

"Sound" in superfluid liquids

B. N. Esel'son, M. I. Kaganov, É. Ya. Rudavskii, and I. A. Serbin

*Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
Institute of Physical Problems, USSR Academy of Sciences
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The existence of two sounds of different natures in the superfluid liquid is one of the best known and most thoroughly investigated properties of helium. An enormous mass of experimental and theoretical material on wave processes in helium has accumulated during the years that have passed since the discovery of second sound. Interest in these problems has especially increased in the last decade, first, because of the discovery of third and fourth sound (oscillations that propagate when the normal component of the liquid is clamped), and second, because of the possibility of investigating ultrashort sound waves by neutron and photon scattering. In this article the fundamental theoretical concepts and experimental results concerning the propagation of sound waves of the several types in superfluid liquid (in He^4 and $\text{He}^3\text{-He}^4$ solutions) are reviewed. Much attention is given to quantitative, as well as qualitative, comparison of the experimental results with theory. The bibliography has been made as complete as possible. The following topics are not discussed, although they touch directly upon the subject under review: the propagation of sound near the lambda point, the passage of waves across liquid-liquid and liquid-vapor interfaces, and the propagation of large-amplitude sound waves.

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INTRODUCTION

Thanks to its unique properties, which have naturally aroused great interest, liquid helium is now one of the most thoroughly investigated substances in nature.

Although the superfluidity of He^4 , discovered by P. L. Kapitza^[1], was explained in principle by L. D. Landau more than 30 years ago^[2], helium continues to be very intensively studied and new properties of the quantum liquid, such as quantized vortices, the motion of electric charges in helium, etc., are continually being recognized.

A notable feature of the study of helium and its properties, which characterizes the level of our understanding of the processes taking place within it, is that the theory describing some phenomenon or other is confirmed, as a rule, by complete quantitative agreement with experiment. Every deviation from this rule means either that the theory requires further development or that the experimental errors are appreciable. Especially precise measurements, detailed development of the theory, and careful comparison of the various experiments are therefore required.

The existence of two sounds (first and second) in the superfluid liquid is one of the best known anomalies of helium. The attention of investigators was naturally attracted to the study of wave propagation in quantum liquids. An enormous mass of experimental and theoretical material on wave processes in helium has accumulated during the years that have passed since the discovery of second sound. Interest in these problems has especially increased in the last decade, first, be-

cause of the discovery of so called third sound and fourth sound, and second, because of the possibility of investigating ultrashort sound waves by neutron and photon scattering.

In this article we shall attempt to present the fundamental theoretical concepts and experimental results concerning the propagation of sound waves of the several types in superfluid liquids (in He^4 and $\text{He}^3\text{-He}^4$ solutions)^[1].

We feel that it would be useful to mention the sort of questions that have been excluded from the review because of space limitations. These include the propagation of sound near the λ point^[2] (this is a special topic which would require a detailed review of the properties of helium near the second-order phase transition point and an exposition of current ideas concerning phase transitions) and the passage of waves across liquid-liquid, liquid-vapor, and liquid-solid interfaces.

1. HYDRODYNAMIC EQUATIONS AND ACOUSTIC MODES

When the mean free path of the excitations is much shorter than the wavelength of the sound ($l \ll \lambda$) the wave processes are described by the hydrodynamic equations for a superfluid liquid, which express the conservation of the mass and momentum of the liquid, the increase in entropy, and the possibility of superfluid motion^[2,8]

In the linear approximation in small deviations from the equilibrium values, these equations can be written in the form

$$\frac{\partial p}{\partial t} + \text{div } \mathbf{j} = 0 \quad (\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n),$$

$$\begin{aligned} \frac{\partial j_i}{\partial t} + \frac{\partial p}{\partial r_i} &= \eta \frac{\partial}{\partial r_k} \left(\frac{\partial v_{nk}}{\partial r_i} + \frac{\partial v_{nk}}{\partial r_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_{nl}}{\partial r_l} \right) \\ &+ \frac{\partial}{\partial r_i} [\zeta_1 \operatorname{div} (\mathbf{j} - \rho \mathbf{v}_n) + \zeta_2 \operatorname{div} \mathbf{v}_n], \\ \frac{\partial \mathbf{v}_s}{\partial t} + \nabla \mu &= \nabla [\zeta_3 \operatorname{div} (\mathbf{j} - \rho \mathbf{v}_n) + \zeta_4 \operatorname{div} \mathbf{v}_n], \\ \frac{\partial (\sigma p)}{\partial t} + \sigma p \operatorname{div} \mathbf{v}_n &= \frac{\kappa}{T} \Delta T; \end{aligned} \quad (1.1)$$

here ρ_s and \mathbf{v}_s (ρ_n and \mathbf{v}_n) are the density and velocity, respectively, of the superfluid (normal) component; $\rho = \rho_s + \rho_n$ is the density of the helium; P , T , and μ are the pressure, temperature, and chemical potential; σ is the entropy per unit mass of helium; η and κ are the first-viscosity and heat-conductivity coefficients; ζ_1 , ζ_2 , ζ_3 , and $\zeta_4 = \zeta_1$ are "second-viscosity" coefficients; and δ_{ik} is Kronecker's delta.

By neglecting dissipative processes one can easily derive the dispersion equation for the wave propagation velocity u from Eqs. (1.1):

$$u^4 + u^2 \left[\left(\frac{\partial p}{\partial \rho} \right)_\sigma + \frac{\rho_s}{\rho_n} \sigma^2 \left(\frac{\partial T}{\partial \rho} \right)_\rho \right] + \frac{\rho_s}{\rho_n} \sigma^2 \left(\frac{\partial T}{\partial \sigma} \right)_\rho \left(\frac{\partial p}{\partial \rho} \right)_T = 0. \quad (1.2)$$

This equation has two roots [9]:

$$2u^2 = u_{1\sigma}^2 + u_{20}^2 \pm [(u_{1\sigma}^2 - u_{20}^2)^2 + 4u_{20}^2(u_{1\sigma}^2 - u_{1T}^2)]^{1/2}, \quad (1.3)$$

where

$$u_{1\sigma}^2 = \left(\frac{\partial p}{\partial \rho} \right)_\sigma, \quad u_{1T}^2 = \left(\frac{\partial p}{\partial \rho} \right)_T, \quad u_{20}^2 = \frac{\rho_s}{\rho_n} \sigma^2 \left(\frac{\partial T}{\partial \sigma} \right)_\rho.$$

The difference between the adiabatic and isothermal compressibilities is small:

$$\frac{u_{1\sigma}^2}{u_{1T}^2} = \frac{C_P}{C_V} = 1 + \alpha^2 u_{1T}^2 \frac{T}{C_V}$$

(here C_P and C_V are the heat capacities at constant pressure and volume, and α is the thermal expansion coefficient). At $T = 1.5^\circ \text{K}$ the ratio C_P/C_V differs from unity by a quantity of the order of 10^{-3} . Hence from (1.3) we have

$$u_1^2 \approx u_{1\sigma}^2 \left(1 + \alpha^2 \frac{T}{C_V} \frac{u_{20}^2 u_{1\sigma}^2}{u_{1\sigma}^2 - u_{20}^2} \right), \quad (1.4)$$

$$u_2^2 \approx u_{20}^2 \left(1 - \alpha^2 \frac{T}{C_V} \frac{u_{1\sigma}^4}{u_{1\sigma}^2 - u_{20}^2} \right). \quad (1.5)$$

The existence of two roots means that waves of two types can propagate (and with different velocities) in superfluid helium. Landau called these wave processes first and second sound. Besides differing in propagation velocity, first and second sound differ as regards the nature of the oscillations [10]: in first sound it is mainly the pressure (density) that oscillates, and in second sound, the temperature.

In many cases α is small enough to permit the second terms in (1.4) and (1.5) to be neglected; then these equations reduce to the well known expressions for the velocities of first and second sound:

$$u_1^2 = \frac{\partial p}{\partial \rho}, \quad (1.6)$$

$$u_2^2 = \frac{\rho_s}{\rho_n} \sigma^2 \frac{\partial T}{\partial \sigma}. \quad (1.7)$$

We emphasize that the presence of two types of waves in a superfluid liquid is a consequence of the possibility of two types of motion: normal and superfluid. When $\rho_s = 0$, only one type of wave is propagated: first (ordinary) sound.

The dissipative processes described by the viscosity and heat-conduction coefficients naturally lead to attenuation of the sound waves. Moreover, when the dissipative

processes are included the dispersion equation has a third root (which is complex). The wave corresponding to this third root is called the viscous wave. When all dissipative processes except first viscosity (specified by the coefficient η) are neglected, the length of the viscous wave is given by $\lambda_\eta = (2\eta/\omega\rho_n)^{1/2}$. The viscous wave plays an important part in the propagation of sound waves in limited spaces. In particular, the character of the wave processes in He II in limited spaces depends on the ratio of the characteristic linear dimension d of the region occupied by He II to the penetration depth λ_η of the viscous wave. When $d \ll \lambda_\eta$, the normal component is clamped ($\mathbf{v}_n \rightarrow 0$) and oscillations are propagated only in the superfluid component.

The condition $d \ll \lambda_\eta$ is satisfied in a He II film on a solid surface. In this case oscillations analogous to capillary waves on an ordinary liquid propagate on the free surface of the He II film; they are called third sound [11].

The normal component can be clamped by saturating a porous medium with He II, provided the characteristic linear dimension of the channels (pores) of the medium is considerably smaller than λ_η . Then the liquid has no free surface, and density oscillations due to oscillations of the superfluid component propagate along the channel. Such a wave process is called fourth sound [11].

It must be emphasized that third and fourth sound are not new independent acoustic modes of the superfluid liquid; they are the wave processes into which first and second are transformed when the effects of the boundaries become predominant.

He⁴ is not the only superfluid liquid; He³-He⁴ solutions are also superfluid over a wide range of temperatures and concentrations. The hydrodynamic equations for superfluid solutions are similar to those for He⁴, but there must be an additional equation to describe the conservation of the number of atoms of the second component (He³) and the equations themselves must be altered to take into account additional dissipative processes: diffusion, thermal diffusion, and barodiffusion [8, 12]. First and second sound can also propagate in He³-He⁴ solutions. Their propagation velocities are determined from the dispersion equation

$$(u^2 - s_1^2)(u^2 - s_2^2) = \beta^2 \frac{\rho_s}{\rho_n} s_1^2 u^2, \quad (1.8)$$

in which

$$s_1^2 = \left(\frac{\partial p}{\partial \rho} \right)_{\sigma, c}, \quad s_2^2 = \frac{\rho_s}{\rho_n} \left[\bar{\sigma}^2 \left(\frac{\partial T}{\partial \sigma} \right)_{\rho, c} + c^2 \frac{\partial}{\partial c} \left(\frac{Z}{\rho} \right) \right],$$

$$\beta = \alpha \bar{\sigma} \left(\frac{\partial T}{\partial \sigma} \right)_{\rho, c} - \frac{c}{\rho} \frac{\partial \rho}{\partial c}, \quad \bar{\sigma} = \sigma - c \frac{\partial \sigma}{\partial c}, \quad Z = \rho (\mu_3 - \mu_4),$$

and μ_3 and μ_4 are the chemical potentials of He³ and He⁴ in a solution in which the weight concentration of He³ is c . The dispersion equation has two roots, which, for $\beta \ll 1$, are given by

$$u_1^2(c) \approx s_1^2 \left(1 + \beta^2 \frac{\rho_s}{\rho_n} \frac{s_1^2}{s_1^2 - s_2^2} \right), \quad (1.9)$$

$$u_2^2(c) \approx s_2^2 \left(1 + \beta^2 \frac{\rho_s}{\rho_n} \frac{s_1^2}{s_1^2 - s_2^2} \right)^{-1}. \quad (1.10)$$

They correspond to the propagation velocities of first and second sound.

In formulating a theoretical description of the properties of superfluid He³-He⁴ solutions it is usually assumed that the He³ participates only in the normal motion, since in superfluid motion the flux of He³ atoms vanishes³⁾. This assertion corresponds to the current

idea of the normal component as a collection of elementary excitations of the liquid. The motion of each He^3 atom is an elementary excitation (quasiparticle) whose dynamical properties depend on the interaction of the He^3 atom with the He^4 atoms and with the other He^3 atoms. These ideas correspond to the theory of a Fermi liquid, according to the fundamental postulates of which the number of elementary excitations is equal to the number of fermions. Participation of the He^3 atoms in the superfluid motion can be expected only when the He^3 atoms themselves, as a result of pairing, form a superfluid liquid dissolved in the superfluid He^4 .

In view of what was said in the preceding section, we shall not give expressions for the velocities of first and second sound derived on the assumption that the He^3 atoms participate in the superfluid motion [13].

2. FIRST SOUND

Any linear wave process is characterized by a propagation velocity $u(\omega)$ and an absorption coefficient $\alpha(\omega)$, which depend on the frequency ω and on other parameters, e.g., on the temperature, pressure, etc. [4].

The propagation velocity and absorption coefficient are connected to one another by dispersion relations [15] analogous to the Kramers-Kronig relations. In principle, the knowledge of one of these quantities at all frequencies is sufficient to permit the other to be calculated. In practice, however, each of the quantities is known only in comparatively narrow frequency intervals, or even only at definite fixed frequencies. Hence independent measurements (and calculations) of the velocity $u(\omega)$ and absorption coefficient $\alpha(\omega)$ are desirable for a complete description of the wave process.

The pioneering measurements of the velocity of first sound were made in 1938 by Findlay, Pitt, Grayson-Smith, and Wilhelm [16,17], using a standing-wave method. In 1947, Pellam and Squire [18] used a pulse technique to investigate the acoustic properties of He^4 , and this enabled them to measure the absorption of sound.

At present, thanks to the use of various techniques, the acoustic properties of He^4 have been investigated at temperatures from 0.05 K to the critical temperature and at frequencies somewhat higher than 10^{11} Hz (Fig. 1). The velocity and absorption have been measured in the frequency range up to 10^9 Hz using piezoelectric crystals to produce and detect density oscillations in the liquid [16-36]. Various optical detection methods [47-52], including Brillouin scattering of laser light, have made it possible to cover the frequency range up to $\sim 7 \times 10^8$ Hz.

Interference of phonons at frequencies somewhat above 10^{11} Hz has been observed in special acoustic interferometers [53-55]. Information on the most energetic phonons has been obtained from x-ray [56,57] and neutron [58-60] scattering data.

The most accurate absolute values of the velocity of first sound in He^4 at the saturated vapor pressure are apparently those obtained in [23,24,38], and at elevated pressures, those obtained in [37,43]. The most complete data on the frequency dependence of the velocity and absorption of first sound at frequencies up to 12 MHz in [35] are given in [42] for frequencies up to 200 MHz. The absorption at elevated pressures has been investigated at frequencies up to 100 MHz in [44].

a) Experimental techniques. The techniques most

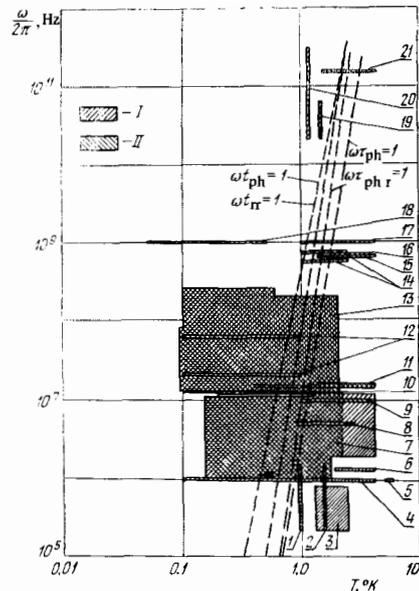


FIG. 1. Regions in which the velocity (I) and absorption (II) of sound in He^4 have been measured at the saturated vapor pressure, and relevant characteristic times [67]. 1-[25], 2-[24], 3-[23], 4-[32,47,48], 5-[34,41], 6-[17], 7-[21,35,38], 8-[31], 9-[37], 10-[22,26], 11-[18,19,28-30,97], 12-[40], 13-[35,36,39,44], 14-[49], 15-[51], 16-[50], 17-[46], 18-[45], 19-[53,54], 20-[60], 21-[58].

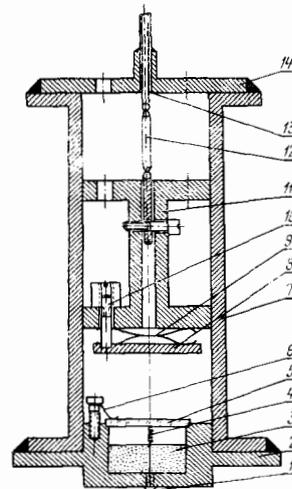


FIG. 2. Sound cell for absolute measurements of the velocity and absorption of first sound in liquid helium [61]: 1—electrical contact, 2—bottom cover, 3—pressed rouge, 4—contact spring, 5—quartz crystal, 6—phosphor bronze spring, 7—cylinder, 8—reflector, 9—spring, 10—adjusting screw, 11—plunger, 12—link, 13—tube, 14—top cover.

widely used in the experimental study of the acoustic properties of liquid helium are various modifications of the pulse method. This is due to the fact that pulse methods involve the dissipation of much less energy in the helium than do resonance methods, so that the measurements, especially relative measurements, can be made with very high accuracy.

