Jamming in Granular Material: Implications for Till

Jesse V. Johnson

University of Montana

Introduction

Jamming–The process where by granular materials become stuck. Familiar in everyday circumstances.

- Hoppers such as those in bulk food isle.
- Industrial processes involving powders, grains.
- Fault gouge. [a]
- Traffic.

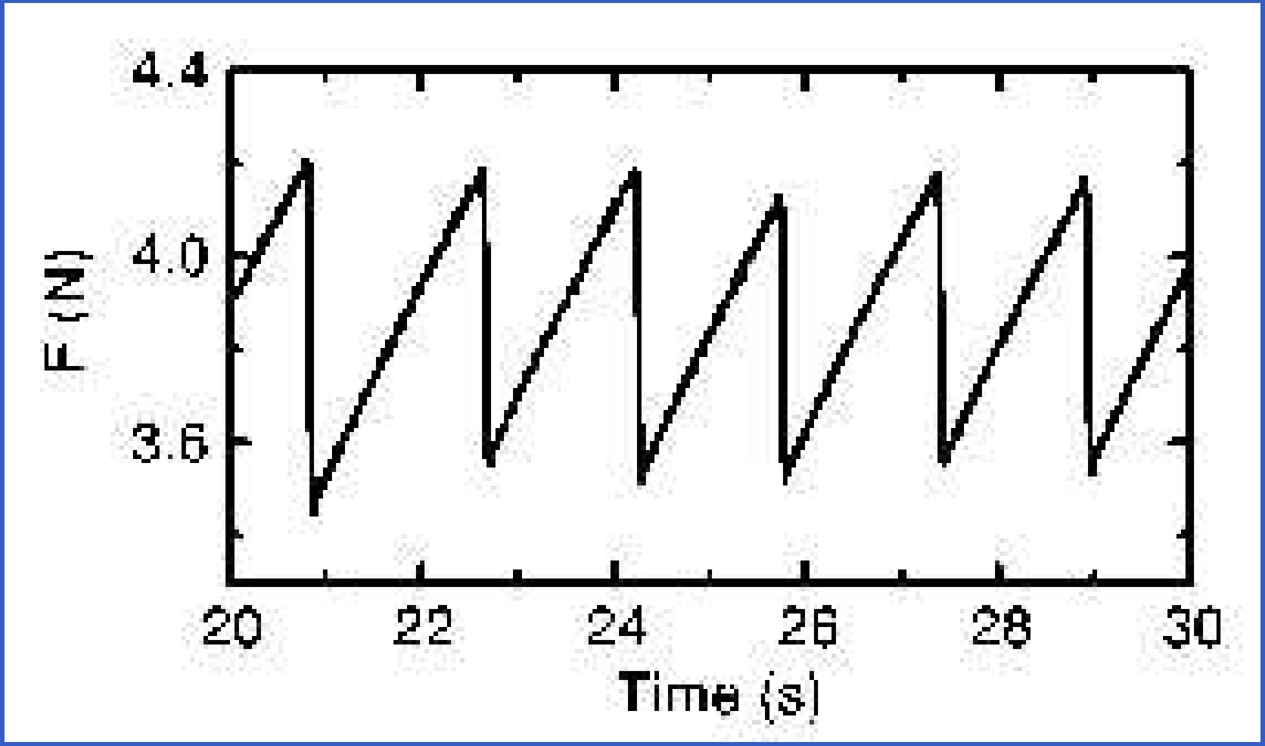
Becoming a focus area of contemporary physics research.

Many publications in the last 5 years.

[a] See [Marone, 1998]

