Modern Numerical Ephemerides of Planets and the Importance of Ranging Observations for Their Creation

E.V. Pitjeva
Institute of Applied astronomy of Russian Academy of Sciences,
e-mail: evp@quasar.ipa.nw.ru

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Abstract. The JPL planetary and lunar ephemerides - DE200/LE200, DE403/LE403, DE405/LE405 and the planetary and lunar ephemerides, EPM87, EPM98, and EPM2000, constructed in the Institute of Applied Astronomy of RAS are described. Common properties and differences of the various ephemerides are given. Graphical comparisons of the DE ephemerides with each other and with the EPM ephemerides are presented. A fairly good agreement of planetary orbits is between DE403, DE405 and EPM98, EPM2000, respectively, over the interval of 120 years (1886-2006) covered by EPM98 and EPM2000. Some differences are explained by a slight disagreement in representing the orbits of Ceres, Pallas and Vesta as they affect the planets. The accurate radar observations of planets and spacecraft make it possible not only to improve the orbital elements of planets but to determine a broad set of astronomical constants as well: km/AU, parameters of Mars' rotation including its precessional rate, the masses of Jupiter, Ceres, Pallas and Vesta, relativistic parameters of the PPN formalism, the variability of the gravitational constant G. These have been obtained in the fitting process of the DE405 and EPM2000 ephemerides to observational data, including nearly 80000 American and Russian radar observations of planets (1961-1997), ranging and doppler to the Viking and Pathfinder landers, and other miscellaneous measurements from various sources and spacecraft.

Keywords: numerical integration, radar observations, planetary ephemerides, solar oblateness, testing of general relativity

1. Historical introduction: mathematical modelling of planet motion

The performance of complicated experiments in space and the introduction of new astrometric methods (radar ranging, lunar laser-ranging, VLBI measurements) now require more precise planetary ephemerides than what is possible with classical analytical theories. On the other hand, these modern data made the creation of such ephemerides possible. The uncertainty of modern ranging observations is only a few meters. That demands the accuracy of calculation of 12 significant figures in calculating the positions of planets. Now this problem is solved more easy by using numerical integration of the equations of the motion of the planets and the moon.

The coordinates of the five outer planets over a 400-year interval were computed by Eckert *et al.*,(1951) using, for the first time, a numerical integration. For the construction of ephemerides with high accuracy the simultaneous numerical integration of the equations of the motion of the major planets and the Moon as well as that of the rotational motion of the Moon is necessary. The first simultaneous numerical integration of major planets and Moon was made by Oesterwinter and Cohen (1972) over the interval 1911–1973.

To ensure space flights in the late 1960's, numerical planetary ephemerides were coming into construction by several groups in the USA and Russia. The American groups worked at the California Institute of Technology/ Jet Propulsion Laboratory (Standish *et al.*, 1976; Newhall *et al.*, 1983), and the Massachusetts Institute of Technology (Ash *et al.*, 1967,1971). In Russia precise numerical ephemerides of planetary motion have been created (Akim *et al.*, 1986) resulting from investigations in the Institute of Applied Mathematics (Akim and Stepan'yants, 1977), Institute of Radioengineering and Electronics (Kislik *et al.*, 1980) and Institute of Theoretical Astronomy, where the numerical ephemerides have been constructed independently by N. I. Glebova (1984) and later by G. I. Eroshkin (Eroshkin *et al.*, 1991) as well as by the group headed by G. A. Krasinsky (Krasinsky *et al.*, 1981,1982). This work was continued in the Institute of Applied Astronomy, where the series of EPM (Ephemerides of Planets and the Moon) has been produced. These EPM ephemerides and the JPL DE ephemerides will be considered further.

2. Similarities and differences between the various ephemerides of DE and EPM

Common to all DE/LE and EPM ephemerides is a simultaneous numerical integration of the equations of motion of the nine major planets, the Sun, the Moon and the lunar physical libration performed in the Parametrized Post-Newtonian metric for the harmonic coordinates $\alpha =\ 0$ and General Relativity values $\beta = \gamma = 1$.

The various ephemerides differ slightly in the

- the modelling of the lunar libration,
- the modelling of the perturbations of asteroids upon the planetary orbits,
- reference frames,
- the sets of observations to which ephemerides are adjusted.

Some characteristics of DE118/LE62, DE200/LE200, DE403/LE403, DE405/LE405, EPM87, EPM98, EPM2000 ephemerides are given in Table I.

