QCD SCALES AND CHIRAL SYMMETRY IN FINITE NUCLEI

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Abstract

We report on our progress in the calculation of nuclear ground-state properties using effective Lagrangians whose construction is constrained by QCD scales and chiral symmetry. Good evidence is found that QCD and chiral symmetry apply to finite nuclei.

Introduction: In 1992 a Dirac-Hartree calculation in mean field approximation was performed by Nikolaus, Hoch, and Madland (NHM) [1] for the nuclear ground-state properties of 57 nuclei and saturated nuclear matter. Their Lagrangian was motivated by empirically based improvements to the Walecka scalar-vector model [2,3], but using contact interactions (point couplings) to allow treatment of the Fock (exchange) terms. The nine coupling constants of the NHM Lagrangian were determined in a self-consistent procedure that solved the model equations for several nuclei simultaneously in a nonlinear least-squares adjustment algorithm with respect to well-measured nuclear ground-state observables. The predictive power of the extracted coupling constants is better than had been expected both for other finite nuclei and for the properties of saturated nuclear matter.

In 1996 Friar, Madland, and Lynn (FML) [4] observed that whereas the nine empirically based coupling constants of NHM span 13 orders of magnitude, if they are instead scaled in accordance with the QCD-based Lagrangian of Manohar and Georgi [5], and the role of chiral symmetry in weakening N-body forces is taken into account (Weinberg [6-7]), then six of the nine scaled coupling constants are *natural*, that is, they are dimensionless numbers of order 1. This is potentially an important result because it may mean that (a) QCD and chiral symmetry apply to finite nuclei and, if so, (b) heretofore unattainable accuracy and predictive power in the nuclear many-body problem may be within reach. Here, it is important to note that our work does not test QCD, or chiral symmetry, but rather *effective Lagrangians* whose construction is *constrained* by QCD and chiral symmetry.

In the next sections the NHM relativistic point coupling model is briefly summarized, the roles of QCD scaling and chiral symmetry are briefly discussed and quantified, a more complete point coupling Lagrangian and first results using it are presented, and the current status is given.

NHM Relativistic Point Coupling Model: The NHM model is a self-consistent Dirac Hartree-(Fock) model utilizing contact interactions (point couplings) in the mean field ($\psi \longrightarrow \langle \psi \rangle$) and no Dirac sea approximations. The model consists of four-, six-, and eight-fermion point couplings leading to scalar and vector densities with both isoscalar and isovector components, derivatives of the densities to simulate the finite ranges of the mesonic interactions, but no explicit mean meson fields; instead, mean nucleon fields in Skyrme-type approximation. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4f} + \mathcal{L}_{hot} + \mathcal{L}_{der} + \mathcal{L}_{em} \ , \quad \text{where} \tag{1}$$

$\mathcal{L}_{free}$ and $\mathcal{L}_{em}$ are the kinetic and electromagnetic terms, and

$$\begin{aligned} \mathcal{L}_{4f} &= -\frac{1}{2}\alpha_S(\bar\psi\psi)(\bar\psi\psi) - \frac{1}{2}\alpha_V(\bar\psi\gamma_\mu\psi)(\bar\psi\gamma^\mu\psi) \\ &\quad -\frac{1}{2}\alpha_{TS}(\bar\psi\vec\tau\psi)\cdot(\bar\psi\vec\tau\psi) - \frac{1}{2}\alpha_{TV}(\bar\psi\vec\tau\gamma_\mu\psi)\cdot(\bar\psi\vec\tau\gamma^\mu\psi) \ , \end{aligned} \tag{2}$$

$$\begin{aligned} \mathcal{L}_{hot} &= -\frac{1}{3}\beta_S(\bar\psi\psi)^3 - \frac{1}{4}\gamma_S(\bar\psi\psi)^4 \\ &\quad -\frac{1}{4}\gamma_V[(\bar\psi\gamma_\mu\psi)(\bar\psi\gamma^\mu\psi)]^2 \ , \ \text{and} \end{aligned} \tag{3}$$

$$\begin{aligned} \mathcal{L}_{der} &= -\frac{1}{2}\delta_S(\partial_\nu\bar\psi\psi)(\partial^\nu\bar\psi\psi) \\ &\quad -\frac{1}{2}\delta_V(\partial_\nu\bar\psi\gamma_\mu\psi)(\partial^\nu\bar\psi\gamma^\mu\psi) \ . \end{aligned} \tag{4}$$