Results of Experiments: Jamming

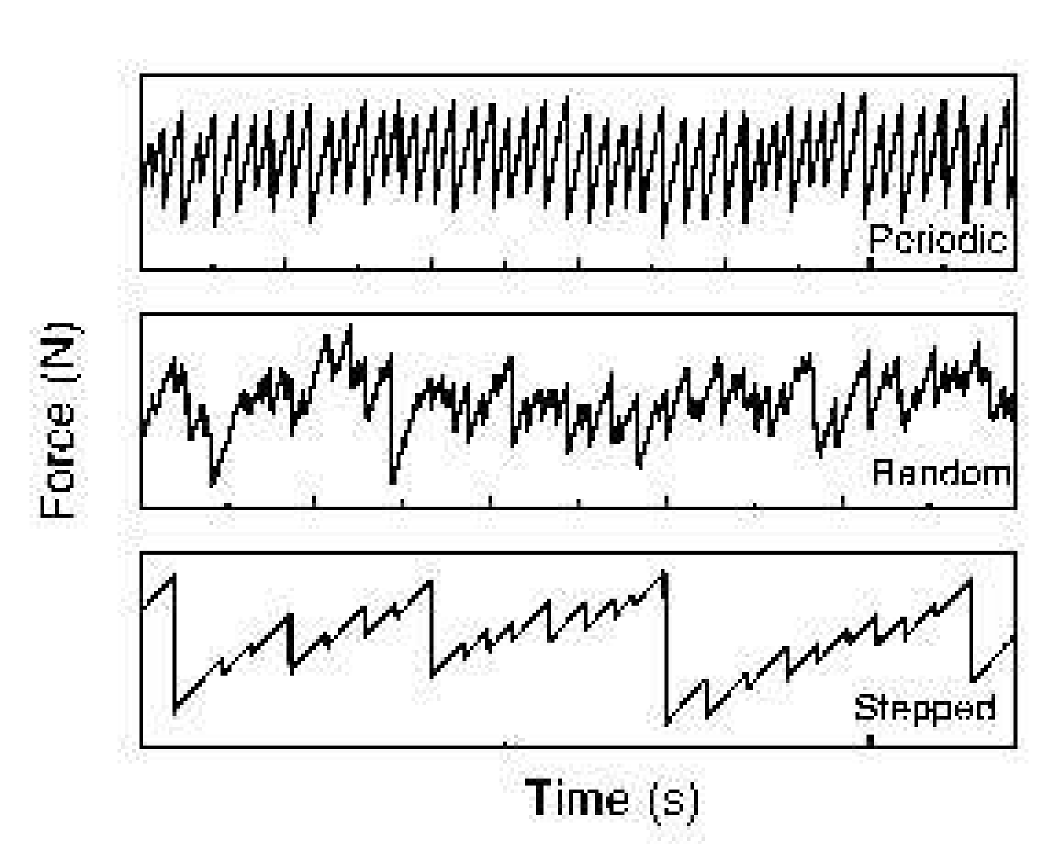
In all experimental setups, jamming is observed in the form of force fluctuations as shear is applied. [a]



[a] Figure from [Albert et al., 2001]

Frequency of Jamming Events

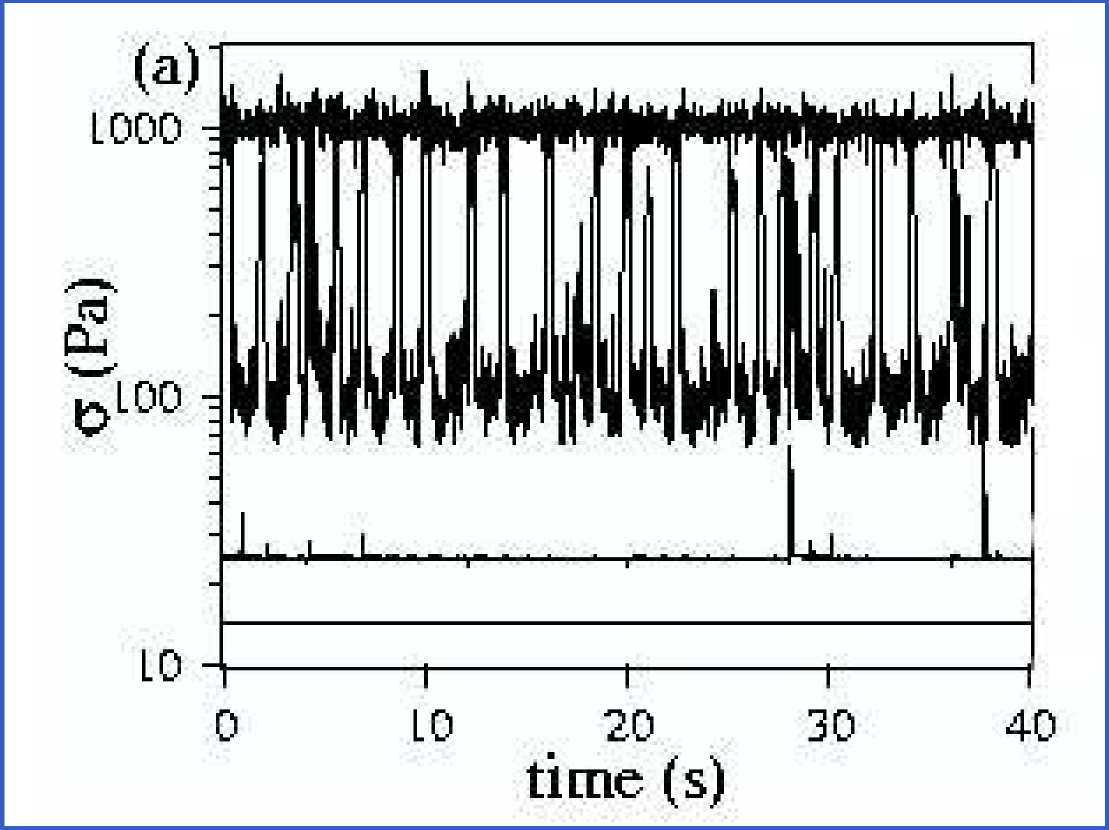
Sometimes the jamming events occur periodically, other times they occur in a random or stepped fashion. [a]



