Title: How Many Jelly Beans Fit Inside?

Brief Overview:

Students will discover that, even with the same perimeter, a rectangle, a square, a circle, and a triangle will have different areas. They will discover the attributes of a shape that maximizes or minimizes the area. The student will find the area of rectangles, graph data, find the area of a circle, and use the Pythagorean Theorem to find the area of a triangle.

Maryland Content Standards:

1.8.1a Recognize, describe, and extend patterns and functional relationships.
1.8.1c Determine whether functions are discrete or continuous.
1.8.1d Determine whether functions are linear or nonlinear when given graphic examples.
3.8.3a Estimate and determine the circumference and area of circles.
3.8.3b Estimate and determine the area of figures by measuring, partitioning, and using formulas
3.8.3d Determine relationships between length, area, and volume and describe how a change in one measure affects the others.

Grade/Level:

Grades 7 - 9.

Duration/Length:

4-5 days of a 45 minute class period, including either homework or extension time in class and an assessment.

Student Outcomes:

Students will:

- Explain how a change in the length of a rectangle will affect the area of the rectangle and graph the relationship.
- Calculate the area of a circle given the circumference.
- Subdivide a triangle into two triangles, apply the Pythagorean Theorem in order to find missing side lengths, and calculate the area of the triangle.
- Students will use necessary materials(string, graph paper) and math tools (calculators, rulers, etc) to decide which shape yields the highest area, given the same perimeter.

Materials and Resources:

- String or chenille stems in lengths of 36 cm
- Tape to secure the string to the desks
- Rulers
- "post-it" notes
- Jelly beans

Development/Procedures:

Lesson 1 Making Rectangles with a Perimeter of 36

Preassessment and Launch–

How many different polygons can you make with a 36 centimeter string? (unlimited number). Hand out a jelly bean to each student. If this were a container for jelly beans, how could you get the most jelly beans contained within the string? Make that shape. Teacher should provide a place for students to sketch and display their estimations.

Teacher Facilitation –

Students should now make only rectangles with their string. As students make rectangles, they should sketch the rectangles on the centimeter grid paper and record their dimensions in the table, It's a String Thing!.

Student Application –

As students work, ask them to stop and observe their data. Is there anything interesting in the numbers?

Embedded Assessment –

Check that students continue the table to the end of the paper and include all rectangles. Use the answer key to check all values.

Reteaching/Extension –

- Check student tables and guide them to see a pattern.
- Ask students what if one of your sides measures 2.4 cm? What would the others be? What would the area be?

Lesson 2 Graphing our Rectangle Data

Preassessment –

Review the questions following the tables on observing It's a String Thing! In addition, ask the students how does the length change? How does the width change? How does the perimeter change? The area? Find a numerical pattern that describes how the area changes. (Each step goes up by odd numbers.)

Launch –

Use the data from the table. On the board, draw two axes of a graph. The *x*-axis is the length and the *y*-axis is the width of the rectangles from Lesson 1. (Intervals of 1, from 0 to 18). Have student volunteers come up and plot each point on the graph. Ask the students what does the graph look like? What shape is it? Is there a point called (18,0) on the graph? What about (0, 18)? Or (0,0)? Is there a point called (-1, 19)? Why not? This is a linear function. Students identify the independent and dependent variables.

Teacher Facilitation –

As volunteers come up and plot points on the graph, students graph the information on the activity page that is entitled Picture It! The length should be plotted on the *x*-axis and the area should be plotted on the y axis.

Student Application

After plotting the points from the table, students examine the graph to find the dimensions of the rectangle that gives the maximum area.

Embedded Assessment

Teacher observes students as they construct their graphs and asks questions that lead students to determine the relationship between the data and the graph.

Reteaching/Extension

What would the maximum point of the graph of a 40 cm perimeter square be? (10cm,100 sq cm) A 4 cm square? (1cm, 1 sq cm) A 100 cm square? (25cm, 625 sq cm)

Ask the students is this a discrete or continuous function? A discrete function has value only at the point graphed. Are there points between the ones we plotted? (Yes). Although the function looks discrete, it is a continuous. Have students draw the curve that connects all of the points. Estimate three other coordinates of length and area.

Lesson 2 ***(TI 73 Calculator Extension Option)***

Graphing our Rectangle Data

Preassessment –

Review questions following the tables. How does the length change? How does the width change? How does the perimeter change? The area? Find a numerical pattern that describes how the area changes. (Each step goes up by odd numbers.)