In these equations, ψ is the nucleon field, the subscripts "S" and "V" refer to the scalar and vector nucleon fields, respectively, and the subscript "T" refers to isovector fields containing the nucleon isospin $\vec\tau$. The physical makeup of $\mathcal{L}$ is that $\mathcal{L}_{4f}$ is a four-fermion interaction, while $\mathcal{L}_{hot}$ contains six-fermion and eight-fermion interactions, and $\mathcal{L}_{der}$ contains derivatives in the nucleon densities. There are a total of nine coupling constants.

Minimizing the expectation value of the Hamiltonian corresponding to Eq. (1) in the space of Slater determinants $|\phi\rangle$ leads to the Dirac-Hartree equations containing the following potentials:

$$V_S = \alpha_S\rho_S + \beta_S\rho_S^2 + \gamma_S\rho_S^3 + \delta_S\Delta\rho_S \ , \tag{5}$$

$$V_V = \alpha_V\rho_V + \gamma_V\rho_V^3 + \delta_V\Delta\rho_V \ , \tag{6}$$

$$V_{TS} = \alpha_{TS}\rho_{TS} \ , \ \text{and} \tag{7}$$

$$V_{TV} = \alpha_{TV}\rho_{TV} \ , \tag{8}$$

where Eq. (5) is the isoscalar-scalar potential corresponding to σ meson (fictitious) exchange, Eq. (6) is the isoscalar-vector potential corresponding to ω meson exchange, Eq. (7) is the isovector-scalar potential corresponding to δ meson exchange, and Eq. (8) is the isovector-vector potential corresponding to ρ meson exchange. In these latter equations the scalar density is given by $\rho_S = \langle\phi|\bar\psi\psi|\phi\rangle$, the vector density is given by $\rho_V = \langle\phi|\bar\psi\gamma_0\psi|\phi\rangle$, the isovector-scalar density is given by $\rho_{TS} = \langle\phi|\bar\psi\tau_3\psi|\phi\rangle$, and the isovector-vector density is given by $\rho_{TV} = \langle\phi|\bar\psi\tau_3\gamma_0\psi|\phi\rangle$.

The nine coupling constants of the NHM Lagrangian were determined in a self-consistent procedure that solves the Dirac-Hartree equations for several nuclei simultaneously in a non-linear least-squares adjustment algorithm of Levenberg-Marquardt type with respect to well-measured nuclear ground-state observables. The well-measured observables used are (a) the ground-state masses (binding energies), (b) the rms charge radii, and (c) the spin-orbit splittings of the least-bound neutron and proton spin-orbit pairs. The spherical closed-shell nuclei ^{16}O, ^{88}Sr, and ^{208}Pb were chosen for the determination of the coupling constants (12 observables to determine 9 coupling constants). The NHM coupling constants are given in Table 1 where the first four coupling constants refer to Eq. (2), the next three refer to Eq. (3), and the remaining two refer to Eq. (4). They span 13 orders of magnitude.

Table 1: Optimized Coupling Constants for the Relativistic Point Coupling Model

Coupling Constant	Magnitude	Dimension
α_S	-4.508×10^{-4}	MeV^{-2}
α_{TS}	7.403×10^{-7}	MeV^{-2}
α_V	3.427×10^{-4}	MeV^{-2}
α_{TV}	3.257×10^{-5}	MeV^{-2}
β_S	1.110×10^{-11}	MeV^{-5}
γ_S	5.735×10^{-17}	MeV^{-8}
γ_V	-4.389×10^{-17}	MeV^{-8}
δ_S	-4.239×10^{-10}	MeV^{-4}
δ_V	-1.144×10^{-10}	MeV^{-4}

With these nine coupling constants one can calculate the following for spherical closed-shell nuclei: (a) single-particle Dirac wave functions and eigenvalues for both protons and neutrons, (b) nuclear ground-state mass and binding energy, (c) proton and neutron densities and their moments, (d) nuclear charge density and its moments, and (e) isoscalar- and isovector-, scalar and vector, potentials. For example, Table 2 gives the average absolute deviations of calculated binding energies and rms charge radii from the measured values for a number of cases. This is an encouraging result, especially if one notes that the corresponding rms deviations are even smaller.

