pressure need not be considered at present. The range of movement required and freedom from vibration must be secured by the use of strong elements well supported.

The apparatus described herein is suggested as a basis for further experimental study. It has not been constructed, but the principle of Mr. Dines's successful engraving instrument has been followed, and essential parts already have been used to a limited extent in instruments produced by Richard and other manufacturers. Referring to figures 14, 15 and 16, the pressure-element (B) is composed of two helical Bourdon tubes secured in a light frame (A), so that their free ends move in opposite directions when there is a change of pressure. A single tube could be used, but the double-tube element is preferred for the reason that thereby may be secured greater compactness and rigidity. One tube carries the record-plate (C), and the other the temperature-element. The latter consists of two or more strips of very thin bronze (T, T), connected by spring hinges and mounted in a light invar frame (D), in such manner that changes of length corresponding to changes of temperature are engraved upon the record-plate by the style (E). The strips (T, T) are insulated from their support.

The inner ends or edges of the plate-carrier (C) and the frame (D), are secured, under tension, to spring hinges in the center of the tube (F), and therefore restrict the motions of the pressure tubes to an arc whose axis is the center of the tube (F). By this means longitudinal movements of the pressure tubes are prevented and there are no pivots with the variable friction inevitable when Bourdon tubes of this kind are mounted in the usual way. Another application of this device, in the construction of a simple thermograph without pivots, is shown in figures 17 and 18. Here, circular motion about the center of the coiled element (T), is obtained by securing to its free end the frame (D). Adjustment for range is accomplished by changing the position of (D) as shown by the dotted lines.

A GENERAL THEORY OF HALOS.

By CHARLES S. HASTINGS.

[Yale University, May, 1920.]

SYNOPSIS.

The general theory of halos developed in this paper rests on the assumptions that two kinds of simple ice-crystals—elongated hexagonal rods and hexagonal plates—are occasionally present in a tolerably transparent atmosphere; moreover, that these crystals subsiding in quiescent air would necessarily fall into four groups.

The first portion of the paper establishes the validity of the assumptions by reference to well-recorded observations.

The second portion is devoted to a development of the consequences from the presence of each of these groups for various altitudes of the sun. It is there shown that all the authenticated features of complex halos are naturally explained (excepting certain rare multiple concentric circles) as inevitable consequences of the hypotheses. In addition, this portion gives a new means of classifying the various phenomena, showing unsuspected relationships as well as essential diversity in certain other cases where common origin was formerly assured.

I.

During the 72 years which have elapsed since Bravais published his celebrated and comprehensive work on halos many observations have been accumulated—some even by means of photography—and much has been written in the effort to improve questionable points in the theory presented by that admirable writer. As regards the efforts of the theorists it does not seem unfair to say that they have been quite futile; at least, no solution of a difficulty left by Bravais, as far as known to me, has ever commanded general acceptance. The elaborate mathematical discussions by Pernter of the tangent arcs to the 46° circle and of the, so called, arcs of Lowitz, perfectly illustrate the rather sweeping statement: Each of these is a logical conclusion from premises which no instructed meteorologist can possibly accept.

Before advancing any new views regarding the highly complex phenomena involved it will be well to summarize what was known when Bravais finished his work. The number of features which he considered and attempted to account for was about twenty. Of them we may ignore one or two as not being sufficiently authenticated, but we must add two which are of unquestionable authenticity; thus the total number remains nearly the same. Unfortunately, a small minority only of these were satisfactorily explained. We may catalogue these here and escape an undue lengthening of this paper by unnecessary repetition.

(1) The ordinary circle about the sun of 22° radius, attributed by Mariotte to the action of ice crystals suspended in the air and having faces inclined at 60°, the directions of their crystallographic axes being entirely fortuitous. This explanation of the commonest of all halos is thoroughly satisfactory and universally accepted.

(2) The 22°-parhelia, often called sun-dogs, are prismatic images of the sun right and left of it and at the same altitude. With a low sun they are at the angular distance named, but at a higher altitude the angular separation increases. They are not seen higher than 50°. At high latitudes they are more frequently noted than any other feature and the explanation—also first advanced by Mariotte—as due to hexagonal ice crystals with persistently vertical axes leaves nothing to be desired.

(3) The parhelic circle—a faint, colorless circle everywhere equally distant from the zenith and passing through the sun. This was attributed by Thomas Young to reflection from the faces of hexagonal prisms falling vertically. Bravais improved this theory by the remark that crystals with their principal axes persistently horizontal would also contribute to this feature. I shall show that probably only such reflection as is total, hence from the interior of the crystals, is generally effective.

(4) Upper and lower tangent arcs to the 22°-circle These are due to the presence of crystals whose principal axes are horizontal, the lateral faces having any direction in space. As the sun rises to an altitude of about 45° these two arcs unite and form a ring inclosing the 22°-halo and touching it at its highest and lowest points. At very high sun this ring, called the circumscribed halo by Bravais, approaches more and more a true circle. This ring may exist alone. Admirable photographs taken at New Haven, Conn., and at Chester, Pa., of the halo of March 20, 1915, have been published in the MONTHLY WEATHER REVIEW.[1] Bravais gave a very complete analysis of these features with tables which may be used to find the position of any desired point

[1] May, 1915, 43: 213-216 and October, 1915, 43: 498-499.

Photo by Paul Schultz.

THE SUN DOGS.

FIG. 7.—A reproduction of the remarkable photograph by Paul Schultz in Archdeacon Stuck's book "Ten Thousand Miles with a Dog Sled," page 388, Scribner's, 1914.

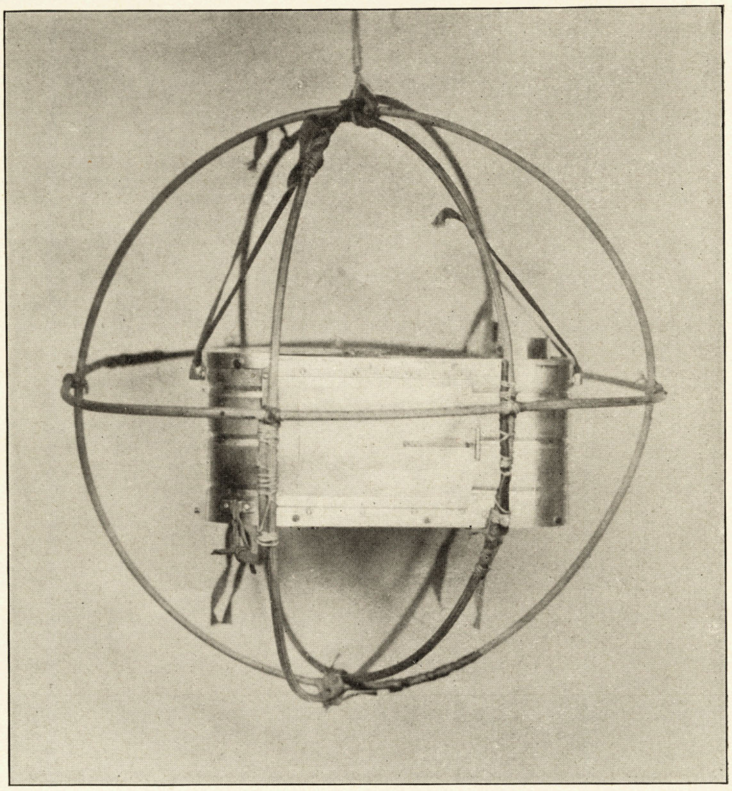

Fig. 19.—Meteorograph in basket.

