Neutrino-nucleon scattering can occur via charged current processes where a $W$ boson is exchanged:

$$\nu + N \rightarrow \ell^- + X \tag{1}$$

$$\overline{\nu} + N \rightarrow \ell^+ + X \tag{2}$$

and via neutral current processes where a $Z^-$ boson is exchanged:

$$\nu + N \rightarrow \nu + X \tag{3}$$

$$\overline{\nu} + N \rightarrow \nu + X \tag{4}$$

In the naive quark parton model, the neutrino scatters from a quark in the nucleon and the allowed scattering processes are constrained by charge conservation and the helicity dependent couplings of the weak bosons.

In particular, charged current neutrino interactions can occur only from left-handed d-type quarks of charge $-1/3$ or right handed u-type antiquarks of charge $-2/3$. Anti-neutrino interactions select left-handed u-type quarks and right-handed d-type antiquarks. Scatters such as $\nu - \mathrm{d}$ in which the helicities cancel are isotropic while scatters such as $\overline{\nu} - \mathrm{u}$ in which the helicities sum to 1 have angular factors of $[(1 + \cos\theta_{CM})/2]^2$.

Figure N illustrates a scatter from a $d$ quark within a proton. M is the proton mass, $-Q^2$ is the momentum transfer squared, $\nu$ is the energy transfer in the lab frame and $y$ is the fractional energy transfer $\nu/E_\nu$. The variable $x = Q^2/2M\nu$ can be interpreted as the momentum fraction carried by the struck quark. $(1-y) = (1 + cost\theta_{CM})/2$ is the helicity factor mentioned above.

The differential cross section for charged current neutrino and anti-neutrino scattering from a proton in the naive quark parton model can be written as:

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{4M_W^4 G_F^2 M E_\nu x}{\pi(Q^2 + M_W^2)^2}\left[[\hat{\mathrm{d}}(x) + \hat{\mathrm{s}}(x)] + [\hat{\overline{\mathrm{u}}}(x) + \hat{\overline{\mathrm{c}}}(x)](1-y)^2\right] \tag{5}$$

$$\frac{d\sigma^{\overline{\nu} p}}{dxdy} = \frac{4M_W^4 G_F^2 M E_\nu x}{\pi(Q^2 + M_W^2)^2}\left[[\hat{\overline{\mathrm{d}}}(x) + \hat{\overline{\mathrm{s}}}(x)] + [\hat{\mathrm{u}}(x) + \hat{\mathrm{c}}(x)](1-y)^2\right] \tag{6}$$

where û, d̂, ŝ ... are the probabilities for finding a quark with the right helicity carrying momentum fraction $x$ inside the nucleon.

For the proton, there are two valence us, one valence d and a large number of 'sea' quarks which occur in quark anti-quark pairs. The s fraction is believed to be around 1/2 the $u$ and $d$ antiquark fractions while charm quarks are substantially suppressed.

These equations can describe both polarized and unpolarized scattering if one remembers that the probability of a quark being left handed in an unpolarized proton is 1/2 and q̂ is thus $\mathrm{q}(x)/2$ where $\mathrm{q}(x)$ is the total probability of finding such a quark in the proton. Similarly, the probability of finding a left-handed quark in a polarized proton is $\hat{\mathrm{q}}x) = 1/2(\mathrm{q}(x) + \Delta\mathrm{q}(x))$ where $\Delta\mathrm{q}(x)$ is the difference between the probability densities for quarks polarized parallel and anti-parallel to the proton spin. Spin effects will be discussed later, for now we will consider only unpolarized targets.

Most heavy targets are approximately isoscalar with equal numbers of protons and neutrons. Isospin invariance implies that

$$\mathrm{u}(x)_p = \mathrm{d}(x)_n, \overline{\mathrm{u}}(x)_p = \overline{\mathrm{d}}(x)_n \tag{7}$$

$$\mathrm{d}(x)_p = \mathrm{u}(x)_n, \overline{\mathrm{d}}(x)_p = \overline{\mathrm{u}}(x)_n \tag{8}$$

$$\mathrm{s}(x)_p = \mathrm{s}(x)_n, \overline{\mathrm{s}}(x)_p = \overline{\mathrm{s}}(x)_n \tag{9}$$

$$\mathrm{c}(x)_p = \mathrm{c}(x)_n, \overline{\mathrm{c}}(x)_p = \overline{\mathrm{c}}(x)_n \tag{10}$$

and the cross section for scattering from an isoscalar target can be written in terms of probabilities for parton densities within the proton.