Launch –

Use the data from the table. On the board, draw two axes of a graph. The x axis is the length and the y axis is the width of the rectangles from Lesson 1. (Intervals of 1, from 0 to 18). Have student volunteers come up and plot each point on the graph. What does the graph look like? What shape is it? Is there a point called (18,0) on the graph? What about (0, 18)? Or (0,0)? Is there a point called (-1, 19)? Why not? This is a linear function.

Teacher Facilitation –

Use the TI-73 to plot the relationship between the length and the area.
Press LIST. Enter the lengths from table in L1. Enter the areas into L2.
2^{ND} QUIT
2^{ND} PLOT, ENTER
Select ON and ENTER.
Select the Scatter/Line graph (first graph option). ENTER
XList: L1 YList L2 Mark (any)
WINDOW
Ask students what the range of the lengths is (1 – 17).
Ask students what the range of the areas is (17 – 81).
These are suggested ranges of the graph.
Xmin: 0
Xmax: 20
ςX: (leave as it stands)
Xscl: 2
Ymin: 0
Ymax: 100
Yscl: 10
GRAPH

Student Application –

Students should compare their estimates of what the graph was going to look like and what it actually came out to be.

Use the TRACE key to find the maximum point on the graph. What are the coordinates of that point? What does that mean? (This is the point where the length creates the maximum area.) What is significant about the the length of 9? What is the width when the length is 9? (They are both 9, so the rectangle is a square). Questions are included in the worksheet entitled Picture It!

Embedded Assessment

Do students understand that the point (9,81) represents a length of 9 and an area of 81? Can they name the width of that rectangle? Do students recognize that a square is a rectangle? Can they explain why? Can they explain where the 9 came from in the first place? (36 cm divided by 4 sides.)

Reteaching/Extension

What would the maximum point of the graph of a 40 cm perimeter square be? (100 sq cm) A 4 cm square? (1 sq cm) A 100 cm square? (625 sq cm)

Is this a discrete or continuous function? A discrete function has value only at the point graphed, and not in between. Are there points between the ones we plotted? (Yes, a rectangle can have a length of 4.8 cm, which means a width of 13.2 cm, and an area of 63.36 sq cm.). Although the function looks discrete, it is a continuous function. Have students draw in the curve that connects the origin and all of the points of the curve.

Sketching the Curve on the TI-73

2^{ND} STAT arrow to the CALC menu. Choose QUAD REG. ENTER.

ENTER. Displays:

$Y = ax^2 + bx + c$

$a = -1$

$b = 18$

$c = 0$

Press Y=

$-x^2 + 18x$

GRAPH

Arrow down to change from the first graph to the linear function. TRACE to find decimal values of length and area.

Lesson 3 Circle Round 36

Preassessment – Ask the students the following questions:

What do you know about the circumference of a circle? Do you know an equation? What if you knew the circumference of the circle, what information can you get from it? (radius) What does π mean?

Launch –

Have the students use their 36 cm of string to create a circle on their desk. Use a ruler to estimate the diameter and compare their measurement with a neighbor. Why are the answers different? What would be your radius?

Teacher Facilitation – Facilitate a discussion with the students using the following questions:

Do you have enough information to calculate the area? What is the formula? ($A=\pi r^2$) Where can you get the radius from a perimeter of 36? ($C=2\pi r$). Students should make an estimate of the circle's area before calculating.

Student Application –

As a group, students should complete the review activity entitled Circle Round for practice manipulating equations about circles. The final question guides students to calculate the area of a circle with a perimeter of 36.

Embedded Assessment –

Circulate, looking for problems manipulating formulas. Ask students to evaluate the area of the circle from number16 and 17 (our 36 cm problem). Ask them questions, such as: Is it reasonable? Is there anything unusual about it? (It's quite large compared to the rectangle.)

Reteaching/Extension

Compare the area of the circle with circumference of 36 to the area of the square with perimeter of 36. Is it smaller or larger?

Lesson 4 Triangle Trial

Preassessment –

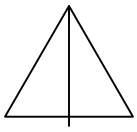
Display a square of any dimension. What is its area? Draw a diagonal. What is the area of each triangle created? What information do you need to find the area of a triangle? (base and height)

Launch –

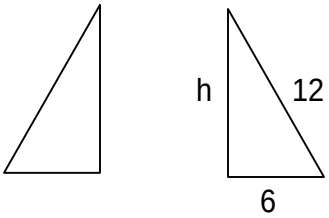
Use the 36 cm of string. Tape one vertex to the desk. or, use a push pin to anchor one point of the string to a soft surface. This will be a stable vertex. Students will use their fingers to form a variety of triangles with the string. They will practice classifying by angle (acute, right, obtuse) and by side (equilateral, scalene, isosceles).

