Characterization of irregularly shaped bodies

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ABSTRACT

The shape of asteroids can at best be described as irregular. However, for certain target opportunities, it is often that a complete characterization the shape may not be necessary for the purpose of mosaicking. In case of slow spinning objects, a simple rectangular bounding is sufficient. Eigenvectors of the scatter matrix from the boundary points of an object can be used to determine the orientation of the bounding rectangle, These eigenvectors correspond physically to the directions about which the 21) projection of the object has maximum and minimum moments of inertia. An optimal mosaic size can then be determined from the aspect ratio of the bounding rectangle, and the size of the rectangle can be used to assist us in determining the starting mosaicking time. in a simple asteroid flyby scenario which the spacecraft travels in a linear trajectory with constant speed, the apparent size of the asteroid can be parametrized in a closed form, The parameter estimation can be solved by a least-squares fit using the size information derived from images taken when the angular diameter of the asteroid is less than the camera's field of view.

Keywords: rectangular bounding, asteroid flyby, mosaicking

1 INTRODUCTION

Unlike most of the planets and their moons, asteroids which are believed to represent '(leftovers" from early planetary formation are likely to be irregularly shaped. A mosaic sequence designed to acquire high resolution asteroid images during flyby should be derived from the estimated information on shape of the asteroid, Without any a prior knowledge of an asteroid, it is difficult to infer its shape from images alone, However, for mosaicking, complete shape characterization may not be necessary under the assumption that the asteroid has a slow spin rate. In this case, a simple rectangular bounding of the asteroid can be helpful in generating a mosaic pointing sequence autonomously. A bounding rectangle allows the spacecraft to monitor and predict the apparent size of the asteroid, which can be used in determining the appropriate time to start the mosaicking operation. Furthermore, the aspect ratio of the rectangle can be used to cleter-mine the mosaic size. In this paper, we show how to compute the bounding rectangle of the target body from the image and apply it to design a mosaic sequence for asteroid flyby. We also discuss the role of local feature matching[1] in reducing estimation errors. These techniques have been tested in a simulated asteroid flyby using the 31) graphic testbed.[2]

2 RECTANGULAR BOUND] NG OF ASTEROIDS

During the near encounter of an asteroid flyby, the boundary points of the asteroid can be extracted using standard segmentation or edge detection techniques.[3] Given a set of the boundary points, $P_i = [x_i \ y_i]^T$, $i = 1, \ldots, n$, the corresponding scatter matrix is defined as

$$S = \sum_{i=1}^{n} (P_i - \bar{P})(P_i - \bar{P})^T,$$

where $\bar{P}$ is the center of gravity of the boundary points, $\bar{P} = \frac{1}{n}[\ \sum_{i=1}^{n} x_i \ \sum_{i=1}^{n} y_i\ ]^T$. The eigenvectors of S correspond physically to the directions about which the figure described by the boundary points Pi has maximum and minimum moments of inertia and they agree closely with our intuitive notion the orientation of the figure.[4] Let u and v be the straight, lines passing through the center of gravity and align with the eigenvector corresponding to the smaller eigenvalue and the larger eigenvalue respectively. Mathematically, the sum of squares of the perpendicular distance from the points to a straight line is minimized for u. The bounding rectangle which aligns with these eigenvectors can be determined by finding those boundary points whose distances to u and v are the maximum.[5] Figure 1 shows an example of bounding an irregular shaped object with a rectangle.

Figure 1: The boundary points and the corresponding bounding rectangle of tile Saturnian moon Epimetheus imaged by the Voyager spacecraft.

3 TRACKING OF TARGET S1 ZE

For the subsequent mosaicking operation and feature tracking, we need the knowledge of the apparent object size at every frame. We consider the simple case of an asteroid flyby that the spacecraft travels in a linear trajectory with constant speed. Furthermore, we assume that the object has no significant spin during the brief flyby period. Then it is possible to estimate the size from the observations during the near encounter period. Let $l(t)$ be the distance from the imaging camera to the object and the closest encounter distance be d (assuming that d is much greater than the object size) which occurs at $t = T$. Then

$$l(t) = \sqrt{d^2 + v^2(t - T)^2} .$$

Let R be the actual size of the object and r the size in the image. Then using tile pinhole camera model,[4]

$$\frac{R}{r} = \frac{l}{f},$$

where f is the focal length. (Both r and f are expressed in pixels:

$$f = \frac{W/2}{\tan(\theta/2)},$$

where W is the width of the camera pixel resolution, and θ is the angular field of view.)

We have

$$\begin{aligned} r(t) &= \frac{Rf}{l(t)} \\ &= \frac{Rf}{\sqrt{d^2 + v^2(t - T)^2}} \\ &= \frac{1}{\sqrt{A^2 + B^2(t - T)^2}} , \end{aligned} \tag{1}$$

where $A = d/Rf$ and $B = v/Rf$. It can be reduced to

$$\rho(t) = a\,t^2 + b\,t + c\,, \tag{2}$$

where $\rho = 1/r^2$, $a = B^2$, $b = -2\,B^2\,T$, and $c = A^2 + B^2\,T^2$.