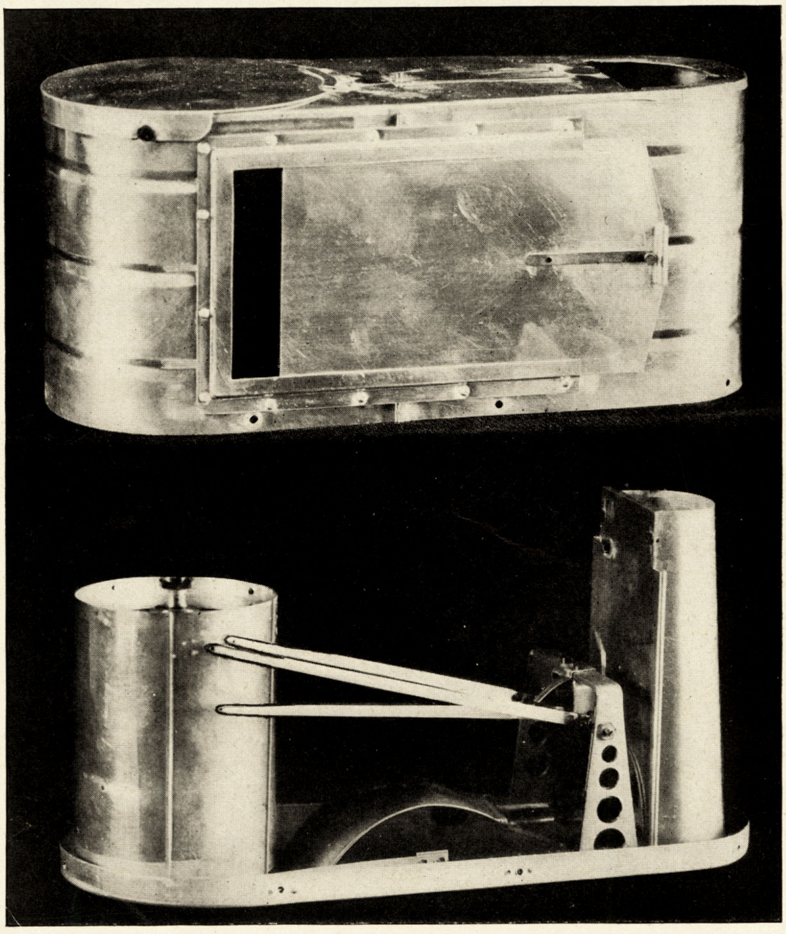

Fig. 20.—Meteorograph with cover removed, showing mechanisms.

with any assigned altitude of the sun. In the calculations which are at the basis of the solutions given below I have saved myself unnecessary labor by recourse to these tables.

(5) Upper and lower tangent arcs to the 46°-halo. These are exactly tangent only in the cases of a solar altitude of 22.1° for the former and 67.9° for the latter, but they may be conspicuous when the sun departs several degrees from these stations. Their forms will appear in certain of the solutions given in this paper. The theory and analysis of this feature by Bravais appears to be faultless.

All theorists since the time of Bravais would doubtless include as a sixth feature of complex halos finally and completely accounted for by him the 46°-halo.

This feature was explained by Cavendish as similar in origin to that of the 22°-halo except that it is produced by prisms having faces at angles of 90° instead of 60°, such as would be formed by a base and any side of a simple hexagonal prism. These refracting edges are assumed to have purely fortuitous directions in space and, as in the case of the 60° prisms, only those which happen to be near the position of minimum deviation are effective. This explanation has been universally accepted and it certainly is recommended by its simplicity; it presents, however, difficulties which are not easily to be disposed of. If it were true, one would expect to see the 46°-circle almost as frequently as the 22°, at least when the latter is brilliant; but this is far from being the case. Again, this accepted explanation would make the appearance as likely with the sun high in the heavens as when the sun is low, which is also very far from true. There is no record of the 46°-circle having attained the zenith or even come very near it—in short, one might assert with considerable confidence that this feature does not appear with the sun more than 32° above the horizon. Again, were this explanation correct, the circle in its ordinary exhibitions ought to appear uniformly bright, as does the 22°-circle. Such a condition may indeed occur, but not according to my rather limited experience. In every case which I have observed the brightness has been far from uniform, but has—a very significant fact—been distributed in arcs symmetrically placed as regards the vertical through the sun. We shall see in Parry's famous record an exactly similar condition. In view of these facts I prefer an explanation offered below which, however, does not exclude the possibility that fortuitously directed crystals play a part in some cases unlike those which have come under my observation.

It will not be profitable to dilate upon the reasons why the explanations of other features of complex halos are held by me as untenable. The only reason of moment is, of course, the belief that the explanations offered here are better, and any reader can easily resolve any doubt as to that by recourse to easily accessible works; I shall, however, permit myself to call attention to those cases where an unforced explanation of a feature as derived in this paper stands without any alternative.

The method adopted by all investigators has been to establish the existence of a feature by reference to the records and then to invent a form of ice crystal which was though adequate to produce it. These inventions are very numerous and some of them of admirable ingenuity. There are two serious objections to such procedure. One is perfectly obvious, namely, the objection to assuming at any particular time the presence of rare, or even unrecognized crystal forms, in such overwhelming numbers as to be effective. The other is a little less evident although even more formidable; the presence of any crystal not immediately effective in the production of a given feature obscures it. The conditions of development of rainbows and of halos are, in a sense, antithetic. In the former the more opaque the background, provided that the opacity is due to raindrops alone, the more brilliant the phenomena; in the latter a considerable transparency of the sky is a primary requisite. Thus economy in the number of types of crystals assumed in any theory is of the highest importance.

In the theory presented here only two form of crystals are postulated both of which are the simplest types of ice crystals and both of which are familiar to observers. The first is that of a hexagonal prism with a short principal axis—in short, a hexagonal plate of which the thickness is a small fraction of its diameter. Such crystals I shall style the A type. The other form is that of a hexagonal prism of which the length is much greater than the diameter: it might be described as a hexagonal rod. This second form I shall designate as the B type. Both types have dihedral edges of 90° and of 120°, the former being the angle at which the basal and side faces meet and the latter that of the lateral faces: but optically (as regards transmitted light) they yield only prisms of 90° and of 60°, the prism of smaller angle being truncated by the plane which forms the intermediate face of the hexagonal crystal.

Such small bodies falling through a quiescent resisting medium would have a decided tendency to assume positions such as to offer a maximum resistance to the relative motion, hence the A crystals would tend to remain horizontal—that is, with their short principal axes vertical—while the B crystals would maintain their principal axes horizontal. This assumption is exactly opposite to that of Bravais and his followers who supposed that such bodies would set themselves so as to meet with the minimum resistance. These assumptions constitute the fundamental difference between the old theories and the present one and should therefore be carefully noted by the critical reader. It is difficult to understand how the earlier view could be taken, notwithstanding the lack of a knowledge of the mechanical principles involved which we now possess, for every one knew of the difficulties met in keeping an elongated projectile in end-on flight.

This tendency to stability of direction of the principal crystalline axis of the two types is far from being a strong one; its effect is as remote as possible from producing a pendulum-like oscillation such as Pernter has assumed as the basis of certain of his explanations. In the latter kind of motion the moment of restitution increases proportionally with the displacement from the position of equilibrium, while in this case of fluid constraint the moment of restitution decreases with departure from equilibrium and wholly vanishes at some indeterminate angle. Thus, although we may have recurrent motion, simulating harmonic motion, in cases restricted to very small amplitudes, the usual result would be a continuous rotation about some major axis. Examples of such motion are familiar to every observer, e. g., as shown by the fall of petals of fruit blossoms, by that of small bits of paper.