In describing experiments we shall limit ourselves to relatively simple apparatus and circuits that permit adequate accuracy to be achieved. In some cases we shall describe the apparatus that was used to obtain the most accurate measurements.

Figure 2 is a drawing of a sound cell intended for absolute measurements of the velocity and absorption of first sound in liquid helium [61]. The X-cut quartz crystal served to generate ultrasonic pulses in the helium, and to detect the reflected signals between these pulses. To suppress spurious signals, a fine powder was pressed

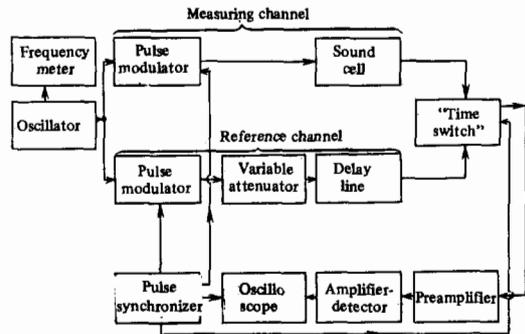


FIG. 3. Simplified block diagram of an ultrasonic comparator [62].

into the bottom cover below the quartz crystal. The grain size of the powder was so chosen as partially to clamp the normal component in the interstices between the grains; this results in strong sound absorption (this is discussed in more detail in Sec. 6) and practically completely suppresses the spurious signals.

The velocity of sound is determined from the transit time of the pulse over the known acoustic path, and the absorption is determined from the change in the amplitude of the first echo signal when the reflector is moved from one extreme position to the other.

Figure 3 is a block diagram of an ultrasonic comparator with which, using the phase comparison method, relative measurements of the velocity of sound can be made with a relative accuracy $\sim 10^{-7}$, and absorption measurements, with a resolution of 0.02–0.1 dB [62].

The signal from the oscillator is split into two; one of the signals is fed to the measuring channel, which contains the sound cell, and the other, to the reference channel, which contains a variable attenuator and a variable delay line. At the input to each channel there is a pulse modulator, which transforms the sinusoidal oscillations into radio pulses. After traversing the measuring and reference channels, the pulses are added, and, passing through the "time switch," they are amplified, detected, and displayed on the oscilloscope. The time switch opens the amplifier-detector immediately before the signals arrive from the measuring and reference channels, the delay line being so adjusted that the two signals arrive simultaneously. Thus, the time switch prevents saturation of the amplifier by spurious signals arriving before the desired signals and thereby reduces the measurement errors.

To make a measurement at some temperature T_0 (ordinarily the lowest attainable) one adjusts the variable attenuator and delay line so that the signals from the reference and measuring channels have the same amplitude and differ in phase by $(2n+1)\pi$ ($n=0, 1, 2, \dots$), i.e., so that a zero signal appears on the oscilloscope. Then one repeats the operation at another temperature. The difference between the attenuator readings gives the change in the sound attenuation due to the change in temperature, and the change in the delay line adjustment gives the change in sound velocity.

b) Velocity, absorption, and dispersion of first sound in He⁴. The use of the hydrodynamic equations (1.1) limits calculations of the velocity and absorption coefficient of sound to the low frequency limits, in the limit $\omega \rightarrow \infty$, we have

$$u_1^2(\omega) = \left(\frac{\partial p}{\partial \rho} \right)_\omega, \quad (2.1)$$

and

$$\alpha_1(\omega) = \frac{\omega^2}{2\rho u_1^2} \left(\frac{4}{3} \eta + \zeta_2 \right). \quad (2.2)$$

The observed frequency dependence of the velocity of sound and the deviation of the frequency dependence of the absorption coefficient from the quadratic law are due to two effects, which can reasonably be called time dispersion and space dispersion.

Time dispersion is a consequence of relaxation processes in helium, which are fairly complex because of the peculiar energy spectrum of the elementary excitations:

$$\epsilon(p) = sp(1 - \gamma p^2) \quad (\text{for small } p), \quad (2.3)$$

$$\epsilon(p) = \Delta + \frac{(p-p_0)^2}{2m_1^*} \quad (\text{for } p \approx p_0), \quad (2.4)$$

where p and s are the phonon momentum and velocity, and γ , Δ , p_0 , and m_1^* are parameters.

According to [63–68] the most probable processes (for $\gamma > 0$) are the following:

1) Roton-roton scattering. This process leads to rapid establishment of local equilibrium in the roton gas; we denote its characteristic time by τ_{rr} .

2) Four-phonon small-angle scattering. This process has a high probability because of the almost linear dispersion law for phonons and leads to the establishment of energy equilibrium for phonons moving in a given direction; characteristic time, $\tau_{ph\text{ph}}$.

3) Four-phonon large-angle scattering. This process must be taken into account when $T < 0.9$ K; characteristic time, $\tau_{ph\text{ph}}$.

4) Five-phonon conversion (conversion of three phonons into two). This process leads to establishment of equilibrium as regards number of phonons; characteristic time, $\tau_{3 \rightarrow 2}$.

5) Phonon-roton scattering. This is a relatively slow process and governs the establishment of energy equilibrium between the roton and phonon gases; characteristic time, $\tau_{ph\text{r}}$.

Table I shows how all the relaxation times depend on the temperature, and lines representing the equations $\omega\tau_i = 1$ for the several scattering processes are included in Fig. 1.

The partial relaxation times characterize the probabilities of these scattering processes. For example, in the time τ_{rr} equilibrium is established in the roton gas but not in the phonon gas, and in the time $\tau_{3 \rightarrow 2}$ equilibrium is established in both the phonon and roton gases, but not between them.

The most general and detailed calculations of the velocity and attenuation of sound were made by Khalatnikov and Chernikova [67, 68] using the kinetic equation, while considerations of partial equilibrium are used first, to select the approximation, and second, to assist in the physical interpretation of the results.

In presenting the results of the kinetic calculations of the velocity and absorption of first sound (Table II) it is convenient to distinguish four temperature intervals in such a manner that the equilibration mechanisms become operative or inoperative on passing from one interval to another, depending on which of the relaxation times τ_i are governing (Fig. 1). We also distinguish between high frequencies ($\omega\tau_{ph\text{ph}}, \omega\tau_{ph\text{r}} \gg 1$) and low

TABLE I

$$\begin{aligned} \tau_{\text{r}}^{-1} &= 2\rho_0 m_{\text{r}}^* \frac{|V_0|^2}{\hbar^4} N_{\text{r}} \approx 2.2 \cdot 10^{-10} N_{\text{r}} = 1.2 \cdot 10^{13} \sqrt{T} e^{-\Delta/T} \text{ (sec}^{-1}\text{)}, \\ \tau_{\text{phph}}^{-1} &= \frac{4.15}{192\pi^3} \frac{(u+1)^4}{\rho^2 s} \left(\frac{kT}{\hbar s}\right)^7 \frac{x^2}{\gamma} \left(1 + \frac{14.7}{x}\right) \approx \\ &\approx 4.6 \cdot 10^{43} \frac{x^2 T^7}{\gamma} \left(1 + \frac{14.7}{x}\right) \text{ (sec}^{-1}\text{)} \quad \left(x = \frac{ps}{kT} \gg 1\right), \\ \tau_{\text{phph}}^{-1} &= \frac{8.66}{192\pi^3} \frac{(u+1)^4}{\rho^2 s} \left(\frac{kT}{\hbar s}\right)^7 \frac{x^3}{\gamma} \approx 9.6 \cdot 10^{43} \frac{x^3 T^7}{\gamma} \text{ (sec}^{-1}\text{)} \quad (1 \gg x \gg \sqrt{ps^2}), \\ \tau_{\text{phph}}^{-1} &= \frac{9.131}{(2\pi)^7 \cdot 2^{13}} \frac{(u+1)^4 \hbar^3}{\rho^2 s} \left(\frac{kT}{\hbar s}\right)^9 \approx 3.8 \cdot 10^7 T^9 \text{ (sec}^{-1}\text{)}, \\ \tau_{3-2}^{-1} &= \Lambda \frac{kT^{12}}{3N_{\text{ph}}} \approx 7.8 \cdot 10^7 T^9 \text{ (sec}^{-1}\text{)}, \\ \tau_{\text{phr}}^{-1} &= 4\pi^3 \Gamma \frac{\rho_0^2}{\rho^2 s} \left(\frac{kT}{\hbar s}\right)^4 N_{\text{p}} \approx 1.3 \cdot 10^{12} T^{9/2} e^{-\Delta/T} \text{ (sec}^{-1}\text{)}, \\ N_{\text{ph}} &= \frac{2.4}{2\pi^2} \left(\frac{kT}{\hbar s}\right)^3 \approx 2 \cdot 10^{18} T^3 \text{ (cm}^{-3}\text{)} - \text{phonon density} \\ N_{\text{r}} &= \frac{2\rho_0^2 (m_{\text{r}}^* kT)^{1/2}}{(2\pi)^{3/2} \hbar^3} e^{-\Delta/T} \approx 5.4 \cdot 10^{22} T^{1/2} e^{-\Delta/T} \text{ (cm}^{-3}\text{)} - \text{roton density} \\ u &= \frac{\rho}{s} \frac{\partial s}{\partial \rho} = 2.84, \quad s = 2.383 \cdot 10^4 \text{ cm/sec}, \quad \rho = 0.145 \text{ g/cm}^3 \\ \Delta &= 8.65 \text{ }^\circ\text{K}, \quad p_0 = 2.02 \cdot 10^{-10} \text{ g-cm/sec}, \quad m_{\text{r}}^* = 1.06 \cdot 10^{-24} \text{ g}, \\ |V_0|^2 &= 6.3 \cdot 10^{-78} \text{ (erg/cm}^3\text{)}^2, \quad \Lambda = 3.4 \cdot 10^{43}, \\ \Gamma &= \frac{2}{9} + \frac{1}{25} \left(\frac{p_0}{m_{\text{r}}^* s}\right)^2 + \frac{2}{9} \frac{p_0}{m_{\text{r}}^* s} A + A^2 = 2.6, \\ A &= \frac{\rho^2}{\rho_0 s} \frac{\partial^2 \Delta}{\partial \rho^2} + \frac{p_0}{m_{\text{r}}^* s} \left(\frac{\rho}{\rho_0} \frac{\partial p_0}{\partial \rho}\right)^2 = -0.1 \text{ (estimate [4])} \end{aligned}$$

TABLE II. Velocity and absorption of first sound in He⁴ ([68])

	T, °K:	0,6	0,9	1,2
		$\tau_{\text{phph}} \ll \tau_{\text{phr}}$	$\tau_{\text{phr}} \sim \tau_{\text{phph}}$	$\tau_{\text{phph}} \gg \tau_{\text{phr}},$ $\tau_{3-2} \gg \tau_{\text{phr}}$
Low frequencies		$u_1 = u_{10} + \frac{s(3u+1)^2}{4} \frac{\rho_{\text{ph}}}{\rho}$	$u_{10} + \frac{s}{2} \left(3u+1 + B \frac{S_{\text{r}}}{S_{\text{ph}}}\right)^2 \frac{S_{\text{ph}} \rho_{\text{ph}}}{C \rho}$	u_{10}
		$\alpha_1(\omega, T) = \frac{\omega^2}{2\rho u_1^3} \left(\frac{4}{3} \eta_{\text{ph}} + \zeta_2\right) \sim \omega^2,$		
		$\eta_{\text{ph}} = \frac{1}{5} s^2 \rho_{\text{ph}} \left(\frac{1}{\tau_{\text{phr}}} + \frac{1}{\tau_{\text{phph}}}\right)^{-1},$		
		$\zeta_2 = m_{\text{r}}^* s^4 \frac{(3u+1)^2}{9kT} \left(1 - \frac{C_{\text{ph}}}{C}\right) \rho_{\text{ph}} \tau_{\text{phr}}$		
High frequencies		$u_1 = u_{10} - \frac{\rho_{\text{ph}}}{\rho} \left\{ \frac{3}{4} (u+1)^2 \ln \left[\frac{1}{2\omega\tau_{\text{phph}}} + \frac{1}{2\omega\tau_{\text{phr}}} + \frac{3}{2} \frac{B_3}{B_2} \gamma \left(2\pi \frac{kT}{s}\right)^2 \right] + 3u + 2 \right\},$		
		$\alpha_1(\omega, T) = \frac{3\pi}{8} (u+1)^2 \frac{\rho_{\text{ph}} \omega}{\rho s} \left\{ 1 + \left[\frac{3}{2} \frac{B_3}{B_2} \gamma \left(2\pi \frac{kT}{s}\right)^2 \omega \right]^{-1} \left(\frac{1}{\tau_{\text{phph}}} + \frac{1}{\tau_{\text{phr}}}\right) \right\}$		
		$u_{10}^2 = \left(\frac{\partial \rho}{\partial p}\right)_T \left(\text{if } T < 0.6 \text{ }^\circ\text{K } u_{10} = s \left[1 - \frac{3}{2} \left(u^2 - \frac{1}{4} \frac{\rho^2}{s} \frac{\partial^2 s}{\partial \rho^2}\right) \frac{\rho_{\text{ph}}}{\rho} \right] \right),$		
		$u = \frac{\rho}{s} \frac{\partial s}{\partial \rho}, \quad B = 1 - \frac{1}{2} \frac{\partial \ln(p_0^2 m_{\text{r}}^*)}{\partial \ln \rho} + \frac{\Delta}{T} \frac{\partial \ln \Delta}{\partial \ln \rho} \frac{1}{3} + \frac{(2\Delta/T)}{3}.$		
		The subscripts ph and r on C, S, and ρ_{ph} indicate the phonon and roton parts, respectively; $B_1 = 1/30$ and $B_2 = 1/42$ are Bernoulli numbers.		

frequencies ($\omega \tau_{\text{phph}}, \omega \tau_{\text{phr}} \ll 1$). For all frequencies, however, it is assumed that equilibrium is established in the roton gas as regards number and energy of the rotos as a result of roton-roton collisions, and that energy equilibrium for phonons traveling in a given direction obtains in the phonon gas, i.e., that $\omega \tau_{\text{r}} \ll 1$ and $\omega \tau_{\text{ph}} \ll 1$.

The general expressions in Table II for the different temperature intervals⁵⁾ can be considerably simplified by taking into account the relations between the various τ_i and bearing in mind that the phonon contributions to the thermodynamic characteristics are small when $T > 0.9$ °K and the roton contributions are negligible when $T < 0.6$ °K. For example, for $T < 0.6$ °K we have $c = c_{\text{ph}}$ and hence $\zeta_2 = 0$, and since $\tau_{\text{phph}} \ll \tau_{\text{phr}}$, we have $\eta = s^2 \rho_{\text{ph}} \tau_{\text{ph}} / 5$.

In the low frequency limit, the absorption is correctly given by the ordinary hydrodynamic expression (2.2), as should be expected, but thanks to the consideration of specific equilibration mechanisms, all the kinetic coefficients (including their numerical values) have been calculated. There are no adjustable parameters in these calculations.

For high frequencies, interpolation formulas are given in Table II. At very high frequencies,

$$\frac{1}{2\omega} \left(\frac{1}{\tau_{\text{phph}}} + \frac{1}{\tau_{\text{phr}}}\right) \ll \frac{3}{2} \frac{B_3}{B_2} \gamma \left(2\pi \frac{kT}{s}\right)^2,$$

where the phonon spectrum deviates considerably from linearity, the terms containing $\omega \tau_{\text{phph}}$ and $\omega \tau_{\text{phr}}$ drop out of the expressions for u_1 and α_1 . At frequencies that are high, but not too high, however, so that terms con-

TABLE III. Parameter values for the phonon energy spectrum

	$\gamma, \text{sec}^2/\text{g}^2\text{cm}^2$	$\delta, \text{sec}^4/\text{g}^4\text{cm}^4$	$s, \text{m}/\text{sec}$
X-ray scattering [17]	$-(5.7 \pm 0.3) \cdot 10^{37}$ or $(0.16 \pm 0.36) \cdot 10^{37}$		
Neutron scattering [60]	$(0.0 \pm 0.2) \cdot 10^{37}$	$2.4 \cdot 10^{75}$	230.7
Heat capacity [70]	$-4.1 \cdot 10^{37}$		238.3 ± 0.1^{38}
Heat capacity [70], as analyzed in [71]	$-8 \cdot 10^{37}$		

aining γ are negligible, the velocity of first sound exhibits a logarithmic frequency dependence and the absorption is proportional to the frequency.

In considering the dispersion of first sound it must not be forgotten that the difference between phonons and first-sound waves is essentially terminological. At $T = 0^\circ\text{K}$, quantized sound waves are indeed phonons⁶⁾, the phonon dispersion law being a consequence of the atomic structure of the liquid. This is especially clearly evident from the relation $\epsilon(p) = p^2/2m_4S(p)$ established by Feynman^[69] between $\epsilon(p)$ and the structure factor $S(p)$ (here m_4 is the mass of an He^4 atom). The dispersion of sound (of phonons) is thus due to the nonlinearity of the relation between energy and momentum, and it is natural to speak of space dispersion.