Table I. Ephemerides DE and EPM.

Ephemeris	Interval of integration	Ref. frame	Mathematical model		data base	
DE118	1599→2169	FK4	integrating of	optical	44755	1911-79
(1981)			Sun,Moon,9 planets	radar	1307	1964-77
⇒DE200		dyna-	+perturbations from	sp.craft	1408	1971-80
		mical	3 asteroids	LLR	2954	1970-80
		frame	(Keplerian ellipses)	total	50424	1911-80
EPM87	1700→2020	FK4	integrating of	optical	48709	1717-80
(1987)			Sun,Moon,9 planets	radar	5344	1961-86
			+perturbations from	sp.craft	–	–
			5 asteroids	LLR	1855	1972-80
			(Keplerian ellipses)	total	55908	1717-86
DE403	-1410→3000	ICRF	integrating of	optical	26209	1911-95
(1995)			Sun,Moon,9 planets	radar	1341	1964-93
⇒DE404	-3000→3000		+perturbations from	sp.craft	1935	1971-94
			300 asteroids	LLR	9555	1970-95
			(mean elements)	total	39057	1911-95
EPM98	1886→2006	DE403	integrating of	optical	–	–
(1998)			Sun,Moon,9 planets,	radar	55959	1961-95
			5 aster.+ perturb.	sp.craft	1927	1971-82
			from 295 asteroids	LLR	10000	1970-95
			(mean elements)	total	67886	1961-95
DE405	1600→2200	ICRF	integrating of	optical	28261	1911-96
(1997)			Sun,Moon,9 planets	radar	955	1964-93
⇒DE406	-3000→3000		+perturbations from	sp.craft	1956	1971-95
			300 asteroids	LLR	11218	1969-96
			(integrated)	total	42410	1911-95
EPM2000	1886→2006	DE405	integrating of	optical	–	–
(2000)			Sun,Moon,9 planets,	radar	58255	1961-97
			300 asteroids	sp.craft	24587	1971-97
				LLR	13500	1970-99
				total	96348	1961-99

3. DE118/LE62 (1981) and DE200/LE200 (1982) ephemerides

The various versions of the DE ephemerides have been created since the 1960's. The ephemerides DE118/LE62 were constructed in September 1981 (Standish, 1990). The Keplerian orbits of the three most massive asteroids (Ceres, Pallas, Vesta) have been computed and their perturbations upon the orbits of the planets have been taken into account during the numerical integration. The parameters of the ephemeris were determined by the adjustment to over 50000 positional measurements. These data included transit and photographic optical data mainly from the U.S. Naval Observatory covering the time span 1911-1979, American radar ranging of planets, spacecraft ranging, and lunar laser-ranging. The ephemerides DE118/LE62 have been aligned onto the FK4 reference frame by transit measurements of the planets and the Sun. The ephemerides DE118/LE62 have been transformed into DE200/LE200 by precessing and then rotating DE118/LE62 onto the dynamical equator and equinox of J2000, the latter determined by performing a least-squares fit to the motion of the instantaneous node and obliquity of the orbit of the Earth-Moon barycenter about the Sun (Standish, 1982). The DE200/LE200 ephemerides have been the basis of the published ephemerides for nearly all of the major almanac offices since 1984.

4. The EPM87 ephemerides (1974-1987)

The development of EPM (Ephemerides of Planets and the Moon) began in 1974. The most complete version, designated EPM87, was created in 1987 (Krasinsky *et al.*, 1993, Pitjeva 1993). The main difference between EPM87 and DE200 was due to the fact that the parameters of our ephemeris were fit to a wide variety of observational data for the time span XVIII-XX centuries.

G. A. Krasinsky proposed a procedure of sequential processing of different types of observations, allowing the complete realization of their precision and inherent information. There are some distinguishing features of this procedure. At the first stage of processing the orbital elements of the inner planets and the Moon were determined from Russian and American radar observations of planets and lunar laser ranging. These precise ephemerides made possible the use of historical observations of the inner planet transits through the solar disk in order to continue the atomic scale into the past till 1715 by the correction of the adopted Brouwer (1952) system of differences between the ephemeris and universal (UT) time. The new time scale turned out to be in use for the treatment of other classical observations during this interval of time. So, the meridian observations of the outer planets were used for determination of new elements of these planets and the value of the lunar tidal deceleration was obtained from the analysis of lunar occultations: $\dot{n}_M = (-22.''9 \pm 0.''8)/cy^2$ (Krasinsky *et al.*, 1985). The ephemerides EPM87 were adjusted onto the 1950 FK4 reference frame by the use of transit measurements of the inner planets and the Sun from the USNO as well as by the precise observations of the occultations of fundamental stars by Venus (1959) and Mars (1976) (Krasinsky, 1985). It was a unique ephemeris of major planets and the Moon, surpassing DE200 in accuracy for the outer planets over the XVIIIth and XIXth centuries. However, this ephemeris was implemented on the Russian BESM-6 computer of domestic design that was soon after dismantled. Fortunately the developers of the ephemeris had an opportunity to use the gained experience in their subsequent work.

