prediction is empirically confirmed. Additionally, the model provides an explanation for S-shaped diffusion.

Key words: textiles, creative destruction, technology locks, S-shaped diffusion and productivity dispersion.

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I. Introduction

Many economists believe that competition weeds out the inefficient firms and plants, providing a force for convergence in productivity levels over time. Yet, recent empirical work seems to contradict this hypothesis; persistent dispersion in productivity levels is observed in many industries over a long time interval. Nevertheless, the more productive plants do grow faster and are less likely to exit.[1] This paper seeks to resolve this puzzle, by identifying the forces working against convergence.[2] The result is a workable model of industry evolution that is broadly consistent with the facts. It predicts how industries with rapid technical change differ and yields an explanation for the phenomenon of S-shaped diffusion. The model's implications are tested through a data set of over 13,000 plants from 21 different four-digit textile industries over 16 years, i.e., an extract of the Longitudinal Research Database (LRD).

The model assumes the existence of a "best practice" that evolves exogenously over

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[1]See for example Dhrymes (1991), Bartelsman and Dhrymes (1991), Olley and Pakes (1992), Baily, Hulten, and Campbell, (1992) and Dwyer (1994). This work is made possible by the availability of plant level data at the Census Bureau's Center for Economic Studies.

[2]Recently, there has been much interest in why income per capita and aggregate productivity levels across countries do not converge faster (cf. Parente and Prescott, 1994; Kremer and Thomson, 1993; Bernard and Jones, 1993). Perhaps identifying the forces working against convergence within an industry will be useful in understanding the process of convergence across countries.

time. A plant can only adopt the best practice by spending a fixed cost; otherwise a plant's productivity level remains fixed.[3] Plants can expand their capital stock gradually through convex costs of adjustment. In any instant, plants exhibit diminishing returns scale, but returns to scale are constant in the long run. This results in an industry evolution characterized by new or retooled plants continually coming on line with a cutting edge technology, gradually expanding and then exiting or retooling when they cease to recover their variable costs.

This model is designed to be broadly consistent with the following empirical facts. Plant productivity levels differ by a factor of two to three within a narrowly defined industry and time period (Dhrymes, 1991; and Dwyer 1994).[4] The productivity level of a given plant has a large permanent component (Bartelsman and Dhrymes, 1991; Baily, Hulten and Campbell, 1992; and Dwyer, 1994). Productivity growth is largely an aggregation phenomenon (Bartelsman and Dhrymes, 1991; Olley and Pakes, 1992; and Baily, Hulten and Campbell, 1992), i.e., the more productive plants receive larger weights when computing the aggregate level of productivity. Entry and exit by plants plays only a minor role in aggregate productivity growth (Baily, Hulten, and Campbell, 1992). Within an industry, some plants expand while others contract (Dunne, Roberts, and Samuelson, 1989; and Davis and Haltiwanger, 1992). These findings conflict with theories in which plants produce at a fixed

[3]Therefore, costs of adjusting a plant's technology are strongly concave. This is a model of plant vintages rather than capital vintages. Capital is homogenous.

[4]Specifically, when plants are grouped into deciles according to total factor productivity (TFP), the ratio of the mean productivity of the ninth decile to second decile, hereafter the TFPratio, commonly ranges from two to three.

optimal size.

This model confirms, in the context of a very classical model, what evolutionary economists have long known: "today's cross-sectional dispersion (in productivity), its width and its expected durability, should be recognized as an essential element of the productivity growth process" (Nelson, 1981, page 1045). In the absence of technical change it predicts near convergence in productivity levels. Industries with large entry costs, large adjustment costs and rapid technical change are predicted to exhibit more widespread dispersion in relative productivity levels. Finally, this model can yield the phenomenon of S-shaped diffusion curves.

The model is applied to the textile industry, which has rapid productivity growth in spite of low R&D expenditures by firms. It is my intention to use this application as a benchmark of comparison for future studies of industries in which firm level R&D and technical change are closely linked. Additionally, the assumptions of constant returns to scale and price taking are not obviously false in the textile industry.[5] Through the LRD, each plant's total factor productivity (TFP) is measured for 21 different four-digit textile industries, which allows for comparisons across industries.

Much of the observed dispersion in productivity levels turns out to be the product of transitory idiosyncratic shocks and the mis-measurement of labor inputs; nevertheless, there is

[5]R&D expenditures as a percentage of sales for R&D performing companies was a mere 0.4 per cent in 1980 in Textiles and Apparel (National Science Foundation, 1984). Productivity growth comes from new machinery developed in Germany, Switzerland, and Japan, and new synthetic materials developed by the chemical industry (Cline, 1990). With regards to price taking, Bailey states that "industry concentration is not high when compared to major durable goods producers such as auto and steel" (1988, page 4). I measure close to constant returns to scale for the 21 different textile industries (Dwyer, 1994a).

a sizeable persistent component to dispersion in productivity levels (Dwyer, 1994). Indeed, the more productive plants do grow faster and are less likely to exit (Table 2). Furthermore, industries with rapid technical change exhibit more widespread dispersion in relative productivity levels as predicted by the theory (Figure 4). This supports the argument that widespread dispersion in productivity levels is intrinsically linked with the productivity growth process.

Relation to the Literature

Many models have been offered to explain why plants differ in equilibrium. Market power stemming from product differentiation has been used to explain persistent differences in profit rates (cf. Pakes' review of Mueller, 1987). Selection and technical change, in contrast, have been used to explain why some plants expand while others simultaneously contract within one industry (Jovanovic, 1982; Davis and Haltiwanger, 1992; Hopenhayn, 1992a; Caballero and Hammour, 1994). Absent from these recent models of industry equilibria is a concept of firm expansion; they assume that conventional inputs can be quickly adjusted and therefore plants always produce at their optimal size.[6] These models rely on

[6]Ericson and Pakes (1994), and Jovanovic and MacDonald (1994), construct models in which firms invest to improve their technology and firms with better technologies are bigger: "firms can be described as 'larger' and more 'technologically' advanced interchangeably (Jovanovic and MacDonald, 1994, page 29)." My model, in contrast, predicts the existence of *dinosaurs*, i.e., large old plants which are marginally profitable and downsizing in terms of employment. In Jovanovic (1982), firm growth is the process of a firm learning its optimal size. Davis and

either increasing marginal costs (sometimes an arbitrary capacity constraint) or a downward sloping demand curve to generate a unique maximizing level of output for each plant and an equilibrium in which plants differ.[7]

Economists abstracted away from firm expansion because models with expansion by the representative firm were unable to explain the size distribution of firms (see Jovanovic, 1982). Early empirical studies supported Gibrat's law--firm size seemed to be independent of firm growth. Therefore, models of firms with constant returns to scale, equal productivity levels, and convex costs of adjustment seemed consistent with empirical knowledge (see Lucas, 1978). Other empirical work, however, showed that small firms grow faster but are more likely to fail (Mansfield, 1962). This made the span of control viewpoint--the viewpoint that managerial talent differed and each manager had a finite span of control, i.e., diminishing returns to scale--appealing--because it generated a non-degenerate distribution of production inputs across managerial quality (Lucas, 1978; Calvo and Wellisz, 1978; and Jovanovic, 1982).

No one believed, however, that firms instantly reached their optimal size. Lucas noted that his model is a "limiting case of a model in which there are adjustment costs of rearranging assets among managers" (Lucas, 1978, page 513). Furthermore, he notes that his theory "predicts the size distribution of firms, but only *given* the distribution of persons by

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Haltiwanger (1992), however, find that passive learning only accounts for 11-13 per cent of job-reallocation. Therefore, it seems unlikely that passive learning alone can account for a substantial proportion of productivity growth.

[7]Papers that employ product differentiation and a downward sloping demand curve include Segerstrom (1991) and Caballero and Jaffe (1993). Examples of increasing marginal costs include, Caballero and Hammour (1994) and Hopenhayn (1992b).

managerial talent" (Lucas, 1978, page 510). This paper demonstrates that if the distribution of managerial talent, in my case plant productivity levels, is not fixed over time, then Lucas's limiting case may never be reached *even approximately*. Consequently, it is no longer necessary to assume diminishing returns to scale to yield a non-degenerate distribution of production inputs across productivity levels/managerial talent.

In the real world, many plants do not appear to be at their optimal size. Plants with higher *levels* of productivity grow faster in terms of real value added, total employment and capital stock (Table 2). Furthermore, the fact that share effects--changes in a plant's relative size--play an important role in determining aggregate productivity growth suggests that expansion by more productive plants plays an important role in the evolution of an industry. Arguing that this phenomenon is only the product of passive learning is problematic because entry and exit only play a minor role in determining aggregate productivity growth (Baily, Hulten and Campbell, 1992).

In practice we observe close to constant returns to scale when measuring production functions (cf. Dwyer 1994). In some industries the approximation of price taking seems appropriate. These two assumptions imply that a firm supplies either zero, indeterminate, or infinite output, which implies that either all plants are equally productive, or that adjustment of production inputs is not instantaneous. Yet, plants differ in all industries. Therefore, incorporating convex costs of adjustment into models in which firms differ is a natural extension of the literature, because it allows one to model firm differences in a competitive industry with constant returns to scale technology.

There have been other attempts to model investment in the context of technical change. Nelson and Winter (1982), Iwai (1984a and 1984b), and Klepper and Graddy (1990), create similar models which endogenize technical change while modeling the rate of firm expansion as an increasing function of its earnings, i.e, an arbitrary investment rule.[8] Performing comparative analysis on models with arbitrary decision rules is problematic, because the results may not be robust to economic agents adapting their decision rule to the new economic environment. To my knowledge this paper is the first to solve and characterize a competitive industry equilibrium, with technology locks, in which plants choose when to enter, when to exit, how much to invest in capital, and how much to produce in order to maximize the present discounted value of profits.[9]

The next section develops a model of industry evolution in which convex costs of adjustment determine how resources are reallocated towards the more productive plants and characterizes the model's long run equilibria under the assumption of zero productivity growth. Section III further specifies the model, proves the existence of an equilibrium that turns out to be a balanced growth path, and derives several comparative dynamics. Section IV demonstrates that the model can predict an S-shaped diffusion curve. Section V confirms

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[8]Nelson and Winter model technical change as the outcome of a search for either labor or capital saving innovations. Iwai models both innovation and imitation. Klepper and Graddy create a selection model consistent with empirical regularities regarding the evolution of new markets. In these models firms do not base their expansion decisions on forward looking rational expectations.