Teacher Facilitation –

Ask students to create an equilateral triangle. Sketch. How long is each side? (12 cm) Ask students to calculate the area. Why is there not enough information yet? Bisect the triangle.



Students should recognize that there are two triangles.



The height of the triangle can be found, but it is missing. Model the Pythagorean Theorem to solve this problem.
$h^2 + 6^2 = 12^2$ $h = (144 - 36)$ $h = (108)$ $h = 10.39$

Student Application –

As a group, ask students to describe how the Pythagorean Theorem was used to calculate the height of the triangle.

Embedded Assessment –

Asks students to make two other isosceles triangles using the 36 cm string. Sketch each one, label their dimensions, calculate the height, and then the area. Use the worksheet Triangle Trial! Student will practice finding the area of a triangle when only its perimeter is known

Reteaching

Each student is given a “post-it” that is in the shape of a square.

Students measure the length and width of the “post-it”.

Teacher asks students to determine the area of the “post-it”.

Teacher then asks students to cut it along one of its diagonals so as to form two triangles. Students should realize that the area of each triangle is half the area of the square that each triangle is an isosceles triangle. Students verify the area of the triangle by taking measurements the height and the base of the triangle and calculating the area.

Summative Assessment:

The final assessment of this unit requires students to determine which shape will allow them to gather the most jelly beans within the boundaries of a 48 cm string. They should use a table, numbers, written descriptions, algebra, a graph, and drawings to illustrate their conclusions. Use Student Resource Sheet 6, My Area is the Highest!

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Name _____ Student Resource 1

It's a String Thing!

You have 36 centimeters of string. How many different rectangles can you with the string?

Complete the table with the lengths and widths of the rectangles. Record the shapes on centimeter grid paper.

Length	Width	Area	Perimeter
1 cm	**17 cm**	**17 cm^2**	**36 cm**
2 cm	**16 cm**	**32 cm^2**	**36 cm**

Name _____ Student Resource 2

It's a String Thing!

Examine the data in your table and then respond to the following questions.