With this enlarged view as to the phenomena presented by small, regularly formed bodies in falling through quiescent air, we find that we have, with two types of ice crystals only, four different groups. These may be designated and defined as follows:

A group; those hexagonal plates which fall with their principal crystaline axis continuously vertical.

A′ group; similar plates which, in falling, rotate continuously about a major diagonal.

B group; those elongated hexagonal crystals which fall not only with their principal axes continuously horizontal but also their maximum cross-section horizontal.

B′ group; crystals like the last but rotating continuously about their principal crystaline axis. Some, or even many, of these may be assumed to have a motion about their centers of mass so that these axes describe cones in space, always, however, of small angular opening.

With these simple postulates the problem of halos divides into two parts, first, to demonstrate from the records the occasional existence of these four groups of crystals in our atmosphere; and, second, to deduce all of the optical consequences from the presence of such crystals and compare these consequences with recorded observations.

There exists one record (only one, unfortunately) which admirably meets our requirements for the first step; it is the halo figured and described in Parry's First Voyage,[2] p. 164–165. This record is peculiar in several respects; first, it was observed in common by two skilled observers, Parry and Sabine, and had a complexity involving three of the groups of crystals named above; and, second, because of the remarkable duration of slowly changing phases due to the high latitude, which was 74° north.

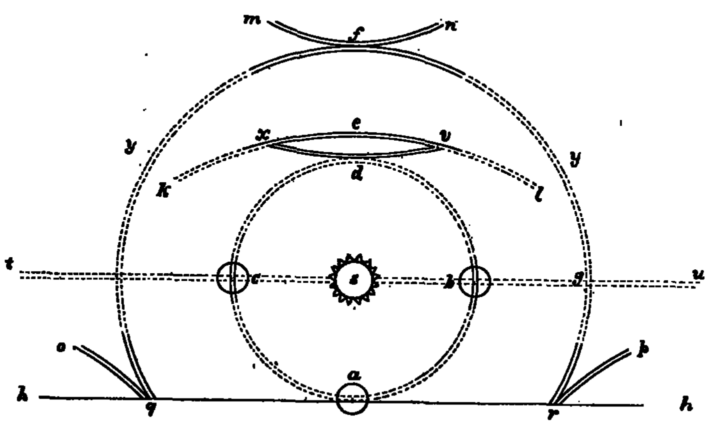

FIG. 1.—The halo of Parry and Sabine.

The diagram and description as given by Parry and Sabine are as follows:

From half-past six till eight A. M., on the 9th, a halo, with parhelia, was observed about the sun, similar in every respect to those described on the 5th. At one P. M. these phenomena re-appeared, together with several others of the same nature, which, with Captain Sabine's assistance, I have endeavoured to delineate in the annexed figure.

s, the sun, its altitude being about 23°, h, h, the horizon.

t, u, a complete horizontal circle of white light passing through the sun.

a, a very bright and dazzling parhelion, not prismatic.

b, c, prismatic parhelia at the intersection of a circle a, b, d, c, whose radius was 22½° with the horizontal circle t, u.

x, d, v, an arch of an inverted circle, having its centre apparently about the zenith. This arch was very strongly tinted with the prismatic colours.

[2] John Murray, London, 1821.

k, e, l, an arch apparently elliptical rather than circular, e being distant from the sun 26°; the part included between x and v was prismatic, the rest white. The space included between the two prismatic arches, x e v d was made extremely brilliant by the reflection of the sun's rays, from innumerable minute spiculae of snow floating in the atmosphere.

q f r, a circle having a radius from the sun, of 45°, strongly prismatic about the points f q r, and faintly so all round.

m n, a small arch of an inverted circle, strongly prismatic, and having its centre apparently in the zenith.

r p, q o, arches of large circles, very strongly prismatic, which could only be traced to p and o; but on that part of the horizontal circle t u, which was directly opposite to the sun, there appeared a confused white light, which had occasionally the appearance of being caused by the intersection of large arches coinciding with a prolongation of r p, and q o.

The above phenomenon continued during the greater part of the afternoon; but at six P. M., the distance between d and e increased considerably, and what before appeared an arch, x, d, v, now assumed the appearance given in fig. 12, plate 287, of Brewster's Encyclopaedia, resembling horns, and so described in the article "Halo," of that work. At 90° from the sun, on each side of it, and at an altitude of 30° to 50°, there now appeared also a very faint arch of white light, which sometimes seemed to form a part of the circles q o, r p; and sometimes we thought they turned the opposite way. In the outer large circle, we now observed two opposite and corresponding spots y, y, more strongly prismatic than the rest, and the inverted arch m, f, n, was now much longer than before, and resembled a beautiful rainbow.

This sketch is not drawn according to any geometric projection and does not admit of quantitative analysis; but if we accept the position angles as referred to the sun and the angular distances as proportionel to the linear distances, we shall be able to redraw it according to any system of projection preferred, since the scale is given by the known angular dimensions of the inner circle. The projection which I shall choose for all of the diagrams in this paper is that known as the spherical projection. Moreover, with a single exception, I shall choose the plane of the paper as that of the horizon, the center of the circle representing the visible horizon being the projection of the zenith. The advantages of this particular system are many; every circle in the heavens is represented by a circle on the plane (which becomes a straight line when a great circle passing through the zenith) and the angle of all intersections is preserved unaltered. Moreover, the coordinates of every point as represented by azimuth and zenith distances are readily found, the first by direct reading from horizon circle and the second by means of a simple trigonometric formula or by a scale constructed for that purpose. The only serious fault is that of a considerable distortion in the neighborhood of the horizon.

Figure 2 represents the observations of Parry and Sabine thus reduced to a spherical projection. I shall proceed to construct a halo, according to the same laws of projection, which would result as a consequence of the fundamental assumptions above. A comparison of the two may be expected to validate these assumptions or the contrary. To do this it is necessary to define certain constants and symbols.

The acepted mean index of refraction of ice is 1.31, which is the value adopted by Bravais and his followers. It is that of yellow-green light, the most brilliant portion of the spectrum. This constant yields 49.76° for the critical angle of interior reflection, 45.74° for minimum deviation for 90° prisms and 21.84° for that of 60° prisms.

In calculating the optical effect of a prism, the prism will be regarded as placed at the center of the celestial hemisphere of which the area within the circle of the horizon is the projection; the aspect of the prism with respect to the celestial sphere will be defined by the positions of the poles of its eight faces. A convenient notation which will be adopted is the following: o and *o* mark the poles of one base and its opposite; p, p′, p″ give the places of the poles of three successive lateral

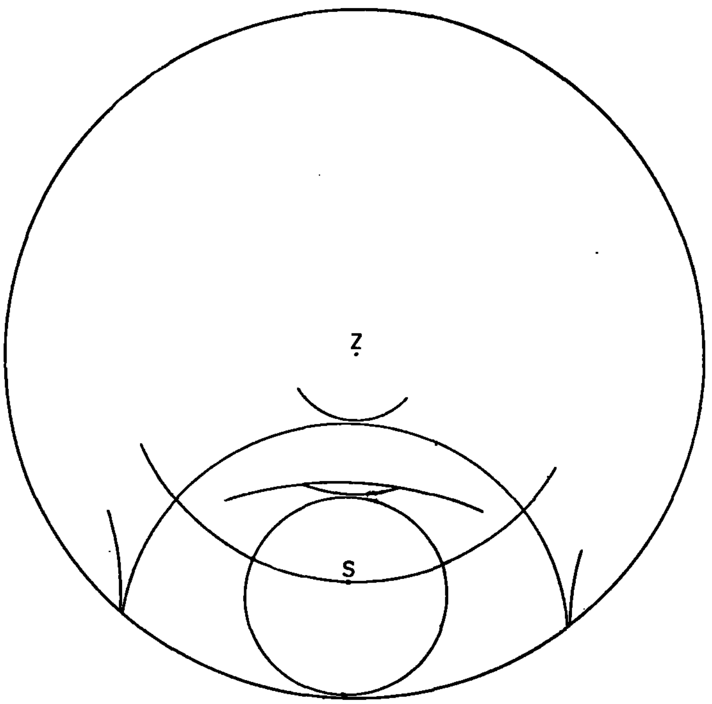

FIG. 2.—Halo of Parry and Sabine drawn in spherical projection with zenith at pole of plane of projection.