In view of the relation that the phonon spectrum bears to sound, we present the basic results concerning the deviation of the phonon energy spectrum from linearity, using the following interpolation formula, which contains two adjustable parameters, γ and δ (in addition to s):

$$\epsilon(p) = sp(1 - \gamma p^2 - \delta p^4). \quad (2.5)$$

The results are presented in Table III. It will be seen that the experimental data are consistent with the condition $\gamma \leq 0$, and a negative value for γ is evidently the more probable.

If γ is negative, then excitation decay processes—processes in which one phonon breaks up into two—are allowed at $T = 0^\circ\text{K}$. However, the phonon lifetime $\tau_\gamma(p)$ calculated on the basis of this mechanism⁷⁾ is very long ($\hbar/\tau_\gamma(p) \ll \epsilon(p)$), so we may regard the phonons as “good” quasiparticles. At finite temperatures the relaxation mechanisms described earlier prove to be more important, and the instability associated with a negative value of γ does not appear at all. At finite temperatures, the difference between sound waves and phonons is associated, as a rule, with the means of excitation. When waves are excited by the inelastic scattering of a neutron one speaks of phonon production, and when a coherent flux of phonons is produced one speaks of the excitation of sound waves.

In investigating the dispersion of sound it must be borne in mind that it is not easy to distinguish experimentally between time dispersion and space dispersion: this can be done only by comparing the experimental results with theoretically predicted dependences of the velocity and absorption on the frequency, temperature, pressure, and other parameters.

First sound in He^4 has been investigated experimentally in considerable detail. Figure 4 shows the “small scale” temperature dependence of u_1 : the monotonic decrease of u_1 with rising temperature (except for the anomaly near the λ point) reflects the temperature dependence of the density of the liquid.

In the high temperature region ($T \gtrsim 1.2^\circ\text{K}$) no appreciable

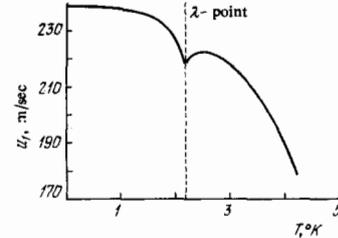


FIG. 4. Temperature dependence of the velocity of first sound in He^4 at the saturated vapor pressure.

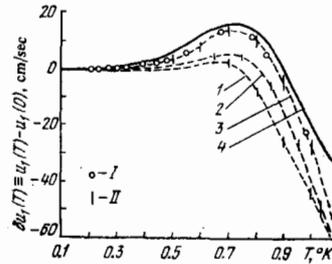


FIG. 5

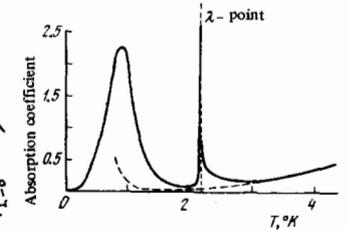


FIG. 6

FIG. 5. Temperature dependence of the quantity δu_1 , I—12 MHz^[42]; II—^[32]: 1—1.00 MHz, 2—3.91 MHz, 3—11.9 MHz; 4—curve calculated in accordance with ^[68] for 12 MHz.

FIG. 6. Absorption of first sound in He^4 at 12 MHz. The full curve represents the experimental results, and the dashed curve, classical viscous absorption (Eq. (2.7) with $\kappa = 0$)^[5].

able frequency dependence of the velocity of first sound has been detected at frequencies up to 10^9 Hz (to be sure, the $\sim 1\%$ accuracy of the measurements is not high). Measurements of the velocity of first sound in a helium film in the $(2-6) \times 10^{10}$ Hz frequency region made with a spin-phonon interferometer revealed a frequency dependence of the velocity of first sound^[53,54],

$$\delta u_1(\omega) \equiv \frac{u_1(\omega) - u_1(0)}{u_1(0)} = (6.3 \pm 0.7) \cdot 10^{-13} \frac{\omega}{2\pi}, \quad (2.6)$$

which is apparently associated with time dispersion, since other experiments^[72] made under conditions in which time dispersion should not appear did not reveal this dependence⁸⁾.

At temperatures somewhat lower than 1.2°K the dispersion becomes appreciable even at frequencies of 10^6-10^7 Hz. Figure 5 shows the quantity $\delta u_1(T) \equiv u_1(T) - u_1(0)$ as a function of T for various frequencies; the frequency and temperature dependences of $\delta u_1(T)$ agree well with the Khalatnikov-Chernikova theory^[68] for frequencies up to 12 MHz.

The absorption of first sound in He^4 depends on the temperature in a complicated manner: in addition to the λ anomaly there is a maximum near 1°K .

In HeI the absorption should be determined by the

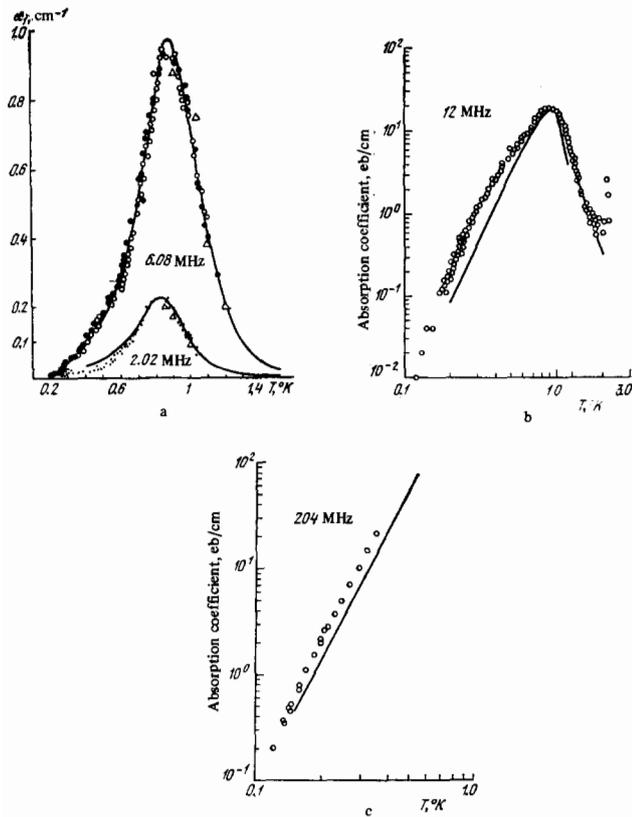


FIG. 7. Absorption of first sound in He⁴: a) from [35], b,c) from [42]. The curves were calculated [68].

viscosity and the heat conduction, as in an ordinary liquid:

$$\alpha_1(\omega, T) = \frac{\omega^2}{2\rho u_T^2} \left[\frac{4}{3} \eta + \kappa (C_V^2 - C_P^2) \right]. \quad (2.7)$$

As is evident from Fig. 6, the experimental data for temperatures above 3 °K agree well with this formula; the discrepancy below 3 °K is associated with features of the λ transition that are not taken into account in Eq. (2.7).

The λ anomaly does not extend so far in HeII as in HeI, but even at a considerable distance from T_λ the viscosity contribution to the absorption of sound is small, the predominant contribution coming from second viscosity, in accordance with the formulas listed in Table II. The maximum absorption takes place when $\omega\tau_{ph}r \sim 1$; the absorption peak shifts toward the higher temperatures as the frequency increases.

Beyond the dispersion region ($\omega\tau_{ph}r \ll 1$) the absorption is proportional to the square of the frequency up to the highest frequency (10⁹ Hz) at which the absorption of first sound has been measured [52].

Figure 7 shows some experimental data on the absorption of first sound at various frequencies, together with $\alpha_1(\omega, T)$ curves calculated in [68]. It will be seen that the theoretical curves agree well with the experimental data at low frequencies over the entire temperature range and at 12 MHz for $T > 0.6$ °K.

In the 12–36 MHz frequency range and 0.2–0.6 °K temperature range, the experimental data satisfy the power law $\alpha_1(\omega, T) \propto \omega^3 T^4$ predicted theoretically, but the experimental values are 2–2.5 times higher than the calculated values. This discrepancy cannot be at-

tributed to four-phonon scattering processes since their contribution to the absorption is proportional to $\omega^3 T^4$. At low temperatures there is still another discrepancy between experiment and theory [68]: whereas $\partial u_1 / \partial \omega$ is positive at frequencies below 12 MHz, according to [42] it is negative at frequencies between 12 and 84 MHz.

Many papers [71, 75–90] have been published in which the velocity and absorption of first sound are calculated in an attempt to explain these discrepancies. The region concerned is attractive from the theoretical point of view because there all the properties of He⁴ are due to long-wavelength phonons alone, the roton density being exponentially small.

In almost all of these theoretical papers, three-phonon processes (the direct absorption of an acoustic phonon by thermal phonons) were discussed; we denote the characteristic time for this process by $\tau_{2 \rightarrow 1}$. Kawasaki [75] was the first to call attention to the possibility of three-phonon processes in the case of a spectrum like (2.3) (even when γ is positive!). When $\gamma > 0$, three-phonon processes do not satisfy the energy and momentum conservation laws. However, the thermal phonons collide with one another, and their mean lifetime τ is finite. This results in an energy uncertainty of the order of \hbar/τ , and if this uncertainty exceeds the quantity $3\bar{p}^2\hbar\omega$ (here \bar{p} is the mean thermal momentum of a phonon) by which energy conservation is violated in three-phonon processes, such processes will be allowed. If $\gamma < 0$, such three-phonon processes are allowed even in the limit as $\tau \rightarrow \infty$.

Taking three-phonon processes into account in the second nonvanishing order of perturbation theory leads to the following expressions for the velocity and absorption of first sound (for $\gamma = 0$) [83–85]:

$$u_1(\omega, T) = u_1(0) + s \frac{3(u+1)^2}{8} \frac{\rho_{ph}}{\rho} \Phi(\omega, T), \quad (2.8)$$

$$\alpha_1(\omega, T) = \frac{3\pi}{8} (u+1)^2 \frac{\rho_{ph}}{\rho} \frac{\omega}{s} F(\omega, T); \quad (2.9)$$

here

$$\Phi(\omega, T) = \ln [1 + (2\omega\tau_{2 \rightarrow 1})^2] + 2 \frac{(2\omega\tau_{2 \rightarrow 1})^2}{1 - (2\omega\tau_{2 \rightarrow 1})^2} - 4 \frac{\text{arctg}(2\omega\tau_{2 \rightarrow 1})}{2\omega\tau_{2 \rightarrow 1}}, \quad (2.10)$$

$$F(\omega, T) = \frac{2}{\pi} \left\{ \text{arctg}(2\omega\tau_{2 \rightarrow 1}) + \frac{2\omega\tau_{2 \rightarrow 1}}{1 + (2\omega\tau_{2 \rightarrow 1})^2} + \frac{\ln [1 + (2\omega\tau_{2 \rightarrow 1})^2]}{2\omega\tau_{2 \rightarrow 1}} \right\}.$$

The values of $\alpha_1(\omega, T)$ calculated with $F(\omega, T)$ turn out to agree well with the measured absorption coefficient of first sound in He⁴ at the “unfavorable” frequency 36 MHz. The maximum of $F(\omega, T)$ corresponds to the condition $\omega\tau_{2 \rightarrow 1} = 1$. Thus, there is dispersion of sound in the phonon gas as a result of the slowness of the three-phonon scattering processes.

Basing his calculations on the numerical solution of the kinetic equation for three-phonon small-angle scattering processes and an $\epsilon(p)$ spectrum with $\gamma = -8 \times 10^{37} \text{ sec}^2/\text{g}^2 \text{ cm}^2$, Maris [87] calculated the frequency dependence of the relative velocity δu_1 and the absorption of sound. In Fig. 8 the results of this calculation are compared with the experimental data for $T = 0.35$ °K (this is the temperature at which the discrepancy between experiment and the Khalatnikov-Chernikova theory is greatest). As the figure shows, the calculated and measured values of $\alpha_1(\omega)$ are in agreement and, what is the most attractive, the frequency dependence of δu_1 also agrees with experiment: $\partial \delta u_1 / \partial \omega$ is positive in the 1–10 MHz range and negative in the 10–100 MHz range, although there remains some numerical discrepancy.

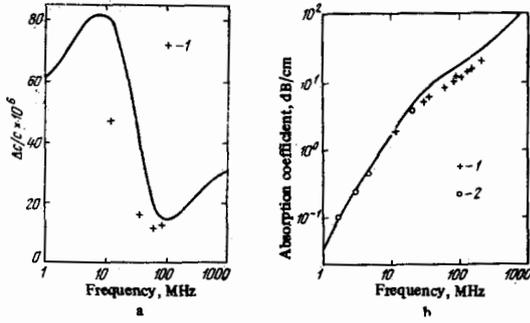


FIG. 8. Frequency dependence of the relative velocity (a) and absorption (b) of first sound in He⁴ [71]; experimental data from [42] (1) and [40] (2).

Thus, phonon-roton and three-phonon scattering processes lead to time dispersion of first sound, which appears when $\omega\tau_{\text{ph}r} \sim 1$ and $\omega\tau_2 \rightarrow 1$. The frequency dependence of the velocity, and at low temperatures ($T < 0.6^\circ\text{K}$) the absorption, of first sound is due both to time dispersion and to space dispersion (the deviation from linearity of the momentum dependence of the phonon energy). A number of results favor, although not always unambiguously, a spectrum of the type $\epsilon(p) = sp(1 - \gamma p^2 - \delta p^4)$ with $\gamma < 0$ and $\delta > 0$.

The remaining discrepancies between theory and experiment apparently have nothing to do with our basic ideas concerning the nature of the relaxation processes in HeII, but are due to inadequate accuracy of the analytic calculations, or even of the numerical calculations.

c) He³-He⁴ solutions. The experimental data on the acoustic properties of solutions of the helium isotopes are much less abundant than the data for He⁴, and the theoretical treatment of the solutions is less complete.

The velocity of first sound in He³-He⁴ solutions has been measured over the entire concentration range (including pure He³) at temperatures from 0.5 to 4°K [37, 91-94]. The absorption of first sound has been investigated at frequencies up to ~15 MHz in the 0.4-2°K temperature range for He³ concentrations up to 11% [95-97], and it has been measured at higher temperatures (1.4-3.5°K) in solutions containing up to 20% He³ [61].

Figure 9 shows the concentration and temperature dependence of the velocity of first sound, $u_1(c, T)$, for solutions. The curves for superfluid solutions, as for He⁴, have characteristic anomalies at the λ points, which gradually smooth out as the concentration increases. At temperatures below 0.9°K the curves for solutions of different concentrations merge into a common curve, which is a peculiar sort of solubility curve in $u_1(c, T)$ - T coordinates.

Expression (1.9) for the velocity of first sound in He³-He⁴ solutions simplifies when $s_1^2 \gg s_2^2$:

$$u_1^2(c, T) = \left(\frac{\partial p}{\partial \rho}\right)_{c, T} \left(1 + \bar{c}^2 \frac{\rho_s}{\rho_n}\right), \quad \bar{c} = \frac{c}{\rho} \frac{\partial \rho}{\partial c}. \quad (2.11)$$

It is easy to explain the linear dependence of u_1 on concentration that obtains in the region of superfluid solutions (Fig. 9, b) on the basis of this relation [93].

In the hydrodynamic (limiting) case, the absorption of first sound in superfluid solutions does not depend only on η and ζ_2 , as in the case of He⁴ (provided the thermal expansion coefficient be neglected), but also

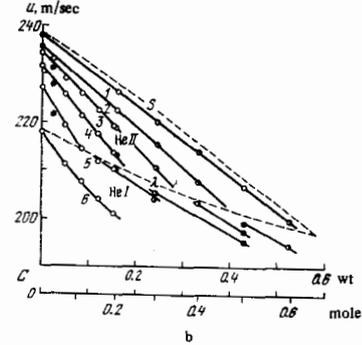
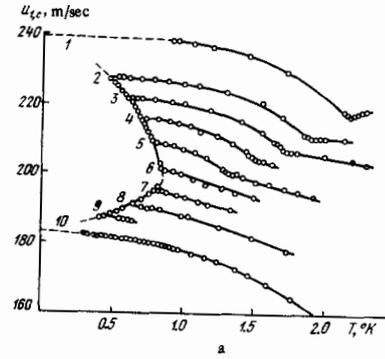


FIG. 9. Velocity of first sound in He³-He⁴ solutions [92, 93]. Plot a): 1-He⁴, 2-20% He³, 3-30.1%, 4-39.8%, 5-50%, 6-59.7%, 7-69.9%, 8-79.8%, 9-89.7%, 10-100% He³; plot b): 1-1.0°K, 2-1.4°K, 3-1.6°K, 4-1.8°K, 5-2.0°K, 6-3.0°K (S is the solubility curve, and λ is the λ curve).

depends on κ , ζ_1 , and ζ_3 . According to [98],

$$\alpha_1(c, T, \omega) = \frac{\omega^2}{2\rho u_1^2} \left(1 + \bar{c}^2 \frac{\rho_s}{\rho_n}\right)^{-1} \left[\left(\frac{4}{3}\eta + \zeta_2\right) \left(1 - \bar{c} \frac{\rho_s}{\rho_n}\right)^2 + 2\rho \bar{c} \frac{\rho_s}{\rho_n} \left(1 - \bar{c} \frac{\rho_s}{\rho_n}\right) + \rho \zeta_3 \left(\bar{c} \frac{\rho_s}{\rho_n}\right)^2 + \frac{\kappa}{C} \bar{c}^2 \rho \frac{\rho_s}{\rho_n} \frac{\partial T}{\partial \sigma} \frac{\partial^2}{\partial u_1^2}\right]. \quad (2.12)$$

In addition to this, diffusion processes give rise to additional absorption, which is given by [98]

$$\alpha_{1,D}(c, T, \omega) = \frac{\omega^2 c^2 D}{2u_1 c^2} \left[\left(1 + \bar{c}^2 \frac{\rho_s}{\rho_n}\right) \frac{\partial Z}{\partial c} \frac{Z}{\rho}\right]^{-1} \left[c \frac{\rho_s}{\rho_n} \frac{\partial}{\partial c} \frac{Z}{\rho} \left(c + \frac{k_T \sigma}{C}\right) + u_1^2\right]^2 \quad (2.13)$$

(D and Dk_T are the diffusion and thermal-diffusion coefficients); here the first term in the second pair of brackets is due to diffusion and thermal diffusion, and the second term, to barodiffusion.