1. What do you know about the perimeter?

   _____

   _____

2. What do you know about the area?

   _____

   _____

3. Are there any other patterns?

   _____

   _____

   _____

4. What is the maximum area? Explain.

   _____

   _____

   _____

5. What would change if the length were 2.6 cm.? Explain.

   _____

   _____

   _____

Bonus

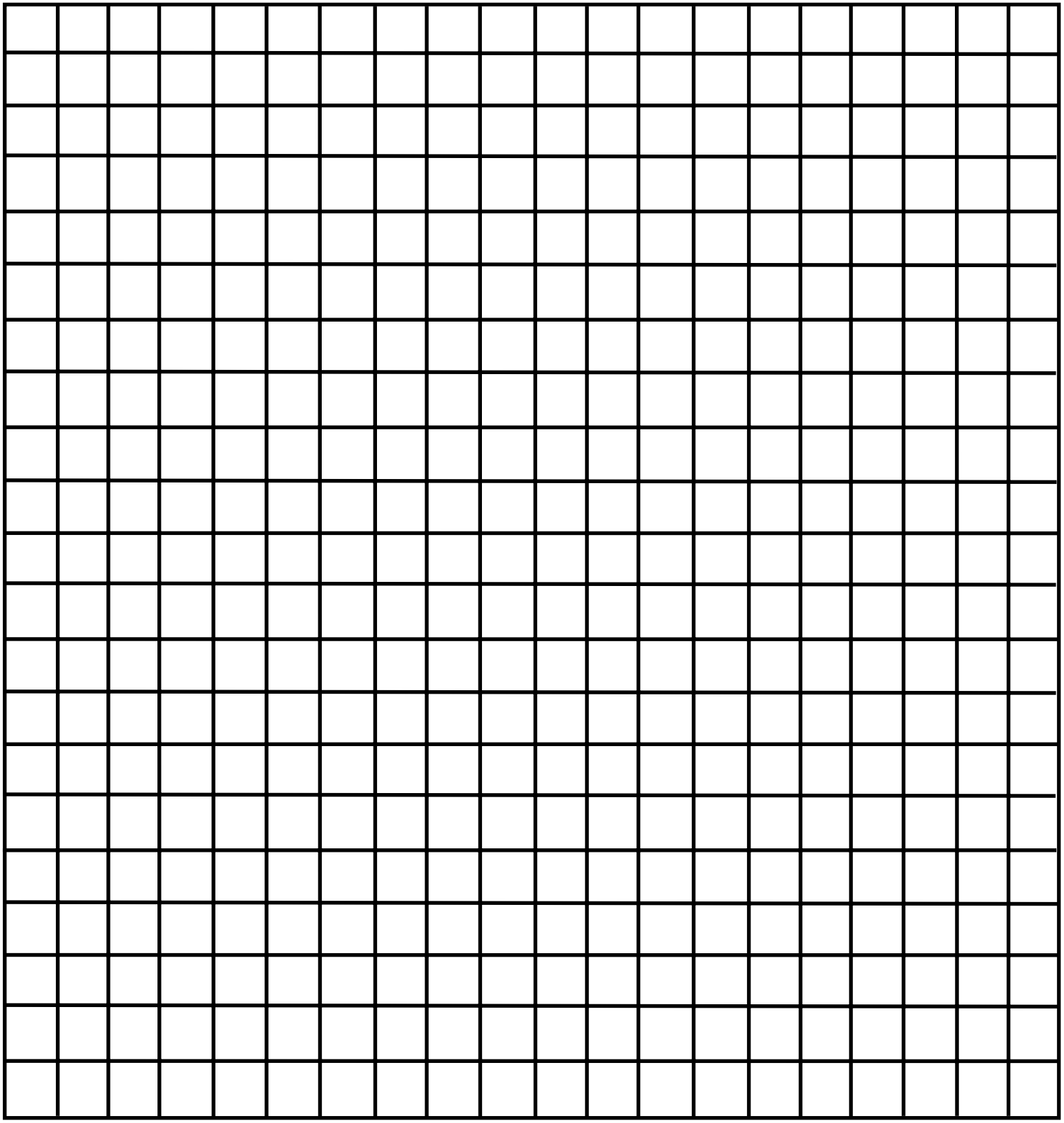
6. Describe what you think the graph of the length and width would look.. Would it be a straight line? A curve?