Table 2: Average Absolute Deviations of Calculated Observables from Measured Observables for the Relativistic Point Coupling Model

Observable	Avg. Abs. Deviation	Number of Cases
E_B	2.52 MeV	34
$< r^2 >^{1/2}_{charge}$	0.020 fm	17

QCD Scales and Chiral Symmetry: An $SU(2) \times SU(2)$ Lie algebra is generated by the commutation rules of vector and axial charges. Assuming that axial currents are approximately conserved, the resulting symmetry is called *chiral symmetry*. In the exact chiral limit quarks are massless and the Goldberger-Treiman relation, connecting the strong and weak interactions, is exact:

$$g_A(0) = \frac{G_A}{G_V} = g_{\pi NN}\frac{f_\pi}{m_N} , \tag{9}$$

where g_A is the strong axial-vector coupling constant, G_A and G_V are the weak axial-vector and polar-vector coupling constants, respectively, $g_{\pi NN}$ is the effective pion-nucleon coupling constant, f_π is the pion decay constant, and m_N is the nucleon mass. In 1990 Weinberg [6] addressed the (broken) chiral symmetry and introduced chiral perturbation theory into nuclear physics and showed that chiral Lagrangians predict the suppression of N-body forces. He accomplished this by constructing the most general possible chiral Lagrangian involving pions and low-energy nucleons as an infinite series of allowed derivative and contact interaction terms and by using QCD mass scales and dimensional power counting to categorize the terms of the series according to their characteristic (average) momentum or energy scales. He concluded that N-body forces were a series in the ratio of a small momentum scale to a large one, leading to a systematic suppression of N-body forces. That is, the infinite series is not physically infinite.

Consider the generic Lagrangian for pions ($\vec{\pi}$) and nucleons (ψ) and containing derivatives, (∂^μ), used in dimensional power counting by Manohar and Georgi [5], and later refined by Weinberg [7] and Lynn [8]:

$$\mathcal{L} \sim -c_{lmn} \left[\frac{\overline{\psi}\psi}{f_\pi^2 \Lambda}\right]^l \left[\frac{\vec{\pi}}{f_\pi}\right]^m \left[\frac{\partial^\mu, m_\pi}{\Lambda}\right]^n f_\pi^2 \Lambda^2 \tag{10}$$

where f_π and m_π are the pion decay constant, 92.4 MeV, and pion mass, 139.6 MeV, respectively. If the theory is *natural* [5,8], this Lagrangian should lead to dimensionless coefficients c_{lmn} of order unity for each order in the QCD large-mass scale, $\Lambda = 1$ GeV. The chiral constraint is given by

$$\Delta = l + n - 2 \geq 0, \tag{11}$$

which guarantees that no Λ appears in the numerator of $\mathcal{L}$, and hence the Lagrangian is a series in Λ^{-1} and therefore converges. Thus, all information on scales ultimately resides in the c_{lmn}. If they are natural, QCD scaling works.

As a first test, the nine coupling constants of the NHM Lagrangian are again shown in Table 3, both in dimensional (identical to Table 1) and dimensionless form, the latter obtained by equating Eqs. (2)–(4) and Eq. (10). [Note that we use $\vec{\tau}$ in Eq. (2) and $\vec{t} = \frac{1}{2}\vec{\tau}$ in Eq. (10)]. One sees that the nine terms of the NHM Lagrangian represent portions of three different orders in the large-mass QCD scale Λ, namely, Λ^0, Λ^{-1}, and Λ^{-2}. However, while the nine

Table 3: Optimized Coupling Constants for the Relativistic Point Coupling Model and Corresponding Dimensional Power Counting Coefficients and Chiral Expansion Order