[9]Lach and Rob (1993) create a discrete analogy to my model; innovations are discrete improvements upon the previous generation and there are a finite number of firms who compete in a Cournot oligopoly setting. Lach and Rob, however, endogenize technical change and assume that installation costs are zero. They solve for the case in which innovations are drastic enough that no one chooses to invest in an old technology, that is existing firms choose not to expand. My model, in contrast, shows that plants will invest in old technologies provided it is costly to adopt the most recent technology.

the model's prediction that textile industries with more rapid technical change will exhibit more widespread dispersion in productivity levels. Concluding remarks finish out the paper. The first appendix provides a comprehensive table of symbols used in the paper. The second appendix provides the mathematical details of the proofs and formalizes the plant's optimal control problem.

II. A Model of Industry Evolution Through Creative Destruction

This section first describes a model of industry evolution and defines its equilibrium path. It then demonstrates several properties of the equilibrium price path and the plant's maximization problem assuming an equilibrium exists. Finally, it characterizes the model's long run equilibria in the absence of technical change. The most important element of the model is the specific concept of technical change. The other element of the model that is not entirely standard is the concept of plant expansion. The remaining elements are standard and necessary to define an industry equilibrium. Each element is described in turn.

The technology frontier is assumed to move outward according to an exogenously specified path. Let b_t represent the cutting edge productivity level at time t; b_t is a nondecreasing and continuous function from $\mathbb{R}$ to $\mathbb{R}_+$; time is continuous and begins at negative infinite. In order to get to the cutting edge productivity level a plant has to pay a fixed cost, $F \geq 1$ and for this fixed cost a plant receives one unit of capital; $K_{tt} = 1$, where K_{tt} is the capital stock of a plant born at date t at date t. Both the price of capital and the initial size of a plant are normalized to one. Therefore, a new plant pays 1 for the capital and F-1 for the technical known-how associated with adopting the cutting edge technology. A plant's productivity level remains fixed over time until it retools; let b_s denote the productivity level of a plant born or retooled at date s. There are a continuum of plants in this model.

In order for a plant to expand its capital stock, it must pay for the new capital and pay a cost associated with disrupting the factories operations, i.e., an adjustment cost. In order for a plant to expand at the rate of $I = dK/dt$, it must pay a cost of $I(1+\gamma(I/K))$ per unit time, where 1 is the price of capital and γ is the per unit cost of installing a unit of capital. It is assumed that the disruptiveness of installing a new unit of capital is in proportion to the size of a plant; two plants expanding at a rate of 10% per year pay the same cost of adjustment per unit capital regardless of their relative size. Total adjustment costs, I((I/K), are assumed to be convex. This specification closely resembles that of Blanchard and Fischer (1989, pages 58-61). Note that a plant pays a cost for adopting its new technology that it is in proportion to its initial size. In order to expand it does not have to pay this cost again. Therefore, to expand slowly costs strictly less than building a new plant; a plant will typically want to expand following entry.

A plant produces with a constant returns to scale production function that employs labor, L, and capital, K. Technical change is assumed to be Hicks neutral; therefore b_s represents total factor productivity. Let $y_{st} = b_s M(K_{st}, L_{st})$ be the output per unit time of a plant with productivity b employing K units of capital and L units of labor, where the subscripts s and t denote the date of entry/retooling and the current time period respectively. M is assumed to be constant across plants and time. Let w and * be the per unit rental cost of labor and per unit operating cost of capital per unit time, respectively. Both w and * are assumed to be constant. The per unit cost of capital is an operating cost that can only be avoided by shutting down the plant; investment is irreversible.[10] A plant maximizes the present discounted value of cash flows for a given discount rate, r.

There is an inverse downward sloping industry demand curve, p=D(Y), where p is price and Y is industry output. This completes the description of the model. The assumptions

[10]This specification is chosen because it yields an intuitive endogenous exit condition; a plant exits when it ceases to recover its variable costs. Other specifications are possible.

on functional forms are specified and number below. We are now ready to state the plant's problem and define an equilibrium path.

Assumptions on Functional Forms:

A1. The inverse demand curve slopes downward: D' < 0.

A2. Plants enter with 1 unit of capital, $K_{tt} = 1$.

A3. M(K,L) is homogeneous of degree 1, twice differentiable and concave;
$\lim_{L60}$ MM/ML = 4; and $\lim_{L64}$ MM/ML = 0.

A4. Investment is irreversible, I_{st} $ 0.

A5. Total adjustment costs are convex:
if I=0 then ((0) = 0;
if I > 0 then ((I/K) > 0 and ('(I/K) > 0;
$\lim_{I/K64}$ ((I/K) = 4; and (, (', and (" are continuous.

A6. The technology frontier shifts outward in a continuous manner:
b_t is a continuous and weakly monotonic function on ú.

The Plant's Problem:

A plant with productivity level b_s chooses labor inputs (L_{st}), an investment path (I_{st}), and an exiting date (T_s) to maximize the present discounted value of profits for a given price path:

$$V_{st}=Max_{[T,L_\tau,I_\tau]}\int_t^T\left(p_\tau b_s\Phi(K_\tau,L_\tau)-\delta K_\tau-wL_\tau-I_\tau-I_\tau\gamma\left(\frac{I_\tau}{K_\tau}\right)\right)e^{-r(\tau-t)}d\tau, \qquad \text{(M1)}$$

$$subject\ \ to\ I\ \geq 0\ \ and\ I\ =\ \frac{dK}{dt}.$$

The solution to M1, provided it exists, implies a labor demand curve (L_{st}*), an optimal investment function (I_{st}*), an optimal capital stock (K_{st}*), an optimal supply curve (y_{st}*) and an optimal exit date (T_s). Note that under this specification, a plant can expand slowly for less than the cost of building a new plant. Therefore, plants will want to expand following entry.

Equilibrium:

In equilibrium there will be a rate of plants per unit time entering the industry. Let C_t be a function ($C_t: \mathbb{R} \to \mathbb{R}_+$) representing the number of new plants per unit time entering the industry, where C_t is defined as the derivative of the cumulative mass of entry at any given t (if E_t denotes the total mass of plants that have entered in the time interval $(-\infty,t]$, then $C_t \equiv dE_t/dt$).[11] For an exogenous b_t, an equilibrium path is a p_t, C_t, and T_s such that for all t:

(1) All plants in an industry produce, invest and exit according to the solution to M1.

(2) The value of an existing new plant is equal to or less than the fixed cost of entry, and equal to if entry is positive: $V_{tt} \leq F$ and $F = V_{tt}$ if $C_t > 0$.

(3) The quantity produced by all plants clears the output market: $p_t = D(Y_t)$.

Y_t is the sum of the optimal level of output of all plants in existence at time t. A property of the equilibrium, provided it exists, is that the date of exit is weakly monotonic in s (Property 3). Therefore, Y_t can be written as:

[11] This definition assumes that such a derivative exists, which implies that a positive mass of plants cannot enter in an instant.

$$Y_t = \int_{t-\bar{a}_t}^{t} C_s \, y^*_{st} \, ds = \int_0^{\bar{a}_t} C_{t-a} \, y^*_{t-at} \, da,$$
$$\text{where} \quad a = t-s,$$

i.e., the integral of plant output over the birth dates of plants or over the age of plants ($\bar{a}_t$ being the age of the oldest plant still in operation at date t).

Note that if F = 1, the per unit cost of building or retooling a plant is less than or equal to the cost of expanding an existing plant. Because a new or retooled plant is more productive, entry or retooling has a strict advantage over expanding an existing plant when F=1. Therefore, when F is 1 this model collapses into a model in which plant size has an arbitrary bound and all new capital is allocated to the cutting edge productivity level. That is, when F=1 the model becomes a vintage capital model that closely resembles Phelps (1962) and Caballero and Hammour (1994). When necessary I assume that F is greater than one, and these are the cases where the model's implications differ from those of a vintage capital model.

I am now ready to state four properties of the solution to M1, and a proposition about the equilibrium. They regard the recursive nature of the plant's problem. The plant chooses its labor inputs, L, to maximize instantaneous profits. It then chooses its exit date, T, to avoid negative profits. Finally, it chooses its investment path, I_J, to maximize the present discounted value of future cash flows.

Property 1:

L_{sJ}* maximizes

$$p_\tau b_s \Phi(K_\tau, L_\tau) - \delta K_\tau - wL_\tau.$$

This property is immediate since L is an unconstrained variable that only affects the instantaneous rate of profits.

Property 2:

There exists a function B(p,s) such that

$$\pi(p,s)K = p b_s \Phi(K, L^*) - \delta K - wL^*.$$

i.e., instantaneous returns on capital net of investment are independent of the size of the plant.

Proof:

Standard application of Euler's theorem which states that the partial derivative of a function that is homogenous of degree 1 is homogeneous of degree 0.

Property 3:

T_s is nondecreasing in b_s.

i.e., a plant does not exit before a less productive plant.

Proof:

Immediate application of the fact that the return on capital, B, is increasing in productivity, b_s (see Appendix II). This implies that T_s is nondecreasing in s, provided b_s is strictly increasing in s. In the case that $b_s = b_{s'}$, it is convenient to assume that T_s is nondecreasing in s.

Proposition 1:

Under assumptions A1-A6, provided an equilibrium exists, p_t is continuous and nonincreasing.