The temperature dependence of the absorption coefficient for first sound in solutions is similar to that for He⁴, but the concentration dependence is different for superfluid and nonsuperfluid solutions. In HeI the absorption increases linearly with increasing concentration, mainly on account of the concentration dependence of the quantity ρu_1^3 , whereas in HeII the second component reduces the absorption, rather than increasing it (Fig. 10). This is explained by the fact that the main contribution to the absorption in the region under consideration comes from second viscosity, which decreases with increasing He³ concentration because the presence of He³ results in more frequent roton-phonon collisions. The decrease in $\tau_{\text{ph}r}$ leads on the one hand to a shift of the absorption peak (at $\omega\tau_{\text{ph}r} \sim 1$), and on the other hand, to a decrease in the second viscosity, and consequently in the absorption.

Assuming that the equilibrium as regard numbers of rotons and phonons is established by the slowest processes, Andreev [99] calculated the change in the second-viscosity coefficient ζ_2 resulting from three-

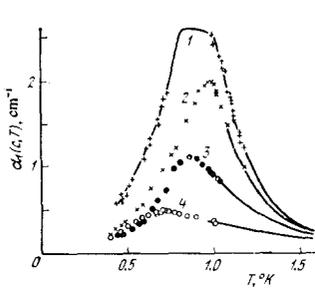


FIG. 10

FIG. 10. Absorption of first sound at 14.0 MHz in He^3 - He^4 solutions [97]: 1- He^4 , 2-0.32% He^3 , 3-1.2% He^3 , 4-5.2% He^3 .

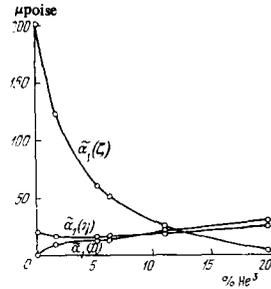


FIG. 11

FIG. 11. Reduced absorption coefficients $\tilde{\alpha}_1(\eta)$, $\tilde{\alpha}_1(D)$, and $\tilde{\alpha}_1(\Delta)$ at 1.4 °K vs. He^3 concentration [61].

phonon scattering from the impurity and phonon-roton conversion at the impurity. Then from the experimental data on dilute solutions he evaluated several parameters $\Gamma_{\text{ph r i}}$ and $\Gamma_{\text{ph i}}$ characterizing the rate of change of the number of rotons and phonons.

Figure 11 shows the concentration dependence of the reduced absorption coefficients (the absorption coefficients divided by $\omega^2/2\rho u_1^3$) associated with first and second viscosity and diffusion: $\tilde{\alpha}_1(\eta)$, $\tilde{\alpha}_1(\zeta_1)$, and $\tilde{\alpha}_1(D)$. The values of $\tilde{\alpha}_1(\eta)$ and $\tilde{\alpha}_1(D)$ were calculated directly from Eqs. (2.12) and (2.13), and $\tilde{\alpha}_1(\zeta)$ was defined as the difference between the measured absorption coefficient $\tilde{\alpha}_1$ and $\tilde{\alpha}_1(\eta) + \tilde{\alpha}_1(D)$, i.e., $\tilde{\alpha}_1(\zeta) = \tilde{\alpha}_1 - \tilde{\alpha}_1(\eta) - \tilde{\alpha}_1(D)$; estimates indicate that the contribution from heat conduction is negligible. If one assumes that $\tilde{\alpha}_1(\zeta) \approx \tilde{\alpha}_1(\zeta_2)$, one can estimate ζ_2 from $\tilde{\alpha}_1(\zeta)$. The values obtained in this way for ζ_2 at He^3 concentrations exceeding 7% turn out to be considerably smaller than the values calculated from Andreev's equation using his values of the parameters $\Gamma_{\text{ph r i}}$ and $\Gamma_{\text{ph i}}$.

d) **The effect of pressure.** By applying pressure to liquid helium one can vary its properties within certain limits, including its acoustic properties—the velocity and absorption of sound.

The pressure dependence of the velocity of sound has been investigated in the temperature range 1–4.2 °K, both for He^4 [17,20,37] and for solutions [37]. Special measurements were made for He^4 at low temperatures (below 0.1 °K) [43], and the Grüneisen constant $u = \partial \ln s / \partial \ln \rho$ and the quantity $w = (\rho^2/s)(\partial^2 s / \partial \rho^2)$ were evaluated from the results. These constants serve as a measure of the intensity of the phonon-phonon interaction in the third and fourth orders in an expansion in the density oscillations. It was found that $u = 2.84$ and $w = 8.26$ in the limit $P \rightarrow 0$, and $u = 2.21$ and $w = 4.94$ at $P = 25$ atm; i.e., the phonon-phonon interaction weakens somewhat as the pressure increases.

The pressure dependence of the absorption of first sound in He^4 at moderate frequencies (below 15 MHz), which has been measured in the 0.2–1.8 °K temperature interval [26–29] (Fig. 12), can be explained by the fact that the second viscosity, and with it the sound absorption, decreases as the quasiparticle collisions become more frequent as a result of increasing density. However, the agreement between the experimental data and the calculated [100] values of ζ_2 proves to be only qualitative.

Measurements of the absorption at high frequencies (15–256 MHz) in the 0.1–1.0 °K range at pressures up

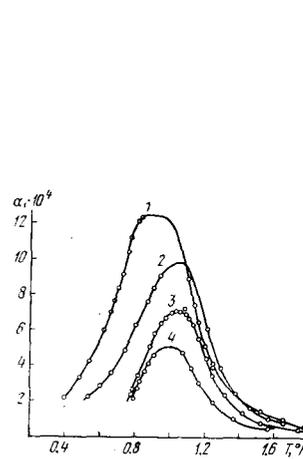


FIG. 12

FIG. 12. Temperature dependence of α_1 for He^4 at 14.4 MHz and various pressures [29]: 1—saturated vapor pressure, 2—8.4 atm, 3—16.4 atm, 4—24.7 atm.

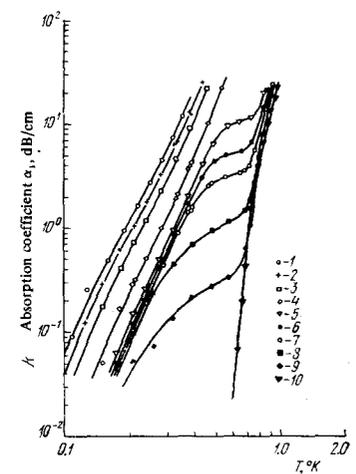


FIG. 13

FIG. 13. Temperature dependence of the absorption of first sound in He^4 at 105 MHz and various pressures [44]: 1—saturated vapor pressure, 2—1.1 atm, 3—3.0 atm, 4—8.4 atm, 5—14.0 atm, 6—15.5 atm, 7—16.4 atm, 8—17.8 atm, 9—19.0 atm, 10—24.7 atm.

to 25 atm led to the discovery of unexpected regularities (Fig. 13): when $P \leq 8.4$ atm, $\alpha_1(T) \propto T^4$, but at higher pressures and temperatures above 0.6 °K, the $\alpha_1(T)$ curve has a form close to $\exp(-\Delta/T)$.

A process that has come to be called partially allowed three-phonon scattering has been proposed [101] in order to explain such a complicated pressure and temperature dependence of sound absorption. In essence, this process is based on the fact that for a spectrum like (2.5) with $\gamma < 0$ and $\delta > 0$, there is a limiting momentum $p_C = -3\gamma/5\delta$ such that $\partial \epsilon / \partial p \geq s$ when $p \leq p_C$. Hence an ultrasonic phonon can be absorbed by a thermal phonon only if $\bar{p} \leq p_C$, where \bar{p} is the mean thermal momentum of the phonon. Calculations lead to the expression

$$\alpha_1(\omega, P, T) = \alpha_{2 \rightarrow 1} \frac{I(z)}{I(\infty)}, \quad (2.14)$$

where

$$I(z) = \int_0^z x^4 f(x) [1 - f(x)] dx,$$

$f(x)$ is the Bose function, $z = (sp_C/kT)$, and $\alpha_{2 \rightarrow 1}$ is the absorption due to three-phonon scattering. When $\gamma \neq 0$, $\alpha_{2 \rightarrow 1}$ is given in the first nonvanishing order of perturbation theory [81] by Eq. (2.9) with $F(\omega, T)$ replaced by $F_1(\omega, T)$:

$$F_1(\omega, T) = \frac{2}{\pi} [\arctg(2\omega\tau_{2 \rightarrow 1}) - \arctg(3\gamma\bar{p}^2\omega\tau_{2 \rightarrow 1})].$$

Thus, at low temperatures ($z \gg 1$), $\alpha_1 \approx \alpha_{2 \rightarrow 1} \propto \omega T^4$, and at high temperatures ($z \ll 1$), $\alpha_1 \approx \alpha_{2 \rightarrow 1} z^3/3 \propto \omega T$.

At low pressures the values of γ and δ are such that p_C is large ($z \gg 1$) and the partially allowed three-phonon processes reduce to fully allowed processes. As the pressure rises, p_C decreases and the partially allowed three-phonon processes begin to appear as a weak temperature dependence of the absorption: $\alpha_1 \propto T$.

The experimental data can be reconciled with calculations of the absorption of first sound based on Eq.

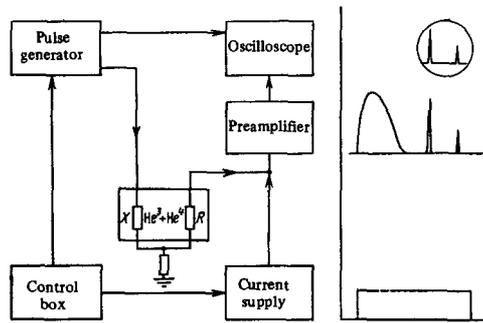


FIG. 14. Block diagram of apparatus for investigating second sound by the solitary thermal pulse method. [132]

(2.14) by choosing values of the parameters γ and δ (i.e., of p_c) for various pressures. The roton component, which is responsible for the exponential dependence of the absorption on the reciprocal temperature, becomes important when $T > 0.6$ °K.

3. SECOND SOUND

The peculiar wave process predicted by Landau [2] and known as second sound was detected by Peshkov [102,103] after Lifshitz [10] had elucidated its physical nature. In the many subsequent studies, the velocity of second sound in He³ has been measured at temperatures from 0.1 °K to the λ point [102-119], and its absorption, at frequencies from 0.2 to 270 kHz and temperatures from 0.7 °K to the λ point [120-126]. Second sound in He³-He⁴ solutions has been less thoroughly investigated [127-132].

a) Experimental techniques. The velocity and absorption of second sound have been measured both by resonance methods and by pulse methods. As an example we shall describe the system used by Sandiford and Fairbank [132] to investigate He³-He⁴ solutions (Fig. 14). Its principal advantage, which is very important for helium solutions, is that very little heat is injected into the liquid. The measurements were made as follows. First a long (50 msec) rectangular dc pulse was applied to the receiver R. Because of the long (5 msec) rise time of this pulse and the ~10 kHz low-frequency cutoff of the preamplifier, the latter did not saturate and dc conditions effectively prevailed after ~20 msec. The oscilloscope time base was triggered 10 msec before the end of this pulse, and a short pulse was fed to the transmitting resistor X ~10 μ sec later. This pulse produced a pulse of second sound and also appeared on the oscilloscope screen because of the drop across the resistance r of the ground lead. After a time $t = u_2/l$ (l is the distance from X to R) the second-sound pulse reached the receiver, was amplified, and also appeared on the oscilloscope screen. Thus, the velocity could be determined from the transit time of the solitary pulse.

The absorption of second sound can be determined either from the decrease in the pulse height on increasing the acoustic path or from the width of the resonance curve obtained by varying the frequency of the sound while keeping the distance between the emitter and detector fixed.

Temperature sensitive phosphor bronze wires were used as detectors in early experiments with second sound; later more sensitive transducers were developed, which employed thin carbon films or powders (the fabrication of these detectors is discussed, for example, in [133,134]). It has been suggested [135] that the susceptibility of a

paramagnetic salt be used to record temperature oscillations. Semiconducting materials [136], films of superconducting materials and alloys [137-140], and carbon bolometers [141,142] have recently begun to be used for this purpose. A condenser microphone with a porous membrane as the vibrating element [143-145] has proved to be a very efficient emitter and detector of second sound.

In addition to the "direct" methods described above, second sound has been studied by Brillouin scattering of light from second-sound waves [146-148] and by schlieren photography [149].

b) Velocity, absorption, and dispersion of second sound in He⁴. Detailed analysis of the various mechanisms for the establishment of equilibrium in the excitation gas, the basic ideas of which were presented in the preceding section, enabled Khalatnikov [8,65,150-152] and Khalatnikov and Chernikova [67,68] to investigate the dispersion and absorption of second sound. The results of these studies are presented in Table IV where, as in the case of first sound, we distinguish between low frequencies ($\omega \tau_{ph} r \ll u_2/u_1$) and high frequencies ($\omega \tau_{ph} r \gg u_2/u_1$).

In the low-frequency limit the absorption of second sound is given by the ordinary hydrodynamic formula, but the formula for the velocity of second sound differs from the ordinary hydrodynamic expression because the isobaric expansion coefficient, which is proportional to $3u + 1 + (BS_T/S_{ph})$, is taken into account.

As in the case of first sound, one can use not only the experimental values of the kinetic coefficients in analyzing the experiments, but also the values calculated on the basis of studies of the relaxation processes. We emphasize once more that there are no adjustable parameters at all in this comparison.

It is interesting that for $T < 0.6$ °K, when the properties of He⁴ are due to phonons alone⁹⁾, all the kinetic coefficients vanish except the phonon part of the viscosity, and the absorption of second is given by the formula

$$\alpha_2(\omega, T) = \frac{2\sqrt{3}}{15} \frac{\omega^2 \tau_{ph} r}{s} \sim \omega^2 T^{-9},$$

while the velocity of second sound tends to the well known limit $u_{20} \rightarrow s/\sqrt{3}$ as $T \rightarrow 0$.

Estimates show that when $T > 0.6$ °K, the main contribution to $\alpha_2(\omega, T)$ comes from heat conduction.

At high frequencies, the velocity of second sound is given by the hydrodynamic formula with the complete thermodynamic quantities replaced by their respective roton contributions; this is the so called roton second sound. In this case the absorption coefficient $\alpha_{2\infty}$ is frequency independent.

Thus, on increasing the frequency while holding the temperature constant one should observe a gradual transition from ordinary equilibrium second sound to roton second sound. This transition involves a decrease in the velocity from u_{20} to $u_{2\infty}$.