   _____

   _____

   _____

CENTIMETER GRID PAPER

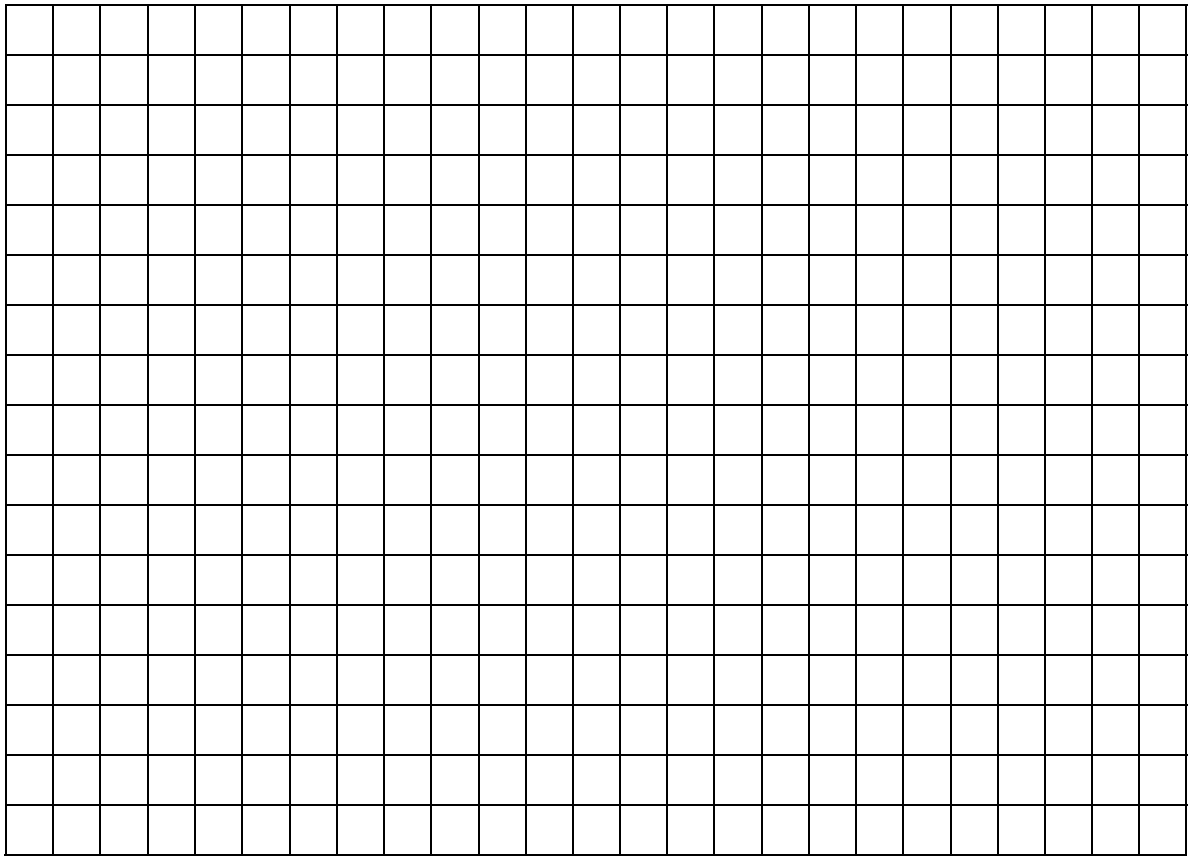


Student Resource 3

Picture It

Independent variable _____range_____ interval_____

Dependent variable _____range_____ interval _____



Describe your graph of length vs area. Is it a straight line? Do you know anything about that shape? Explain.

What are the coordinates of the highest point on the curve?

What is the significance of that point? What does it mean? What is the width of that rectangle?

Would the graph look the same if you graphed the width to the area? Explain.

Name _____ Student Resource 4

Circle Round!

Round to the Hundredths place.

Find the radius.

1. d = 10 _____
2. d = 12.2 _____
3. d = 152 _____

Find the circumference of these circles.
$C = 2\pi r$

4. r = 72 _____
5. r = 140 _____
6. r = 10.6 _____

Find the area of these circles.
$A=\pi r^2$

7. r = 3 _____
8. r = 12 _____
9. r = 9.1 _____

Find the radius.

10. 2 = 2πr _____
11. 64 = 2πr _____
12. 84.2 = 2πr _____
13. 144 = 2πr _____

Explain how you solve for r.

_____

_____

_____

_____

Find the radius.

14. C = 72 _____

Find the area of this circle.

15. _____

Find the radius.

16. C = 36_____

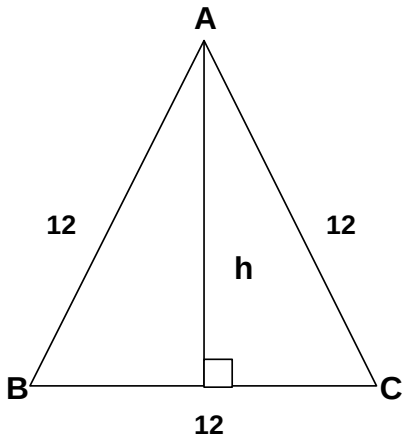
Find the area of this circle.

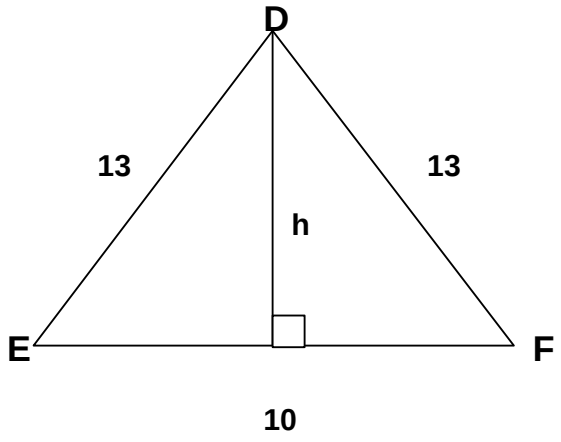
17. _____

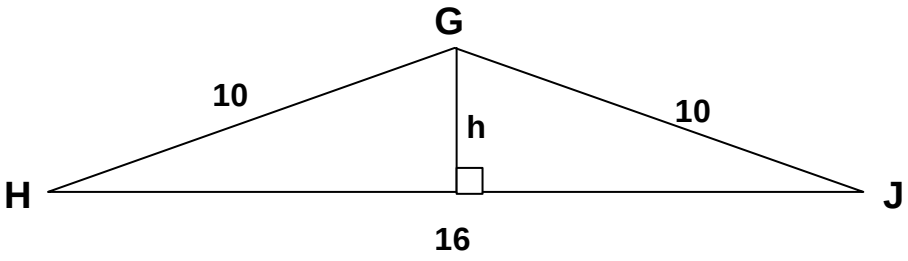
Use a compass and ruler to sketch the last circle from number 16 and 17. Compare the circle to your 36 cm segment of string.