5. DE403/LE403 ephemerides (1995)

In 1995 a new version of the JPL ephemerides - DE403/LE403 - appeared (Standish *et al.*, 1995). Its basic differences from DE200/ LE200 are as follows:

• The improved values for the planetary masses determined from various space mission were used.

• Modern high precision measurements of all types (optical, radar, laser and VLBI spacecraft) obtained during fifteen years (1981-1995) were added to the observational data set. Some early radar observations of the planets were excluded from the adjustment.

• The ephemerides DE403/LE403 are based upon the International Celestial Reference Frame (ICRF), which is defined by the directions of compact extragalactic radio sources, observed with VLBI method by several national and international programs. Adjustment of the DE403 ephemerides onto the ICRF was automatically accomplished by the inclusion of VLBI measurements of spacecraft near Mars and Venus and from a determination combining VLBI and LLR observations (Folkner *et al.*, 1994).

• As experience shows, it isn't enough to take into account perturbations from only the several most massive asteroids upon Mars' orbit. Therefore, the modelling of the asteroid perturbations upon the planetary orbits has been considerably improved. At first, using mean elements received from Williams (1989), positions were computed for the 300 asteroids which had the largest effect upon the orbit of Mars. Then, the asteroids were handled in two different ways. The forces of Ceres, Pallas and Vesta were included in the equations of motion, knowing the asteroids' positions and masses. The remaining 297 asteroids (with smaller masses) were grouped according to three taxonomic classes with assumed different densities, their masses were calculated from estimated diameters. Then the perturbations upon the Earth, Mars and the Moon were computed for each asteroid throughout the interval of integration and summed over each of the three classes.

All of the improvements listed above have caused rather significant improvements in the orbits of all of the planets (1-2 km for the inner planets, 100 km for Jupiter, and up to 400000 km for Pluto). Figure 1 shows the results of comparison of geocentric distances of the inner planets, the Sun, the Moon and heliocentric distances of the outer planets between DE403/LE403 and DE200/LE200 over the 1800-2050 time interval. The original graphics of comparison of planetary distances between DE403 and DE200 as well as between DE403 and DE405 have been given in the papers by Standish *et al.* (1995), Standish (1998). These papers were used here for the review of ephemerides.

6. The EPM98 Ephemerides (1998)

The new stage of the construction of the EPM ephemerides was started about 1995. Along with it the numerical ephemeris of motion of the Moon has been created by E. Yu. Aleshkina, G. A. Krasinsky and M. V. Vasilyev (1997) in the fitting process to the 25-year set of lunar laser ranging with an accuracy close to that of LE403.

The ephemerides of the planets and the Moon were produced by simultaneous numerical integration of the equations of motion of nine planets, the Sun, the Moon, lunar physical libration and five most massive asteroids (Pitjeva, 1998). The initial elements of these minor planets have been taken from "Ephemerides of minor planets for 1992" (1991) and then were improved from a century's worth of observations. In comparing with the previous versions of our ephemeris the dynamic model of planetary motion was supplemented with modelling of the perturbations from three taxonomic classes of asteroids (295 objects), whose perturbations were taken into account in the same manner as in DE403. Numerical integration of the equations of motion in the barycentric coordinate frame of J2000.0 has been carried out by the Everhart method (1974) of nineteenth order over the interval JD 2410000.5 - 2454000.5 (1886 – 2006). The integrator for the modelling of planetary motion within the frame of programming system ERA (Ephemeris Researches in Astronomy) has been developed by M. V. Vasilyev (Krasinsky and Vasilyev, 1997). After the completion of the integration process a set of Chebyshev polynomials for positions and velocities of all considered objects was constructed over the given interval of time. Intervals of approximation and degree of the polynomials were taken so that the accuracy of approximation for the coordinates of the inner planets was retained within one meter. Provisional initial conditions for numerical integration and values of parameters of the dynamic ephemeris were taken from the JPL Ephemerides DE403. Internal accuracy of the ephemeris EPM98 was controlled by comparison of the results of forward and backward integrations over the 50-year interval. Discrepancies of the coordinates of the Sun, the Moon and planets (except for Mercury) were below 1 meter. The error of longitude for Mercury was growing proportionally to the time interval at the rate dl= 5 m/cy and at the rate of 0.12 m/cy for the Moon.