[a] Figure from [Albert et al., 2001]

Intensity of Jamming Events

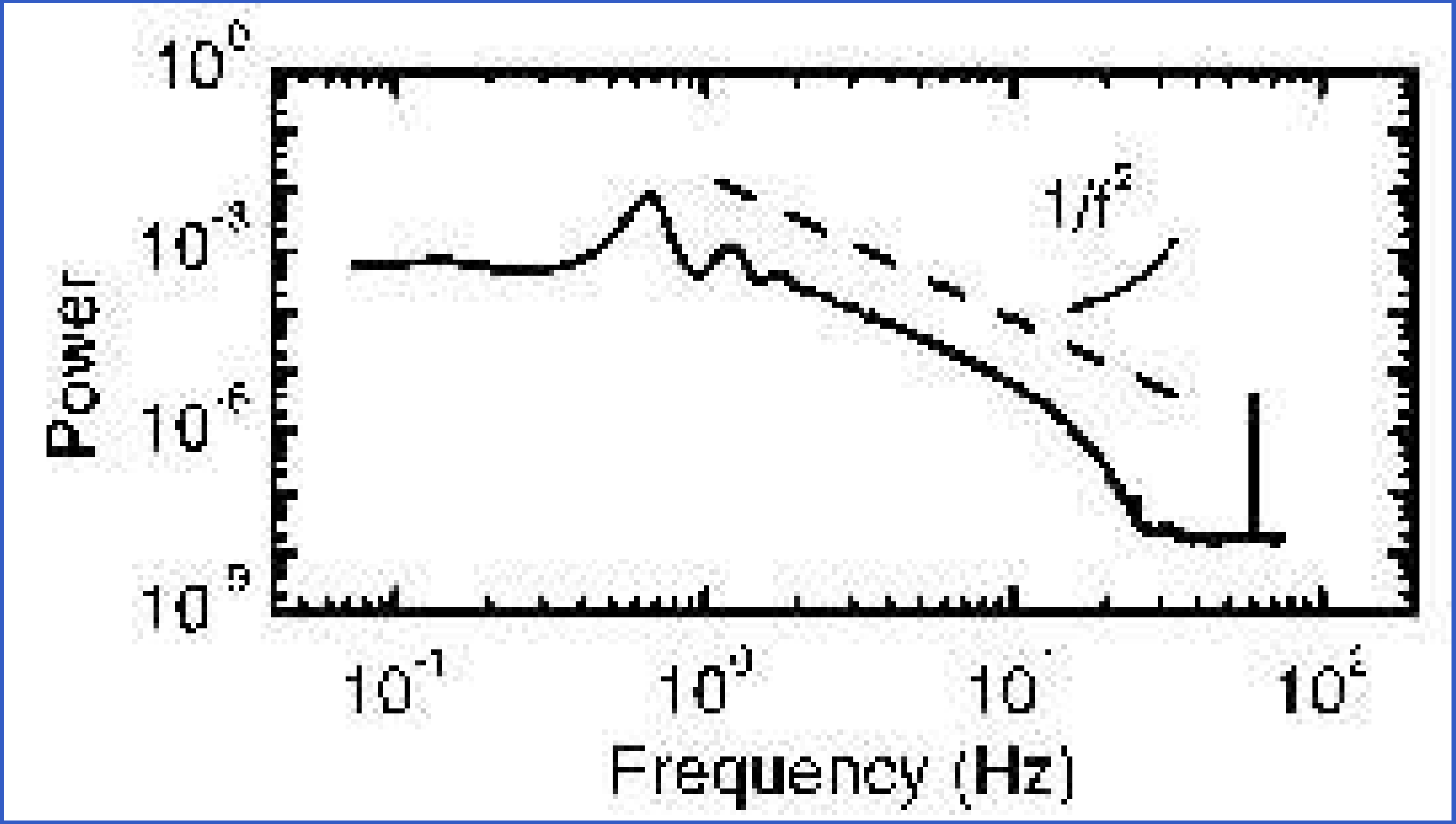
Jamming events may produce up to and order of magnitude change in the stress response. [a]