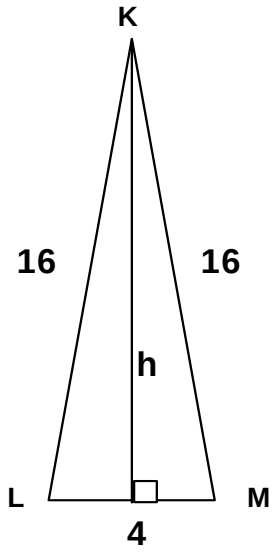
Name _____ Student Resource 5

Triangle Trial









1. Use the Pythagorean Theorem to find the area of each triangle.

	Height	Area
△ABC	_____	_____
△DEF	_____	_____
△GHJ	_____	_____
△KLM	_____	_____

2. Which triangle has the greatest area? Describe it.

3. Recall from the first day of this lesson which special rectangle has the greatest area. Describe it.

4. What was the area of the circle?

5. What do these three shapes have in common?

Name _____ Student Resource 6

My Area is the Highest!

You have 48 cm of string. Use your string to design a shape that will contain the most jelly beans. Use what you know about perimeter and area to design your shape. Sketch and label the dimensions of your shape.

Use a table, shapes, a graph, algebra, numbers, symbols, and mathematical words, etc., to support your answer.

An excellent answer will illustrate at least four shapes, show the areas of all shapes so that comparisons can be made, and justify the conclusion in writing.

Name _____

Answer key Teacher Resource 1

It's a String Thing!

You have 36 centimeters of string. How many different rectangles can you make with that length of string?

Complete the table with the lengths and widths of the rectangles.

Length	Width	Area	Perimeter
1 cm	**17 cm**	**17 cm^2**	**36 cm**
2 cm	**16 cm**	**32 cm^2**	**36 cm**
3 cm	**15 cm**	**45 cm^2**	**36 cm**
4 cm	**14 cm**	**56 cm^2**	**36 cm**
5 cm	**13 cm**	**65 cm^2**	**36 cm**
6 cm	**12 cm**	**72 cm^2**	**36 cm**
7 cm	**11 cm**	**77 cm^2**	**36 cm**
8 cm	**10 cm**	**80 cm^2**	**36 cm**
9 cm	**9 cm**	**81 cm^2**	**36 cm**
10 cm	**8 cm**	**80 cm^2**	**36 cm**
11 cm	**7 cm**	**77 cm^2**	**36 cm**
12 cm	**6 cm**	**72 cm^2**	**36 cm**
13 cm	**5 cm**	**65 cm^2**	**36 cm**
14 cm	**4 cm**	**56 cm^2**	**36 cm**
15 cm	**3 cm**	**45 cm^2**	**36 cm**
16 cm	**2 cm**	**32 cm^2**	**36 cm**
17 cm	**1 cm**	**17 cm^2**	**36 cm**