faces and *p*, *p′*, *p″* those of their opposite faces, respectively. The spherical coordinates of points on the sphere are the zenith distance, symbol z, and the dif-

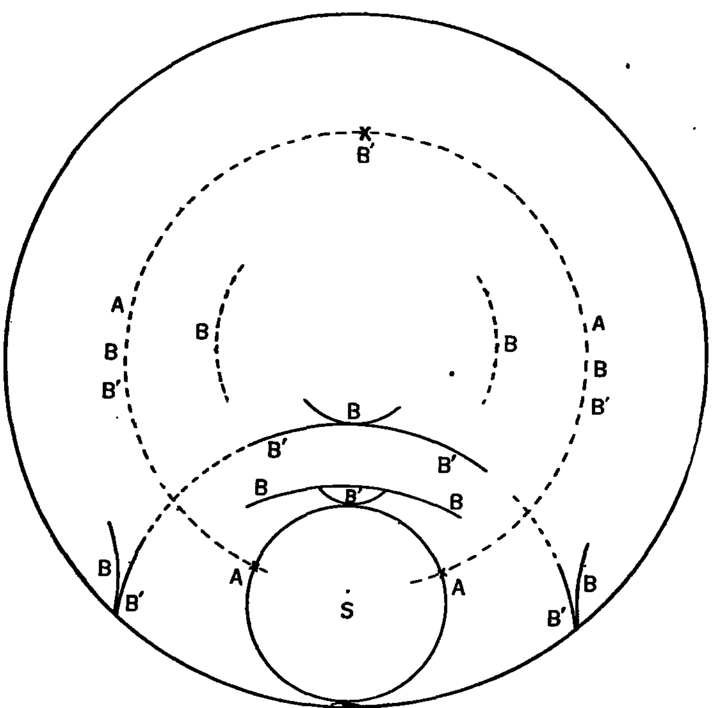

FIG. 3.—Halo of Parry and Sabine according to theory here presented. The letters indicate particular group of crystals involved.

ference in azimuth between that of the point to be defined and that of the sun. The latter angle will be called the amplitude and designated by the symbol u. The accompanying figure 3 shows the results of such calculations; it includes two features which are outside the limits of the drawing by Parry and Sabine but are described in the text. These are the faint luminous spot opposite the sun in the parhelic circle called the anthelion, and the pair of faint arcs between the zenith and the parhelic circle having a mean amplitude of approximately 90°. The origin of each of the details is indicated by the lettering which shows the group or groups of crystals concerned in its production. The close resemblance between figures 2 and 3, the former being merely a record of eye observations and the latter a mathematical deduction from the fundamental assumptions for a sun altitude of 23°, is a quite sufficient proof that of the groups of crystals assumed all but the A′ group are occasionally effective in causing halos. The proof as regards this last group will appear later.

A somewhat complete analysis of this solution embodied in figure 3 will save many words in descriptions of corresponding features characteristic of higher altitudes.

The effects of the A group are rather insignificant in this particular manifestation for, besides the familiar 22°-parhelia, they only add a small portion of the total light in the parhelic circle and to the inverted arc above the 46°-circle.

The B group, particularly prominent in this halo, although far from rare in other cases, is the cause of the arc whose highest point is about 39.7° from the zenith, equivalent to 5.5° above the vertex of the 22°-circle; of the brilliant spot of light at the horizon which is merely the complement of the preceding feature; of all of the light in the oblique arcs springing from the horizon at the lower ends of the interrupted 46°-circle; of most of the light in the upper tangent arc to the 46°-circle; of the faint arcs between the zenith and parhelic circle which we shall later find cause for denominating the higher oblique arcs through the anthelion; and, finally, to much of the light which comes from the parhelic circle.

The B′ group produces the upper and lower tangent arcs to the 22°-circle, the latter being mostly below the horizon; of the three brilliant portions of the 46°-circle which are tangent to three arcs mentioned above as due to B crystals and, possibly, to the totality of the 46°-circle; of much of the parhelic circle; and, finally, of the faint spot of light opposite the sun in the parhelic circle. These various details of the halo in question will be taken up in the order named, ignoring, however, the 22°-circle, concerning which everyone is agreed.

That the A group is relatively inconspicuous in the Parry halo is proved by the moderate intensity and extension of the 22°-parhelia, very different in these respects from the famous halo of Hevelius. In the latter, which will be discussed later, the presence of the A′ group, here wanting, will be demonstrated.

The crystals of the B group have the upper and lower lateral faces persistently horizontal; in other words, they subside in the atmosphere with a maximum cross-section constantly horizontal. The two basal planes are constantly vertical. Light from the sun which falls upon the upper face, p, will emerge after refraction through the surface *p′*, provided that the amplitude of the pole o of the base is not too far removed from ±90°. A sufficient number of places of images of the sun produced thus with different values of the amplitude of o were calculated so that the long arc depicted between the 22° and the 46° circles could be accurately constructed. Since none of my predecessors has considered this highly important feature I venture to call it Parry's upper arc, and hereafter I shall refer to it under that name.

Light which enters the same B crystals at a p″ face and emerges also at a p' face forms an arc below the sun convex upward and mostly below the horizon. Its vertex is 23° from the sun and very brilliant. At first thought it appears contradictory that the observers noted this as white, but a recognition of the facts that the less refrangible portion of its spectrum is combined with a more refrangible portion of the ordinary lower tangent arc of the 22°-circle, and that the distinctive colors of short wave lengths are invisible on account of falling below the horizon, readily disposes of the contradiction. This arc, which will recur in other cases with a higher sun, will be called Parry's lower arc.

In this particular halo the arc tangent at the vertex of the 46°-circle is chiefly due to the B group, although in many cases only the A group is concerned, as in the Hevelian halo, which follows; indeed, Bravais emphasizes the relation of this arc to the 22°-parhelia inasmuch that they occasionally exist together as the whole of the manifestation. Of course that writer, recognizing neither the A group nor the B group as defined here, but only the elongated prisms assumed to fall with vertically directed axes as providing horizontal rectangular edges, was obliged to regard them as always associated, whereas in this particular halo the association is only partial. However, given such persistently horizontal refracting edges of 90°, the theory of Bravais is complete and the topic might be left without further discussion were it not for a significant remark in the description which is worth consideration. The observers remark extraordinary purity of the colors in this arc when the sun was much lower, although during the earlier period the incidence of the light was almost exactly that corresponding to minimum deviation and maximum brightness. The reason is not far to seek. The visible spectrum produced by a right angle prism of ice is a short one—we may rate it at about 1.5° in neglecting the fainter terminal colors; but the diameter of the sun is much too considerable a portion of this angle to yield a spectrum approaching purity of colors. This effect due to angular magnitude of the source diminished with increasing angle of incidence and the spectrum becomes an absolutely pure one at the limit of 90° incidence. As this effect is reversed when the angle of incidence is less than that proper to minimum deviation it is likely that the arc has been more frequently recorded with excessive incident angles than when the emergence angles were equally in excess.

