Role of Order Parameter Supression on Quasiparticle Properties of Disordered d-wave Superconductor

UNIVERSITY OF FLORIDA

W.A. Atkinson,[a] P. J. Hirschfeld[a] and A.H. MacDonald[b]

[a]Department of Physics, University of Florida, Gainesville, FL 32611, USA

[b]Department of Physics, Indiana University, Swain Hall, W. 117, Bloomington, IN 47405 USA

ABSTRACT

We report on a numerical study of disorder effects in 2D d-wave BCS superconductors. We compare exact diagonalization calculations of the density of states $\rho(E)$ with the standard self-consistent T-matrix approximation. Local supression of the order parameter near impurity sites is found to lead to asymptotic power law behavior $\rho(E) \sim |E|^\alpha$, with disorder-dependent exponent α. Scaling studies show that this novel behavior is not associated with localization, and is therefore fundamentally different from recent power-law predictions.

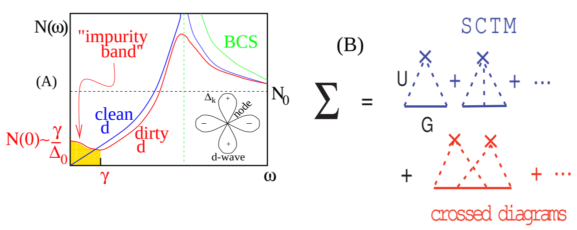
1 Introduction

1.1 Density of states of disordered d-wave system

- Standard prediction for strong scatterers in d-wave superconductors from self-consistent T-matrix approx. (SCTM):

$$\rho(E \to 0) \sim \gamma \simeq \sqrt{\Gamma \Delta_0} \tag{1}$$

where Γ is normal state scatt. rate, Δ_0 gap max. Usual picture (A)



- Nersesyan, Tsvelik, Wenger 1994:[1] T-matrix series singular in 2D–need to sum crossed diagrams(B).

  Mapped usual model → Dirac fermions with random gauge fields, $\rho(E \to 0) \sim |E|^\alpha$, α dependent on # of nodes, internode scattering.

- Hettler, Hirschfeld, Ziegler (1996,98)[2]: Lower bounds on $N(0)$ claimed for wide class of disorder models.
- Senthil,Fisher (1998,99)[3]: Localization drives $N(0)$ to zero with $\alpha = 1$

1.2 Localization

- P.A. Lee (1993)[4]: States in d-wave superconductor localized below mobility edge

$$E_0 = \gamma \tag{2}$$

- Franz et al. (1996)[4]: Confirm localization by calculating inverse participation ratio for exact diag. on finite size systems. Claim mobility edge similar to that predicted by Lee.[4]

1.3 Questions

- Is density of states $N(0)$ finite?
- Over what range is SCTM useful, if at all?
- What is role of order parameter supression?
- Are there localized quasiparticle states?

2 Method: solve BdG equations

- Hamiltonian:

$$\mathcal{H} = -t\sum_{\langle i,j\rangle}\sum_\sigma c^\dagger_{i\sigma}c_{j\sigma} - \sum_{i,\sigma}[\mu - U_i]c^\dagger_{i\sigma}c_{i\sigma} - \sum_{\langle i,j\rangle}\{\Delta_{ij}c^\dagger_{i\uparrow}c^\dagger_{j\downarrow} + h.c.\},$$

with $U_i = u_0$ at a fraction n_i of sites, 0 elsewhere, and $\Delta_{ij} = -V\langle c_{j\downarrow}c_{i\uparrow}\rangle$ materials.

- T-matrix and self-energy

averaged self-energy $\Sigma_k(E) = n_i T_{kk}(E)$, where T is

$$T_{ij}(E) = U_{ij} + \sum_{R,R'} U_{iR}G(R-R',E)T_{R'j}(E), \tag{3}$$

and Green's function

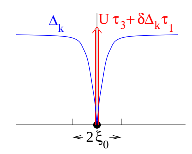
$$G_k(E) = \left[E\tau_0 - \epsilon_k\tau_3 - \Delta_k\tau_1 - \Sigma_k(E)\right]^{-1}, \tag{4}$$

where ϵ_k is tight-binding band.