Name _____

Answer key Teacher Resource 3

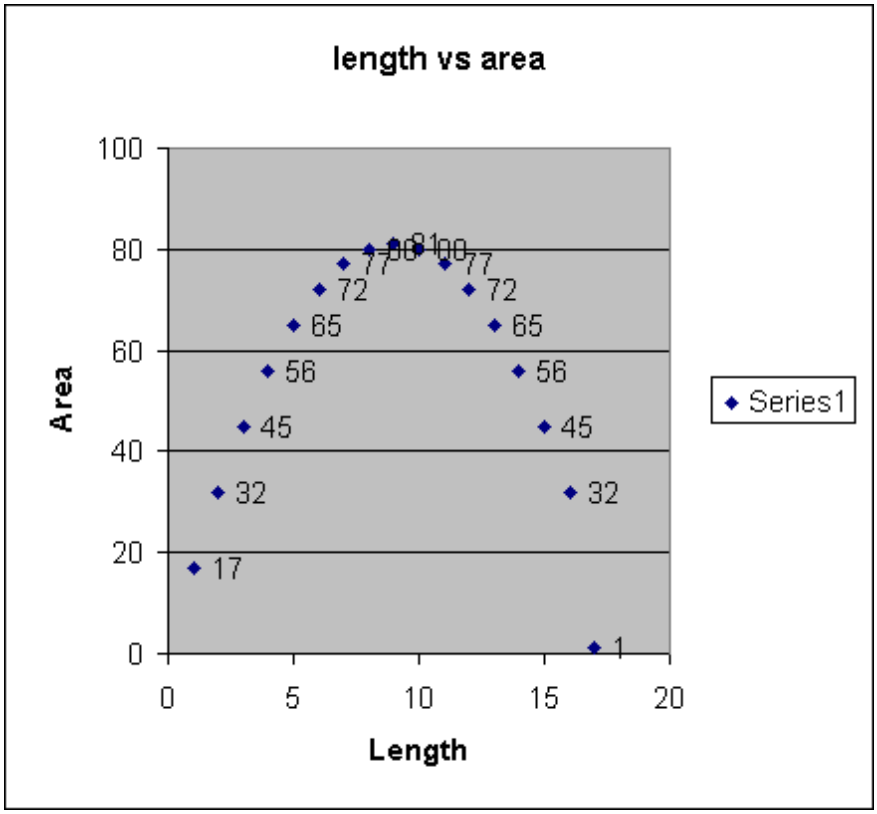
"Picture It"

Answers may vary

Independent variable _____**length**_____ **range**___**0-17 cm**_____ **interval**__**5 cm**___
Dependent variable _____**area**_____ **range**___**0-81 cm^2**____ **interval** _**10 cm^2**__

Answers may vary

1	17
2	32
3	45
4	56
5	65
6	72
7	77
8	80
9	81
10	80
11	77
12	72
13	65
14	56
15	45
16	32
17	1



Describe your graph. Do you know anything about the shape of the graph?

(Answers may vary—As the length increased from 1 cm to 9 cm the area increased from 17cm^2 to 81 cm^2 and as the length increased from 10 cm to 17 cm the area decreased. The shape of the graph is a parabola.)

Name _____

Answer key Circle Round! Teacher Resource 4

Find the radius.

1. d = 10 _____ (5)

2. d = 12.2 _____ (6.1)

3. d = 152 _____ (76)

Find the circumference of these circles.

$C = 2\pi r$

4. r = 72 _____ (452.16)

5. r = 140 _____ (879.2)

6. r = 10.6 _____ (66.57)

Find the area of these circles.

$A=\pi r^2$

7. r = 3 _____ (28.26 sq.units)

8. r = 12 _____ (452.16 sq.units)

9. r = 9.1 _____ (260.02 sq units)

Find the radius.

$10.2 = 2\pi r$ _____ (0.32)

$11.64 = 2\pi r$ _____ (10.19)

$12.84.2 = 2\pi r$ _____ (13.41)

$13.144 = 2\pi r$ _____ (22.93)

Name _____

Explain how you solve for r.

To solve for "r", I divided the diameter by 2

Find the radius.

14. C = 72 _____(11.46)

Find the area of this circle.

15. _____ (412.74 sq. units)

Find the radius.

16. C = 36 _____ (5.73)

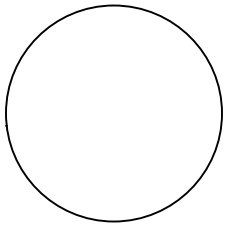
Find the area of this circle.

17. _____ (103.18 sq. units)

Use a compass and ruler to sketch the last circle in number 16 and 17. Compare the circle to your 36 cm segment of string.