The curious arcs springing from the horizon and tangent to the 46°-circle come from light which, incident on a base of a B crystal, emerges from a lateral face. It is easy to see that the conditions necessary for their production are very unusual, and they would also be very evanescent, except in polar regions; they have not been considered, as far as known to me, by any previous writer except Bravais, who classed them with certain tangent arcs due to B′ group. In this he was certainly in error, as will appear when I discuss the features attributable to the latter group. They may be called Parry's lateral tangent arcs to the 46°-circle.

A portion of the light which enters the upper face of a B crystal would fall on a basal plane, and after reflection from that plane would emerge from the p' surface. In those cases in which the amplitude of the crystal is such that this interior reflection is total this light is significant. Such is the origin of the arcs near the zenith and drawn as broken lines in figure 3. With a higher sun this feature is sometimes conspicuous and it will be convenient to defer a theoretical consideration until such cases come under review. They may be styled Upper oblique arcs passing through the anthelion.

As a final feature, due in part only to the B group, should be named the parhelic circle. All light reflected from the bases of both groups B and B′ would appear to come from the parhelic circle as also that from the sides of the A group; but especially important would be that portion which has been totally reflected from the interior. Such total reflection ceases at amplitudes not far from 130°. Beyond these limits the circle would be fainter, a character which is not noted in the description unless the extreme faintness of the anthelion clearly implies it.

The effects of the B′ group in the immediate vicinity of the 22°-circle are so perfectly understood from the discussion of Bravais that it is not necessary to consider them further here, especially as all the solutions of these details in this paper are deduced from the tables given by him.

The contributions of the B′ group to the features near the 46°-circle can not be so easily dismissed. They consist of a number of arcs, more or less perfectly tangent to the 46°-circle, sometimes concave toward the sun, but occasionally having the opposite curvature. The conditions of their appearance are easily defined. Suppose a crystal of this group at the center of the celestial sphere; change the amplitude of its principal axis, at the same time rotating it about this axis, until a principal plane of a rectangular edge passes through the sun, then, if the angle of incidence of the sunlight is that, or nearly that, corresponding to minimum deviation there will result an image of the sun at a point in the 46° circle or just outside of it. In this case all crystals having nearly the orientation defined would contribute to the formation of an arc passing through this image no point of which could be nearer the sun, hence the arc would appear to be tangent to the 46°-circle.

There is another highly instructive method of attaining the result as applied to the Parry and Sabine halo. Having shown that crystals of the B group, which have a single degree of freedom as regards orientation, produce three tangent arcs to the 46°-circle and knowing that the crystals which are supposed to produce the 46°-circle have complete freedom in this respect, that is, three degrees, it follows at once that the B′ group, possessing two degrees of freedom, must form arcs more closely adjusted to the circle than the former arcs. The observations are in complete accord.[3]

It will be noted that these three arcs, although departing but little from the 46°-circle, are limited in extent and leave portions of this circle vacant. If, however, we attribute to some of this group a moderate rocking motion about the center of mass of the character described above, this vacancy would disappear and the circle would appear unbroken although not uniformly bright. Calculation shows that crystals departing 14° from the horizontal would perfectly replace the hypothetical random crystals to which the accepted theory attributes the 46°-circle, while half this angular deviation would be quite sufficient to give rise to a ring which could not be distinguished from a circle except by careful measurement. These considerations lead me to prefer this explanation of the 46°-circle to the current one which has long been accepted. It is adequate—even to ex-

[3] The previous theories of these arcs are somewhat tangled. Bravais gives his theory in three lines, a theory which is not clear to me. Pernter rejects this theory and replaces it by another which so acute a critic as Besson finds untenable: nor does the latter theory appear to me to accord with the mechanical laws to which falling crystals are subject or the records.

plaining the unique observation of Besson, who found a visible separation between the circle and the closely-agreeing tangent arc—and also answers the puzzling question as to why the circle has never been seen complete; that is, wholly above the horizon.

There remain the short arcs through the anthelion and the anthelion itself. These I attribute to the action of the B′ group and explain as follows: Imagine a crystal of this kind at the center of the celestial sphere with its p face vertical, the p'' and p′ being respectively above and below it, and the sun near the horizon. Light from the sun entering this face near the end of the prism would, after successive reflection from the vertical base and opposite side in either order, emerge at the surface of entry as coming from a point in the parhelic circle exactly opposite the sun, in short, from an anthelion. This would be true for all angles of incidence, but in those cases where both interior reflections are partial the returning light would be entirely insignificant. On the other hand, when the reflection from the basal surface is total the quantity of light returned would be vastly greater. If, however, the angle of incidence is small the dimensions of the reflected beam of light would be small on account of the foreshortening of the totally-reflecting surface; as this angle increases, the quantity of light would continuously increase until it reached its maximum at the critical angle of interior incidence. With a higher sun, approximating to 30° for example, light entering at the p'' face and emerging, after having experienced a similar double reflection, at the p′ face would also appear to come from the anthelion. But the assumed position of the crystal is not a stationary one according to the mechanical principles governing the falling of light bodies through a resisting medium, hence the effects produced would be less simple than this. In fact, the rotating B′ crystals would yield two arcs passing through the anthelion; the outer edges would be tolerably well defined and much the brightest portions, so that they would appear as two short arcs crossing at the anthelion under a determinate angle depending upon the altitude of the sun. With the sun at the horizon calculation shows that this angle would be about 20°, the angle increasing rapidly with increasing altitude. With the altitude of the sun at 30° the short arcs due to light entering and emerging at different faces the angle of inclination may be rated at 70°. With a certain range of intermediate altitudes both pairs of arcs may coexist, although such occurrences must be infrequent.

A review of the last paragraph shows that the present theory does not allow for a true anthelion, that is, there is no stationary image produced by a host of crystals of widely varying orientations; on the other hand, there is often a brighter portion of the parhelic circle exactly opposite the sun which is the locus of the intersection of three or of five arcs, as the case may be, and which still may bear the name. This node is accentuated by the fact that the short arcs are brightest just at their middle points, which fall on the parhelic circle. Necessary deductions are that these phenomena are only associated with a low sun. The explanation is advanced with some confidence not only because it seems to fit admirably with the records but also because there is no alternative other than one which attributes the arcs to diffraction from highly fantastic crystals which have never been observed and which are supposed to fall edgewise.

The famous halo of Hevelius, observed and recorded by that astronomer in 1661, presents a new set of phenomena. Unfortunately the record is very imperfect, since the drawing and the description are very discordant just where we demand precision. But in the admirable collection of recent observations described by Dr. Louis Besson [4] there is one, a drawing by Orin Parker, of a halo seen by him at Bentonville, Ark., November 1, 1913, which is almost a replica of that recorded by Hevelius except that the paranthelia and the oblique arcs through them are placed in their proper positions, namely, at amplitudes of 120°, plus and minus, respectively. At any rate, the following explanation will rest upon the assumption of the essential identity of the two.