[a] Figure from [Lootens et al., 2003]

Probability Distribution of Frequency

The frequency of events follows a power law distribution. [a]

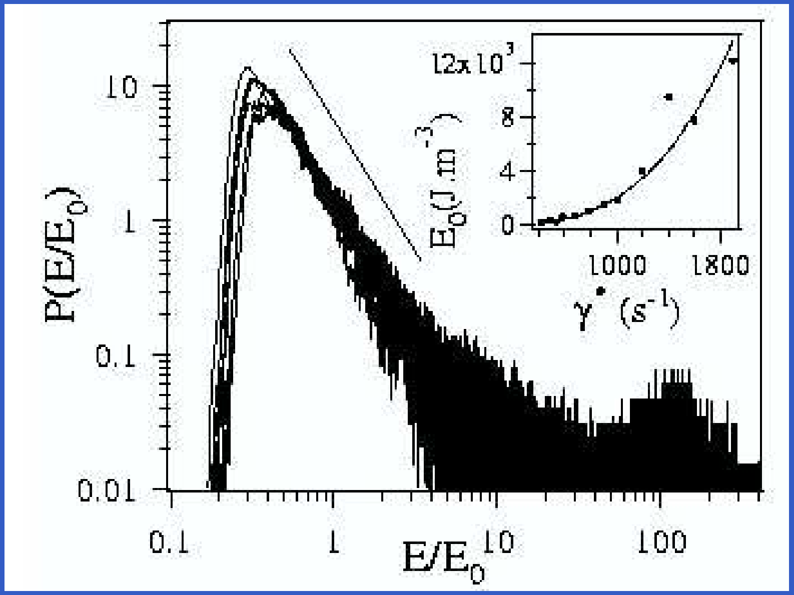


[a] Figure from [Albert et al., 2001]

Distribution of Energy Released

The energy release of events also follows a power law distribution. [a].

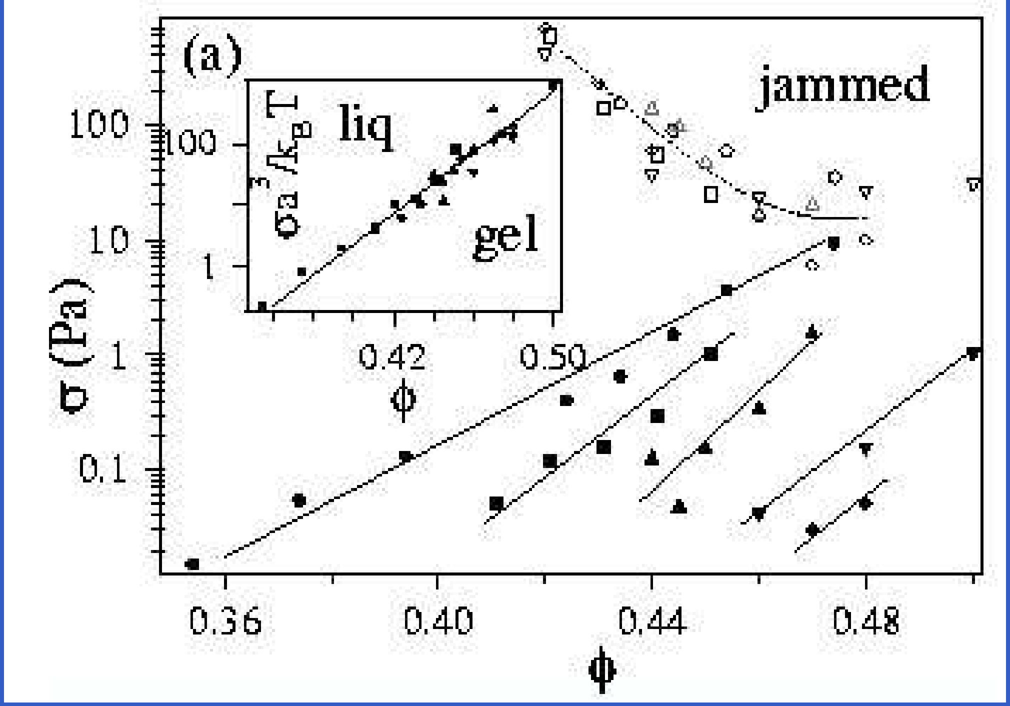
$$E = \int_{peak} \left[\sigma(t') - \sigma_{max}\right] \dot{\gamma} dt' \qquad (1)$$



[a] Figure from [Lootens et al., 2003]

