Dynamics of rotating Boson stars

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We study dynamics of head-on collisions of initially rotating Boson stars. For Newtonian case, we solve a fully consistent Schrödinger-Poisson system numerically assuming axial symmetry about the axis of collision. For GR, ... **This is NOT a draft of a paper. It should be simply considered as a working "note".**

PACS numbers:

Introduction – Physics to look at
*Primary goals: Collapse to form blackholes (from an unstable branch of a rotating boson star!), Critical phenomena
*Secondary goals: Head-on collisions (two regimes–solitonic, merging ; bh formation?), Understand Merging regime!

Computational Model – Dynamics of Boson stars in Newtonian limit is described coupled equations of Schrödinger-Poisson equations (we will ignore nonlinear self-interaction term for now),

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_g\psi \tag{1}$$

where m is the mass of scalar field ψ. The gravititational potential V_g satisfies the Poisson equation,

$$\nabla^2 V_g = 8\pi G m^2|\psi|^2 \tag{2}$$

The equations (1) and (2) can be derived from the energy function, (well, I already used the scaling introduced below),

$$E[\psi] = \int\{\frac{1}{2}\nabla\psi^*\nabla\psi + V_{ext}|\psi|^2 + \frac{C}{2}|\psi|^4\} - \frac{1}{2}\int\int\frac{|\psi(\vec{r})|^2|\psi(\vec{r'})|^2}{|\vec{r}-\vec{r'}|}dVdV' \tag{3}$$

Given the fact,

$$V_g(\vec{r}) = -\int\frac{|\psi(\vec{r'})|^2}{|\vec{r}-\vec{r'}|}dV' \tag{4}$$

the total energy integral is given by

$$E[\psi] = \int\{\frac{1}{2}\nabla\psi^*\nabla\psi + V_{ext}|\psi|^2 + \frac{C}{2}|\psi|^4\} + \frac{1}{2}\int|\psi(\vec{r})|^2V_g(\vec{r})dV \tag{5}$$

Rescaling by the relations, $x = 10^{\frac{3}{2}}\frac{\hbar}{m}\tilde{x}, t = 10^3\frac{\hbar}{m}\tilde{t}, \psi = \frac{1}{10^3}\frac{m^{3/2}}{\hbar\sqrt{u}}\tilde{\psi}, V_g = \frac{1}{10^3}m\tilde{V}_g$, (**need to correct this!!**) gives the following unitless equations (now dropping $\sim$'s without confusion),

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + V_g\psi \tag{6}$$

$$\nabla^2 V_g = |\psi|^2 \tag{7}$$

In the calculations of collisions, I assume axi-symmetry about the axis of collision, say z-axis, so that the equations depend only on ρ and z in the cylindrical coordinate system. Eqns (6) and (7) are discretized on a grid using 2nd order finite differencing scheme. Crank-Nicholson method [3] is used to integrate time evolution of wavefunction, ψ, and multigrid method [4] is used to solve for V_g. To confirm integrity of numerical solutions, I verified 2nd order convergence of the numerical solutions as the grid spacing, h, approaches zero as well as total energy conservation.

Initial Data – I first discuss how to set up a single rotating boson star solution. Initial set-up of double condensates for a collision problem is discussed below. To obtain an stationary state of the ground state rotating boson star, I assume an *ansatz*,

$$\psi(t,\mathbf{x}) = \Phi(\mathbf{x})e^{i\omega t} \tag{8}$$

where $\Phi(\mathbf{x})$ is a function of space only. Note that ω is an Energy eigenvalue. Solution to the equation (eigenfunction) is stationery only when the ω is precisely the eigenvalue.

Eqns (6) and (7) become,

$$\begin{aligned} -\omega\Phi(\mathbf{x}) &= -\frac{1}{2}\nabla^2\Phi(\mathbf{x}) + V_g\Phi(\mathbf{x}) && (9)\\ \nabla^2 V_g &= |\Phi(\mathbf{x})|^2 && (10)\end{aligned}$$

Further assume in spherical coordinate system,

$$\Phi(\mathbf{x}) = \phi(r,\theta)e^{im\varphi} \tag{11}$$

where $\phi(r,\theta)$ is a *real* function of (r,θ) only. Note that m is an integer to satisfy the condition that the function Φ is a single-valued.

Eqns (9) becomes,

$$\begin{aligned} &-\omega\phi - V\phi = \\ -\frac{1}{2}\frac{1}{r^2\sin\theta}&[\sin\theta\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}\phi) + \frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}\phi) - \frac{m^2}{\sin\theta}\phi] = \\ -\frac{1}{2}\frac{1}{r^2}&[\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}\phi) + \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\phi + \frac{\partial^2}{\partial\theta^2}\phi - \frac{m^2}{\sin^2\theta}\phi] \end{aligned} \tag{12}$$

Eqns (10) becomes,

$$\begin{aligned} &\phi^2 = \\ \frac{1}{r^2\sin\theta}&[\sin\theta\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}V) + \frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}V)] = \\ \frac{1}{r^2}&[\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}V) + \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}V + \frac{\partial^2}{\partial\theta^2}V] \end{aligned} \tag{13}$$

Here we introduce compactified coordinates $(\tilde{r}, s)$ defined by

$$\begin{aligned} \tilde{r} &= \frac{r}{1+r} && (14)\\ s &= \cos\theta && (15)\end{aligned}$$

Then, Eqns (12) and (13) become

$$\begin{aligned} &-\omega\phi - V\phi = \\ -\frac{1}{2}(\frac{1-\tilde{r}}{\tilde{r}})^2&[(1-\tilde{r})^2\frac{\partial}{\partial\tilde{r}}\tilde{r}^2\frac{\partial}{\partial\tilde{r}} + \frac{\partial}{\partial s}(1-s^2)\frac{\partial}{\partial s} - \frac{m^2}{1-s^2}]\phi && (16)\\ \phi^2 = (\frac{1-\tilde{r}}{\tilde{r}})^2&[(1-\tilde{r})^2\frac{\partial}{\partial\tilde{r}}\tilde{r}^2\frac{\partial}{\partial\tilde{r}} + \frac{\partial}{\partial s}(1-s^2)\frac{\partial}{\partial s}]V && (17)\end{aligned}$$

where $\phi = \phi(\tilde{r}, s)$ with $0 \le \tilde{r} \le 1$ and $-1 \le s \le 1$.