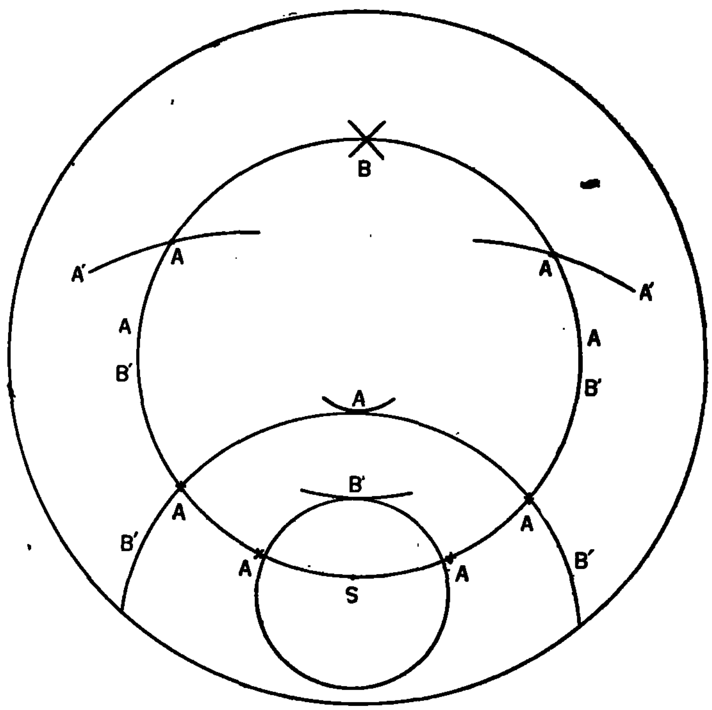

FIG. 4.—Halo as deduced from present theory, similar to that of Hevelius. The B group supposed to be absent. Zenith distance of sun 65°.

Both the halos are characterized by the brilliancy of the effects due to the A type of crystals, the other type being represented only by the B′ group. The evidence in favor of this statement lies in the intense brightness of the 22° parhelia and the absence of all traces of the Parry arcs and of their attendant consequences. As the B′ group produces nothing not already considered in the Parry halo, there is no reason for discussing their effects—the geometric drawing of figure 4 will enable one not only to compare the solution with Parker's drawing, but to find coordinates for all points desired with sufficient precision.

Let an A crystal be placed at the center of the celestial sphere and consider the course of sunlight falling upon it. Most of the light entering the upper surface will emerge from the under surface in an unchanged direction, but a portion will fall upon a vertical face and be reflected—totally reflected if the zenith distance of the sun is not too great—thence, emerging from the lower base, the light would come from some point in the parhelic circle. In many cases, however, a portion of the light reflected from the vertical face would, before being transmitted through the lower base, fall on an adjacent face; such light would also appear to come from the parhelic circle, but from one of two points only, each at 120° from the sun. This assertion does not require proof here because it is contained in the familiar theory of the kaleidoscope, but it is equiva-

[4] MONTHLY WEATHER REVIEW July, 1914, 42: 436-446.

lent to a statement that these common and puzzling features are an immediate consequence of our theory. Nor is it merely a few of the crystals which contribute light to the paranthelia of 120°, for theoretically just half of them are thus involved, although those of importance at any one instant are a smaller portion. It is clear that these paranthelia can not appear when the sun is very near the horizon or the zenith, whence we may fairly conclude that they appear more frequently at mid altitudes, from 25° to 50°, for example, a conclusion wholly accordant with the records. The only other theoretical explanations of the paranthelia known to me involve columnar crystals with their principal axes continuously vertical, in one case demanding stellate cross sections, and in the other two interior partial reflections. They appear quite untenable.

If a sufficient number of the crystals belong to the A′ group, there results a curious addition to the 120°-paranthelia, namely, a short oblique arc passing through each of them. These are due to that portion of the A′ group which rotates about that axis, which is equally inclined to those faces which, when the hexagonal plate is horizontal, would contribute to the paranthelion itself. I have calculated the form of these arcs for a sun altitude of 30° and represented them in the figure 4. The sun appears as sensibly at the center of these arcs, since the 120°-paranthelia being at 97.2° from the sun, the arcs from 21.0° below the parhelic circle to 12° above are more distant by the inappreciable amount of two and a half degrees.

Without doubt these arcs constitute the famous 90° halo of Hevelius which has baffled all explanation. The obvious objections to this declaration are, first, that they are fragmentary arcs—but that is true of the drawing by Hevelius; second, that the recorder described them as 90° from the sun, but his sketch places them at 78°, which is a notably greater divergence.[5]

Figure 4 shows the general halo for the zenith distance of 60° for the sun in the absence of the B group, but the effective presence of the other three; it may be compared with the figures of Hevelius and of Parker.

II.

The first section of this paper may be accepted as demonstrating the occasional existence in the atmosphere of some or all of four groups of crystals, these groups being necessary consequences of the two types of familiar crystal forms which are known to exist. The present section will concern itself with an investigation of the extension of the theory to halos accompanying the sun at higher altitudes. This does not involve a great deal of description, since the illustrative figures are all geometrical, so the amplitude of any point on the diagram can be determined directly, by means of a protractor and the zenith distance, from the formula

$$Z = R \, tg \, 1/2 \, z$$

where R is the radius of the circle representing the horizon, Z the linear distance from the center of the circle to the point in question, and z the zenith distance. It will be noted that, as in the preceding projections, the lines show the loci of that particular color corresponding to a refraction index of 1.31 and neglect the angular dimensions of the sun. The modifications necessary to involve other colors and the dimensions of the sun are easily supplied; moreover, I shall give later a list of the uncolored features of halos, the others being prismatic.

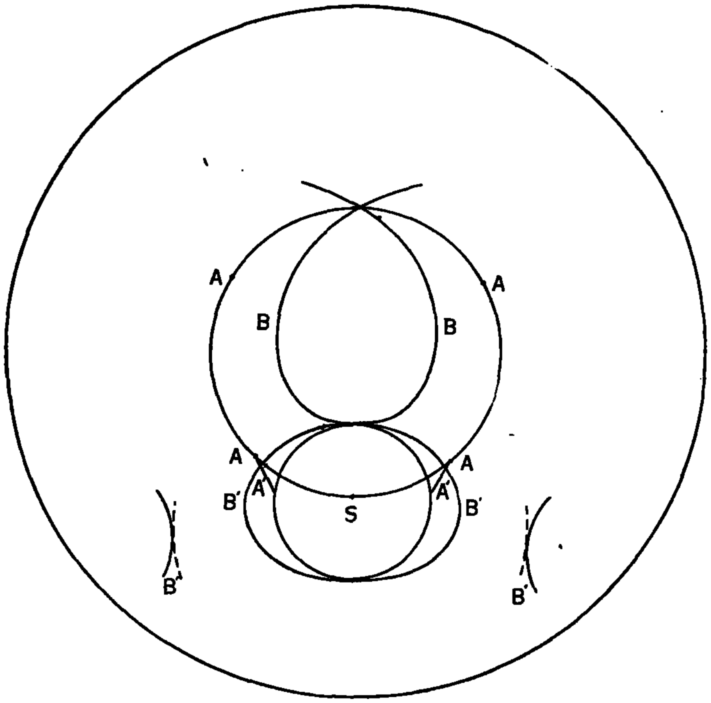

FIG. 5.—A highly developed halo with sun at a zenith distance of 45°, closely resembling the St. Petersburg halo recorded by Lowitz.

Figure 5 shows a possible halo with the sun at an altitude of 45° when all four groups are effective; the 22°-circle is added for comparison, although not necessarily present. The letters attached to each detail indicate the origin with sufficient definiteness with the exception of the pair of long arcs crossing at the anthelion and the pair connecting the ordinary parhelia with the 22°-circle. The former pair I shall style the Upper Oblique Arcs passing through the anthelion, in order to distinguish them from another pair of similar origin which occasionally attend the sun at very high altitudes; the second pair has already been named the arcs of Lowitz from the observer and recorder of the famous St. Petersburg halo of 1790. The long arcs have heretofore been confused with the short arcs confined to lower altitudes of the sun and produced by a different group of crystals, while the arcs of Lowitz have given rise to much theoretical discussion. Let us consider them in the order named.