The dispersion of second sound discussed above should be observed for

$$\omega \tau_{ph} r \sim \frac{u_2}{u_1},$$

when the phonon mean free path l_{ph} becomes comparable with the wavelength λ_2 of second sound. At

TABLE IV. Velocity and absorption of second sound in He⁴ ([⁶⁸])

	T, °K:		1,2
	0,6	0,9	
	$\tau_{\text{phph}} \ll \tau_{\text{phr}}$	$\tau_{\text{phph}} \sim \tau_{\text{phr}}$	$\tau_{\text{phph}} \gg \tau_{\text{phr}}$ $\tau_{3-2} \ll \tau_{\text{phr}}$
Low frequencies	$u_2 = \frac{s}{\sqrt{3}} \left[1 - \frac{3}{4} \times (3u^2 + 2u + 1) \frac{\rho_n \text{ph}}{\rho} \right]$ $\alpha_2(\omega, T) = \frac{\omega^2}{2\rho u_{20}^3} \frac{\rho_s}{\rho_n} \left[\frac{4}{3} \eta_{\text{ph}} (\zeta_2 + \rho^2 \zeta_3 - 2\rho \zeta_1) + \frac{\rho_n}{\rho_s} \frac{\rho \times \text{ph}}{C} \right]$ $\eta_{\text{ph}} = \frac{1}{5} S^2 \rho_n \left(\frac{1}{\tau_{\text{phph}}} + \frac{1}{\tau_{\text{phr}}} \right)^{-1}$ $\times_{\text{ph}} = 0$	$u_{20} \left[1 - \left(3u + 1 + B \frac{S_r}{S_{\text{ph}}} \right)^2 \frac{S_r}{2C} \frac{\rho_n \text{ph}}{\rho} \right]$ $\zeta_2 + \rho^2 \zeta_3 - 2\rho \zeta_1 = \left(1 - \frac{3S}{C} \right)^2 \frac{m_4^* s^2}{9kT}$ $\times_{\text{ph}} = s^2 S_{\text{ph}} \tau_{\text{phr}} \left(1 - \frac{TS}{\rho_n s^2} \right)^2$	u_{20}
High frequencies	$u_{2\infty} \rightarrow 0,$ $\alpha_{2\infty} \rightarrow \infty$	$u_{2\infty} = \left[\left(1 - \frac{\rho_n r}{\rho} \right) \frac{TS_r}{\rho_n r C_r} \right]^{1/2},$ $\alpha_{2\infty} = \frac{1}{2u_{2\infty} \tau_{\text{phr}}} \frac{C_{\text{ph}}}{C_r} \frac{3kT}{m_4^* s^2}$	
$u_{20} = \frac{\rho_s}{\rho_n} \frac{TS^2}{\rho C},$ and C_r and S_r are the heat capacity and entropy of the roton gas, ρ_{nr} is the roton contribution to the density of the normal component.			

higher frequencies $l_{\text{ph}} \gg \lambda_2$, and equilibrium can be established during a period of the sound oscillations only in the roton gas, since it is assumed that $\omega \tau_{\text{r}} \ll 1$.

When the temperature is low enough, however, the quantity $(3kT/m_4^* s^2)(C_{\text{ph}}/C_r)$ may become so large that roton second sound will be damped out in a distance of the order of the wavelength of second sound, the dispersion being "dragged," so to speak, up to higher frequencies:

$$\omega \tau_{\text{phr}} \sim \frac{3kT}{m_4^* s^2} \frac{C_{\text{ph}}}{C_r} \gg \frac{u_2}{u_1}.$$

At very high frequencies, $\omega \tau_{\text{phr}} \gg (3kT/m_4^* s^2)(C_{\text{ph}}/C_r)$ and second sound again becomes weakly attenuated; then its velocity and absorption are given by the formulas presented in Table IV for the high-frequency region.

Despite the fact that dispersion should set in at lower frequencies for second sound than for first sound, second sound has not been investigated in its dispersion region because of experimental difficulties.

Figure 15 shows calculated $u_{20}(T)$ and $u_{2\infty}(T)$ curves together with corresponding experimental data. At high temperatures, the measured velocities of second sound agree perfectly with the theoretically calculated u_{20} values. At temperatures below ~ 0.6 °K, however, the discrepancies are large. This may be attributed to the fact that the mean free path of the excitations (phonons) becomes comparable with the dimensions of the apparatus at low temperatures. Under these conditions one does not measure the equilibrium value of u_2 , but a much larger quantity, which can approximate the phonon propagation velocity s .

The experimental data on the absorption of second sound in He⁴ (Fig. 16) agree well with the theoretical values.

c) He³-He⁴ solutions. One of the peculiarities of second sound is its high sensitivity to the presence of He³.

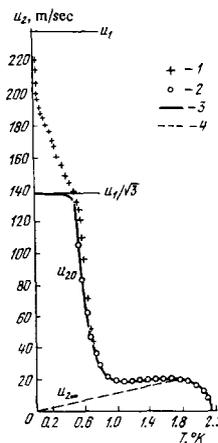


FIG. 15

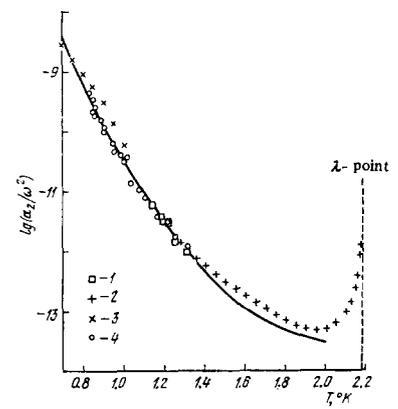


FIG. 16

FIG. 15. Temperature dependence of the velocity of second sound in He II: 1—data from [¹¹⁹], 2—[¹²⁶], 3— u_{20} , 4— $u_{2\infty}$, calculated in accordance with [⁶⁸].

FIG. 16. Absorption of second sound in He⁴: 1—data from [¹²²], 2—[¹²³], 3—[¹²⁴], 4—[¹²⁵].

Pomeranchuk^[153] first called attention to this in connection with his theory of dilute He³-He⁴ solutions. Even slight additions of He³ can considerably alter the velocity u_2 of second sound, and the $u_2(T)$ curves for solutions that are not dilute (Fig. 17) differ markedly from the corresponding curve for He⁴. The $u_2(c, T)$ curves for fixed He³ concentration tend toward zero as the λ point of the solution is approached, and at low temperature they merge into a common curve corresponding to a branch of the solubility curve for superfluid solutions in u_2-T coordinates.

To calculate $u_2(c, T)$ in the hydrodynamic approximation for solutions of arbitrary concentration we use Eq. (1.10), from which, neglecting thermal expansion, we easily derive the following expression for the case $s_1^2 \gg s_2^2$:

$$u_1^2(c, T) = \frac{\rho_s}{\rho_n} \left[\bar{c}^2 \left(\frac{\partial T}{\partial \sigma} \right)_{P, c} + c^2 \frac{\partial}{\partial c} \left(\frac{Z}{\rho} \right) \right] \left(1 + \bar{c}^2 \frac{\rho_s}{\rho_n} \right)^{-1}. \quad (3.1)$$

This expression is not well suited for a comparison of the calculated $u_2(c, T)$ values with experiment because in the general case no explicit expression is known for the potential Z , and experimental data on the excess chemical potentials of He^3 and He^4 in the solution, from which Z could be calculated, are available only in a limited range of temperatures and concentrations.

An expression for Z can be derived from various solution models, or for dilute solutions by expanding in the concentration. In the latter case we obtain [3]

$$u_1^2(c, T) = \frac{\rho_s}{\rho_n} \left[\frac{\partial T}{\partial \sigma} \left(\sigma_{40} + \frac{kc}{m_3} \right)^2 + \frac{kTc}{m_3} \right]; \quad (3.2)$$

here σ_{40} is the entropy per gram of He^4 and m_3 is the mass of a He^3 solute atom.

The expression for $u_2(c, T)$ simplifies even further if we assume not only that the solutions are dilute, but also that the temperature is so low that the phonon and roton contributions to the thermodynamic properties can be neglected. Under these conditions, using the dispersion law $\epsilon = p^2/2m_3^*$ for the impurity excitations, we obtain

$$u_1^2 = \frac{c_p}{c_v} \left(\frac{\partial p}{\partial \rho} \right)_T = \frac{5}{3} \frac{kT}{m_3^*}. \quad (3.3)$$

This result has a simple physical meaning: when the thermodynamic properties of the solutions are due mainly to the solute particles, second sound propagates in this solute gas—"extrinsic second sound"—and its velocity is the velocity of first sound in an ideal gas of particles having the mass m_3^* .

The absorption coefficient of second sound in He^3 - He^4 solutions has been calculated for dilute solutions by Khalatnikov [7] and for solutions of arbitrary concentration, by Sanikidze and Karchava [98]. In the most general case, the part of the absorption coefficient due to heat conduction and viscosity is given by

$$\alpha_2(c, \kappa, \eta, \zeta) = \frac{\omega^2}{2\rho u_1} \frac{\rho_s}{\rho_n} \left(1 + \bar{c}^2 \frac{\rho_s}{\rho_n} \right)^{-1} \left[\left(\frac{4}{3} \eta + \zeta_2 \right) (1 + \bar{c})^2 + 2\rho \zeta_1 (1 + \bar{c}) + \rho^2 \zeta_3 + \frac{\rho_n}{\rho_s} \frac{\kappa}{c} \frac{\bar{c}^2}{u_1^2} \frac{\partial T}{\partial \sigma} \right]. \quad (3.4)$$

Apart from heat conduction and viscosity, there are other dissipative mechanisms for solutions: diffusion, thermal diffusion, and barodiffusion. The part of the absorption coefficient due to these processes can be written in the form

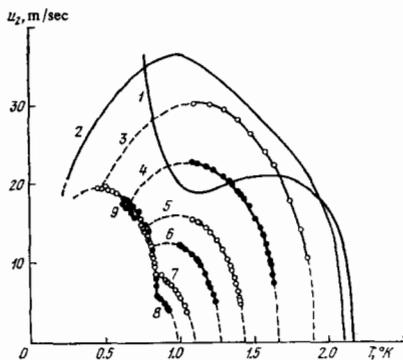


FIG. 17. Velocity of second sound in He^3 - He^4 solutions [129]: 1- He^4 , 2-4.3% He^3 , 3-18.4%, 4-31.4%, 5-43.9%, 6-50.5%, 7-59.4%, 8-63.9% He^3 ; 9-solubility curve.

$$\alpha_2(D) = \frac{\omega^2}{2u_1^2} D \frac{\rho_s}{\rho_n} \left[c^2 \frac{\partial}{\partial c} \frac{Z}{\rho} \left(1 + \bar{c}^2 \frac{\rho_s}{\rho_n} \right) \right]^{-1} \times \left[c \frac{\partial}{\partial c} \frac{Z}{\rho} \left(c + \frac{kT\bar{c}}{C} \right) - \bar{c}^2 u_1^2 \right]^2; \quad (3.5)$$

here the first term in the second pair of brackets (the numerator) is associated with diffusion and thermal diffusion, and the last term, with barodiffusion.

Unfortunately, the absorption of second sound in He^3 - He^4 solutions has been investigated experimentally only in very dilute solutions (He^3 concentration below 0.03%) [131]. The resulting data were used to determine the heat conductivity κ on the assumption that in such dilute solutions, as in He^4 , the absorption of second sound is due entirely to heat conduction. The temperature dependence of κ found in this way is in satisfactory agreement with the calculations of Khalatnikov and Zharkov [154] for $T > 0.6$ °K. There is considerable discrepancy between theory and experiment for $T < 0.6$ °K, i.e., where the rotons are "frozen out."

d) **Effect of pressure.** In addition to the measurements of the velocity of second sound in He^4 at the saturated-vapor pressure, there are data on the T dependence of u_2 at elevated pressures up to 27 atm [106, 116, 118]. It was found that at temperatures at which the roton contribution is predominant, u_2 decreases with increasing pressure (as the He^4 energy spectrum changes), while at temperatures at which the phonon contribution is predominant, u_2 increases with increasing pressure (much as does u_1).

Similar data have been obtained for solutions of He^3 in He^4 in the 0.2-1.1 °K temperature interval [30, 132]. The parameters of the Bose branch of the spectrum (Δ , ρ_0 , and m_3^*) have been calculated from measured values of u_2 for a 0.32% solution of He^3 in He^4 ; the results proved to be in good agreement with the values found for He^4 from the neutron scattering. In addition, the pressure dependence of the effective mass m_3^* of the impurity excitations was elucidated.

4. PECULIARITIES OF THE EXCITATION AND PROPAGATION OF FIRST AND SECOND SOUND

The concept of first and second sound as oscillations of pressure and temperature, respectively, gives only a crude picture of the real situation, in which certain details are lost.

Calculations [155] show that, neglecting dissipation, the relations

$$\left(\frac{v_n}{v_s} \right)_1 = 1 + \frac{\rho}{\rho_n} \frac{\beta}{1-\beta}, \quad \left(\frac{\sigma'}{v_s} \right)_1 = \frac{\rho_s}{\rho_n} \frac{\sigma}{u_1} \frac{\beta}{1-\beta}, \quad (4.1)$$

$$\left(\frac{P'}{v_s} \right)_1 = \frac{\rho u_1}{1-\beta}, \quad \left(\frac{c'}{v_s} \right)_1 = \frac{\rho_s}{\rho_n} \frac{c}{u_1} \frac{\beta}{1-\beta},$$

obtain in a first-sound wave in an He^3 - He^4 solution, and the relations

$$\left(\frac{v_n}{v_s} \right)_2 = -\frac{\rho_s}{\rho_n} \frac{1-\beta}{1+\beta(\rho_s/\rho_n)}, \quad \left(\frac{\sigma'}{v_s} \right)_2 = -\frac{\sigma}{u_2} \frac{\rho_s}{\rho_n} \frac{1}{1+\beta(\rho_s/\rho_n)}, \quad (4.2)$$

$$\left(\frac{P'}{v_s} \right)_2 = \rho u_2 \frac{\rho_s}{\rho_n} \frac{\beta}{1+\beta(\rho_s/\rho_n)}, \quad \left(\frac{c'}{v_s} \right)_2 = -\frac{c}{u_2} \frac{\rho_s}{\rho_n} \frac{1}{1+\beta(\rho_s/\rho_n)};$$

obtain in a second-sound wave. Here the prime indicates the variable part of the corresponding thermodynamic quantity. (The relations for pure He^4 are obtained by setting $c = 0$.)

Thus, first and second sound are two independent acoustic modes that propagate with different velocities. The quantities v_n , v_s , P , σ , and c all oscillate in each of these modes, but in a first-sound wave the entropy

(temperature) oscillations are relatively small, as are the concentration oscillations in the case of solutions, whereas in a second-sound wave it is the pressure oscillations that are small.

The excitation of first and second sound takes place in accordance with what has just been said. For example, if a cylindrical tube of radius r be filled with superfluid He^4 , then temperature oscillations of amplitude T_0 will excite pressure oscillations, which will propagate with velocities u_1 and u_2 , and whose amplitudes (with allowance for the viscosity coefficients) will be given by [156]

$$\left(\frac{P'_1}{T_0}\right) = \rho u_1 u_2 \left\{ \alpha + \frac{C}{TS} \left(\frac{u_2}{u_1}\right)^2 \left[\frac{i\omega}{\rho u_1^2} \left(\frac{4}{3}\eta + \zeta_2\right) + (i+1) \frac{\rho_n \lambda \eta}{\rho r} \right] \right\} \quad (4.3)$$

$$\left(\frac{P'_2}{T_0}\right) = \rho u_2^2 \left\{ \alpha + \frac{C}{TS} \left[\frac{i\omega}{\rho u_2^2} \left(\frac{4}{3}\eta + \zeta_2\right) + (i+1) \frac{\rho_n \lambda \eta}{\rho r} \right] \right\} \quad (4.4)$$

In these expressions the first terms in the brackets describe the effect of viscosity throughout the entire volume, while the second terms describe the wall effect.

As a rule, the viscosity contribution to P'_1 is small as compared with the contribution from thermal expansion (proportional to α), but the contribution from bulk viscosity to P'_2 at $T < 1.4^\circ\text{K}$ and $\omega > 10$ kHz is of the same order as the contribution from thermal expansion. Thermal expansion plays a special part above 1.4°K at low frequencies. The ratio of the intensity of the first sound to that of the second sound excited simultaneously by temperature oscillations is of the order of 10^{-3} over a wide temperature range.

If one end of the tube be closed by a membrane that is impermeable to He^4 and vibrates with amplitude v_0 , then not only will pressure oscillations be excited in the He^4 , but also temperature oscillations, which propagate with velocity u_2 :

$$\frac{T'_2}{v_0} = u_2^2 \frac{T}{C} \left[\alpha + \frac{C}{TS} (i+1) \frac{\rho_n \lambda \eta}{\rho r} \right]; \quad (4.5)$$

here we give only the principal terms, the corrections for η and ζ_2 being small. The intensity ratio of the first and second sound simultaneously excited in this manner does not exceed 10^{-6} , even under the most favorable conditions.

Similar results have been obtained [157] by numerical solution of the hydrodynamic equations for He^4 , including nonlinear terms in the velocities but neglecting dissipative terms. It was found that a pressure pulse $P' = 1$ bar excites a temperature pulse $T' \approx 5 \times 10^{-8}^\circ\text{K}$, but that a temperature pulse $T' = 10^{-3}^\circ\text{K}$ excites an easily detectable pressure pulse $P' = 0.2-0.4$ bar ($T = 1.5^\circ\text{K}$).

Both the pressure oscillations P'_1 and P'_2 excited by temperature oscillations, and the temperature oscillations T'_2 excited by pressure changes have been investigated experimentally. The pressure oscillations were detected either by condenser microphones [156, 158] or by light scattering [146, 147]. The temperature oscillations were produced with the aid of carbon or gold films on massive backings. The same carbon films served to detect the temperature oscillations, while in this case the pressure change was produced by a condenser microphone [156].