Comparison of the osculating, heliocentric elements and solar-system-barycentric positions, velocities, distances of planets and the Sun between EPM98 and DE403 over the time interval 1954-2001 was made. It showed fairly good agreement for these ephemerides: the discrepancies for semi-major axes are in the range from 0.4 m (Mercury) to 344 m (Pluto), the mean longitudes of all planets (except for Mars) differ by less than 0.027 mas. By way of illustration, the differences for osculating, heliocentric ecliptic elements of Mercury, Venus and Mars given by the ephemerides DE403 and EPM98 are shown in Figures 2-4. Small letters a, b, c, d, e, f in these figures indicate corresponding differences for a, sin$i$cosΩ, sin$i$sinΩ, $e$cosπ, $e$sinπ, λ, where a - the semi-major axis, i - the inclination of the orbit, Ω - the ascending node, e - the eccentricity, π - the longitude of perihelion, λ - the mean longitude. Time expressed in years (1954 - 2001) is given along the abscissa. The scale of ordinates is given in meters for semi-major axis (Figure a) and in milliarc-seconds (mas) for other elements (Figures b-f). Maximum differences (of about 500 m) are obtained for Mars over the interval 1964-1973. They are explained by a slight disagreement of the dynamic models of DE403 and EPM98, which is attributed to the more correct computation of the orbits of Ceres, Pallas and Vesta by the ephemeris EPM98 over the present epoch. When we changed our integrator so that the perturbations from all asteroids were computed as in DE403 (ephemeris EPMTEST), the differences decreased to 10.3 m for the barycentric distances (Table II) and the jumps were no longer observable (Figure 5). The disagreements between the orbits of other planets were decreased appreciably as well. The maximal differences (ΔR) for the barycentric positions and distances between DE403 and EPM98 are given in meters for all planets in Table II; ΔR^*, given in the last column of Table II, are the differences for the barycentric distances between DE403 and EPMTEST.

Table II. Maximal differences of the solar-system–barycentric positions and distances between DE403 and EPM98 (in m); ΔR* is for the distances between DE403 and EPMTEST.

planet	Δ X	Δ Y	Δ Z	Δ R	Δ R*
Mercury	4.9	5.7	2.5	3.3	0.2
Venus	16.4	14.2	6.5	3.9	0.3
Earth-Moon	13.0	14.8	7.8	3.8	0.5
Mars	458.9	455.8	216.8	80.6	10.3
Jupiter	76.1	68.2	28.3	32.8	12.9
Saturn	75.7	60.2	23.3	29.6	5.4
Uranus	69.8	82.5	34.7	59.5	18.1
Neptune	44.0	140.7	55.9	96.5	41.7
Pluto	151.4	183.2	22.5	118.1	54.0
Sun	2.8	2.6	1.3	3.1	1.2

The numerical ephemerides EPM98 were created using the set of American and Russian radar observations of planets obtained during 1961-1995 (nearly 60000 observations), range measurements of martian landers Viking-1,-2 and Mariner-9 tracking data.

7. The DE405/LE405 Ephemerides (1998)

At present a new set of the JPL ephemerides, DE405/LE405, has been realized (Standish, 1998). This version differs from the previous DE403 by the inclusion of new observations into the adjustment; in particular, photoelectric and CCD measurements of the outer planets have led to the improvement of those planetary orbits. Additional VLBI observations of the spacecraft have improved the orientation of DE405 onto the ICRF. The procedure for calculating the asteroid positions has also been improved. Now the orbits of all asteroids are computed by numerical integration. The accuracy of orienting these ephemerides onto ICRF, using VLBI, and the accuracy of the inner planet ephemerides, limited primarily by the uncertainties in the masses of asteroids, are estimated to be on the order of 1 km (Standish, 2000).