Phase Diagram for Jamming

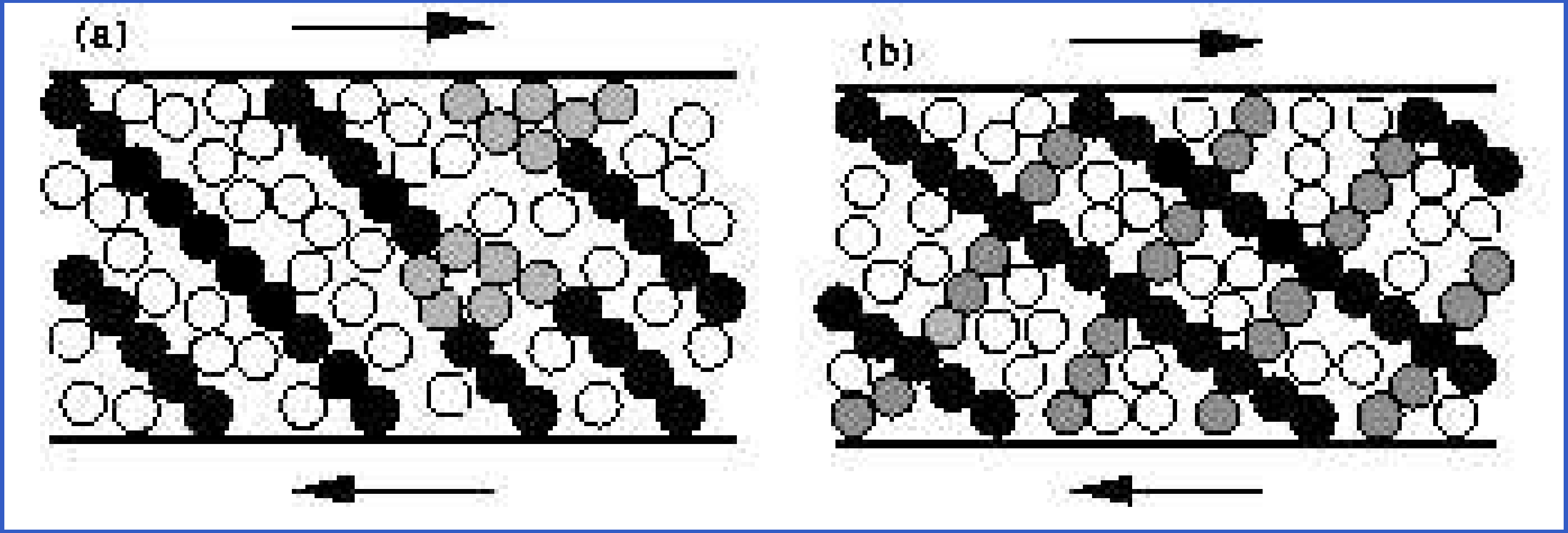
Experimental data gives a phase diagram of this sort. [a]



[a] Figure from [Lootens et al., 2003]

Theoretical Approach: Force Chains

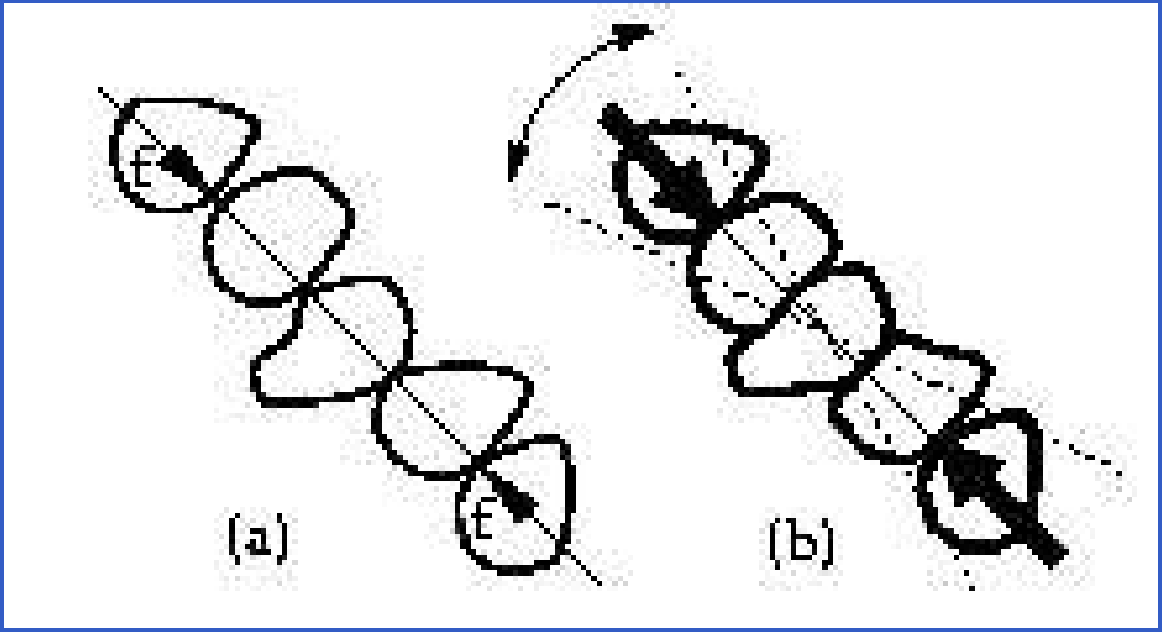
The source of jamming is believed to be the formation of ‘force chains’. They form normal to the applied shear and propagate through the substance. [a]. In the idealized case (b), other force chains form normal to the original ones.