Coup. Const.	Magnitude	Dimension	c_{lmn}	Order
α_S	-4.508$\times 10^{-4}$	MeV^{-2}	-1.98	Λ^0
α_{TS}	7.403$\times 10^{-7}$	MeV^{-2}	0.0128	Λ^0
α_V	3.427$\times 10^{-4}$	MeV^{-2}	1.48	Λ^0
α_{TV}	3.257$\times 10^{-5}$	MeV^{-2}	0.56	Λ^0
β_S	1.110$\times 10^{-11}$	MeV^{-5}	0.28	Λ^{-1}
γ_S	5.735$\times 10^{-17}$	MeV^{-8}	9.28	Λ^{-2}
γ_V	-4.389$\times 10^{-17}$	MeV^{-8}	-7.10	Λ^{-2}
δ_S	-4.239$\times 10^{-10}$	MeV^{-4}	-1.84	Λ^{-2}
δ_V	-1.144$\times 10^{-10}$	MeV^{-4}	-0.49	Λ^{-2}

original coupling constants span 13 orders of magnitude, the dimensional-power-counting coefficients c_{lmn} are six of order (1), two of order (10), and one of order (10^{-2}). Given that the c_{lmn} should be of order unity if they are natural this is a surprisingly good result since it has been obtained with an incomplete mix of terms from three orders in Λ and the pions have been ignored, that is, assumed to cancel out. Presumably, the absent terms are represented by unphysical (unnatural) values of some of the existing c_{lmn} and this introduces yet other unphysical consequences. The question is can we improve on this situation?

Revisit the Relativistic Point Coupling Model: In our subsequent calculations studying the NHM coupling constants we observed that while only one of the two isospin-dependent terms is natural, namely, $c_{\alpha_{TV}}$, the sum of the two, namely, ($c_{\alpha_{TS}} + c_{\alpha_{TV}}$), appears to be natural. [We connect the 1st and 4th columns of Table 3 with this notation]. We also observed that while both eight-fermion interactions are unnatural, their sum ($c_{\gamma_S} + c_{\gamma_V}$) appears to be natural. These two behaviors persist throughout an exploration of the χ^2 space occupied by the nine coupling constants of the NHM Lagrangian. Therefore, we performed a more detailed grid-search on $c_{\alpha_{TS}}$ and $c_{\alpha_{TV}}$ that resulted in a total of three separate minima corresponding to three different sets of these coupling constants. These are, approximately, the sets {0.01,0.56} (original NHM set), {-0.37,0.92}, and {-1.50,1.88}. In the latter two cases the individual coupling constants are natural and their sums are natural whereas in the former case only one coupling constant is natural, but the sum is natural. Note that the three sums are similar having the values, respectively, of 0.57, 0.55, and 0.38. The calculations corresponding to these three minima are labeled (a), (b), and (c), respectively, in Table 4. [The * appearing for calculation (a), that of the original NHM set, refers to the fact that the spin-orbit pair weights here are more stringent than for the remainder of the table so the χ^2/pt is correspondingly higher]. In this table, $\langle\delta\mathrm{BE}\rangle$ and $\langle\delta\mathrm{RMS}\rangle$ are the average absolute deviations of calculated from measured binding energies and rms charge radii, respectively (as in Table 2).

Table 4: Summary of Important Calculations to Date

Calc.	# Coup. Const.	# Natural	χ^2/pt	$\langle\delta\mathrm{BE}\rangle$ (MeV)	$\langle\delta\mathrm{RMS}\rangle$ (fm)
a	9	6	*8.94	2.52	0.020
b	9	7	4.10	2.86	0.021
c	9	7	4.33	3.78	0.031
d	10	8	4.94	2.60	0.022
e	9	9	57.61	8.90	0.125
f	10	10	68.92	6.31	0.118

Calculations (b) and (c), in comparison to (a), show that although naturalness occurs for 7 of 9 coupling constants in two cases, neither of these has better predictive power than the original NHM set with only 6 natural coupling constants. However, it is clear that the predictive power of calculation (b) is only slightly less good than that of calculation (a).

These results suggest that the incompleteness of the NHM Lagrangian (four terms of order Λ^0, one term of order Λ^{-1}, and four terms of order Λ^{-2}) should be addressed. In particular, there are only two terms containing isospin (both of order Λ^0) and only one term of order Λ^{-1}.