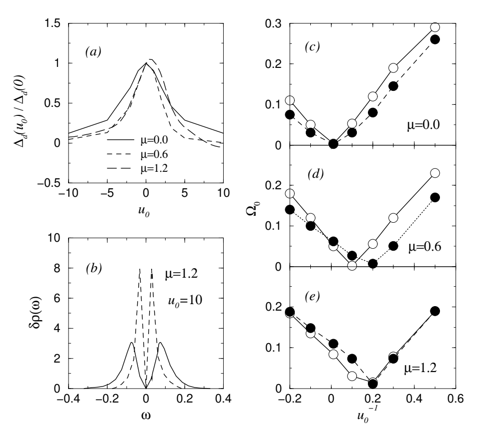
3 1-Impurity Problem

3.1 Background

- Balatsky et al.(1995)[6]: Bound state in resonant limit at $\Omega_0 = \Delta_0\,(2N_0u_0\log 8N_0u0)^{-1}$
- Joynt (1997)[7]: Particle-hole asymmetry shifts resonances
- Byers & Flatté (1999)[8]; Hettler & Hirschfeld (1998),[9] Shnirman et al. (1999)[10]: include order parameter supression (OPS)



3.4 1-impurity results

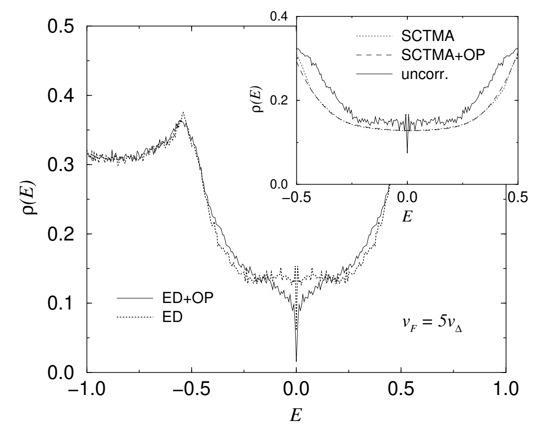


(a) d-wave OP at impurity site relative to bulk value; (b) shift in resonance in $\delta\rho(E)$ for $u_0 = 10$ and $\mu = -1.2$ due to OPS; (c)-(e) Peak position of resonance as function of u_0 for different μ. Closed (open) symbols indicate peaks with (without) OPS.[11]

- For tight-binding band $\mu \neq 0$, unitarity limit $\Omega_0 = 0$ and strong scattering limit $u_0 \to \infty$ are not identical.
- Addition of OPS can move resonance toward Fermi level[10] or away.

4 Disorder: Density of States

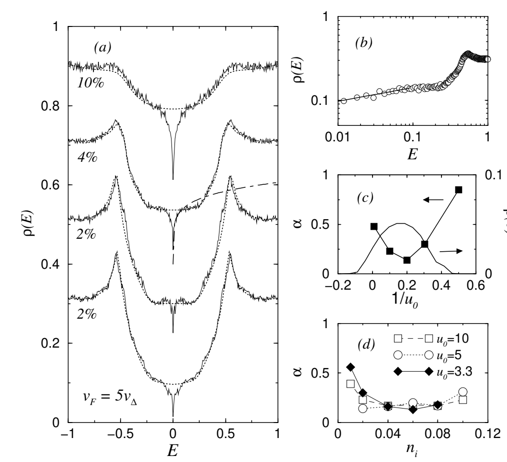
4.1 Comparison with T-matrix



Density of states for n_i=0.04 and $u_0 = 10$, with (ED+OP) and without (ED) self-consistent calculation of Δ_{ij}. Inset: T-matrix calculations with (SCTM+OP) and without (SCTM) OPS. Also shown (uncorr.): model with uncorrelated diag. & off-diag. potentials.

- OPS appears to supress $\rho(E=0)$ substantially, perhaps to zero.
- This behavior not captured by SCTM with or without OPS
- T-matrix excellent approximation if OPS is ignored. Sharp "pseudogap" around $E = 0$ appears to be mesoscopic effect, scales as $1/(\rho L^2)$.