The solar oblateness $J_2 = 2.0 \cdot 10^{-7}$, obtained from some astrophysical estimations has been taken into account. Figures 6-7 are given as an illustration of the influence of the solar oblateness on the orbital elements of the planets. The solar oblateness causes a secular trend in the planetary elements except for the semi-major axis and eccentricity. The values of the secular trends are inversely proportional to the square of the semi-major axis with the maximum secular trend existing for Mercury's perihelion. According to Brumberg (1972) the rate of the secular trend for the perihelion is given by the formula

$$\frac{d\pi}{dt} = \frac{3}{2}\left(\frac{R_\odot}{a_i}\right)^2 \frac{n_i}{(1-e_i^2)^2} J_2.$$

If $J_2 = 2 \cdot 10^{-7}$ then for Mercury:

$$\frac{d(ecos\pi)}{dt} = -5.12 \; mas/century,$$
$$\frac{d(esin\pi)}{dt} = 1.04 \; mas/century.$$

Differences for the heliocentric ecliptic elements (the same as in Figures 2-5) of Mercury and Mars in the ephemerides DE403 and DE405 over the 1600-2200 time interval are presented in Figures 6-7. The secular trends for the Mercury elements, visible in Figures 6b-6f, agree with Brumberg's estimations and therefore are explained almost completely by the introduction of the solar oblateness into the ephemerides DE405. As for the Mars elements these trend are not visible.

Figure 8 presents the comparison of the geocentric distances of the inner planets, the Sun, the Moon and heliocentric distances of the outer planets between DE403/LE403 and DE405/LE405 over the same time interval. If comparing differences of planetary ephemerides DE403-DE200 (Figure 1) and DE403-DE405 (Figure 8) it may be seen that the distinctions between the ephemerides become considerably less. For the outer planets they are less almost by one order.

8. The EPM2000 Ephemerides (2000)

At last, our new EPM2000 ephemerides are in progress. The numerical integration for EPM2000 (Pitjeva, 1999) extended over a 120-year time interval and included the equations of motion for 311 bodies and the lunar physical libration. For the 16 main bodies (the 9 planets, the Sun, the Moon, and 5 selected asteroids), the equations of motion included their mutual perturbations, general relativity, the effect of the solar oblateness, and the perturbations from another 295 selected asteroids. For those 295 asteroids, the equations of motion included just the newtonian gravitational effect of the 16 main bodies and the effect of the solar oblateness. The 5 selected asteroids were Ceres, Pallas, Vesta, Iris, and Bamberga, chosen because of their strong perturbations on Mars over the latter half of the 20th century. The other 295 were selected (Williams, 1988) for the same reason though their perturbations are decidedly less. The masses of planets and asteroids as well as the provisional initial positions and velocities of planets correspond to the ephemerides DE405. Since the mean asteroid elements, computed by Williams (1989), cover such a long time interval, orbits computed using mean elements, as in DE403 and EPM98, differ significantly from their true orbits; therefore, it is necessary to integrate the orbits of each of the 300 asteroids. Orbits of the most massive asteroids (Ceres, Pallas, Vesta, Eunomia, Iris, Bamberga) and Andromache and Tatjana, for whom positions computed with mean elements differ from those integrated using osculating elements by more than 1 AU, have been improved from the set of optical observations. Positions of other asteroids have been calculated from their elements taken from "Ephemerides of minor planets for 1992" (1991).

The orbits of Ceres, Pallas and Vesta are computed as before in different ways in the ephemerides DE405 and EPM2000; therefore, differences for the planetary positions and elements given by these ephemerides are the same as between the DE403 and EPM98 ephemerides.

The ephemerides EPM2000 were fitted to data totaling nearly 80000 American and Russian radar observations of planets (1961-1997), ranging and doppler measurements of the Viking and Pathfinder landers, various measurements of spacecraft at Jupiter (1973–1997), and VLA positional measurements of Jupiter's thermal emission. The ephemerides of the Moon were adjusted to 13500 lunar laser ranging (1970-2000) by E.Yu. Aleshkina. Residuals of LLR measurements are close to their *a priori* standard deviations (Aleshkina *et al.*, 2000).

9. Determination of parameters of the constructed ephemerides

The parameters of the orbital motion and rotation of the Moon are determined from lunar laser ranging. The parameters of planetary ephemerides are obtained from two large classes of measurements, optical and radio engineering data.