Proof:

See Appendix II. While tedious, the basic idea is simple. A point of discontinuity in price implies a discrete change in supply. A discrete change in supply can only be brought about by a positive mass of plants exiting, causing an increase in price. An increase in price implies either that some plants should not have exited or they should have exited earlier.

Property 4:

T_s solves B(p_T,s) = 0, and is therefore independent of the optimal investment path I_{st}*, that is a plant exits when it no longer recovers its variable costs.

This property immediately follows from price being nonincreasing and B being increasing in price. Once B = 0, in the future B # 0 which implies that exiting does at least as well as staying in business.

Therefore, M1 can be rewritten as

$$V_{st} = \underset{[I_\tau]}{Max} \int_t^{T_s} \left(\pi(p_\tau, s) K_\tau - I_\tau - I_\tau \gamma\left(\frac{I_\tau}{K_\tau} \right) \right) e^{-r(\tau - t)} d\tau, \tag{M2}$$

$$subject \ \ to \ I \geq 0 \ and \ I = \frac{dK}{dt}.$$

Proposition 2:

For a given price path, provided that T_s is finite, a unique solution to M2 exists.

Proof:

In order to simplify the mathematics, I add the additional constraint:

$I_\tau \leq \bar{I}_\tau$, *where* $\bar{I}_\tau$ *solves*:

$$F = 1 + \gamma\left(\frac{I_\tau}{K_\tau}\right) + \left(\frac{I_\tau}{K_\tau}\right)\gamma'\left(\frac{I_\tau}{K_\tau}\right).$$

This constraint is innocuous because in equilibrium it will never bind; the value of an installed unit of capital is bounded above by the fixed cost of entry. Existence is now a standard application of the Filippo-Cesari theorem. The solution is unique because eq #A.1 (see Appendix II) has a unique solution.[12] For clarity, this constraint is suppressed throughout the rest of the paper.

Appendix II characterizes the solution to M2 as an optimal control problem. As is standard in the investment literature, q is the value of an installed unit of capital. If q # 1 then I = 0, but if q > 1 then investment is positive and increasing in q. If B # r then q # 1. That is, if the current rate of return on capital is less than or equal to the discount rate, then the value of an installed unit of capital is less than or equal to 1 because B_{st} is nonincreasing in t. Since a new plant has a value of F and is 1 unit of capital, a new plant has a q of F and positive investment occurs following entry provided F > 1.

In the absence of technical change, Proposition 3 shows that this industry has a set of

[12]Theorem 8, page 132; and Theorem A.5, page 415; respectively in Seierstad and Sydsæter (1987).

stationary equilibria--stationary in the sense that no one enters, exits, or invests and therefore price and quantity produced remain constant--in which the most productive plant earns a return on capital equal to the discount rate, π=r, and the least productive plant recovers its variable costs. That is, provided that the economy reaches one of these equilibria, it stays there. Since it is not necessarily the case that anyone will choose to enter with the cutting edge technology, I adopt the following notation. Let B and $\bar{B}$ denote the productivity level of the most productive plant in operation and the most productive technology available, respectively, at date t. Let p* solve $\pi(p^*,B) = r$ and $\bar{a}$ solve $\pi(p^*,b_{t-\bar{a}}) = 0$. Finally, let $W(\bar{B},p)$ denote the value of a new plant with productivity $\bar{B}$, under the assumption that price remains constant at p^*.

Proposition 3:

Under assumptions A1-A6, if

$F \geq 1$,

$D(\int_{t-\bar{a}}^{t} C_s y_{st}\, ds) = p^*$,

$b_\tau \in [B, \bar{B}]$ for all $\tau > t$, and

$W(\bar{B},p^*) < F$,

then an equilibrium exists characterized by no exits, no entrants, $p_\tau = p^*$, $\pi(B) = r$, and $I_{s\tau} = 0$, for all $\tau > t$. That is, the industry is in a stationary equilibrium at t.

Proof:

I must show that: (1) it is optimal for all plants not to exit; (2) it is optimal for all plants not to invest; (3) it is optimal for potential plants not to enter; and (4) supply remains constant.

(1) Observe that B(b_s, p_J) $ 0 for all J $ t and s $ t-6. Plants born before t-6 will have exited before or at t, since price is nonincreasing and continuous. Therefore, any plant in operation at t is at least indifferent between remaining in operation and exiting in the future.

(2) I^* = 0 for all plants because B # r which implies q # 1, for all plants.

(3) By assumption. Note that B = & is sufficient to guarantee this condition because V_{JJ} = $*^{4}_{t+x}$ $re^{-r(J-t-x)}$dJ = 1 # F, because B(b_J) = r and I^*_{Jv} = 0, for all J and v > t, therefore not entering does at least as well as entering.

(4) Supply remains fixed, because no one exits, enters or invests. Q.E.D.

Clearly there are no other stationary equilibria with the property that the returns on capital exceed the discount rate, B > r, for the most productive plant in operation.[13] Equilibria with B < r for the most productive plant in operation, while stationary, imply that some plant behaved irrationally. Therefore, the above set is the set of interesting stationary equilibria. Heuristically, in the absence of technical change, plants will enter as long as it is profitable, and the most productive plant will continue to expand as long as B > r. These two effects will lower the price and force some plants out of business. The rate of investment, however, approaches zero as B approaches r; and therefore, the set of stationary equilibria is approached asymptotically.[14] Observe that if F > 1, then entry will have ceased before t, and the stationary equilibria are approached by existing plants accumulating capital and shifting

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[13]Suppose not. If B > r forever then q > 1 and I > 0 and price is falling implying that this equilibrium is not stationary.

[14]Proving these statements first requires that a solution to M2 exists for T = 4 and B approaching a limit of r asymptotically, which remains an open question.

the aggregate supply curve outward. This capital accumulation drives the price down, forcing some plants to exit after entry has ceased. Note that the ratio of the most productive plant's to the least productive plant's productivity levels in operation in any of these stationary equilibria is independent of F.[15]

In this section, I defined an equilibrium and proved that if it exists it exhibits a nonincreasing continuous price path, and in the absence of technical change there exists a set of stationary equilibria. Whether or not assumptions A1-A6 are sufficient conditions for an equilibrium to exist, however, remains an open question.

III. Existence of an Equilibrium: A Balanced Growth Path

This section proves the existence of an equilibrium--it turns out to be a balanced growth path--for a specific set of functional forms and proves several propositions characterizing this equilibrium. The equilibrium yields a time invariant distribution of plants and capital across relative productivity levels. Proposition 5 demonstrates that more rapid technical change leads to more widespread dispersion in relative productivity levels. Additionally, Propositions 6, 7 and 8 show that the greater the cost of entry and expansion, the larger the dispersion in productivity levels.

I assume a constant returns to scale Cobb-Douglas production function, a constant elasticity of demand and a constant growth rate of the cutting edge productivity level.[16]

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[15]Strictly speaking, the ratio of B to the lower bound of the least productive plant in operation is independent of F. I have not shown that a plant entered with productivity b_{t-a}.

[16]It is straightforward to let $p = Y^{-1/F}e^{Nt}$, that is, to include aggregate demand growth. Everything that follows still holds except $\hat{c} = FN+(F-1)\mu$ and $\hat{Y} = F(N+\mu)$ in Proposition #3.

A1! p $= Y^{-1/F}$

A3! $M(K,L)$ $= K^{\$}L^{"}$, with $" + \$ = 1$.

A6! b_t $= b_0 e^{\mu t}$ $= e^{\mu t}$.[17]

It is straightforward to show that A3' implies

$$\pi(s,p) = (p b_s)^{\frac{1}{\beta}} A - \delta,$$

where A = ("/w)$^{"/\$}$ \$; and,

$$y^*(s, p_t, K_{st}) = b_s^{\frac{1}{\beta}} \left(\frac{\alpha p}{w} \right)^{\frac{\alpha}{\beta}} K_{st}.$$

The key to the existence proof (Proposition 4) is the following observation. If price falls at the rate of technical progress, then the plant's profits become a function of its age only. Therefore, I will define the following functions of a, where a represents the age of a plant provided $p_t = p_0 e^{-\mu t}$. Under this assumption,

$$\pi_{st} = (b_s \, p_t)^{\frac{1}{\beta}} A - \delta = p_0^{\frac{1}{\beta}} A e^{\frac{(s-t)\mu}{\beta}} - \delta.$$

Letting a = t-s,

$$\pi_{t-a t} = p_0^{\frac{1}{\beta}} A e^{-\frac{a\mu}{\beta}} - \delta \equiv \pi_a.$$

Additionally B_{aG} = 0 implies

[17]Without loss of generality, I have set $b_0 = 1$.

$$\bar{a} = \frac{\log\left(p_0\left(\frac{A}{\delta}\right)^{\beta}\right)}{\mu} = \frac{\log\left(\left(\frac{\pi_0+\delta}{\delta}\right)^{\beta}\right)}{\mu},$$

where $\bar{a} = T_s - s$, the life span of a plant, which is independent of its birth date. Note that π_0 is the return on capital for any new plant, hereafter the initial return on capital. Observe that the life span of the plant approaches infinite as δ approaches zero and the life span of plant is zero when β is zero. The life span of a plant becomes infinite when there are no operating costs of capital, because the marginal product of capital goes to infinite as output goes to zero. The life span becomes zero when labor is the only input into the production process, because the most productive plant instantaneously expands to dominate the industry. Let K_a* be the capital function associated with the solution to:

$$V_0 = Max_{[I_a]} \int_0^{\bar{a}} \left(\pi_a K_a - I_a - I_a \gamma\left(\frac{I_a}{K_a}\right)\right) e^{-ra} da, \tag{M3}$$

$$subject\ \ to\ K_0 = 1\ ;\ \ I \geq 0\ ;\ \ and\ I = \frac{dK}{dt}.$$

Proposition 4 (Existence of a Balanced Growth Path):

Under assumptions A1', A2, A3', A4, A5, and A6', If $K_{s0} = K_{-s}$* on $[-\bar{a}, 0]$ and $C_s = C_0 e^{\mu(\sigma-1)s}$ on $[-\bar{a}, 0]$, then there exists an equilibrium path defined by:

(1) $\hat{Y}/\sigma = -\hat{p} = \hat{C}/(\sigma-1) = \mu$,[18] and

(2) C_0 and p_0 that simultaneously solve

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[18] $\hat{x}$ denotes (dx/dt)/x.

$$p_0^{-\sigma} = \int_0^{\bar{a}} \left(C_0 e^{-\mu\left(\sigma - 1 + \frac{1}{\beta}\right)a} K_a^* \left(\frac{\alpha p_0}{w} \right)^{\frac{\alpha}{\beta}} \right) da, \qquad \text{(initial condition \#1)}$$

and

$$F = V_0, \qquad \text{(initial condition \#2)}$$

for all t.