[a]Figure from [Cates et al., 1998]

Fragile Matter

Jammed material clearly strong in one direction and very weak in others. [a]. This leads to the term ‘fragile matter’.



[a] Figure from [Cates et al., 1998]

Fixed Principal Axes Model

- Consider force chains only in terms of their impact on the stress tensor:

$$\sigma_{ij} = \Lambda_1 n_i n_j + \Lambda_2 m_i m_j + \Lambda_3 l_i l_j \tag{2}$$

- Further, assume that the nonparallel directors, $\mathbf{n}, \mathbf{m}, \mathbf{l}$ lie on the principal axes.
- with some simplification one can now write

$$\frac{\sigma_{xx} - \sigma_{yy}}{\sigma_{xy}} = \frac{\sin^2\phi - \cos^2\phi}{\sin\phi\cos\phi} = \text{a constant} \tag{3}$$

- Where ϕ is the angle between the major compression axis and the y axis.
- Experimental evidence supports this conclusion.

Molecular Dynamics Approach

- Using the well known form for molecular dynamics code, with appropriate forces. Integrate forces with Verlet algorithm.
- Instead of a Leonard-Jones type potential, use the explicit force given by:

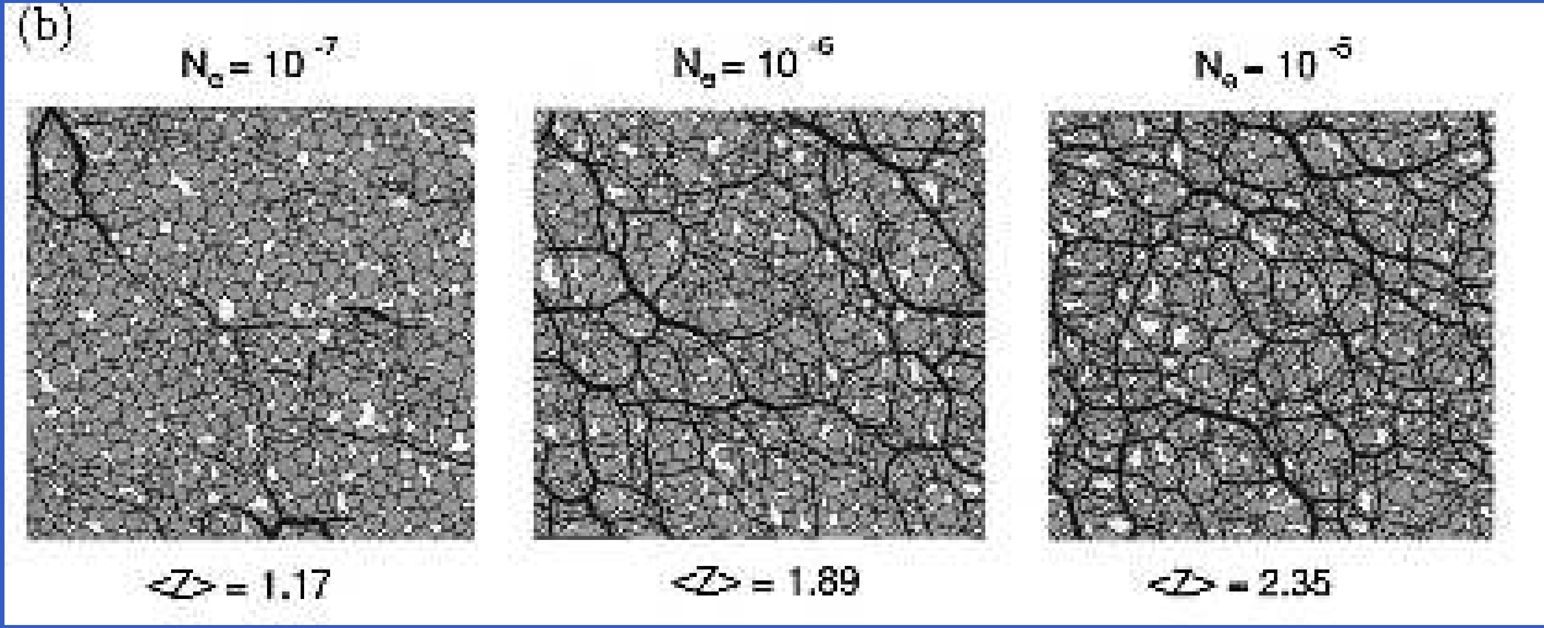
$$F_{ij}(t) = [k_n(R_i + R_j - r_{ij}) - \gamma m_{ij}(\dot{\mathbf{r}}_{\mathbf{ij}} \cdot \hat{\mathbf{n}})]\hat{\mathbf{n}} \quad (4)$$