The upper arcs passing through the anthelion are simply the development of those faint arcs which we found in the halo of Parry and Sabine between the zenith and the parhelic circle. They are produced by light which, falling near the ends of the B crystals and undergoing total reflection, emerges through the same surface as that effective in the upper Parry's arc. The ends toward the sun are at the middle of the latter arc, although it is obvious that it would be impossible to trace them very near that point; they would ordinarily be confounded with the Parry arc, as is so well exemplified in figure 8 in Besson's paper cited above. The theoretical limits in the opposite direction are set by the approach to the critical angle of incidence on the emergent surface, although it is easy to see that the visible limit must be reached before that.

The arcs of Lowitz are of special theoretical interest on account of their extreme rarity with unquestionable authenticity and the fact that theorists have given them

[5] One might add to these objections the fact that Bravais has cited three more recent observations of a 90° halo, but I have, by referring to the original sources, persuaded myself that the citations were founded upon misapprehensions. Since one of the references was to the account of Perry and Sabine, printed in full above, the reader may judge for himself in respect to that one.

so much attention. According to the theory here presented, they are caused by a certain portion of the A′ group. Such crystals rotate about one of three major diagonals; for suppose a host of such crystals at the center of the celestial sphere and we confine our attention to those of them which when horizontal contribute to the light of a 22°-parhelion. Dominate the face of entry by p and that of exit by p″, then one-third of these crystals will rotate about an axis which causes p and p″ to alter their direction at an equal rate; a second third causes displacements of p and p″ in opposite directions, the latter being at the greater rate, and, finally, the remainder cause displacements of unequal rate but that of p being greater. These may be called the first, second, and third modes. The first mode does not produce a visible effect at this altitude of the sun, but, as shall be proved later, it is the occasional cause of a curious feature with the sun at the horizon. So, too, the third mode is ineffective, but the second mode gives rise to the short arcs connecting the parhelia with the 22°-circle and which are in closest agreement with the record of Lowitz. The fact that the first mode alone produces tangent arcs to the 22°-circle when the sun is at the horizon, while only the second is effective at the altitude of 45° is very suggestive of the reason why these arcs are so rare and are not seen at intermediate altitudes.

At first thought it appears improbable that so small a number of effective crystals could produce a visible effect, but calculation shows that an enormous change in the angle of rotation about the axis shifts the image of the sun along the arc by a very small amount. Thus the condition is an approximation to the "stationary" state of a parhelion and the brightness is correspondingly enhanced.

Aside from these points, the diagram may be regarded as explaining itself except, perhaps, the apparent fragments of the 46°-circle—not always present, it is true—which are attributable to that fraction of the B′ crystals which have a restricted oscillation about their centers of mass.

With a zenith distance much less than that of figure 5 a new feature appears which has been rarely seen in the latitudes of northern Europe but less uncommonly in the United States and which has not yet been discussed. This feature consists of a pair of arcs lying chiefly outside of the parhelic circle but crossing at the anthelion point and from there curving toward the zenith. These arcs have been observed by Lea, Melville, and probably by Meriwether, all of whom are cited in the work by Bravais; but by far the best record known to me is that by H. W. Crawley, described and figured in the Report Brit. Assn. 1861 (2), p. 63. One suspects that only the B and B′ groups were present in this interesting phenomenon and that the circle drawn about the sun was in reality the circumscribing oval, otherwise the darkness within it could not have so impressed the observer. This observation seems to have escaped the notice of meterologists. This feature under discussion is due to the B group of crystals and are therefore complimentary to the oblique arcs exhibited in figure 5. The accompanying figure 6 shows the form which these arcs have when the zenith distance of the sun is 30°. They may be styled the Lower Oblique Arcs through the anthelion since they are produced chiefly by light which emerges from the lower horizontal surface after refraction at a p′ surface and total reflection at a basal surface. The figure shows only these curves due to the B crystals and the ordinary 22°-circle to afford a ready scale of dimensions. It will be recalled that the upper oblique arcs, as shown in figure 5, are due to light which enters the upper horizontal face. In this particular case the inner extension from the anthelion are also due to light entering this face and are therefore properly a portion of the upper oblique arcs. It is easy to state in general terms what the added features would be if the other groups were present. The A group would yield its share of the light to the parhelic circle and possibly exhibit paranthelia, although these would be unquestionably very faint; if the arc below the 46°-circle were apparent, as it often is with the sun at a somewhat greater altitude, this too would be in part due to the A group. The B′ group, if present, would also contribute to the parhelic circle and form a circumscribed oval about the 22°-circle lying very close to it in all its parts.

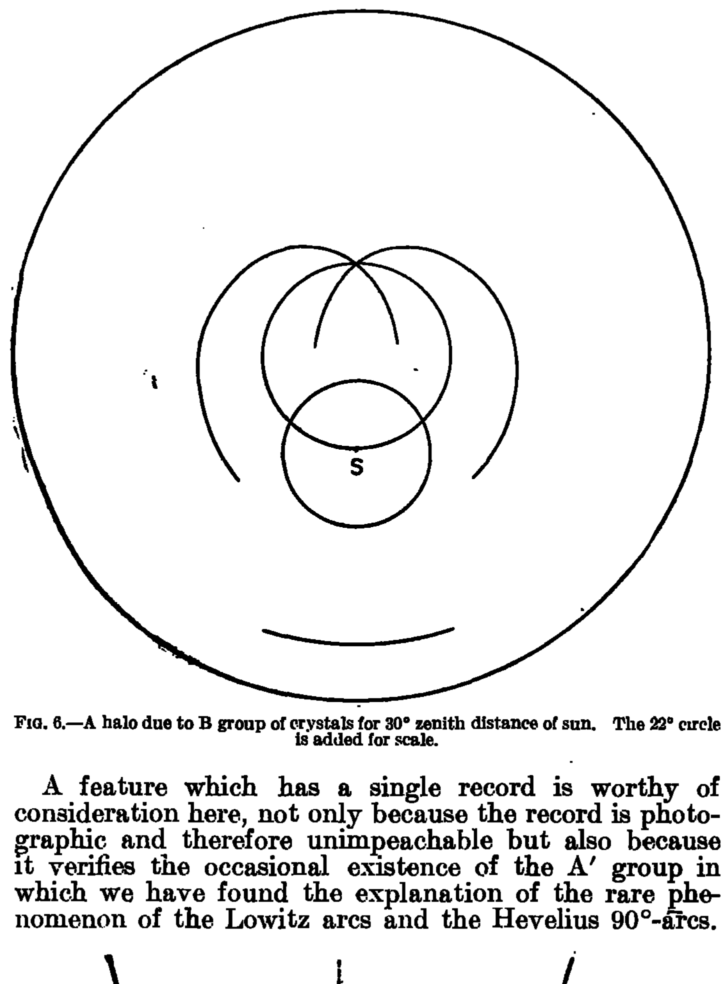

FIG. 6.—A halo due to B group of crystals for 30° zenith distance of sun. The 22° circle is added for scale.

A feature which has a single record is worthy of consideration here, not only because the record is photographic and therefore unimpeachable but also because it verifies the occasional existence of the A′ group in which we have found the explanation of the rare phenomenon of the Lowitz arcs and the Hevelius 90°-arcs.

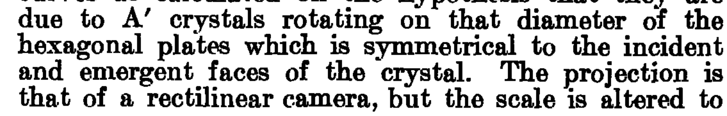

FIG. 8.—Halo of Schultz according to theory. The projection is that of a rectilinear camera so that it may be compared directly with the photograph.