We note that experiments have been made, both using the pulse method and using the standing wave method. In comparing the experimental results with theory one must either normalize the experimental data

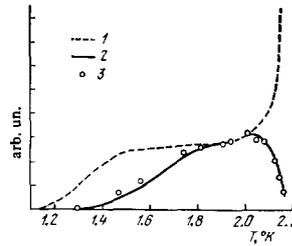


FIG. 18

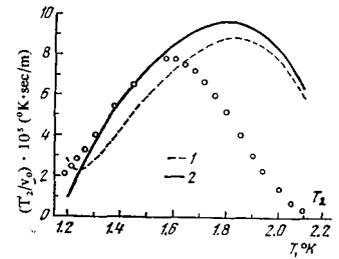


FIG. 19

FIG. 18. Temperature dependence of the amplitude of density oscillations in a second-sound wave [147]: 1—calculated from Eq. (4.4); 2—density oscillations p'_2 averaged over the width d of the laser beam; 3—experiment. $\bar{p}'_2 = \bar{p}'_2(T) \{1 - \exp[-\alpha_2(T)d]\} / \alpha_2(T)d$.

FIG. 19. Temperature oscillations T_2 generated by a membrane vibrating with amplitude v_0 [156]: 1—calculated with Eq. (4.5), 2—from Eq. (4.5) with $\eta = 0$. The experimental data were normalized at 1.4°K .

to the theoretical value at some fixed temperature or calibrate the transducers independently.

Figure 18 shows the temperature dependence of the amplitude of density oscillations in a second-sound wave in He^4 . The experimental points were obtained by measuring the scattering of light from a neon-helium laser beam. The experimental data agree well with the theory when the change in the amplitude of the density oscillations in the second-sound wave over the width of the laser beam is taken into account. Equally good agreement between experiment and theory is also obtained for the temperature dependence of the amplitude ρ'_1 of the density oscillations in a first-sound wave.

Similar data on ρ'_1 have been obtained by detecting the pressure wave directly, using a condenser microphone [157]. A discrepancy between theory and experiment noted in [157] was explained in [159] as resulting from perturbation of the acoustic field by the recording transducer itself, a factor that was not taken into account in reducing the experimental data.

Figure 19 shows the results of measurements [156] of the temperature dependence of the amplitude T'_2 of the temperature oscillations in a second-sound wave excited by a vibrating membrane. The reason for the discrepancy between theory and experiment at high temperatures is not known (perhaps the discrepancy is due to incorrectly chosen boundary conditions).

The peculiarities of the excitation and propagation of first and second sound in superfluid He^3 - He^4 solutions are basically described by the second term in the expression for the parameter β :

$$\beta = \alpha \bar{c} \frac{T}{C} - \bar{c}, \quad \bar{c} = \frac{c}{\rho} \frac{\partial \rho}{\partial c}.$$

Because of this term, β can be large. For example, for $c = 0.2$, we have $\beta \approx \bar{c} \approx -0.15$. Over a wide temperature range, we have $v_n = (1.2-1.5)v_s$ in a first-sound wave, and $v_n = -(0.6-0.8)(\rho_s/\rho_n)v_s$ in a second-sound wave. The difference from the case of pure He^4 , for which $v_n \approx v_s$ and $\rho_s v_s \approx \rho_n v_n$ for first and second sound, respectively, is the greater, the higher the He^3 concentration: for highly concentrated solutions, $\bar{c} \approx -0.4$.

Because of the large values of β , both pressure and temperature oscillations are present in both first- and second-sound waves, and their amplitudes (in com-

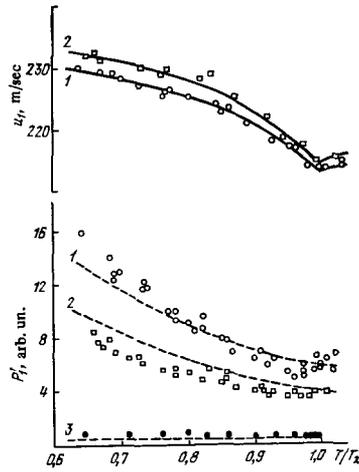


FIG. 20. Temperature dependence of the amplitude of the pressure oscillations excited by a heat pulse [¹⁶⁰]: 1—0.085 He³, 2—0.046 He³, 3—He⁴. The dashed curves were calculated.

parable units) can differ only by a small factor. This appears under conditions in which first and second sound are excited in superfluid He³-He⁴ solutions. If the temperature of an immobile membrane that is impermeable to He II is caused to fluctuate, the ratio of the intensities of the first- and second-sound waves thereby simultaneously excited will be given by

$$\frac{J_1}{J_2} = \beta^2 \frac{\rho_n u_1}{\rho_n u_2} \quad (4.6)$$

and for a solution with $c = 0.2$ at $T \approx 1.5$ °K will be of the order of 0.1. If the membrane is caused to vibrate, however, while its temperature is held constant and there is no heat flux through it, then

$$\frac{J_2}{J_1} = \beta^2 \frac{\rho_s u_2}{\rho_n u_1}, \quad (4.7)$$

and for $c = 0.2$ and $T < 2$ °K, this ratio is close in order of magnitude to 10^{-4} . (Equations (4.6) and (4.7) are also valid for He⁴, but for that case one must put $c = 0$ in the expression for β .)

The simultaneous excitation of first- and second-sound waves in superfluid He³-He⁴ solutions by temperature oscillations (the most favorable case) has been confirmed experimentally [¹⁰⁶]. In these experiments the temperature oscillations were induced by an electric current pulse through a thin carbon film. The emitter was mounted at one end of a cylindrical cell, and a first-sound detector (a device similar to a condenser microphone) or a second-sound detector (a phosphor bronze wire) was mounted at the other end.

The temperature dependence of the amplitude of the pressure oscillations excited by a heat pulse is shown in Fig. 20. The pressure oscillations propagated with the velocity of first sound in the solution; hence it is the temperature dependence of P'_1 that was measured. The dashed curves in the figure show the calculated value of $P'_1 \approx (\partial u_1 / \partial T) q$, where q is the heat flux at the heater. The experimental data were matched to the theoretical curve for $c = 0.085$ at $T = 1.8$ °K. The agreement between theory and experiment is evidently satisfactory.

The simultaneous excitation of first and second sound must be taken into account when investigating the absorption of second sound in He³-He⁴ solutions, since in this case the fraction of the power supplied to

the emitter that is expended in generating second sound depends on the temperature and concentration.

5. WAVE PROCESSES IN He II FILMS; THIRD SOUND

As a result of superfluidity, He II films have a number of unusual properties, including that of supporting the propagation of sound waves. The general picture of wave processes in helium films is very complicated; it depends essentially on the ratio of the thickness d of the film and the penetration depth $\lambda_\eta = (2\eta_n / \omega \rho_n)^{1/2}$ of the viscous wave. If $d \ll \lambda_\eta$, i.e., if the normal component of the liquid is fully clamped ($v_n \equiv 0$), a weakly damped wave, which has come to be called third sound, can propagate along the film. Third sound was first predicted by Atkins [¹¹¹] and was later detected and investigated experimentally [¹⁶¹⁻¹⁶⁴]. The character of third sound, in turn, changes sharply on going to very thin (unsaturated) films not more than 50 atomic layers thick [¹⁶⁵⁻¹⁶⁸].

If $d \sim \lambda_\eta$ or $d > \lambda_\eta$, not only the superfluid component, but also the normal component of the helium participates in the propagation of oscillations along the film, and this leads to a peculiar dispersion of third sound [¹⁶⁹⁻¹⁷²] and its conversion at $d \gg \lambda_\eta$ into ordinary capillary or capillary-gravitational waves.

In discussing the propagation of sound in He II films it is convenient to begin with the very general case of arbitrary d (arbitrary frequencies ω). We assume that the normal motion in the film is partially clamped and introduce an effective clamping force proportional to the velocity of the normal component:

$$f_{\text{clamp}} = -R v_n \equiv \omega \rho v_n. \quad (5.1)$$

Strictly speaking, to take the slippage of the normal component into account one must solve the entire set of hydrodynamic equations with the correct boundary conditions (as is done in treating fourth sound—see Sec. 6), but the physical picture of the propagation of the oscillations will look much simpler if we take the slippage of the normal component into account phenomenologically by introducing the parameter R or r , which is a complicated complex function of the oscillation frequency and film thickness [¹⁷²]:

$$r = -i \frac{\rho_n}{\rho} \frac{\text{tg } k_3 d}{k_3 d - \text{tg } k_3 d}, \quad (5.2)$$

where $k_3 = (i\omega \rho_n / \eta_n)^{1/2}$ is the wave vector of the viscous wave.

This approach also allows us to neglect the variations of the velocity and other quantities throughout the thickness of the film, i.e., we may assume that all the thermodynamic quantities (v_n , v_s , P' , T' , ρ' , and c') depend only on the time t and the coordinate x measured along the film. In addition, in this approximation we need not consider the components of v_n and v_s normal to the surface of the film ($v_n \equiv v_{nx}$, $v_s \equiv v_{sx}$).

To derive the equations we make use of the conservation of mass, energy, and momentum, and note that the local thickness of the film, $\xi = \xi(x, t)$, must be included among the unknown functions. Mass conservation leads to the equation

$$\rho \frac{\partial \xi}{\partial t} + \rho_s d \frac{\partial v_s}{\partial x} + \rho_n d \frac{\partial v_n}{\partial x} + K T' = 0. \quad (5.3)$$

The last term in (5.3) represents the change in the mass

of the film due to local vaporization or condensation of helium.

Denoting the heat of vaporization by L , we obtain the energy conservation equation in the form

$$\rho_s d\sigma T \frac{\partial v_s}{\partial t} - d\rho\sigma T \frac{\partial v_n}{\partial x} - \rho dC \frac{\partial T'}{\partial t} - KLT' = 0. \quad (5.4)$$

In writing the equation for superfluid flow (see Eq. (20.3) in [3]) it is convenient to express the pressure change P' in terms of the variables T' and ζ :

$$P' = \beta T' + \rho f \zeta; \quad (5.5)$$

here $\beta = \partial P / \partial T$, the derivative being taken along the vapor pressure curve, and f is the restoring force per unit mass of film, which we shall discuss later.

Thus, the equation for superfluid flow takes the form

$$\frac{\partial v_s}{\partial t} + f \frac{\partial \zeta}{\partial x} + \left(\sigma - \frac{\beta}{\rho} \right) \frac{\partial T'}{\partial x} = 0. \quad (5.6)$$

Taking the clamping force (5.1) into account, we can write the momentum conservation equation [169] in the form [10]:

$$\frac{\partial v_n}{\partial t} + f \frac{\partial \zeta}{\partial x} + \left(\frac{\rho_s}{\rho_n} \sigma - \frac{\beta}{\rho} \right) \frac{\partial T'}{\partial x} = - \frac{\rho}{\rho_n} \omega r v_n. \quad (5.7)$$

We note again that using the phenomenological coefficients r , K , and L enables us to avoid solving the two-dimensional problem with the correct boundary conditions.

Assuming as usual that v_n , v_s , T , and ζ in Eqs. (5.3), (5.4), (5.6), and (5.7) are harmonic functions of x and t , i.e., that they are proportional to $\exp[-i(\omega t - kx)]$, we obtain the dispersion equation for the velocity $u = \omega/k$ of the wave. The equation for u is biquadratic, and this means that oscillations of two types can propagate in the film. The first root corresponds to a surface van der Waals wave, and the second to a thermal wave.

The analytic expressions for the velocity and damping ($\text{Im}k$) of each of these oscillations are complicated and their implications are not immediately evident. Figure 21 shows the velocity and absorption coefficient of the oscillations of the first type as functions of film thickness and frequency. For thin films ($d \lesssim 10^{-5}$ cm for $\omega = 2\pi \times 10^3 \text{ sec}^{-1}$) one obtains Atkins' result (third sound); in this region the oscillations are only slightly damped. As d increases the absorption rises sharply, reaches a maximum, and then decreases as a new oscillation regime becomes established. Thus, Fig. 21 depicts the dispersion of surface sound resulting from the clamping of the normal component. The second root of the dispersion equation corresponds to a strongly damped wave. Calculations [172] indicate that the minimum attenuation coefficient at 1.8 °K in the frequency range $10^2 - 10^8$ Hz for films from 10^{-6} to 10^{-4} cm thick is of the order of 10^3 cm^{-1} . Oscillations of the second type are very difficult to investigate experimentally because of the strong absorption.

The propagation velocity of oscillations of the first type when $d \ll \lambda_\eta$ (third sound) is easily obtained directly from Eqs. (5.3), (5.4), and (5.6) by setting $v_n = 0$:

$$u_3^2 = \frac{\rho_s}{\rho} df \left(1 + \frac{T\sigma}{L} \right). \quad (5.8)$$

As will be seen from Eq. (5.8), the velocity of third

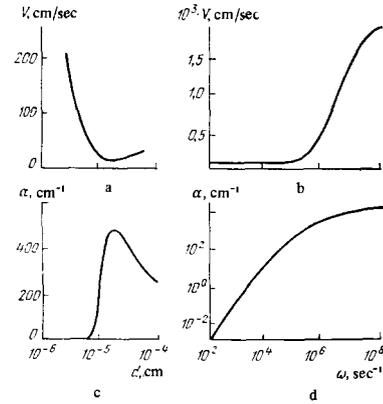


FIG. 21. Theoretical curves [172] showing the velocity and absorption coefficient of surface van der Waals waves at $T = 1.8^\circ \text{ K}$, as functions of film thickness (a, b) at $\omega = 2\pi \cdot 10^3 \text{ sec}^{-1}$ and of frequency (c, d) at $d = 5 \times 10^{-6} \text{ cm}$.

sound depends on the restoring force f , the characteristics of which are determined by the thickness of the film. If the film is thick enough, the restoring force will be determined by the van der Waals forces between the film and the wall. For a film that has crept up the wall to a height h under the action of these same van der Waals forces (with a potential proportional to d^{-n}), the expression for the velocity of third sound assumes an especially simple form:

$$u_3^2 = n \frac{\rho_s}{\rho} gh \quad (5.9)$$

(g is the acceleration of gravity). The exponent n depends on the film thickness [173]: $n = 3$ for a film no thicker than about 14 atomic layers [174], and n approaches 4 as the film thickness increases.

For thicker films (but with $d \ll \lambda_\eta$ as before) in which the restoring force is associated with surface tension, third sound becomes modified [11] capillary waves, for which (see Sec. 25 in [3])

$$\omega^2 = \frac{\rho_s}{\rho} \frac{\gamma}{\rho} k^3 \tanh kd, \quad (5.10)$$

where γ is the surface tension.

Whenever sound is excited in a film, both modes are usually present, but of course only the weakly damped wave (third sound if the film is thin enough) reaches the detector.

The relation between the amplitudes of the various quantities for an arbitrary value of d/λ_η is very complicated, but it simplifies considerably for the case of third sound:

$$\frac{\zeta}{d} = \frac{\rho_s}{\rho} \frac{v_s}{u_3}, \quad \frac{T'}{T} = -i \frac{\rho_s}{\rho} \frac{\rho d \sigma \omega}{KL} \frac{v_s}{u_3}. \quad (5.11)$$

The imaginary unit in the second of Eqs. (5.11) shows that the P' and v_s oscillations are 90° out of phase. Estimates indicate that for $T = 1.2^\circ \text{ K}$ and $\omega = 2\pi \times 10^3$ Hz, a relative change $\zeta/d = 0.25$ in the film thickness corresponds to an amplitude $T'/T \approx 3 \times 10^{-6}$ for the temperature oscillations, and the temperature oscillations are even weaker at lower frequencies. In other words, the temperature oscillations in a third-sound wave are very small.

Everitt, Atkins, and Denenstein [161, 162] observed third sound in saturated He II films 200–900 Å thick, using a

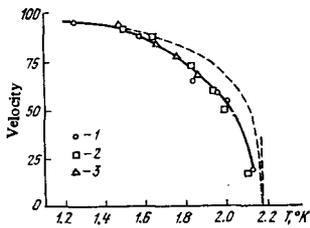


FIG. 22. Velocity of third sound vs. temperature [162] for various heights of the film above the helium surface: 1—0.44 cm, 2—12.3 cm, 3—13 cm. The dashed curve was calculated from Eq. (5.8) and normalized to the experimental data at 1.3° K.

thermo-optical excitation method. The sound was generated as a result of periodic vaporization of a narrow strip of the film by infrared radiation and was detected by an optical interference system that was sensitive to local variations in the film thickness.

The measured temperature dependence of u_3 is presented in Fig. 22. One cannot very well compare the measured u_3 values with values calculated directly from Eq. (5.8), because neither d nor f is known. One must normalize the theoretical curve to the experimental value at some point. The dashed curve in the figure represents the quantity $(\rho_S/\rho)^{1/2}[1+(TS/L)]^{1/2}$ calculated on the assumption that ρ_S/ρ is the same in the film as in the bulk of the liquid. The evident discrepancy between theory and experiment may be due, first, to a temperature dependence of f and d , and second, to the effect of oscillations of the second type.

The technique used in [163] made it possible simultaneously to measure the velocity and absorption coefficient of third sound. The experimental values obtained for the absorption spread widely and were 2–3 orders of magnitude larger than predicted by Atkins' theory [161], which is based on the assumption that the principal attenuation mechanism is connected with the vaporization and condensation of helium in the film associated with the temperature oscillations in the third-sound wave.