Optical observations have been used for the solution of three problems:

- Orientation of coordinate system (now it is deduced from VLBI and LLR observations completely)
- Improvement of the orbits of objects not adequately observed by radio methods (outer planets, for example); and,

• Study of the century-long variations of elements, in virtue of the extended time interval of the optical observations.

At present, radiometric observations of planets serve for the solutions of many different tasks:

• Centimeter-to-meter wavelengths penetrate planetary clouds and cometary comae and permit the investigation of the structure, bulk density and concentrations of metals and ice.

• Radar allows the construction of topographical maps of planets and asteroids.

• Environment between the Earth and the observable planet influences the propagation of radio signals. As a result, some characteristics of plasma of the solar corona are measured by these signals.

• VLBI observations of Phobos, Magellan and Ulysses spacecraft at Mars, Venus, and Jupiter determine the orientation of the ephemerides onto the International Celestial Reference Frame (ICRF).

Radiometric observations are also used for the following tasks, closer to our concern here:

• Improvement of the orbital elements of the planets;

• Determination of the parameters of rotation of the planets;

• Estimation of the masses of objects of solar system;

• Testing of gravitational theories, variability of the gravitational constant, and the solar oblateness.

More than 80000 Russian and American observations of planets, spacecraft and Mars landers obtained during 1961-1997 have been used for the solution of these tasks.

Reductions of measurements include relativistic corrections, effects of propagation of electromagnetic signals in the Earth troposphere and in the solar corona with simultaneous evaluation of parameters of the corona model. Great attention was given to the reduction of radar observations for the topography of planets. For Mars and Venus the correction was carried out with the help of hypsometric maps. The topography of Mercury is expanded in the set of Legendre functions to the second order, the coefficients being estimated from radar observations, as was done by Anderson et al. (1996).

It is necessary to note, especially, the extremely precise observations of the martian Viking (1976-1982) and Pathfinder (1997) landers. The uniqueness of such observations is that they are free from the uncertainties due to planetary topography, which do remain in radar ranging despite the modeling of topography. The lander observations are the most accurate of all planetary position data. The positions of the landers are computed, taking into account the precession, nutation and seasonal terms of the Mars rotation. (Folkner *et al.*, 1997; Pitjeva, 1999)

The Viking-1, -2, and Pathfinder ranging and differenced range data, their *a priori* standard deviations σ_{apr} and post-fit rms are given in Table III, where σ_{EMS} is the rms of post-fit residuals received from Standish, σ_{DE} and σ_{EPM} are the rms of post-fit residuals obtained by the author using the ephemerides DE405 and EPM2000, respectively.

Table III. Radar observations of the Viking-1, -2, and Pathfinder martian landers (all σ values are given in m for τ and in mm/sec for $d\tau$). It should be noted that the DE ephemerides were created before the Pathfinder data were available.

Object	type	interval	number	σ_{apr}	σ_{EMS}	σ_{DE}	σ_{EPM}
Viking-1	τ	7.1976-11.82	1078	7-12	9.5	8.5	9.4
Viking-2	τ	9.1976-09.77	80	7-10	-	5.4	6.0
Viking-1	$d\tau$	7.1976-12.78	15030	1.3-27	0.7-4.3	0.9	0.9
Pathfinder	τ	7.1997-09.97	90	10-22	5.3	5.7	2.9
Pathfinder	$d\tau$	7.1997-10.97	7613	0.11	0.1	0.1	0.1

The residuals of radar observations of the martian landers are shown in Figures 9-12. It is seen that all rms are close to each other and do not exceed the *a priori* standard deviations of the data σ_{apr}. The adjusted parameters of the Mars rotation determined from the landers observations by the author are given in Table IV.

Table IV. The parameters of the Mars rotation.

$\dot{V}$ [^{o}/day]	I_q [^{o}]	$\dot{I}_q$ [$''$/year]	Ω_q [^{o}]	$\dot{\Omega}_q$ [$''$/year]
350.891985129	25.1893932	-0.0036	35.437724	-7.5712
$\pm$ 0.000000011	$\pm$0.0000050	$\pm$0.0024	$\pm$ 0.000020	$\pm$ 0.0050

Following formulas of Folkner et al. (1997) and considering the gravity coefficient of Mars J_2, obtained from detection of the Mars gravity field with the use of the Viking orbiter and other tracking data by Konopliv and Sjogren (1995), to be equal to 0.0019586, the corresponding normalized polar moment of inertia of Mars was determined as

$$C/MR^2 = 0.3664 \pm 0.0013,$$

which practically coincides with the value by Folkner et al. (1997)

$$C/MR^2 = 0.3662 \pm 0.0017.$$

Since the orbit of Mars is perturbed by Jupiter and asteroids, the observations of the martian landers provide valuable information concerning the mass of Jupiter, elements of its orbit, and the masses of the three largest asteroids Ceres, Pallas and Vesta (Standish and Hellings, 1989; Pitjeva, 1997). The Sun-Jupiter and Ceres-Sun, Pallas-Sun and Vesta-Sun mass ratios, obtained by the author, are presented in Table V.