$$+\{\mathrm{min}[k_s \Delta s, \mu(\mathbf{F} \cdot \hat{\mathbf{n}})]\}\hat{\mathbf{s}} \quad (5)$$

- Translational and rotational degrees of freedom.
- Shear ($\hat{\mathrm{s}}$) and normal forces ($\hat{\mathrm{n}}$).

Molecular Dynamics Results

Results from the MD approach to modeling granular materials provide an excellent visualization of force chain formation. [a]



[a] Figure from [Cates et al., 1998]

Critical State, Percolation Theory

- As the previous slide shows, force chains increase with density of material.
- Can find critical solid fraction of particles.
- CSL of soil mechanics corresponds to a critical density where jamming is frequent.
- Attractive boundary: tills want to jam.

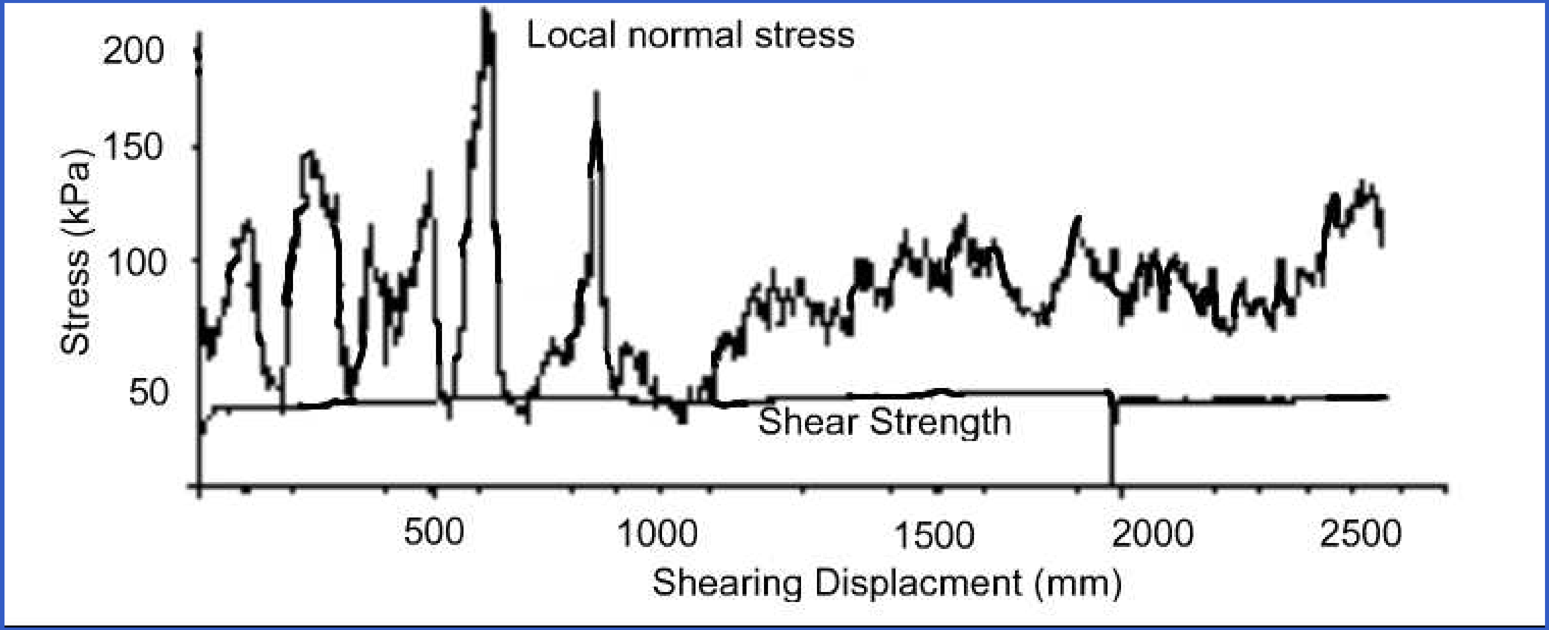
2D Model	0.805	[Aharonov and Sparks, 1999]
3D Extension	0.613	$\nu_{3D} = \frac{4\nu_{2D}^{3/2}}{3\pi^{1/2}}$ [Campbell and Brennen, 1985]
Tills	0.69-0.12$\log(\sigma_n)$	[Tulaczyk et al., 2000]