Let $x = [\; a \; b \; c \;]^T$, $z = [\; \rho_1 \; \rho_2 \; \ldots \; \rho_n \;]^T$, and

$$H = \begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ \vdots & & \\ t_n^2 & t_n & 1 \end{bmatrix} .$$

The least-squares estimate of x is given by

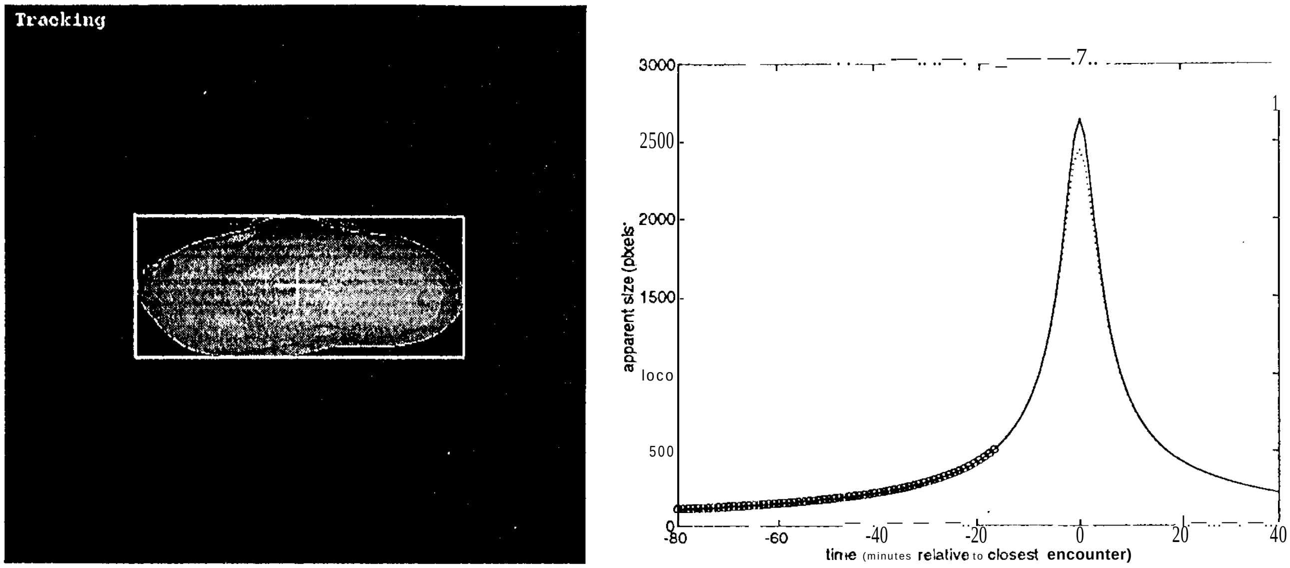
$$x = H^+ z\,,$$

and the covariance of the estimate error is

$$P = H^+ R H^{+T}\,,$$

where R is the error covariance of z and H^+ is the pseudoinverse of H.[6] (H^+ can be robustly computed from the singular value decomposition of H: if $H = U\Lambda V^T$, $H^+ = V\Lambda^+ U^T$).

Given the bounding rectangle, wc can use the length (or perimeter) of rectangle as the size of the object, which is relatively immune to small local variation of the boundary points. By collecting a series of measurements of the object size, wc can determine the parameters a, b and c through least-squares fit. (We do not need to know the absolute time for the measurements only the time interval between frames and the reference time can be arbitrarily chosen.) Then, wc will be able to estimate the size thereafter. And from the estimate wc also obtain the closest encounter time (T) and the closest encounter distance and the velocity relative to the size of object (i.e. d/R and v/it). Figure 2 shows the estimate of the object size (the length of the rectangle) in the trajectory



(a) (b)

Figure 2: The asteroid flyby simulation. (a) The asteroid and the bounding rectangle. (b) The size profile of the asteroid: The solid line represents the actual size of the object in the field of view, the dot line is the estimated size and the thick line is the size measured by the proposed rectangular bounding technqiue.

used in the simulation. (For this simulation, the spacecraft speed is 10 km/s, the closest encounter distance is 2000 km, the asteroid is 180 km in length and the camera field of view is 1.00 with 512x512 pixel resolution.)

The rectangular bound can be calculated only when the entire object is within the field of view. But the dimension information is more accurate when the object is closer to the camera. If the object appears greater than the field of view long before the closest encounter, the predicted size may not he accurate (see Figure 2(b)). Local feature matching may provide additional information on object dimensions[1] which can be incorporated into the previous estimate by recursive update.[6]

4 MOSAIC PLANNING

As an application of rectangle bounding and object size tracking, we describe how the mosaic sequence can be generated autonomously based on the predicted object sizes from the asteroid images.