$$\frac{d\sigma^{\nu N}}{dxdy} = \frac{G_F^2 M E_\nu x}{\pi}\Big[[\mathrm{u}(x) + \mathrm{d}(x) + 2\mathrm{s}(x)] + [\overline{\mathrm{u}}(x) + \overline{\mathrm{d}}(x) + 2\overline{\mathrm{c}}(x)](1-y)^2\Big] \tag{11}$$

$$\frac{d\sigma^{\overline{\nu} N}}{dxdy} = \frac{G_F^2 M E_\nu x}{\pi}\Big[[\mathrm{u} + \mathrm{d}(x) + 2\mathrm{c}(x)](1-y)^2 + [\overline{\mathrm{u}}(x) + \overline{\mathrm{d}}(x) + 2\overline{\mathrm{s}}(x)]\Big] \tag{12}$$

Where the approximation $M_W >> Q^2$ has been used. Data taken on iron [**?**,**?**,**?**] indicate that the integrals of the parton distributions $x\mathrm{q}(x)$ are approximately, (0.26, 0.15, 0.015, $< 0.01$) (0.030, 0.035,0.015, $< 0.01$) for $(x\mathrm{u}, x\mathrm{d}, x\mathrm{s}, x\mathrm{c})$ and $(x\overline{\mathrm{u}}, x\overline{\mathrm{d}}, x\overline{\mathrm{s}}, x\overline{\mathrm{c}})$.

This leads to the approximate expression:

$$\frac{d\sigma^{\nu N}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi}\left[0.44 + 0.065(1-y)^2\right] \tag{13}$$

$$\frac{d\sigma^{\overline{\nu} N}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi}\left[0.41(1-y)^2 + 0.095\right] \tag{14}$$

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi}\left[0.33 + 0.060(1-y)^2\right] \tag{15}$$

$$\frac{d\sigma^{\overline{\nu} p}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi}\left[0.52(1-y)^2 + 0.10\right] \tag{16}$$

The neutrino cross section is thus approximately flat in the inelasticity variable $y$ while the antineutrino cross section favors low values of $y$ and hence high values for the scattered lepton energy.

These expressions are modified by higher-twist processes such as quasi-elastic scattering, non-conserved axial vector currents, resonances, by final state mass effects if a charm particle or $\tau$ lepton appears in the final state, longitudinal components and perturbative Quantum Chromodynamic corrections which introduce a $\log Q^2$ dependence. These additional factors are reasonably small for neutrino energies above 20 GeV and the total cross section can be reasonably approximated by

$$\sigma(\nu + N \to \mu^- + X) \approx 0.68 \times 10^{-38}\mathrm{cm}^2 \times (E_\nu, \mathrm{GeV}) \tag{17}$$

$$\sigma(\overline{\nu} + N \to \mu^+ + X) \approx 0.34 \times 10^{-38}\mathrm{cm}^2 \times (E_{\overline{\nu}}, \mathrm{GeV}) \tag{18}$$

The number of charged current events observed per incident neutrino on an isoscalar target of mass density $\rho$ and length $l$ is:

$$N(\nu + N \to \mu^- + X) = 0.67 \times 10^{-38}\mathrm{cm}^2 \times (E_\nu, \mathrm{GeV})\rho l N_A \tag{19}$$

$$= 1.13 \times 10^{-14}(E_\nu, \mathrm{GeV})\text{events per gr/cm}^2 \tag{20}$$

$$N(\overline{\nu} + N \to \mu^+ + X) = 0.34 \times 10^{-38}\mathrm{cm}^2 \times (E_\nu\ , \mathrm{GeV})\rho l N_A = \tag{21}$$

$$= 0.56 \times 10^{-14}(E_{\overline{\nu}}, \mathrm{GeV}) \text{ events per gr/cm}^2 \tag{22}$$

$$\tag{23}$$

For an integrated flux of $10^{-20}$ 25 GeV neutrinos one can thus expect around 28 million charged current neutrino and 14 million anti-neutrino interactions **per gr/cm$^2$** of target.

For a proton target the rates should be more similar, with a parton model prediction of:

$$\sigma(\nu + p \to \mu^- + X) \approx 0.5 \times 10^{-38} \mathrm{cm}^2 \times (E_\nu, \mathrm{GeV}) \tag{24}$$

$$\sigma(\overline{\nu} + p \to \mu^+ + X) \approx 0.4 \times 10^{-38} \mathrm{cm}^2 \times (E_{\overline{\nu}}, \mathrm{GeV}) \tag{25}$$