The photograph is found in Archbishop Stuck's "Ten Thousand Miles with a Dog Sled," p. 388,[6] and is here reproduced in figure 7. Figure 8 gives the forms of the curves as calculated on the hypothesis that they are due to A′ crystals rotating on that diameter of the hexagonal plates which is symmetrical to the incident and emergent faces of the crystal. The projection is that of a rectilinear camera, but the scale is altered to

[6] Charles Scribner's Sons, New York, 1914.

agree with the photograph and its center is a point on the true horizon supposed to be the place of the sun. The slight eccentricity of the sun in the photograph is due to a fault of direction in the camera. The fainter vertical column of light directly above the sun is a secondary phenomenon due to these same crystals; in other words, it is the sum of images of the two tangent arcs formed by A′ crystals and, of course, colorless. This is the only certainly established secondary phenomenon excepting the parhelia at approximately 90° right and left of the sun, which are represented in figure 5 above and which have frequently been recorded.

The question as to whether a feature is prismatic or without color is easy to answer from the mode of its production. In general, if produced by refraction we may expect attendant color unless the refraction at entry and emergence from the crystal is compensatory, as in the parhelic circle, the paranthelia and the anthelia. Of course, all very faint arcs would fail to betray colors for quite the same reason that the lunar rainbow does so.

In the above theory explanations for all the well authenticated features of halos are given with the single exception of the rarely observed concentric circles about the sun of radii differing from 22°. These may be due to the presence of crystals of which the rhombohedral faces are developed—and indeed, Bravais has attempted to explain them in this way—but such crystals, although next in crystalographic simplicity to the hexagonal prisms assume as the bases of the theory here developed, have never been observed and therefore lie outside the proper scope of this paper.[7]

BEAUTIFUL HALO DISPLAY OBSERVED AT ELLENDALE, N. DAK.

By FRANK J. BAVENDICK, Observer.

[Dated Weather Bureau, Ellendale, N. Dak., Mar. 22, 1920.]

Interesting and unusual solar halo forms were observed at Ellendale, N. Dak., on March 8, 1920. About 11:30. a. m. a 22° halo began forming and by 1:15 p. m. this halo (a a) figure 1, was complete and other optical phenomena were developing. At 1:30 p. m. the sky appeared as in the drawing, figure 1. The arcs (c, c′) were parts of a circumscribed halo. The 22° halo and these arcs had brilliant spectral colors; the red being nearest the sun. The infralateral arcs (i, i′) were 38° long and extended to 7° above the southern horizon. They had rainbow colors with the red nearest the sun. The large white parhelic circle (mm) was well defined and was accompanied by the oblique arcs of the anthelion (r, r′; s, s′). These were also white and well defined, but were not as distinct and did not remain for so long a time as the parhelic circle.

The white ring (xx) was about 32° in radius. Its lower edge was tangent to the 22° halo and other portions were tangent to the arcs (r, r′). The lower half of this circle was about half as bright as the primary parhelic circle (mm) and the upper half was indistinct, but continuous. The intersection of this circle (xx) and the halo (aa) was very brilliant.

The arc (bb) had faint rainbow colors and was about 22° above the halo (aa). Parhelia (e, e′) were observed outside of the halo (aa) and intensified patches were also noticed at the intersections of (aa) and (c, c′) with (mm).

The disappearance of these circles and arcs of circles occurred gradually between 2:30 and 2:45 p. m. The sky was covered with nine-tenths Ci. St. clouds and they were heavier than the usual type of Ci. St. clouds observed when halos are visible here. No precipitation had occurred for five days, but a low was approaching from the northwest. The surface wind at the time of the halo was from the south, the humidity was 65 per cent and the temperature was −4° C. There was a temperature inversion of about 5° C. at 1,000 meters above the surface.

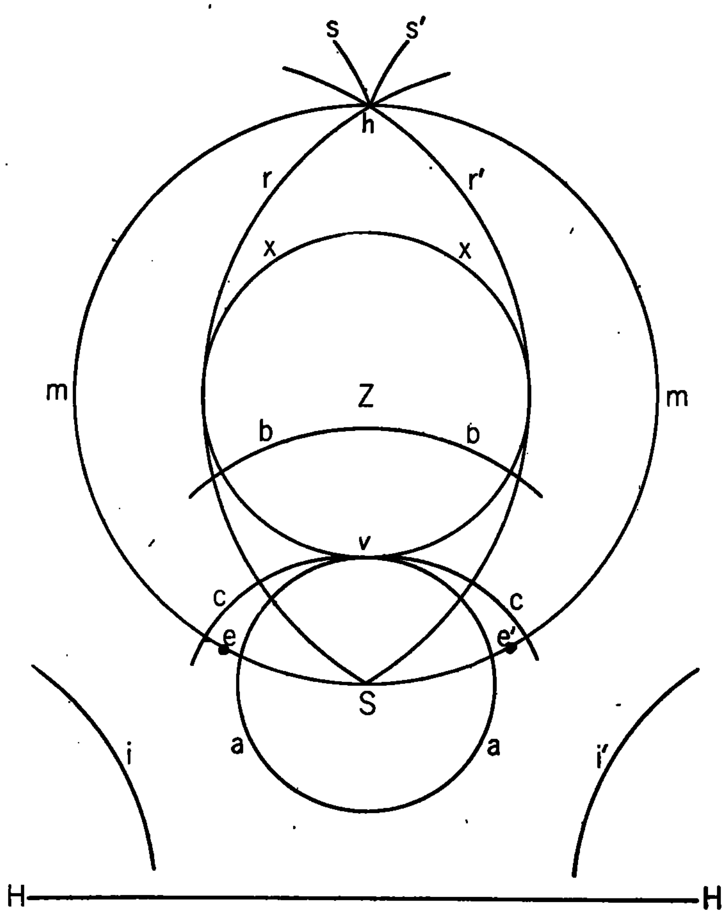

FIGURE 1.—Solar halo phenomena observed at 1:30 p. m., March 8, 1920, at Ellendale, N. Dak., including: Halo of 22° (aa); arc of halo of 46° (bb); arc of circumscribed halo (cc); parhelia of 22° halo (e, e′); anthelion (h); infralateral tangent arcs of 46° halo (i, i′); parhelic circle (mm); two pairs of the oblique arcs of the anthelion (r, r′; s, s′); so-called vertical parhelion of 22° (v); probably secondary parhelic circle (xx). S, sun; Z, zenith; and HH, horizon.

At 3,500 meters the wind was from the west, the humidity about 70 per cent increasing, and the temperature was −15° C.

NOTE.—This description of halo phenomena is of great interest, particularly that portion dealing with the white circle marked (xx). This is presumably what may be called a secondary parhelic circle, induced by the brilliant luminous spot at the summit of the 22° halo; this circle was tangent to the oblique arcs of the anthelion (r, r′). So far as known a complete secondary parhelic circle has never before been observed. In 1896 Rear Admiral A. von Kalmar observed at Pola a portion of this circle,[1] which, if extended, would have been tangent to the oblique arcs of the anthelion.

The observations at Ellendale were made independently by Mr. Bavendick at pilot balloon station "A" and by Mr. Wm. H. Brunkow at the kite house nearby, the angular measurements being determined by means of standard balloon theodolites.

[7] A discussion elucidating some of the more difficult parts of this article will be published in a later issue of the REVIEW.—Editor.

[1] The Different Forms of Halos and their Observation, by Louis Besson. MONTHLY WEATHER REVIEW, July 1914, 42: 444, fig. 20.