Third sound can also be excited in unsaturated He II films, i.e., when the pressure P of the vapor above the film is lower than the equilibrium pressure P_0 of the saturated vapor at the given temperature. Experiments of this type were undertaken by Rudnick, Kagiwada, Fraser, and Guyon [165], who used a thermal method for generating and detecting third sound. The emitter and detector were aluminum films a few hundred Ångströms thick, operating close to the superconducting transition temperature (~1.5 °K) where they are very sensitive to temperature oscillations.

An ordinary transit time method was used to measure the velocity of third sound. The u_3 values obtained for saturated films agree well with the results of [162], but as the pressure difference $P_0 - P$ increases, i.e., as the films become thinner, the velocity rises sharply, reaching ~3000 cm/sec at $P = 1000 \mu\text{m Hg}$ (~5.5 atomic layers) for $T = 1.5 \text{ °K}$.

In unsaturated films, the absorption of third sound becomes almost an order of magnitude lower than in saturated films.

The results of numerical calculations of the absorption of third sound using the entire set of linearized hydrodynamic equations for the case of saturated films are presented in Fig. 23. The general behavior of the absorption coefficient α is as follows: for thin films α decreases with increasing d as $d^{-5/2}$ and is proportional to $\omega^{1/2}$; then the absorption passes through a

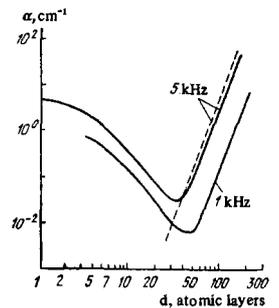


FIG. 23. Absorption of third sound at 1.3° K vs. film thickness. The full curves were calculated by numerical solution of the complete set of hydrodynamic equations for third sound [168], and the dashed line represents the prediction of Atkins' theory [11].

minimum and begins to increase with increasing d as $d^{11/2}$ and becomes proportional to ω^2 . In the latter case the behavior of the absorption as a function of d and ω agrees with the predictions of Atkins' theory [111], but quantitatively, they agree only in order of magnitude. The dependence of α on d presented in Fig. 23 has not yet been confirmed experimentally.

Studies have shown that third sound in He II films can be successfully used to investigate the nature of superfluid films. The method of third sound has made it possible to determine the critical flow velocity of a helium film and to investigate peculiarities of the superfluid transition in films [174–180].

Not only third sound, but also waves that have a classical analog—ordinary capillary waves—can propagate on the free surface of helium (when $d \gg \lambda_\eta$). Such waves have been investigated experimentally in liquid helium, beginning in [162] and then in more detail in [181–182].

In concluding this section we consider still another interesting phenomenon that was recently predicted by Andreev and Kompaneets [183, 184]: surface second sound in liquid helium. These authors called attention to the fact that in liquid helium there are not only bulk excitations, but also surface elementary excitations—capillary waves—and that in superfluid solutions of He³ in He⁴ there are surface impurity levels [185, 186]. Their motion, which is accompanied by transport of mass, energy, entropy, etc., can be regarded as the motion of a surface normal component. Unlike the density of the bulk normal component in He⁴, which is proportional to T^4 , the density of the surface normal component varies as $T^{5/3}$. At sufficiently low temperatures, therefore, one can neglect the effect of the bulk normal component on the surface phenomena and assume that there is only a surface normal component. This appears especially clearly in the case of dilute solutions of He³ in He⁴, which have a dense layer of He³ on the surface at at temperatures of the order of 0.1 °K.

Using a specially derived set of equations for surface hydrodynamics to describe the motion of the surface, Andreev and Kompaneets showed that, in addition to ordinary capillary waves, oscillations analogous bulk second sound can propagate on the surface. Waves of this latter type have been called surface second sound.

The velocity of surface second sound in pure He⁴ is given by

$$u^2 = \frac{63}{40} \frac{\Gamma(7/3) \zeta(7/3)}{\Gamma(5/3) \zeta(5/3)} \left(\frac{\gamma T}{\mu \rho} \right)^{2/3}, \quad (5.12)$$

where $\Gamma(x)$ and $\zeta(x)$ are the gamma function and Riemann's zeta function, respectively. For He³-He⁴ solutions at temperatures above the degeneracy temperature, we have

$$u^2 = \frac{2T}{m_s}, \quad (5.13)$$

corresponding to the velocity of sound in a two-dimensional ideal monatomic gas (m_s is the effective mass of an impurity on the surface levels).

If the solute is highly degenerate, the velocity of surface second sound will be

$$u = \frac{N_s}{m_s^*} \frac{\partial \mu}{\partial N_s}, \quad (5.13a)$$

where N_s is the number of solute atoms on surface levels^[185], μ is the chemical potential of the solute, and m_s^* is an effective mass that differs from m_s because of the Fermi-liquid interaction between the impurities.

At very low temperatures u agrees in order of magnitude with the velocity of ordinary sound, and $u(T)$ has a minimum at a temperature close to the degeneracy temperature of the impurities. The principal mechanism for the attenuation of surface second sound is interaction with the bulk normal component. We note that at low temperatures and frequencies exceeding the reciprocal of the time between impurity collisions at the surface, surface second sound must reduce to oscillations of the type of zero sound in a two-dimensional Fermi liquid.

Surface second sound has not yet been detected experimentally.

6. PROPAGATION OF SOUND IN NARROW CHANNELS; FOURTH SOUND

The behavior of sonic oscillations in narrow channels filled with a superfluid liquid is determined by the ratio $\delta = d/\lambda_\eta$ of the transverse dimension d of the channel (capillary) to the penetration depth λ_η of the viscous wave. When $\delta \gg 1$ (high frequencies and wide channels) the walls have no significant effect: "ordinary" first and second sound can propagate in such a liquid. As δ decreases, the normal component of the helium becomes more and more firmly clamped as a result of friction with the walls, and the propagation of sound suffers an essential change in character. In the limit of infinitely narrow channels, $v_n \equiv 0$, but sound can still be propagated via the superfluid component. Oscillations of this type have been called fourth sound^[11,187].

It is not difficult to calculate the velocity of fourth sound. One simply sets $v_n \equiv 0$ in the linearized hydrodynamic equations for HeII (see, e.g., Sec. 17 of^[3]), eliminating the momentum conservation equation since momentum is transferred to the wall of the capillary:

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2 + \frac{\rho_n}{\rho} u_2^2. \quad (6.1)$$

The second term in (6.1) is considerably smaller than the first term. The transition from the case of thick capillaries, in which first and second sound propagate, to thin capillaries, in which fourth sound propagates, can be traced by analyzing the solution of the complete set of linearized hydrodynamic equations^[188], which expresses the wave process as a sum of waves of three types: first and second sound and the viscous wave. As was shown in^[188], the principal part in the dispersion and absorption of sound in channels is played by first viscosity, which leads to the clamping of the normal component.

Oscillations of two types^[12] can propagate in channels,

as in free helium. The dispersion equation (the relation between the frequency ω and the wave vector k) for arbitrary values of δ can be written in the form^[188]

$$\left(\frac{\omega^2}{k^2}\right)_1 = \frac{u_1^2 - u_2^2 [r\rho_s\rho_n / (\rho_n + ir\rho_s)]^2}{1 + i[r\rho_n / (\rho_n + ir\rho_s)]}, \quad (6.2)$$

$$\left(\frac{\omega^2}{k^2}\right)_2 = \frac{u_2^2}{1 + i(r\rho_s/\rho_n)}. \quad (6.3)$$

The dimensionless parameter r is given by Eq. (5.2); it is advantageous to separate its real and imaginary parts:

$$r = \frac{\rho_n}{\rho} (m_1 + im_2) = \frac{\rho_n}{\rho} \frac{a + i[a^2 - b(1-b)]}{a^2 + (1-b)^2}, \quad (6.4)$$

in which a and b are functions of δ that depend on the shape of the channel cross section^[188,189].

We note that Eqs. (6.2) and (6.3) can be derived phenomenologically^[190] by introducing a clamping force equal to $\omega\rho r u_n$.

The velocity and absorption coefficient of both types of sound can be obtained from Eqs. (6.2), (6.3), and (6.4). We give the results for the case^[3] in which $u_2^2 \ll u_1^2$. For first sound,

$$u_{1\delta}^2 = u_1^2 \left(1 - b \frac{\rho_n}{\rho}\right) \left[1 + \frac{3}{8} \left(\frac{a\rho_n}{\rho - b\rho_n}\right)^2\right], \quad (6.5)$$

$$\alpha_{1\delta} = \frac{1}{2} \frac{\omega}{u_{1\delta}} \frac{a\rho_n}{\rho - b\rho_n}, \quad (6.6)$$

and for second sound,

$$u_{2\delta} = \sqrt{2} u_2 \left[\left(1 - 2 \frac{\rho_s}{\rho} m_2 + \frac{\rho_s^2}{\rho^2} m_1^2\right)^{1/2} + 1 - \frac{\rho_s}{\rho} m_2 \right]^{-1/2}. \quad (6.7)$$

$$\alpha_{2\delta} = \frac{\omega}{\sqrt{2} u_2} \left[\left(1 - 2 \frac{\rho_s}{\rho} m_2 + \frac{\rho_s^2}{\rho^2} m_1^2\right)^{1/2} - 1 + \frac{\rho_s}{\rho} m_2 \right]^{1/2}. \quad (6.8)$$

The transition from first sound to fourth sound as the normal component becomes more firmly clamped, and from second sound to damped thermal waves, has been detected and investigated experimentally^[190,193-195].

The transition from first sound to fourth sound via an intermediate region of strong dispersion is graphically illustrated in Fig. 24. Since the difference between u_1 and u_4 increases with increasing temperature, the dispersion is naturally more apparent at comparatively high temperatures. The attenuation of first sound associated with slippage of the normal component increases with increasing frequency, reaches a maximum corresponding to $\delta \sim 1$, and then decreases with further increase of frequency. The experimental results agree well with the theoretical predictions^[188].

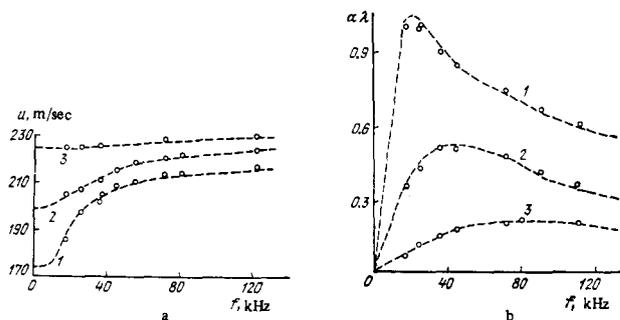


FIG. 24. Dispersion of first sound in He II associated with clamping of the normal component^[194]. Plots a) and b) give the velocity and absorption per wavelength, respectively, of sound as functions of frequency: 1-1.9° K, 2-1.7° K, 3-1.5° K. The dashed curves were calculated from Eqs. (6.5) and (6.6).

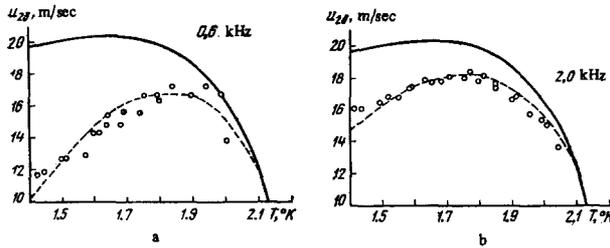


FIG. 25. Dispersion of second sound in narrow channels filled with He II [195]. The plots give the temperature dependence of the velocity for two frequencies: a)—0.6 kHz, b)—2.0 kHz. The full curve gives the velocity of second sound in bulk helium; the dashed curve was calculated from Eq. (6.7) for $d = 6.5 \mu\text{m}$.

The dispersion of second sound in narrow channels was first detected by Pollack and Pellam [190] and was subsequently also observed in [195]. The temperature dependence of the velocity is given in Fig. 25 for two different frequencies. The sound was propagated in straight channels having a mean diameter of $\sim 6.5 \mu\text{m}$. The measured velocities agree well with Eq. (6.7).

The effects discussed above are also possible when sound propagates in narrow channels filled with a superfluid He³-He⁴ solution. The corresponding phenomena have been thoroughly investigated both theoretically [196] and experimentally [197].

The addition of He³ enhances the dispersion of first sound, extends the dispersion region further toward the lower temperatures, and leads to strong absorption of first sound. The dispersion of second sound, on the other hand, should be less marked in solutions than in He⁴.

Fourth sound, which, as was pointed out above, is actually modified first sound, can also propagate in He³-He⁴ solutions [198, 199]. The expression for the velocity of fourth sound in solutions of arbitrary concentration is rather cumbersome. For the case $u_2^2 \ll u_1^2$ it reduces to

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2 \frac{(1 + \bar{c})^2}{1 + (\rho_s/\rho_n)\bar{c}^2} + \frac{\rho_n}{\rho} u_2^2 \left(1 + \frac{\rho_s}{\rho_n} \bar{c}^2\right). \quad (6.9)$$

In the derivation [200] of Eq. (6.9) it was assumed that the He³ participates in the normal motion of the solution (see section 2).

The pressure and temperature simultaneously oscillate in a fourth-sound wave, even if the thermal expansion coefficient be neglected, and in solutions, the concentration also oscillates [200]; in HeII we have

$$P' = -\frac{\rho_s u_1^2}{u} v_s, \quad T' = -\frac{\rho_n}{\rho} \frac{u_2^2}{u_4 \sigma} v_s. \quad (6.10)$$

According to estimates,

$$\frac{T'}{T} \approx 10^{-4} \frac{P'}{P},$$

i.e., the pressure oscillations are much the stronger. Hence the best way to generate fourth sound is to cause the pressure or density of the liquid to vary [201]. However, fourth sound can also be generated by the thermal method [202]. Fourth sound was first detected in HeII by Rudnick and Shapiro [201], and its velocity was later investigated in detail as a function of temperature and He³ concentration [201, 203].

A sound cell such as is usually used to investigate fourth sound is shown schematically in Fig. 26. The

FIG. 26. Sound cell for investigating fourth sound [200] (the upper part is shown disassembled): 1—body, 2—fine grained powder, 3—membrane, 4—massive electrode, 5—insulating gasket, 6—cover.

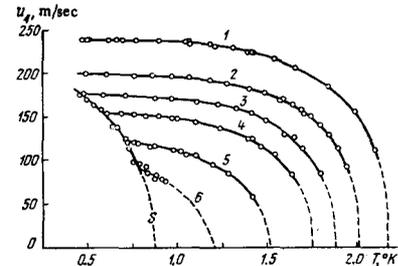
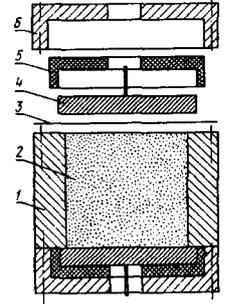


FIG. 27. Temperature dependence of the velocity of fourth sound in He⁴ (curve 1) and in superfluid solutions with the following molar concentrations of He³: 11.05% (2), 19.7% (3), 27.0% (4), 39.9% (5), 53.6% (6). S is the solubility curve.

most suitable device for generating and detecting the sound is a condenser type transducer in which one plate is the stationary electrode (4) and the other movable plate is a membrane (3) consisting of a thin polymer film bearing a metallic coating a few microns thick. The fourth sound propagates in the system of narrow channels formed by tightly pressed fine-grain powder. With such a device one can measure the velocity of fourth sound by both resonance [201] and pulse [200] methods, correcting for multiple scattering of the sound from the powder grains.

We note that in experiments on the propagation of fourth sound, the oscillations in the capillaries are always excited in the bulk helium; strictly speaking, therefore, one excites either first or second sound, which then generates the fourth-sound waves. Estimates [204] show that fourth sound is better excited by first sound than by second sound.

The investigations of the velocity of fourth sound in superfluid He⁴ and He³-He⁴ solutions are summarized in Fig. 27. The experimental data for He⁴ and for dilute solutions agree well with calculations based on Eqs. (6.1) and (6.9). Such a comparison cannot be made for concentrated solutions because of the lack of data on the density of the normal component. In this case, information on ρ_n can be extracted from the measured u_4 values themselves, as was done in [203]. This, in turn, made it possible to make some estimates of the parameters for the Bose and Fermi branches of the energy spectrum of the superfluid solutions.

Fourth sound has also been successfully used to determine the density of the superfluid component of He⁴ in very narrow channels with diameters of a few tens of Ångströms [205-207], and to investigate persistent superfluid currents and critical velocities in rotating helium [208-211].

The hydrodynamic theory of the absorption of fourth sound in HeII was worked out by Sanikidze, Adamenko,

and Kaganov^[212]. The part of the fourth-sound absorption coefficient due to viscosity can be written in the form

$$\alpha_4(\eta, \zeta) = \frac{d^2 \rho_n^2 \omega^2}{6\eta \rho_s} + \frac{\zeta_3 \rho \omega^2}{2u_1^2}. \quad (6.11)$$

The first term in (6.11) is due to incomplete clamping of the normal component. It is curious that this term contains the viscosity in the denominator and so decreases with increasing η . This is due to the fact that a decrease in η increases the slippage of the normal component and thus increases the energy dissipation. Unlike the first term, which can be attributed to surface losses, the second term is due to absorption taking place directly in the volume occupied by He II. Although the second-viscosity coefficient ζ_2 plays the principal part in the absorption of first sound, it is the coefficient ζ_3 that makes the main contribution to the absorption of fourth sound. This is due to the fact that the superfluid component moves relative to the normal component in the propagation of fourth sound.