Circumference 36 cm

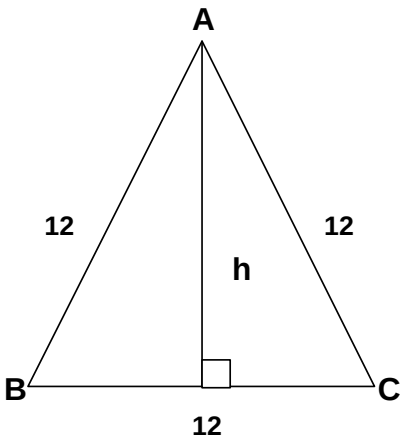
Radius 5.73 cm

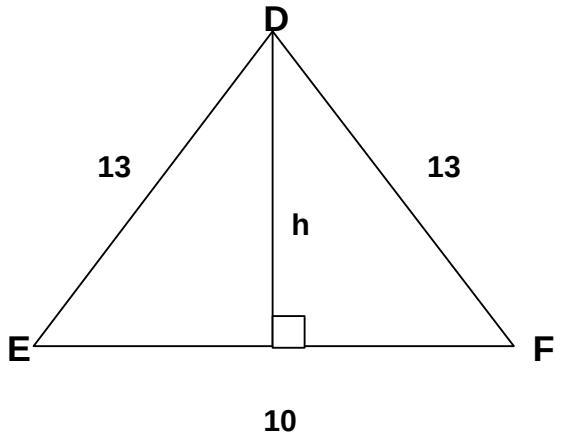


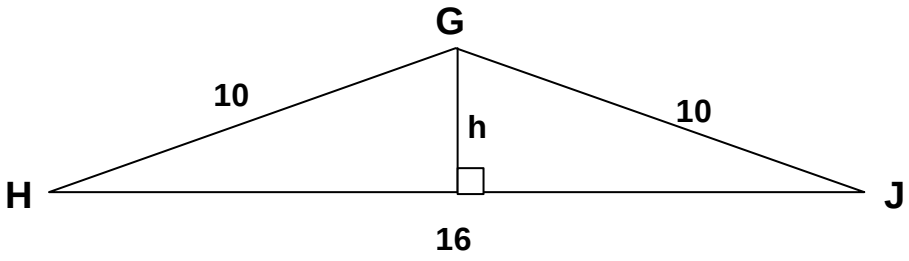
Name _____

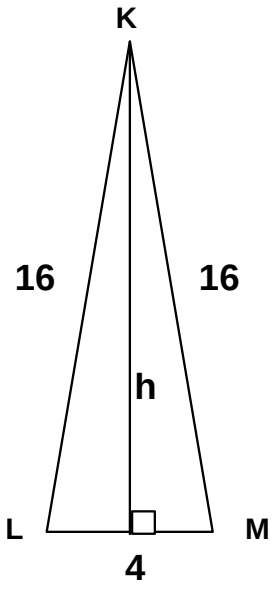
Teacher Resource 5

Triangle Trial (answer key)









1. Use the Pythagorean Theorem to find the area of each triangle.

	Height	Area
△ABC	10.39 units	62.34 sq. units
△DEF	12 units	60 sq. units
△GHJ	6 units	48 sq. units
△KLM	15.87 units	31.74 sq. units

2. Triangle ABC has the greatest area (62.34 cm^2). It is an equilateral triangle.

3. The rectangle with a length of 9 cm and a width of 9 cm had the greatest area. It was a square of 81 cm^2.

4. 103.18 cm^2

5. What these polygons have in common is that they are both equilateral. Triangle ABC is an equilateral triangle and the rectangle is equilateral or a square.

My Area is the Highest!

You have 48 cm of string. Use your string to design a shape that will contain the most jelly beans. Use what you know about perimeter and area to design your shape. Sketch and label the dimensions of your shape. Use a chart, table, graph, algebra, numbers, symbols, mathematical words, etc., to support your answer.

Sample response

Shape	**Dimensions**	**Perimeter/ Circumference**	**Area**
Rectangle	Length 14 cm Width 10 cm	48 cm	140 cm^2
Square	Length 12 cm Width 12 cm	48 cm	144 cm^2
Triangle	Equilateral 16 cm	48 cm	110.85 cm^2
Circle	Radius 7.64	48 cm	183.28 cm^2

The shape that will give me the most jelly beans is the circle because it contains the greatest area.

My Area is the Highest!

Grading Rubic (1-5)

5 Student has found at least four shapes with a perimeter of 48 cm. All calculations are accurate. Student has shown the formulas and steps used for calculations. Written explanation expresses the student's point of view and has counter examples.

4 Student has done a variety of shapes with a perimeter of 48 cm. Calculations are reasonable, if not accurate. Some work may be incomplete, but there is evidence of reasonable calculations. Written explanation is clear, but may not be well developed.

3 Student has found the area of a few shapes, but may have some inaccuracies. Written and graphic explanations are there, but may not be clearly identified.

2 Student has attempted calculations and may have one or two correct. Student may not be able to use calculations to come to a conclusion.

1 Student has attempted the task and shows evidence of thought, but does not arrive at a valid conclusion, and few, if any, accurate calculations.