Since the error of the estimated object size is large near the closest encounter, we require that the entire mosaic operation is completed before the closest encounter. To simplify the operation, wc rotate the camera so that the bounding rectangle aligns with the image scan lines, and the mosaic is rectangular $(M \times N)$ with the same aspect ratio (α) as the bounding rectangle.

The overlap (μ specified as a fraction of the image width) between mosaic frames is not only necessary for constructing of the composite image but also for compensating estimation and pointing errors; so the amount of overlap reflects the confidence of the estimation and the pointing accuracy. Figure 3 is a schematic drawing of a 2x3 mosaic.

If the number of mosaic frames (n) to be taken is given, only the starting time (τ) remains to be determined.

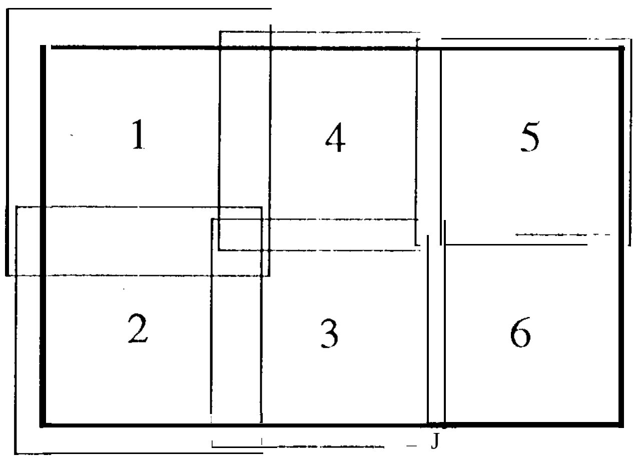


Figure 3: Frame coverage of a 2x3 mosaic. Note that image resolution increases as coverage decreases,

To ensure the coverage of the entire object, we evaluate the expected length (λ_n) of the object in the field view based on the last mosaic frame which has the highest resolution:

$$\lambda_n = N\,W\,(1 - \mu)\,,$$

where W is the image width. Then the expected time to take the last frame can be obtained by solving for t in Equation 2:

$$\frac{1}{\lambda_n} = a\,t^2 + b\,t + c\,;$$

with given frame rate $\Delta t, \tau = t - (n - 1)\Delta t$. (If n is too large for the above equation to have a meaningful solution, n has to be reduced.) The mosaicking sequence is specified by rotation from frame to frame which is calculated from the predicted size of the object. For example, in Figure 3, the rotation $(\Delta\phi)$ from Frame 2 to Frame 3 is:

$$\Delta\phi = \frac{(\lambda_2 + \lambda_3)(1 - \mu)\,\theta}{2\lambda_6}\,,$$

where λ_k is the length at kth frame and θ is the angular field of view of the camera. Note that the overlap between the first few frames is larger than the specified. If N is not given, we impose an additional constraint: the maximum overlap (the overlap between the first two frames). This requirement effectively specifies the ratio of image resolution between the first and last frames, which leads to the determination of both τ and N.

Finally, the knowledge of image resolution of each mosaic frame also helps in constructing the composite image. Figure 4 shows the sequence of mosaic images captured in the 3D graphic simulation of asteroid flyby.

5 CONCLUSIONS

We illustrate how the shape characterization of an asteroid can assist the important spacecraft operation ---- mosaicking. The simple technique of rectangular bounding is shown to be effective in linear flyby scenarios. The future work will devise techniques for a more complete characterization of irregularly shaped objects to handle more complex cases such as rapid spinning asteroids.

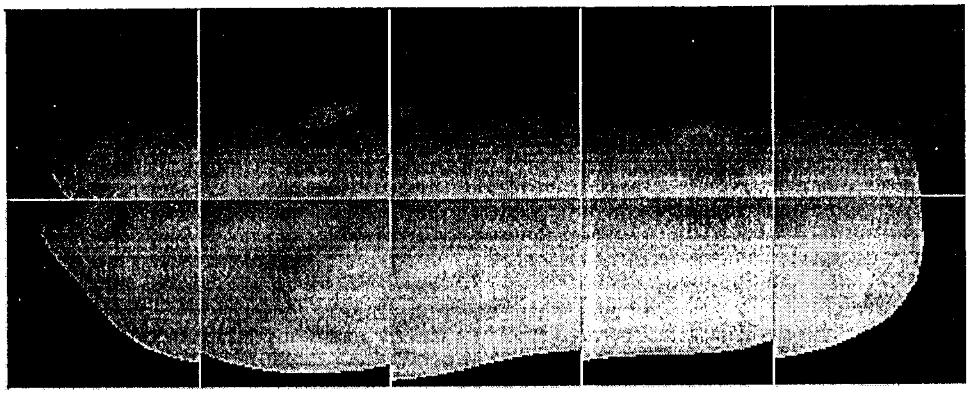


Figure 4: 'I'he mosaic images captured in the 3D graphic simulation of asteroid flyby. 'l'he mosaic size is determined autonomously from the asteroid dimension tracking.

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