To calculate the part of the fourth-sound absorption coefficient associated with heat conduction one must take into account the boundary conditions for the temperature of the wall; these boundary conditions affect even the velocity of the sound, leading to a peculiar type of dispersion^[212]. If the heat resistivity $1/\alpha$ of the boundary is small and the heat conductivity κ_w of the wall is large, the temperature oscillations in the fourth-sound wave will impose their thermal conditions on the wall. In this case the velocity of sound is rigorously given by

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2. \quad (6.12)$$

If the heat flow through the wall is small, however, the velocity of fourth sound will be given by Eq. (6.1).

In the intermediate case of partial heat flow through the wall the second term in (6.1) acquires the weight factor C/C_{eff} , where $C_{\text{eff}} = C + C_w(d_1/d)$. Thus, in narrow channels, dispersion of sound associated with the thermal conditions at the wall can arise.

In He³-He⁴ solutions, the presence of He³ solute atoms in the superfluid helium leads, as was shown in^[196,213], to an increase in the viscous absorption of sound; in addition, still another dissipative mechanism is present to contribute to the bulk losses: diffusion of the solute.

The hydrodynamic approximation used up to now is valid for $l \ll d$, where l is the mean free path of the phonons and rotons. Fourth sound can also propagate in the opposite limiting case. It is easy to understand that Eq. (6.1) will still give the velocity of fourth sound when $l \gg d$ since only the condition of complete clamping of the normal component ($v \equiv 0$) was used in deriving it. When $l \gg d$, direct collisions of the phonons and rotons with the walls of the capillary will assure that the normal component will be clamped.

To calculate the absorption coefficient of fourth sound for $l \gg d$ one should solve the kinetic equation^[214], the principal energy dissipation mechanism being collisions of phonons with the walls of the capillary. Since in narrow capillaries the probability for the elastic scattering of phonons with the wall vanishes except within a small angle $\varphi \approx u_1/u_w$ reckoned from the normal to the wall (u_w is the velocity of sound in the

solid body), the phonons moving within this angle (the "normal" phonons) come into equilibrium with the solid body more rapidly than do the other phonons. Hence one of the dissipative processes under these conditions is the establishment of temperature equilibrium between the "normal" phonons and the other ones (under such conditions two temperatures are required to specify the state of the phonon gas).

7. SOUND IN DEGENERATE He³-He⁴ SOLUTIONS

The phase diagram for He³-He⁴ solutions^[215] shows that the solubility curve does not go to zero as $T \rightarrow 0$: the solubility of He³ in He⁴ remains finite at $T = 0$ ($c \lesssim 6.3\%$). This means that an He³-He⁴ solution at $T = 0$ is a superfluid liquid, and that at low enough temperatures it is a Fermi-Bose quantum liquid in which the principal elementary excitations are the Fermi excitations characteristic of a Fermi liquid.

The treatment of dilute He³-He⁴ solutions at low temperatures as Fermi-Bose quantum liquids makes it possible to describe their thermodynamic properties in a comparatively simple manner^[216] and to investigate the peculiarities of the propagation of sound in them^[217-220]. The theory of Fermi-Bose quantum liquids has been presented in detail in Khalatnikov's monograph^[3] (Chapter 16), so we shall not dwell on it here but shall merely write down the formulas for the velocity of sound in degenerate solutions. As in any superfluid liquid, both first and second sound can propagate in a Fermi-Bose quantum liquid. Their respective velocities are given by^[3]

$$u_1^2 = s^2 \left\{ 1 + \frac{N_3 m_3^*}{\rho [1 + (F_1/3)]} \left[\left(\alpha \left(1 + \frac{F_1}{3} \right) + 1 \right)^2 - 1 \right] \right\}, \quad (7.1)$$

$$u_2^2 = \frac{v_F^2}{3} \left(1 + \frac{F_1}{3} \right) (1 + \tilde{F}_0) \left[1 - \frac{N_3 m_3^*}{\rho [1 + (F_1/3)]} \left(\alpha \left(1 + \frac{F_1}{3} \right) + 1 \right)^2 \right]; \quad (7.2)$$

here

$$\alpha = \left(\frac{\partial \epsilon}{\partial p} \right)_{\epsilon = \mu_3} \frac{\rho}{m_3^* s^2}, \quad s^2 = \rho \frac{\partial^2 E}{\partial \rho^2}, \quad v_F = \left(\frac{\partial \epsilon}{\partial p} \right)_{\epsilon = \mu_3},$$

ϵ , p , and m_3^* are the energy, momentum, and effective mass of a Fermi excitation, N_3 is the number of excitations per unit volume, E is the energy per unit volume of the liquid, and the F_i are the coefficients in the expansion in Legendre polynomials of the correlation function characterizing the Fermi-liquid interaction between the He³ atoms (only the first two harmonics were taken into account in the calculation). In the case of solutions, the interaction of the He³ atoms with the He⁴ atoms results in the necessity of replacing the coefficient F_0 , which occurs in the corresponding expressions for a pure Fermi liquid, by the renormalized constant \tilde{F}_0 :

$$\tilde{F}_0 = F_0 - 3\alpha^2 \frac{s^2}{v_F^2} \frac{N_3 m_3^*}{\rho}.$$

We should call attention to the fact that in the present case second sound is nothing but ordinary sound in the Fermi liquid consisting of the He³ atoms. If it were not for correlation effects, its velocity, as Eq. (7.2) shows, would be $v_F/3$, the velocity of sound in a degenerate Fermi gas. First sound, however, reduces to first sound in He II as the He³ concentration tends to zero.

The question whether zero sound can propagate in degenerate He³-He⁴ solutions has not yet been cleared up; the values of the correlation constants F_0 and F_1 seem to be such that zero sound cannot propagate^([3], Sec. 77).

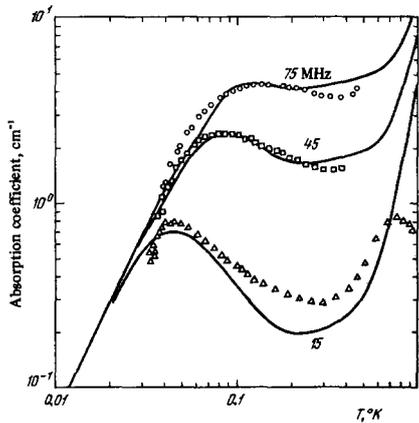


FIG. 28. Absorption of first sound in a 5.5% solution of He³ in He⁴ at T < 1° K. The experimental points are from [222] and the curves were calculated in accordance with [225].

If we estimate the parameters in Eq. (7.1) from the experimental data we find that the velocity of first sound is somewhat lower in degenerate solutions than in pure He⁴. An extremely sensitive and precision technique is required for an experimental study of the temperature and frequency dependence of u₁. Abraham, Eckstein, Ketterson, and Vignos^[62] developed such a technique on the basis of the phase comparison method and used it to make very accurate relative measurements of the velocity and absorption of first sound in degenerate solutions^[221-223].

Studies of the velocity of sound at various frequencies have shown that u₁ varies strongly with frequency, and that this frequency dispersion is accompanied by appreciable absorption. The physical cause of this dispersion and absorption is a relaxation process in the Fermi-excitation gas. According to Baym^[219] this relaxation process is a consequence of ordinary viscosity, which, as in any Fermi liquid, is inversely proportional to the square of the temperature (see, e.g., Chapter 14 in [3]). The characteristic relaxation time of this process is given by the semiempirical formula^[222]

$$\tau_{\eta} = \frac{A}{T^2} \left[1 + B \left(\frac{T}{T_F} \right)^2 \right], \quad (7.3)$$

in which T_F is the Fermi degeneracy temperature and the parameters A and B, which depend on the He³ concentration, are evaluated from experimental data. A theoretical formula for the velocity of first sound^[224] derived with allowance for this relaxation process, which is due to the interaction of the He³ atoms, represents the experimental data very well at temperatures below 0.3 °K.

Measurements of the absorption of first sound in degenerate solutions have led to interesting results. Such measurements were first made^[221] on a 5% solution at frequencies of 20, 60, 100, and 140 MHz and were then repeated^[222] on a 5.5% solution at 15, 45, and 75 MHz. Although in He⁴ the absorption decreases monotonically with decreasing temperature below 0.8 °K, in the solutions, the absorption passes through a minimum at T ≈ 0.3 °K; further, a maximum was found at a lower temperature, which broadens and shifts toward the higher temperatures with increasing frequency (Fig. 28).

These features of the temperature and frequency dependence of the absorption coefficients have been explained by Baym, Ebner, and Saam^[219,226-228], who showed

that at temperatures below 0.3 °K, where the effect of thermal phonons can be neglected, the absorption of first sound in dilute solutions of He³ in He⁴ is due almost entirely to the He³ impurity atoms, and, as was mentioned above, the characteristic relaxation time τ_η is associated with the viscosity of the He³. Calculation of the absorption coefficient of a 5% solution of He³ in He⁴ leads to the simple formula^[219-226]

$$\alpha_1 = 0.57 \cdot 10^{-8} Y(T) \frac{2\omega^2 \tau_{\eta}}{1 + \omega^2 \tau_{\eta}^2}, \quad (7.4)$$

in which

$$Y(T) = 1 + \frac{5\pi^2}{12} \left(\frac{T}{T_F} \right)^2 - \frac{\pi^4}{16} \left(\frac{T}{T_F} \right)^4 + \dots$$

This equation is valid for T << T_F, the relaxation time τ_η being given by Eq. (7.3), in which the second term in the brackets may be neglected.

At temperatures above 0.3 °K thermal phonons, interacting with the He³ quasiparticles, contribute significantly to the absorption of sound. Calculations^[225,227] in which this interaction is taken into account (and which are valid for temperatures up to 0.7 °K) lead to very complicated formulas that are ill suited for study. A numerical calculation is compared with the experimental results in Fig. 28; the agreement is satisfactory^[22].

The study of second sound in degenerate solutions of He³ in He⁴ was begun in 1949 by Pomeranchuk^[153], who treated the impurity particles as an ideal quasiparticle gas with the dispersion law E₀ + (p²/2m₃^{*}) and showed that the velocity of second sound at low temperatures is extremely sensitive to even small additions of He³. It was found that in a fully degenerate Fermi gas the velocity of second sound for a fixed concentration is temperature independent, being determined by the effective mass alone:

$$u_2^2 = \frac{5}{3} \xi_0 \frac{c^{2/3}}{m_3^*}, \quad (7.5)$$

where

$$\xi_0 = \frac{(3\pi^2)^{2/3}}{3} \frac{\hbar^2}{m_3^*} \left(\frac{\rho}{m_4} \right)^{2/3}.$$

The low-concentration limit of the range of applicability of Eq. (7.5) is determined by the condition that the contribution from impurity excitations predominate over the contribution from phonons (at T ~ 4.5 °K, this condition is satisfied when c > 10⁻⁵). The high-concentration limit of the range of applicability is associated with the necessity of taking into account the interactions of the impurity excitations with one another. The possibility of taking these interactions into account arose after an effective He³-He⁴ interaction potential was proposed, as was done in [216], and this led to the appearance of a number of experimental and theoretical papers on second sound in dilute He³-He⁴ solutions at low temperatures^[143,144,230-234].

One peculiarity of the experimental study of second sound in degenerate solutions is associated with the fact that the oscillations propagate in the Fermi component of the liquid and have much in common with first sound in a Fermi liquid. Although here, as before, the temperature oscillations play the principal part in a second-sound wave, at very low temperatures the heater generates second sound, not mainly by the direct mechanism, but as a result of the interaction of the phonons and rotons excited by the heat with He³ atoms. In this connection, one way to observe second sound in a solution is to

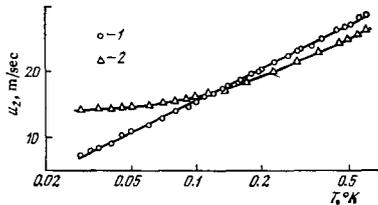


FIG. 29. Temperature dependence of the velocity of second sound in degenerate solutions [144]: 1—0.143% He³ ($T_F \approx 0.029^\circ \text{K}$), 2—6.28% He³ ($T_F \approx 0.33^\circ \text{K}$).

excite it by means of the mechanical vibrations of a plane surface; this should result in the simultaneous generation of first and second sound^[232].

Sherlock and Edwards^[143] developed special mechanical transducers to excite and detect oscillations. These transducers consist of condenser heads in which the vibrating membrane is a porous film permeable only to the superfluid component. Special analysis showed that the fraction of the energy emitted by such a transducer as first sound when $\rho_n \ll \rho$ is negligible. These transducers have been successfully used to investigate second sound in solutions of He³ in He⁴ at low temperatures^[144,233].

The measurements were made in the temperature range 0.03–0.6 °K. In this region, the Fermi excitations in very dilute solutions can be described by Boltzmann statistics, and as is evident from Fig. 29, the "classical" relation $u_2 \propto T^{1/2}$, corresponding to Eq. (3.3), obtains for the 0.143% solution. The effect of Fermi degeneracy of the impurity is manifest for the 6.28% solution: there are deviations from the $T^{1/2}$ law at low temperatures and a tendency toward saturation, in accordance with Eq. (7.5), at $T < T_F$.

The results are in qualitative agreement with a numerical calculation of the velocity of second sound in degenerate solutions^[234] in which the interaction of the impurity excitations was taken into account in the context of the Bardeen-Baym-Pines theory^[216]. The quantitative difference is apparently associated with the values chosen for the parameters in the Bardeen-Baym-Pines theory. In accordance with the calculation, the form of the temperature dependence of the velocity of second sound remains virtually unchanged as the pressure increases; this was also noted in^[132,231].

The absorption of second sound in the degeneracy region still remains virtually uninvestigated. Only estimated upper bounds to the absorption coefficient based on analysis of experimental data have been given^[144].

Up to now we have been considering the propagation of sound under conditions in which the effect of the walls of the container can be neglected. If the solution is contained in narrow channels whose characteristic linear dimension is much smaller than the penetration depth of the viscous wave or than the mean free path of the elementary excitations, fourth sound can also propagate. This possibility has been discussed by Sanikidze and Shaanova^[235].

We also note that fourth sound is possible in pure He³ in the temperature region in which it can become superfluid^[235].

Note added in proof. The propagation of fourth sound has recently been detected in pure He³ in the superfluid phase (at a temperature below $3 \times 10^{-3} \text{K}$)^[236].

After the work on the review had been completed,

there appeared a number of papers^[237–248], which we can merely list.

- ¹It is assumed that the reader is familiar with the principal properties of liquid helium, which have been presented in some detail in monographs [3–6].
- ²This topic is discussed in a recent review article [7].
- ³A rigorous proof of this assertion for low He³ concentrations is given in [14].
- ⁴In addition, any wave is naturally characterized by its polarization and the ratios of the amplitudes of the various physical quantities involved in it. In place of the propagation velocity, one may characterize a wave by its wavelength $\lambda(\omega) = 2\pi u(\omega)/\omega$.
- ⁵For clarity we have considered only temperatures below 1.2° K and have neglected heat conduction.
- ⁶Historical tradition is responsible for the use of the two terms "phonons" and "rotons."
- ⁷Private communication from L. P. Pitaevskii.
- ⁸An attempt has been made [73] to explain this $\delta u_1(\omega)$ relation on the basis of an energy spectrum containing a term quadratic in the momentum, and the numerical value of the constant in Eq. (2.6) was successfully reconciled with the neutron data on $\epsilon(p)$ [60]. However, it was shown in [74] that the spectrum with the quadratic term in the momentum is much less successful than the widely used spectrum (2.5) in accounting for the pressure dependence of the heat capacity and the velocity of first sound. We also note that the neutron "does not know" what causes the dispersion.
- ⁹Hence in this region, second sound in He⁴ is "ordinary" sound in the phonon gas.
- ¹⁰We note that the heat exchange between the film and the substrate and between the film and the vapor above it was not taken into account in deriving Eqs. (5.3), (5.4), (5.6), and (5.7). To take the interaction of the film with the substrate and the vapor rigorously into account one must use not only the hydrodynamic equations for the superfluid film, but also the equations for the temperature distribution in the substrate and for the viscous, acoustic, and thermal waves in the vapor [166, 168]. Such calculations lead to much more cumbersome formulas, which, however, give essentially the same results except in the case of unsaturated films.
- ¹¹With the normal component clamped.
- ¹²As before, we shall call them first and second sound.
- ¹³A similar treatment of wave processes in narrow channels has also been given in [191, 192].
- ¹⁴We note that in [219, 225–228], where the sound absorption problem was solved in the τ approximation, the fermion relaxation time enters as an adjustable parameter. The τ approximation means essentially that only the zeroth and first harmonics in the angular expansion of the distribution function are retained in the collision integral in the kinetic equation. As has been recently shown [229], the contribution to sound absorption from the second harmonic is of the same order as those from the zeroth and first harmonics.

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