Table V. Sun-Jupiter and Ceres-Sun, Pallas-Sun, Vesta-Sun mass ratios.

$M_{\odot}/M_{Jup}$	$M_{Cer}/M_{\odot}$	$M_{Pal}/M_{\odot}$	$M_{Ves}/M_{\odot}$
1047.34866	$4.808 \cdot 10^{-10}$	$0.996 \cdot 10^{-10}$	$1.364 \cdot 10^{-10}$
± 0.00003	$\pm 0.012 \cdot 10^{-10}$	$\pm 0.004 \cdot 10^{-10}$	$\pm 0.010 \cdot 10^{-10}$

Our main task is the determination of the orbital elements of the planets. The accuracy of the estimations of the elements from ranging observations is rather high. For example, formal standard deviations of the semi-major axes of Earth and Mars and the astronomical unit are about 0.1-0.2 m. However, it is well known that the real accuracy is often several times worse than such formal statistics. The resulting value of the astronomical unit is in good agreement with that of DE405

$$AU_{EPM2000} = (149597870691.2 \pm 0.2)\ m,$$

$$AU_{DE405} \ = \ 149597870691\ m.$$

The formal standard deviations of the orbital elements of Mercury, Venus, Earth, Mars and Jupiter are given in Table VI. The elements of Jupiter were improved from VLBI and range observations of spacecraft at Jupiter and the martian landers.

Table VI. The *formal* standard deviations of elements of the planets.

planet	a [m]	$sinicos\Omega$ [mas]	$sinisin\Omega$ [mas]	$ecos\pi$ [mas]	$esin\pi$ [mas]	λ [mas]
Mercury	0.187	3.169	3.329	0.321	0.279	0.826
Venus	0.309	0.606	0.594	0.039	0.040	0.184
Earth	0.102	–	–	0.001	0.001	–
Mars	0.259	0.024	0.025	0.002	0.002	0.008
Jupiter	675.1	4.817	4.086	0.321	0.322	1.164

With high-precision radar observations covering the time interval of over 30 years one may estimate the time derivative of the gravitational constant characterizing the fundamental properties of our physical time-space and representing a test of the relativistic parameters and the solar oblateness. As it is known, the secular relativistic trend of the Mercury perihelion depends on the linear combination of the relativistic parameters β, γ and the solar quadrupole moment (J_2):

$$\dot{\pi}_1 = 42.''98[(2 + 2\gamma - \beta)/3 + 0.296 J_2 \cdot 10^4].$$

Applying the obtained correction to the Mercury perihelion motion and putting $(2 + 2\gamma - \beta)/3 = 1$ (which also agrees with our results), the estimates for J_2 can be computed. The obtained results are given in Table VII.

The residuals of radar observations of Mercury, Venus and Mars are shown in Figures 13-15. The residuals of spacecraft at Jupiter are plotted in Figures 16-17. The circles represent observations in right ascension; triangles, in declination; asterisks, VLBI measurements from the Galileo spacecraft. The rms of post-fit residuals of observations are 1.4 km for Mercury, 0.7 km for Venus and Mars, 8 m for Viking and 4 m for Pathfinder measurements.

Table VII. Relativistic parameters, $\dot{G}/G$ and the solar quadrupole moment.

$\dot{G}/G(10^{-11}yr^{-1})$	$\beta - 1$	$\gamma - 1$	$\Delta\dot{\pi}_{Mer}(''cy^{-1})$	$J_2(10^{-7})$
0.004	0.0004	0.0001	0.0055	2.43
±0.008	±0.0002	±0.0001	±0.0085	± 0.67

10. Conclusions

It should be stressed once again that just the use of ranging observations has ensured the milliarcsecond accuracy of the ephemerides of the inner planets permitting the determination of many other astronomical parameters with high precision.

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