Jamming in Glaciology

- Big changes in numerous estimates for till strength.
- Changes in models such as the undrained plastic bed model.
- Formidable tool for explanations.
  - Stick-slip motion.
  - New source of ‘sticky spots’.
- Extensions to ice pack, mega floods.

Seen it all Before

[Iverson et al., 1996] looked at mudstone samples in a ring shear device. Evidence of jamming. Many authors remove large clasts to prevent jamming ([Kamb, 2001]).



Nagging Concerns

- Is it possible that what is a lab study scale phenomena (cm) can scale to the large (km) scales under glaciers?
  - Fault gouge.
  - [Iverson et al., 1996] states that jammed regions are of the order $\sim$100x grain diameter.
  - Another percolation problem as we consider the fraction of the bed covered with jammed materials.
- Are resistive forces of jammed material significant. Up to order of magnitude, but not necessarily so large.
- Is the rough bottom of the ice sheet rough enough to constantly be breaking fragile matter (force chains).

Future Directions

- Additional experimental evidence specific to tills.
- Modeling based on molecular dynamics seem most productive.
  - Set distribution of particle sizes to fractal

    $$N(r) = \left(\frac{r}{r_o}\right)^{-b} \tag{6}$$

    with b=2.85.[Iverson et al., 1996]
  - Viscous damping.
  - Improved force equation.
- Reconsider existing models.

References

[Aharonov and Sparks, 1999] Aharonov, E. and Sparks, D. (1999). Rigidity phase transition in granular packings. *Physical Review E*, 60:6890–6.

[Albert et al., 2001] Albert, I., Tegzes, P., Albert, R., Sample, J., Barabasi, A.-L., Vicsek, T., Kahng, B., and Schiffer, P. (2001). Stick-slip fluctuations in granular drag. *Physical Review E*, 64:313071–9.

[Campbell and Brennen, 1985] Campbell, C. and Brennen, C. (1985). Computer simulation of granular shear flows. *Journal of Fluid Mechanics*, 151:167–88.

[Cates et al., 1998] Cates, M., Whittmer, J., Bouchaud, J.-P., and Claudin, P. (1998). Jamming, force chains, and fragile matter. *Physical Review Letters*, 81:1841–44.

[Iverson et al., 1996] Iverson, N. R., Hooyer, T., and Hooke, R. L. (1996). A laboratory study of sediment deformation: stress heterogeneity and grain size evolution. *Annals of Glaciology*, 22:167–175.

[Kamb, 2001] Kamb, B. (2001). Basal zone of West Antarctica ice streams and its role in lubrication of their rapid motion. In Stone, D. and Runcorn, S., editors, *The West Antarctic ice sheet: behavior and environment*. American Geophysical Union, Dordrecht.

[Lootens et al., 2003] Lootens, D., Damme, H. V., and Hebraud, P. (2003). Giant stress fluctuations at the jamming transition. *Physical Review Letters*, 90:178301(4).

[Losert et al., 2000] Losert, W., Geminard, J.-C., Nasuno, S., and Gollub, J. (2000). Mechanisms for slow strengthening in granular materials. *Physical Review E*, 61:4060–68.

[Marone, 1998] Marone, C. (1998). Laboratory-derived friction laws and their application to seismic faulting. *Annual Reviews of Earth Planet Science*, 26:463–96.

[Tulaczyk et al., 2000] Tulaczyk, S., Kamb, W., and Engelhardt, H. (2000). Basal mechanics of Ice Stream B, West Antarctica 1. Till mechanics. *Journal of Geophysical Research-Solid Earth*, 105 (B1):463–481.

[Veje et al., 1999] Veje, C. T., Howell, D. W., and Behringer, R. B. (1999). Kinematics of a 2d granular couette experiment at the transition to shearing. *Physical Review E*, 59:739.