In words, if at time 0 plants of age a (alternatively -s) have the capital stock associated with the solution to M3 and the density of plants across productivity levels is consistent with the growth path, then a balanced growth path exists such that: (1) C_0 and p_0 simultaneously solve the market clearing condition and the zero profit to entry condition; (2) output grows at the rate of technical progress multiplied by the demand elasticity; (3) price falls at the rate of technical progress; and (4) the number of new entrants increases at the rate of technical progress multiplied by the elasticity of demand less one.

Proof:

Appendix II demonstrates that the value of the plant is continuous and increasing in p_0, i.e., $V_0(p_0)$ is strictly increasing in p_0, approaches zero as p_0 approaches $(*/A)^{\$}$ and approaches infinity as p_0 approaches infinity. Therefore, by the intermediate value theorem there exists a unique p_0 that satisfies the second initial condition. That is, there exists one initial price, which implies an initial return of capital, that sets the value of a new plant equal to the fixed cost of entry. By the implicit function theorem, there exists an implicit function that solves initial condition 1, $C_0(p_0)$ defined on $((*/A)^{\$},4)$.[19] Consequently, there exists a unique C_0 and p_0 that satisfy initial conditions 1 and 2. Additionally, as $p_0 > (*/A)^{\$}$, the initial return on capital must be positive.

Now I must show that the growth path $\hat{Y}/F = -\hat{p} = \hat{C}/(F-1) = \mu$ always satisfies the 3 equilibrium conditions, for the p_0 and C_0 that satisfy the initial conditions. It is given that price is falling at the rate of technical progress which implies that $B_{st} = B_a$, which is independent of s and t. The life span of a plant is given by $\hat{a}$, which is also independent of s. This implies that M2 and M3 are equivalent maximization problems under the specified growth path. In any given instant $K_{st} = K_{t\text{-}s}$. Furthermore, V_{tt} is a constant, because all new plants face the same maximization problem. This result combined with initial condition 2 implies that equilibrium condition 3 is always satisfied. The value of a new plant always equals the fixed cost of entry.

Equilibrium Condition 2 (market clearing) implies that:

$$p_t^{-F} = p_0^{-F} e^{\mu F t} = Y_t.$$

Applying the specified growth path to the definition of Y_t, pulling terms involving t outside the integral, and invoking initial condition 1 implies:

[19]The derivative of

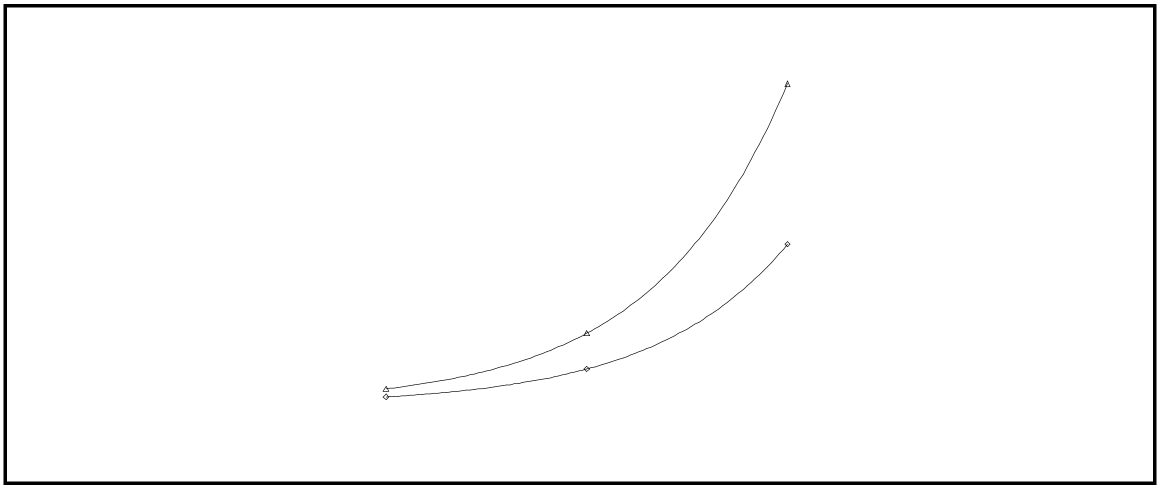
$$p_0^{-\sigma} - \int_0^{\bar{a}} \left(C_0 e^{-\mu\left(\sigma-1+\frac{1}{\beta}\right)a} K_a^{*}\left(\frac{\alpha p_0}{w}\right)^{\frac{\alpha}{\beta}} \right) da = 0,$$

with respect to C_o is always negative and continuous; and the derivative with respect to p_0 exists and is continuous on the given set; therefore, the conditions for the implicit function theorem are satisfied.

$$
\begin{aligned}
Y_t &= \int_0^{\bar{a}} C_{t-a} y^*_{t-at}\, da \\
&= \int_0^{\bar{a}} C_{t-a} b_{t-a}^{\frac{1}{\beta}} \left(\frac{\alpha p_t}{w} \right)^{\frac{\alpha}{\beta}} K^*_a \, da \\
&= \int_0^{\bar{a}} C_0 e^{\mu(\sigma-1)(t-a)} e^{\frac{\mu(t-a)}{\beta}} \left(\frac{\alpha p_0}{w} \right)^{\frac{\alpha}{\beta}} e^{-\frac{\alpha\mu t}{\beta}} K^*_a \, da \\
&= e^{\mu\left(\sigma-1+\frac{1}{\beta}-\frac{\alpha}{\beta}\right)t} \int_0^{\bar{a}_0} C_0 e^{-\mu\left(\sigma-1+\frac{1}{\beta}\right)a} \left(\frac{\alpha p_0}{w} \right)^{\frac{\alpha}{\beta}} K^*_a \, da \\
&= e^{\mu\sigma t} Y_0 \quad = \quad p_0^{-\sigma} e^{\sigma\mu t}.
\end{aligned}
$$

(Recall that "+$ = 1.) That is, the specified growth path implies that Y_t equals Y_0 multiplied by $e^{\mu F t}$, which implies that demand always equals supply. Therefore, under the specified growth path equilibrium condition 2 is always satisfied. Q.E.D.

Figure 1:



Î $b_t = e^{\mu t}$ (the cutting edge plant's productivity)

• $b_{t-a} = (*/A)^{\$}/p_t = ((*/(*+B))^{\$}e^{\mu t}$ (the marginal plant's productivity)

Now, I wish to consider how the rate of technical change affects the dispersion in productivity levels in operation at any given time. Because the life span of a plant is a constant over time, the ratio of the most productive plant to least productive plant at any given time is a constant. Let

$$TFPratio = \frac{b_t}{b_{t-\bar{a}}} = \frac{e^{\mu t}}{e^{\mu(t-\bar{a})}} = e^{\mu\bar{a}} = p_0\left(\frac{A}{\delta}\right)^{\beta} = \left(\frac{\pi_0+\delta}{\delta}\right)^{\beta}.$$

Figure 1 illustrates that although the productivity levels of the cutting edge plant and the marginal plant are both growing over time, the ratio of these two is a constant. Furthermore, the initial return on capital, B_0, is a sufficient statistic for the TFPratio. The initial return on capital is the rate necessary to set the value of a new plant equal to the fixed cost of entry.[20] Observe that the TFPratio goes to 1 as $ goes to zero and goes to infinite as * goes to 0. If labor is the only input in the production process, the most productive plant instantly hires enough labor to drive the competition out of business. If there are no operating costs, a plant remains in operation forever because its marginal product of labor goes to infinite as output goes to zero.

The next proposition demonstrates that more rapid technical change leads to larger TFPratios. The logic is simple; an increase in the rate of technical progress causes a plant's return on capital to erode faster. Attracting entrants, therefore, requires a higher initial return on capital. A higher return on capital for the most productive plant in operation lowers the productivity level of plant that earns a zero return on capital, i.e., the marginal plant. Because the productivity level of the cutting edge plant is exogenously fixed, dispersion increases.

Proposition 5:

Under assumptions A1', A2, A3', A4, A5, and A6', in equilibrium the TFPratio is increasing in

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[20]The initial return on capital, B_0, is a more concrete concept than the initial price, p_0. Therefore, the comparative dynamics that follow will be motivated in terms of B_0. Because the TFPratio is linear in p_0, however, the algebra is less cumbersome if performed in terms of p_0. Therefore, the TFPratio is differentiated with respect to p_0 in the comparative dynamics.

μ.