Accordingly, a new term was added to the Lagrangian, with coupling constant β_{TS}, that is of isovector-scalar character and of order Λ^{-1}. The new Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{4f} &= -\frac{1}{2}\alpha_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \\
&\quad -\frac{1}{2}\alpha_{TS}(\bar{\psi}\vec{\tau}\psi)\cdot(\bar{\psi}\vec{\tau}\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_\mu\psi)\cdot(\bar{\psi}\vec{\tau}\gamma^\mu\psi) \ ,
\end{aligned} \tag{12}$$

$$\begin{aligned}
\mathcal{L}_{hot} &= -\frac{1}{3}\beta_S(\bar{\psi}\psi)^3 - \frac{1}{3}\beta_{TS}(\bar{\psi}\vec{\tau}\psi)\cdot(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\psi) \\
&\quad -\frac{1}{4}\gamma_S(\bar{\psi}\psi)^4 - \frac{1}{4}\gamma_V[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)]^2 \ , \text{ and}
\end{aligned} \tag{13}$$

$$\begin{aligned}
\mathcal{L}_{der} &= -\frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) \\
&\quad -\frac{1}{2}\delta_V(\partial_\nu\bar{\psi}\gamma_\mu\psi)(\partial^\nu\bar{\psi}\gamma^\mu\psi) \ .
\end{aligned} \tag{14}$$

And the corresponding new potentials appearing in the Dirac-Hartree equations are given by

$$V_S = \alpha_S\rho_S + \beta_S\rho_S^2 + \gamma_S\rho_S^3 + \delta_S\Delta\rho_S + \frac{1}{3}\beta_{TS}\rho_{TS}^2 \ , \tag{15}$$

$$V_V = \alpha_V\rho_V + \gamma_V\rho_V^3 + \delta_V\Delta\rho_V \ , \tag{16}$$

$$V_{TS} = \alpha_{TS}\rho_{TS} + \frac{2}{3}\beta_{TS}\rho_S\rho_{TS} \ , \text{ and} \tag{17}$$

$$V_{TV} = \alpha_{TV}\rho_{TV} \ . \tag{18}$$

There are now 10 coupling constants in the Lagrangian. The new term appearing in Eq. (13) results in two additions to the potentials appearing in the Dirac-Hartree equations, one in the isoscalar-scalar potential, Eq. (15), that does not change sign with isospin but does contain the isovector-scalar density ρ_{TS}, and one in the isovector-scalar potential, Eq. (17), that does change sign with isospin and also contains the isovector-scalar density.

The χ^2 minimization process with the new Lagrangian and 10 coupling constants resulted in calculation (d) of Table 4. Eight of the ten coupling constants are natural and the sum of the remaining two, c_{γ_S} and c_{γ_V} of the eight-fermion interaction, is also natural. The predictive power is almost as good as that of the original NHM Lagrangian, and is better than that of calculations (b) and (c). However, the χ^2/pt for this calculation is somewhat larger than those of calculations (b) and (c) whereas one would expect it to instead be somewhat smaller. This behavior is at least partly due to the numerical difficulties associated with having a *self-consistent* Dirac-Hartree solver as the function call in a nonlinear least-squares minimization algorithm for the Dirac coupling constants.

At this point the constraint $c_{\gamma_S} \equiv c_{\gamma_V}$ was invoked and grid-search calculations were performed where the starting values for these eight-fermion interactions were taken as one-half their sum from calculation (b), for the original Lagrangian, and from calculation (d), for the new Lagrangian. The results from the two grid-search calculations are labeled (e) and (f) in

Table 4. With this constraint all of the coupling constants are natural for both the original, Eqs. (2)–(4), and the new, Eqs. (12)–(14), Lagrangians. However, the χ^2/pt values are factors of $\sim$14 and $\sim$15 higher, and the predictive powers are factors of $\sim$3 to $\sim$5 worse. The constraint was then removed, for both Lagrangians, and full-search calculations were performed. These resulted in small changes in the respective sets of coupling constants; no new minima were found.

Conclusions: Our calculations to date, summarized in Table 4, constitute good evidence that QCD and chiral symmetry apply to finite nuclei. The evidence, however, is at this time only partly compelling. The goal is to construct a Lagrangian whose coupling constants are not only all natural, but whose predictive power is superior to the original NHM Lagrangian. This goal has not yet been achieved. To achieve it, additional isospin dependence may be required, tensor terms may be required, and the pions may have to be included. The work will continue.

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