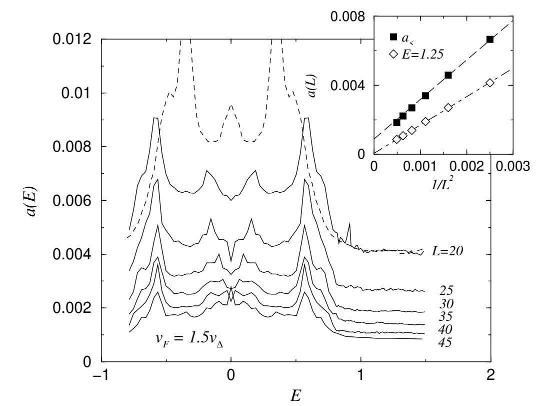
4.2 Dependence on n_i and u_0



(a) ED+OP (solid) and SCTM (dotted) calculations for $u_0 = 5$ for $n_i = 0.10, 0.04, 0.02$ (top to bottom). Also shown (dot-dash) is fit $\rho(E) = A|E|^\alpha$ for $n_i = 0.04$. (b) Logarithmic plot showing asymptotic power law behavior for $n_i = 0.04$, $u_0 = 5$. (c) Dependence of power law on u_0 for $n_i = 0.02$. For comparison, SCTM calculation of $\rho(0)$ vs. u_0 is shown. Unitary limit is $u_0 \approx 5$. (d) Scaling of power law with n_i. Note that when $n_i = 0$, $\alpha = 1$.

- $\rho(E)$ power law strongly depends on disorder parameters
- α differs from Refs. [1](α=1/7),[3] (α=1)

5 Disorder: Localization



Inverse participation ratio $a \equiv x_4/x_2^2$ with $x_m(E_n) = \sum_r[|u_n(r)|^m + |v_n(r)|^m]$ for system sizes L = 20 to 45. Inset: scaling of $a^<$, averaged over energies less than "mobility edge" γ, and $a(E > \gamma)$.

- Note $a(E > \gamma) \sim 1/L^2 \Rightarrow$ states are extended
- No saturation observed in $a^<$ as in Franz et al.[5] $\Rightarrow$ states are not exponentially localized
- Order parameter suppression does not enhance localization. $\Longrightarrow$ changes in $\rho(E)$ are not connected to localization.

6 Conclusions

- In absence of order parameter supression, self-consistent T-matrix approx. excellent down to $1/(\rho L^2)$ (may saturate at localization scale $1/(\rho \xi_L^2)$?)
- Single-impurity resonance shifted by order parameter suppression similar to Shnirman et al,[10] but sign depends on band asymmetry
- Order parameter suppression induces anomalous power-law $\rho(E)$ in 2D d-wave superconductors.
- Power law exponent α appears unrelated to localization: multiple impurity resonances?

References

[1] A. A. Nersesyan, A. M. Tsvelik, and F. Wenger, Nucl. Phys. B **438**, 561 (1995); Phys. Rev. Lett. **72**, 2628 (1994).

[2] K. Ziegler, M. H. Hettler, and P. J. Hirschfeld, Phys. Rev. Lett. **77**, 3013 (1996).

[3] T. Senthil, Matthew P. A. Fisher, Leon Balents, and Chetan Nayak, Phys. Rev. Lett. **4704**, 1998; T. Senthil and Matthew P. A. Fisher, cond-mat/9810238.

[4] P. A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).

[5] M. Franz, C. Kallin, and A. J. Berlinsky, Phys. Rev. B **54**, R6897 (1996).

[6] A. V. Balatsky, M. I. Salkola, and A. Rosengren, Phys. Rev. B **51** 15 547 (1995); A. V. Balatsky and M. I. Salkola, Phys. Rev. Lett. **76**, 2386 (1996).

[7] R. Joynt, J. Low Temp. Phys. **109**, 811 (1997).

[8] M. E. Flatté and J. M. Byers, Solid State Phys. **52**, 137 (1999).

[9] Matthias H. Hettler and P. J. Hirschfeld, Phys. Rev. B **59**, 9606 (1999).

[10] Alexander Shnirman, İnanc Adagideli, Paul M. Goldbart, and Ali Yazdani, Phys. Rev. B **60**, 7517 (1999).

[11] W.A. Atkinson, P.J. Hirschfeld, A.H. MacDonald, cond-mat/9912168