Proof:

From the definition of TFPratio:

$$\frac{d(TFPratio\ )}{d\mu}=\left(\frac{A}{\delta}\right)^{\beta}\ \frac{dp_0}{d\mu}.$$

Initial condition 2 implies that:

$$\left(\frac{\partial V_0}{\partial \mu}\right)d\mu+\left(\frac{\partial V_0}{\partial p_0}\right)dp_0=0 \qquad =>$$

$$\frac{dp_0}{d\mu}\ =\ -\ \frac{\partial V_0/\partial\mu}{\partial V_0/\partial p_0}\ >\ 0.$$

The last step follows because $MV_0/Mp_0 > 0$ and $MV_0/M\mu < 0$, as demonstrated in Appendix II. Therefore,

$$\frac{d(TFPratio\ )}{d\mu}=\left(\frac{A}{\mu}\right)^{\beta}\left(\frac{dp_0}{d\mu}\right)>0.$$

Q.E.D.

The next proposition demonstrates that the larger the entry costs the larger the TFPratio. In words, an increase in the fixed cost of entry requires a higher initial return on capital in order for the value of a new plant to continue to equal the fixed cost of entry. This higher initial return lowers the productivity level of the marginal plant at any point in time.

Proposition 6:

Under assumptions A1', A2, A3', A4, A5, and A6', in equilibrium the TFPratio is increasing in F.

Proof:

Initial condition 2 implies

$$dF = \left(\frac{\partial V_{tt}}{\partial p_0}\right) dp_0;$$

$MV_{tt}/Mp_0 > 0$ implies $dF/dp_0 > 0$, which in turn implies

$$\frac{d(TFPratio)}{dF} = \left(\frac{\partial TFPratio}{\partial p_0}\right)\left(\frac{\partial p_0}{dF}\right) > 0.$$

Q.E.D.

This rather intuitive result differs from Proposition 3, where the dispersion in productivity levels in a stationary equilibrium was independent of F. This observation brings us to the next proposition.

Let B* solve:

$$F = \int_0^{\infty} \left(\pi^* K_t^* - I_t^* - I_t^* \gamma\left(\frac{I_t^*}{K_t^*}\right)\right) e^{-rt} \, dt.$$

That is, the initial return on capital that makes entrants indifferent between entering and not entering provided the return on capital remains constant. Recall that if F > 1 then the value of an installed unit of capital for a new plant is F > 1, implying that q > 1. In order for q to be greater than 1, the return on capital must exceed the discount rate, i.e., B* > r. Recall that in Proposition 3, I proved that return on capital for the most productive plant in operation associated with the set of stationary equilibria would equal r. Under the assumption of a Cobb-

Douglas production function this implies that the TFPratio associated with the stationary equilibria is equal to $(r/*+1)^{\$}$.

Proposition 7:

Under assumptions A1', A2, A3', A4, A5, and A6, in equilibrium, if F > 1 and μ > 0, then the B_0 associated with the balanced growth path is bounded below by $B^*>r$.

Proof:

Suppose not. Then $V_{tt} < F$ for all t which implies $C_0 = 0$. Q.E.D.

Thus the limit of the TFPratio as μ approaches zero, $(B^*/(+1)^{\$}$, is greater than the TFPratio associated with the stationary equilibrium when μ = 0. This is the case because entry must always occur if μ > 0. If μ = 0 then investment continues after entry has ceased (if $V_{tt} = F$, then q = F > 1), which drives down the return on capital and forces some of the less productive plants out of business. Therefore, the limit of the TFPratio as μ 6 0 is greater than the stationary TFPratio associated with no technical change; an industry with arbitrarily slow but consistent technical change will exhibit a larger TFPratio in equilibrium than an industry with zero technical change.

Now I will consider the affects of adjustment costs on the TFPratio. If adjustment costs were larger, one would expect more dispersion in productivity levels along the balanced growth path because the most productive plants would expand more slowly and therefore need to earn a higher initial return on capital to recover their fixed cost. This intuition can be formalized by letting ((I/K) = R'(I/K). An increase in R increases adjustment costs and is expected to increase the TFPratio.

Proposition 8:

Under assumptions A1', A2, A3', A4, A5, and A6', in equilibrium, if F > 1 then the TFPratio is increasing in R.

Proof:

$$\left(\frac{\partial V}{\partial p_0}\right) dp_0 + \left(\frac{\partial V}{\partial \psi}\right) d\psi = 0.$$

Therefore, if MV/MR < 0 then d(TFPratio)/dR > 0, because (MV/Mp_0) and dTFPratio/dp_0 > 0. In Appendix II, it is demonstrated that

$$\frac{\partial V}{\partial \psi} = \int_0^{\bar{a}} -K_a \varphi(q_a)\Gamma(\varphi(q_a)) e^{-ra}\, da \;\le\; 0,$$

and strictly less than zero if F > 1; that is dTFPratio/dR is strictly positive provided it is optimal for new plants to invest at a positive rate. Q.E.D.

In this section, I proved the existence of an equilibrium for a specific set of functional forms. These functional forms are general enough to include, by appropriate parameter choices, industries that are either capital or labor intensive, have elastic or inelastic industry demand curves, and exhibit rapid or slow rates of technical change. The link between the productivity growth, the expansion process of plants, and the dispersion of relative productivity levels is clearly established. Note that Propositions 1, 2, 3, 4, and 5 hold if F=1, i.e., in a vintage capital model. Therefore, an equilibrium will exist and more technical change will result in a larger TFPratio for both vintage plant and vintage capital models. The implications that larger fixed costs and installation costs yield larger dispersion in relative productivity levels, however, require a concept of plant expansion. Furthermore, the discontinuity in the TFPratio at μ = 0 relies on the ability of plants to expand after entry has ceased. Under a vintage capital model, the prediction that more technical change will result in more dispersion in productivity levels applies to machines, not plants. Predicting what one would observe at the plant level requires explaining why plants have different distributions of machinery across vintages.

V S-Shaped Diffusion

Several papers have sought to explain the phenomenon of S-shaped diffusion curves--the diffusion of a new technology begins gradually, speeds up and then slows down (cf. Jovanovic and Lach, 1989; and Jovanovic and Macdonald, 1994). Jovanovic and Macdonald make the point that vintage capital models cannot generate S shaped diffusion curves, except in the unlikely case that the distribution of capital across age is bell-shaped. Consequently, most explanations involve a technological or learning spillover, i.e., one firm benefits from the efforts of another.

In the context of my model, diffusion can be defined as the percentage of capital in operation using a post t=0 technology. At t=0, the diffusion is zero and at t=6 it is complete. I will consider the balanced growth path from Proposition 4 in the special case that the industry demand curve has a unitary price elasticity (F=1). In this case the number of new plants per unit time and the total capital stock are both constants. The diffusion of post t=0 technologies can be writen as:

$$D(t) = \frac{\int_0^t K^*_{st} ds}{\bar{K}} = \frac{\int_0^t K^*_a da}{\bar{K}},$$

Figure 2

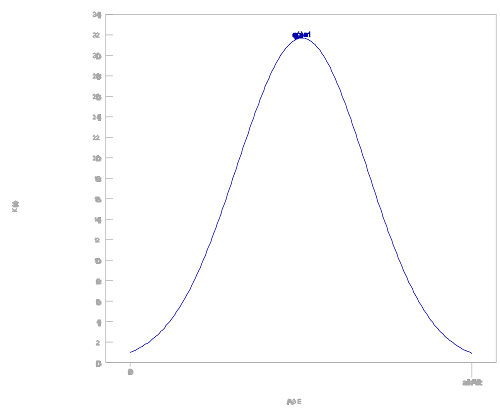
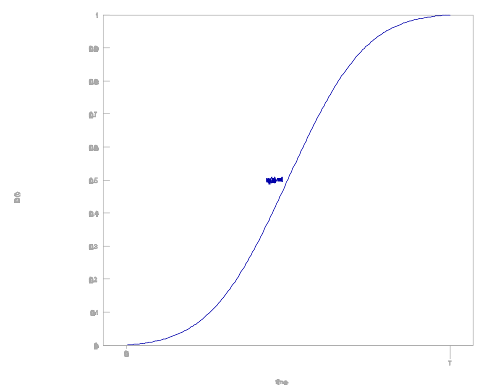


Figure 3:



where & represents the industries aggregate capital stock. (Recall that $_{st}$ indicates the plant's birth date and the time period, respectively; a=t-s is age of the plant.) If the diffusion path is increasing and has an inflection point, it can be said to be S-shaped; in this case, $D'(t) = K^*_t$ must be positive, and $D''(t) = I^*_t$ must start off being positive and become negative before diffusion is complete. If F=1, i.e., a vintage capital model, then K_t is a constant so the rate of diffusion is a constant. If F>1, a plant's level of investment, I^*_a, is initially positive and then falls until it hits zero when $q_t = 1$. Therefore, the rate of diffusion increases until $q_t = 1$, and then remains constant until diffusion is complete.

In order for this model to generate an S shaped diffusion curve, I must relax the irreversibility constraint on investment. It is straightforward to verify that if investment is reversible, then a plant's optimal investment rate is initially positive and slows until q=1 and then investment becomes negative as q falls below one.[21] Therefore, the distribution of plant size (measured in capital) across age is bell shaped (Figure 2) and the diffusion path is S-shaped (Figure 3). A model of plant vintage generates an S-shaped diffusion curve provided investment is reversible and the elasticity of industry demand is unitary. Diffusion begins slowly because it is costly to adopt the new technologies. Diffusion speeds up as expansion by plants with the new technologies becomes increasingly important. Finally, diffusion tapers off as plants with the old technology gradually sell off their capital stock, rather than exiting the market. The phenomenon of S-shaped diffusion can be explained without information externalities and their consequent welfare implications.

VI. An Empirical Application

If technical change in the textile industry is a product of creative destruction, then my theory predicts that the four-digit textile industries with more rapid productivity growth will exhibit more widespread dispersion in relative productivity levels.[22] Testing this hypothesis requires measuring dispersion in productivity levels in specific textile industries as well as the

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[21]It is critical that I is negative and finite when q is less than one. The simplest specification to achieve this is to assume that adjustment costs are convex for I < 0, and that (becomes 1 for a finite and negative I/K.

[22] In this model the only source of productivity growth is creative destruction, i.e., new or re-tooled plants coming on line with the cutting edge technology and forcing out the less productive plants. Appendix III (available from the author upon request) allows existing plants to gradually become more productive, i.e., productivity growth through the representative plant, as another source of productivity growth. It proves the existence of an equilibrium for such a model and demonstrates that dispersion in productivity levels is independent of the magnitude of productivity growth through the representative plant.

rate of productivity growth in these industries.

Through the LRD, the TFP of each plant in the textile industry is measured.[23] Furthermore, the textile industry can be divided into 21 different four-digit industries. From the plant specific measures of productivity, three measures of industry productivity levels are computed: (1) the mean of plant productivity levels; (2) the weighted average of plant productivity levels, where the weights are value added; and (3) the aggregate productivity level, which is an input weighted average of plant productivity levels. The growth rates are then computed by regressing the log of industry productivity onto time. Table 1 reports the growth rates for each four-digit industry according to each of the three measures of industry productivity. Observe that regardless of measure, some industries consistently exhibit rapid productivity growth (Hosiery, Except Socks, 2251; and Tire, Cord and Fabrics, 2296) while others exhibit consistently slow productivity growth (Knit Underwear and Nightwear Mills, 2254; and Cordage and Twine, 2298).

[23]A plant specific measure of total factor productivity can be computed as:

$$TFP_{it} = \frac{RVA_{it}}{(Book_{it})^{\beta}(TE_{it})^{\alpha}},$$

where RVA is real value added, TE is total employment, Book is the book value of capital, " and $ are taken from estimates of a value added Cobb-Douglas production function, and the subscripts $_{it}$ index the plant and time respectively. For further methodological details see Dwyer 1994.

Table 1: Productivity Growth in the Textile Industry 1972-1987

SIC	Growth of mean TFP	Growth of average TFP weighted by value added	Growth of average TFP weighted by inputs	Average TFPratio
2261	0.013	0.013	0.011	3.02
2254	0.014	0.014	0.014	2.88
2298	0.004	0.020	0.016	3.16
2211	0.021	0.024	0.019	2.48
2253	0.022	0.020	0.023	3.05
2252	0.023	0.034	0.025	2.34
2262	0.029	0.031	0.025	2.70
2231	0.041	0.031	0.026	2.69
2283	0.029	0.031	0.030	2.33
2221	0.036	0.031	0.032	2.32
2295	0.037	0.040	0.037	2.89
2269	0.039	0.059	0.037	3.36
2297	0.047	0.034	0.039	2.79
2273	0.044	0.057	0.040	3.79
2257	0.036	0.034	0.040	2.96
2241	0.038	0.038	0.041	2.48
2282	0.051	0.045	0.043	2.72
2258	0.041	0.044	0.046	2.99
2299	0.037	0.046	0.051	3.02
2251	0.074	0.084	0.081	3.38
2296	0.080	0.126	0.088	4.69

In each four-digit industry, plants are grouped into ten ranks on basis of productivity, with each group having the same number of plants in it. That is, plants are ranked into deciles one through ten, with one being the least productive and ten being the most. The TFPratio is computed from the means of the second and ninth decile.[24] The TFPratio typically ranges from between two and three; TFPratios as high as four are not uncommon, and there is no tendency

[24]My theoretical measure of dispersion is the ratio of the most productive to least productive plant in operation at any given instant. This measure of dispersion is chosen because: (1) protecting confidentiality requires the grouping of observations; and (2) the first and tenth deciles are avoided due to outlier problems stemming from faulty measurement, i.e., human error.

towards convergence (Dwyer, 1994). The time mean of the TFPratio is report in Table 1 for each industry.

Table 2: Growth Rates and Exit Rates by Productivity Ranking

Decile	GRVA	GTE	GBOOK	EXIT
1	-0.204	-0.109	-0.070	0.395
2	-0.011	-0.080	-0.074	0.343
3	0.025	-0.067	-0.079	0.314
4	0.033	-0.049	-0.003	0.300
5	0.079	-0.045	-0.056	0.262
6	0.147	0.051	0.001	0.255
7	0.132	0.008	-0.012	0.203
8	0.201	0.031	0.066	0.217
9	0.172	0.050	0.037	0.206
10	0.228	0.106	0.062	0.250

Columns 2-4 report the weighted average of the growth rates of real value added, total employment and book value of capital between census years (between 1972&1977, 1977&1982, and 1982&1987). The growth rate is computed as the difference divided by the average. The deciles are computed on basis of the average of TFP at the beginning and end of the time interval. Each plant is assigned a ranking on basis of its relative standing within its four-digit industry. In computing the exit rates, the plants were assigned into productivity deciles according to their TFP in the beginning of the time interval. A plant was counted as having exited if it was not observed in any industry in the following census year.

First, one should check the model's more obvious implications. Do the more productive grow faster? Are they less likely to exit? The first three columns of Table 2 report the growth rates of real valued added, total employment and capital stock for each decile. The fourth column reports the rate of exit over the next five years. The growth rates are increasing in productivity while the exit rates are falling. This verifies both the model's obvious implications and that these measures of TFP are indicative of the plant's underlying competitive position.

One can test the hypothesis that industries with more rapid technical change exhibit larger TFP ratios by regressing of the mean TFP ratio of each industry on the productivity growth of each industry and one finds a positive and highly significant relationship:

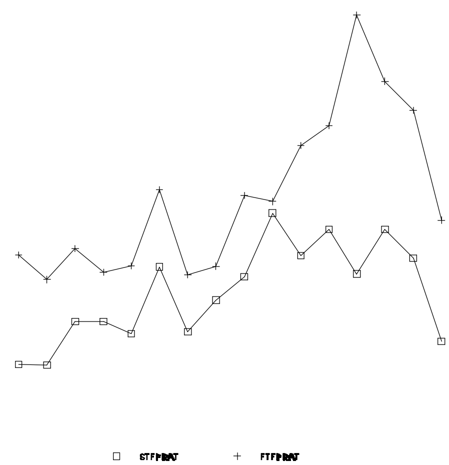
$$\underset{}{TFPratio_{it}} = \underset{(.77)}{2.36} + \underset{(2.38)}{16.26}\ TFPgrowth_{t'}$$

(Number of Observations = 336)

where the subscripts it denote industry and time, respectively, TFPgrowth is the growth of mean TFP, and the standard errors are reported in parentheses.

Such an approach may be problematic. Both the TFPratio and TFP growth are measured with error and errors in measurement may be correlated. Therefore, I group the data to circumvent this problem. The 21 textile industries can be grouped into three groups according to their productivity growth: fast, medium and slow. Under this methodology, the hypothesis that the fast group actually has faster productivity growth than the slow group is reasonable despite the presence of measurement error. We can then compare the average TFP ratio of these two groups in each year (let FTFPratio and STFPratio denote the average TFP ratio for the fast and slow groups, respectively). Furthermore, this grouping can be done according to the three different measures of aggregate productivity growth. Figure 4 presents the average TFP ratio in each year for the fast and slow groups, where the grouping is based on mean productivity growth. The TFPratio of the fast group exceeds the TFPratio of the slow group in every year.

Figure 4: **TFP Ratios of the Fast Productivity Growth Group vs. the Slow Productivity Growth Group, Grouping based on The Growth of the Mean of Industry Productivity**



The extent to which this is a statistically significant relationship can be tested through the following autoregression:

$$TFPratio_{it} = 2.81 + \underset{(.097)}{.49} I_F + \underset{(.053)}{.66} \left(I_F \, (TFPratio_{it-1} - mF) + (1 - I_F)(TFPratio_{it-1} - mS) \right).$$

(Number of Observations = 210)

Here I_F is an indicator variable, taking on the value of 1 if the industry was in the fast group, zero otherwise, and mF and mS are the mean TFPratios for the fast and slow group respectively. Industries with medium productivity growth have been dropped from the regression. The standard errors are reported in parenthesis. The fact that the coefficient on I_F is positive and statistically significant indicates a positive relationship between productivity growth and the TFPratio, as predicted. This is consistent with the hypothesis that productivity growth through creative destruction is causing dispersion in productivity levels.

With regards to alternative explanations, vintage capital models predict this result if the unit of observation is a machine. It is not clear what they predict at the plant level. Both models of productivity growth through the representative plant and models in which plants differ because of product differentiation leading to a downward sloping demand curve predict no relationship between the rate of technical change and productivity dispersion.[25]

The robustness this result was tested by using different measures of total factor productivity as well as labor productivity and the three different measures of industry productivity growth. It is a robust result.

VI. Conclusion

Schumpeter argued that monopoly power is good for productivity growth because it allows the innovator to appropriate the returns from his innovation, thereby increasing the incentive for a firm to perform R&D which in turn increases the rate at which the technology frontier shifts outward. In contrast, developmental economists often argue that monopoly power reduces the level of productivity by allowing the inefficient to remain in operation and thus increasing the dispersion in productivity levels. That is, there is a time consistency problem that has been referred to as the Schumpeterian tradeoff. This tradeoff is intrinsically linked to the time evolution of the distribution of plants across productivity levels.

This paper has characterized the dispersion of productivity levels on the basis of industry

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[25]Specifically, for a model of productivity growth through the representative plant, see Appendix III of this paper. Caballero and Jaffe (1993) develop a model in which productivity is realized through new products of improved quality, which are imperfect substitutes for old products. The distribution of consumption across relative product qualities is determined by only the preferences of the representative consumer.

characteristics under the assumptions of perfect competition and exogenous technical change, i.e., in the absence of a Schumpeterian tradeoff. The technological frontier shifts outward exogenously. Plants are willing to pay sunk costs to get to the frontier because plants at the frontier earn rents. The model is broadly consistent with empirical evidence. It predicts the relationship between technological change and dispersion in productivity levels for one industry. Furthermore, it provides a non-externality based explanation of S-shaped diffusion. This model may provide a useful benchmark of comparison for analyzing how the Schumpeterian tradeoff plays out in different industries.

Provided there is technical change, my model predicts that (1) plants within an industry simultaneously enter, expand and exit; (2) a substantial portion of job creation and destruction is from existing plants changing their size rather than plants exiting and entering; (3) a substantial portion of productivity growth comes from the more productive plants becoming bigger. Therefore, my model is broadly consistent with the empirical evidence. In the absence of technical change this model predicts near convergence in productivity levels.[26] In contrast to models with an arbitrary plant size, the fact that the most productive plant continues to expand after entry has ceased further compresses productivity levels.

This model predicts that industries with more rapid technical change, larger fixed costs and larger adjustment costs will exhibit more dispersion in relative productivity levels. The prediction that four-digit textile industries with more rapid productivity growth will exhibit more widespread dispersion in productivity levels is confirmed. Therefore, large dispersion of

[26]In the Cobb-Douglas case with $b_{t+x} = 1$, Proposition 2 tells us that TFPratio = $(r/*+1)^{\$}$. Letting * = .1, r = .03 and $ = .5 then the TFPratio = 1.14, which is much smaller than anything observed.

productivity levels appears to be evidence of a healthy industry evolving towards higher levels of productivity rather than evidence of a stagnant industry in which the lack of competition has allowed firms to remain inefficient.

Previous models in which plants differ have either departed from the paradigm of fully rational behavior or have relied on increasing marginal costs or a downward sloping demand curve to bound the output of individual plants. Working within the framework of fully rational behavior allows the relationship between the model's parameters and the equilibrium path to be clearly established. Allowing plants to expand provides a richer description of industry evolution. The monotonic relationship between investment and relative productivity levels implies that plants' choose to expand their capital stock until their q falls to 1 and then choose to reverse their investment, if possible, as q continues to fall. As a result, the model can exhibit S-shaped diffusion curves, which are not predicted by models of vintage capital (Phelps, 1962), or models of vintage technology in which a firm is a job is a machine (Caballero and Hammour, 1994). A technological spillover and its implication of a Schumpeterian tradeoff is not necessary to explain S-shaped diffusion.

Appendix I: Table of Symbols

Symbol	Meaning
Mathematical Notation	
F_x	The function F evaluated at x.
MF/Mx	The partial derivative of F with respect to x.
$\hat{x}$	(dx/dt)/x.
%	has the same sign as.
x_{t-} and x_{t+}	The left hand and right hand limit of x at t.
Section II	
b_t	The productivity level of the most productive plant at time t.
b_s	The productivity level of a plant born at date s.
w and *	The per unit cost of labor and capital, respectively.
bM(K,L)	The output per unit time of a plant with productivity b, employing K and L units of capital and labor, respectively.
I	The investment rate, i.e, dK/dt.
((I/K)	The per unit cost of installing a unit of capital.
r	The discount rate.
K_{st}*,I_{st}* and y_{st}*	The optimal stock of capital, rate of investment and level of output chosen by a plant born at s in time t.
F	The fixed cost of entry.
Y_t, p_t	Aggregate output and the price of output at date t.
B_{st}	The rate of profits per unit time per unit capital for a plant born at date s in date t.
T_s	The optimal exiting date of a plant born at s.
C_t	The rate of plants entering per unit time at t.
6_t	The age of the oldest plant in operation at date t.
p*	The price that solves B(b_t,p) = r.

Table of Symbols (Cont.)

Symbol	Meaning
Section III	
F	The elasticity of demand.
µ	The growth rate of the cutting edge productivity level.
" and $	The elasticity of output with respect to labor and capital, respectively. (It is assumed that "+$=1.)
A	$(\text{"}/w)^{\text{"}/\$}\$$, simplifies the profit function.
B_a	The rate of profits as a function of the age of a plant.
K_a* y_a*	The optimal capital stock and output of a plant of age a associated with the solution to M3.
6	The optimal life span of a plant.
R'(I/K)	The per unit cost of installation in Proposition 7.
p**	The price that solves V_0 = F, if price remains constant (see Proposition 6).
TFPratio	The ratio of the productivity levels of the most productive to the least productive plant in operation at any given time.
Appendix II	
f(i)	The density of plants at productivity level i.
	The productivity level of the least productive plant in operation
a_t	The shadow value of capital.
	The shadow value of the nonnegativity constraint on investment.
q	
0	

Appendix II: Proofs of Propositions and the Plants' Optimal Control Problem

Property 3:

T_s is nondecreasing in b_s.

Proof:

Suppose not, then there exist b_s and $b_{s'}$ such that $b_{s'} > b_s$ and $T_{s'} < T_s$, which implies that the present discounted value of profits for $b_{s'}$ would be nonpositive on $[T_{s'}, T_s]$ but is nonnegative for b_s. This is a contradiction because B is increasing in b_s, and both plants can set $I_J = 0$ on $[T_{s'}, T_s]$.

Proposition 1:

p_t is continuous and nonincreasing.

For this proof, it is much more intuitive to compute aggregate output by integrating plant output over productivity levels, rather than over the age of the plant. Let $f(i) = C_s/(db_s/dt)$, where $i = b_s$, i.e., the productivity level of a plant born at date s.[27] f(i) is the number of plants per productivity level at i, i.e., the number of new plants per unit time at date s multiplied by the reciprocal of the change in the maximum productivity level per unit time at time s. Let a_t represent the productivity level of the least productive plant in operation at t. Therefore,

$$Y_t = \int_{a_t}^{b_t} f(i)\, i\, \Phi(K_t, L^*(p_t))\ di$$

[27]In the event that b_t is not strictly monotonic (i.e. db/dt=0 over a finite interval), the notation becomes considerably more cumbersome but the logic is identical.

Proof:

I shall begin by proving that price is continuous:

$$p_{t-} = D\left(\int_{a_{t-}}^{b_{t-}} f(i)\, i\, \Phi(K_{it-}, L^*(K_{it-}, p_{t-}))\, di\right) = D\left(\int_{a_{t-}}^{b_{t}} f(i)\, i\, \Phi(K_{it}, L^*(K_{it}, p_{t-}))\, di\right)$$

$$p_{t+} = D\left(\int_{a_{t+}}^{b_{t+}} f(i)\, i\, \Phi(K_{it+}, L^*(K_{it+}, p_{t+}))\, di\right) = D\left(\int_{a_{t+}}^{b_{t}} f(i)\, i\, \Phi(K_{it}, L^*(K_{it}, p_{t+}))\, di\right)$$

because b_t and K_{it} are continuous.[28] Therefore, if $a_{t-} = a_{t+}$, one p_t solves both equations and p_t is continuous at t. If I can show that $a_{t-} = a_{t+}$ for all t, then p_t is continuous.

Because a_t is increasing, bounded above by b_t, and below by 0, the right and left hand limits of a_t must exist, and therefore the right and left hand limits of p_t must exist. Furthermore, if $a_{t-} \neq a_{t+}$ then $a_{t-} < a_{t+}$ and $p_{t-} < p_{t+}$, that is, a mass of plants must have exited at time t causing an upward jump in price (plants only enter with cutting edge technology, therefore a_t cannot fall). If I can show that $a_{t-} < a_{t+}$ is inconsistent with $i \in (a_{t-}, a_{t+})$ maximizing profits, then price must be continuous.

There exists an ϵ such that for all $\tau \in (t, t+\epsilon)$ either $\pi(i, p_\tau) > 0$ or $\pi(i, p_{t-}) < 0$, because $p_{t-} < p_{t+}$. If $\pi(p_\tau) > 0$ then exiting forgoes positive profits.[29] If $\pi(i, p_{t-}) < 0$, there exists an ϵ such that

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[28]I am adopting the convention that a_{t+} or a(t+) represents the right hand limit of a at t, and a_{t-} or a(t-) represents the left hand limit of a at t. K_t is continuous, because the value of an installed unit of capital is always less the F in equilibrium, which ensures a finite rate of investment.

[29]If $\pi_\tau > 0$ on an interval $(t, t+\epsilon)$ then

$$\int_t^{t+\epsilon}\left(\pi_\tau K_\tau^* - I_\tau^* - I_\tau^*\gamma\left(\frac{I_\tau^*}{K_\tau^*}\right)\right) e^{-r(\tau-t)}\, d\tau \;\geq\; \int_\tau^{\tau+\epsilon} \pi_\tau K_\tau e^{-r(\tau-t)}\, d\tau \;>\; 0.$$

Conversely, if $\pi_\tau < 0$ on an interval $(t, t+\epsilon)$ then for all J 0 (t-,,t), B(i,p_J) < 0 which implies that exiting at t-, avoids negative profits on (t-,,t). Therefore, if p_t jumps upward at t, no plants should exit at t, implying that p_t does not jump upward at t. Therefore, p_t and a_t are continuous functions of time.

I shall now prove that p_t is nonincreasing. Suppose not. There exists p_t and p_{t+*} such that p_{t+*} > p_t, because p_t is not nonincreasing. Furthermore, there exists $p_{t+)}$ = max[p_J] on [p_t,p_{t+*}] with $p_{t+)}$ > p_t which implies that $a_{t+)}$ > a_t because an increase in price can only be brought about by plants exiting. Let i , (a_t,$a_{t+)}$) which implies that T_i 0 (t,t+)), where T_i denotes the date at which i exited. If B_i($p_{T(i)}$) > 0 there exists , > 0 such that B_i(p_J) > 0 for all J 0 [T_i,T_i+,] which implies that i forgoes positive profits by exiting at T_i. If i was making a profit when it exited, it would have made more money by staying in business longer.

If B(i,$p_{T(i)}$) < 0 then there exists an , > 0 such that B_i(p_J) < 0 for all J 0 [T_i-,,T_i], which implies that i suffered negative profits by exiting at T rather than T-,. If i was losing money at T, it would have lost less money by exiting earlier.

If B_i($p_{T(i)}$) = 0 => $B_{a(t+))}$($p_{t+)}$) > 0, because $p_{T(i)}$ # $p_{t+)}$ and i < $a_{t+)}$. Therefore, there exists , > 0 such that $B_{a(t+))}$(p_J) > 0 for J 0 [$T_{a(t+))}$,$T_{a(t+))}$+,] which implies that a(t+)) forgoes positive profits by exiting at $T_{a(t+))}$ = t+) . If i was breaking even when it exited, then a more productive

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$$\int_t^{t+\epsilon}\left(\pi_\tau K_\tau^* - I_\tau^* - I_\tau^*\gamma\left(\frac{I_\tau^*}{K_\tau^*}\right)\right)e^{-r(\tau-t)}d\tau \;\leq\; \int_\tau^{\tau+\epsilon}\pi_\tau K_\tau e^{-r(\tau-t)}d\tau \;<\; 0.$$

The details of this argument will be suppressed for the rest of this proposition.

plant which exiting later when price was higher was making money when it exited. Therefore, this plant would have made more money by exiting later. If $B(i,p_{T(i)})$ is greater than, equal to or less than zero, at least one plant is not exiting at the optimal time. Price must therefore be nonincreasing in order to be consistent with optimizing behavior on part of plants. Q.E.D.

The Plant's Problem

In this section, I develop the plant's problem, M2 and M3, as a standard optimal control problem.

$$V_{st} = \mathbf{Max}_{[I_\tau]} \int_t^{T_s} \left(\pi(\boldsymbol{p}_\tau, s) K_\tau - I_\tau - I_\tau \gamma\left(\frac{I_\tau}{K_\tau}\right)\right) \boldsymbol{e}^{-r(\tau - t)}\, \boldsymbol{d}\tau, \tag{M2}$$

$$\textit{subject to } I \geq 0 \textit{ and } = \frac{\boldsymbol{d}K}{\boldsymbol{d}t}.$$

The corresponding present value Hamiltonian is:

$$H_\tau = \left(K_\tau \pi_\tau - I_\tau - I_\tau \gamma\left(\frac{I_\tau}{K_\tau}\right) + \boldsymbol{q}_\tau I_\tau + \eta\, I_\tau \right) \boldsymbol{e}^{-r(\tau - t)}.$$

The first order conditions are:

$$\frac{\partial H}{\partial K} = \left(\pi_\tau + \left(\frac{I_\tau}{K_\tau}\right)^2 \gamma'\left(\frac{I_\tau}{K_\tau}\right)\right) \boldsymbol{e}^{-r(\tau - t)} = \left(r\boldsymbol{q} - \frac{\boldsymbol{d}\boldsymbol{q}}{\boldsymbol{d}t}\right) \boldsymbol{e}^{-r(\tau - t)}.$$

$$\frac{\partial H}{\partial I} = \left(-1 - \gamma\left(\frac{I_\tau}{K_\tau}\right) - \left(\frac{I_\tau}{K_\tau}\right) \gamma'\left(\frac{I_\tau}{K_\tau}\right) + \boldsymbol{q}_\tau + \eta_\tau \right) \boldsymbol{e}^{-r(\tau - t)} = 0.$$

Because

$$-1-\gamma\left(\frac{I_\tau}{K_\tau}\right)-\left(\frac{I_\tau}{K_\tau}\right)\gamma'\left(\frac{I_\tau}{K_\tau}\right)<-1,$$

either 0 = 0 => I $ 0 and q $ 1 or 0 > 0 and I = 0.

Suppose q > 1, then

$$q = 1 + \gamma\left(\frac{I_\tau}{K_\tau}\right) + \left(\frac{I_\tau}{K_\tau}\right)\gamma'\left(\frac{I_\tau}{K_\tau}\right).$$

By the implicit function theorem, for $I_J/K_J > 0$ there exists n such that I/K = n(q). If q # 1 then let n(q) = 0. Additionally

$$dq = 2\gamma'\left(\left(\frac{I_\tau}{K_\tau}\right) + \left(\frac{I_\tau}{K_\tau}\right)\gamma''\left(\frac{I_\tau}{K_\tau}\right)\right)d\left(\frac{I}{K}\right) \qquad =>$$

$$\frac{dq}{d(I/k)} = \varphi' > 0.$$

That is, the larger the value of an installed unit of capital the greater the optimal rate of investment.

The first order condition with respect to the state variable yields:

$$\frac{dq}{dt} = rq - \pi - \left(\frac{I}{K}\right)^2\gamma'\left(\frac{I}{K}\right). \qquad \text{(eq. \#A.1)}$$

The product rule yields:

$$\frac{\partial(qe^{-r(\tau-t)})}{\partial\tau} = \left(\frac{dq}{dt} - rq\right)e^{-r(\tau-t)}. \qquad \text{(eq. \#A.2)}$$

Substituting eq A.1 into A.2 yields:

$$\frac{\partial(qe^{-r(\tau-t)})}{\partial\tau} = \left(-\pi_\tau - \left(\frac{I_\tau}{K_\tau}\right)^2 \gamma'\left(\frac{I_\tau}{K_\tau}\right)\right) e^{-r(\tau-t)},$$ and

$$\int_t^{T_s}\left(\frac{\partial(qe^{-r(\tau-t)})}{\partial\tau}\right)d\tau = \int_t^{T_s}\left(-\pi_\tau - \left(\frac{I_\tau}{K_\tau}\right)^2 \gamma'\left(\frac{I_\tau}{K_\tau}\right)\right) e^{-r(\tau-t)} d\tau.$$

Performing the integration yields:

$$q_t = \int_t^{T_s}\left(\pi_\tau + (\varphi(q_\tau))^2\gamma'(\varphi(q_\tau))\right) e^{-r(\tau-t)}\, d\tau. \qquad \text{(eq. \#A.3)}$$

Note that I am invoking the transversality condition that $q_{T(s)}e^{-r(T(s)-t)} = 0$, which is given by the fact that T_s has been chosen optimally and the opportunity cost of an installed unit of capital is zero.

Suppose that B_J is a positive constant, property 4 implies that $T_s = 4$, and equation A.3 implies that q_J is a constant. Therefore, equation A.1 implies

$$0 = rq - B - n(q)^2('(n(q)).$$

Note that term $-n(q)^2('(n(q))$ is less than zero for all q > 1 and equal zero for q = 1. Therefore,

If B = r then q = 1 or q > 1, If q > 1 then I/K > 0.

If B > r then q > 1.

If B < r then q < 1 or q > 1. If q > 1 then I/K > 0.

In order to disprove that possibility that q > 1 when B # r, I prove the following claim.

Claim:

If B # r then I/K = 0 earns a larger present discounted value of profits than I/K = R > 0, and therefore q # 1.

Proof:

If I/K = R, then it is straight forward to show that $K_t = K_0e^{Rt}$.

Let V(R) and V(0) denote the present discounted value of profits if I/K = R and I/K = 0, respectively.

$$V(0) = {}^{*4}_{0}\, \mathrm{B}K_0 e^{-rt}dt \ = K_0\mathrm{B}/r.$$

$$V(\mathrm{R}) = {}^{*4}_{0}\,(\mathrm{B}K_t - \mathrm{R}K_t - \mathrm{R}K_t\,((\mathrm{R}))\, e^{-rt}dt,$$

which is non positive if R \$ B and the claim is trivial, therefore from now on I assume that R < B, and it follows that:

$$V(\psi) = \int_0^{\infty}(\pi - \psi - \psi\gamma(\psi))K_0 e^{(\psi - r)t}dt$$

$$= \frac{K_0(\pi - \psi - \psi\gamma(\psi))}{(r-\psi)}.$$

The claim that V(0) > V(R) is equivalent to:

$$\frac{(r-\psi)\pi}{r(r-\psi)} > \frac{r(\pi - \psi - \psi\gamma(\psi))}{r(r-\psi)},$$

which must be true because RB < rR. Therefore,

if B = r, then q = 1;

if B > r then q > 1; and

if B < r then q < 1.

I will now demonstrate how the value of the plant changes as certain parameter value changes in the context of M3. Provided V is differentiable, I can invoke an envelope theorem yielding:

$$\frac{\partial V_a}{\partial p_0} = \int_0^{\bar{a}}\left(\frac{\partial H_a}{\partial p_0}\right)da = \int_0^{\bar{a}}\left(\frac{K_a\, p_0^{\frac{1}{\beta}-1} e^{-\frac{\mu a}{\beta}}A}{\beta}\right)e^{-ra}da \ \geq \ 0,$$ and

$$\frac{\partial V_a}{\partial \mu} = \int_0^{\bar{a}}\left(\frac{\partial H_a}{\partial \mu}\right)da = \int_0^{\bar{a}}\left(\frac{-aK_a\, p_0^{\frac{1}{\beta}} e^{-\frac{\mu a}{\beta}}A}{\beta}\right)e^{-ra}da \ \leq \ 0,$$

where the inequalities become strict provided $\bar{a} > a$.[30]

In the context of Proposition 7 (((I/K) = R'(I/K)),

$$\frac{\partial V_a}{\partial \psi} = \int_0^{\bar{a}} \left(\frac{\partial H_a}{\partial \psi} \right) da = \int_0^{\bar{a}} \left(-K_a \ \varphi(q_a) \ \Gamma(\varphi(q_a)) \right) e^{-ra} da \ \geq \ 0,$$

and strictly greater than zero if $q_0 > 1$. That is it is strictly greater than zero provided it is optimal to have positive investment.

[30]See Seierstad and Sydsæter for the mathematical statement of this theorem (1987, page 216). If one objects to assuming that V is differentiable, then proving continuity and monotonicity of V in the relevant parameters is straightforward, but tedious. The main idea is simple. Suppose there is a change in a parameter value that increases the value of the firm holding the investment path fixed. Because the old investment path is available at the new parameter values, the new investment path must do at least as well as the old investment path.

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