Boundary conditions are given by $\phi(\tilde{r}=1) = 0, V(\tilde{r}=1) = 0$ and quadratic fits on other three boundaries.

Collision between Non-rotating Boson stars – Here I will try to briefly summarize main results for the head-on collision between non-rotating Newtonian Boson stars. *details will be given later.*

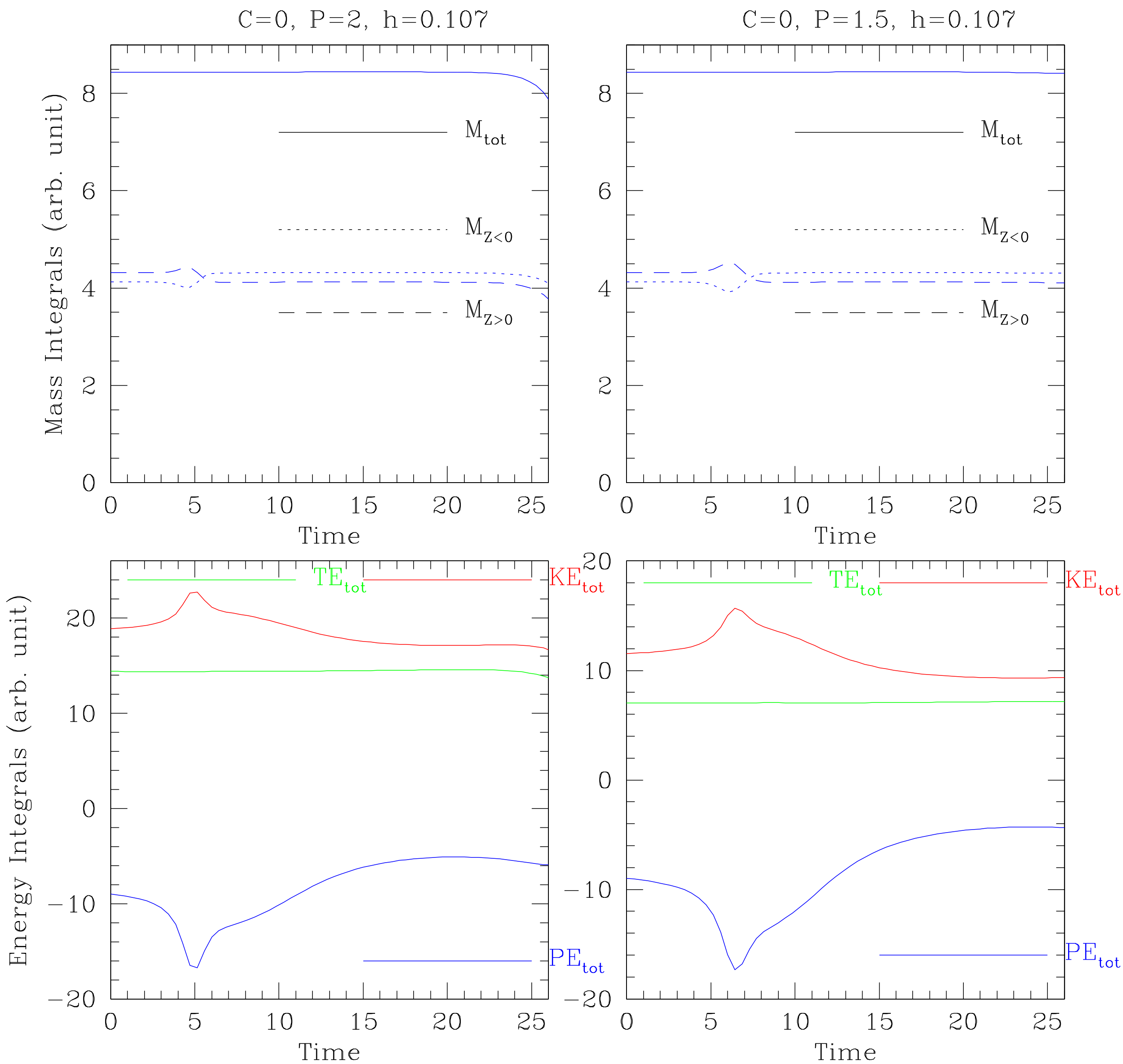

FIG. 1: total mass and total energy conservation for the head-on collision of two non-equal mass Boson stars. Initial momentum is given by $P = \pm 2$(left column), ± 1.5(right column). and initial position is given by $Z_0 = \pm 10$.

Fig. (1) shows that the conversation of total mass and total energy of the binary boson star system. There are the non-rotating boson stars. Central density for the stars are $\phi_0 = 1.0$, and 1.2.

[1] P. Jetzer, Phys. Rep. **220**, 163 (1992).
[2] R. Ruffini and S. Bonazzola, Phys. Rev. **187**, 1767 (1969).
[3] W. H. Press, *et al*, Numerical Recipes: the art of scientific computing. 2nd ed. Cambridge University Press (1992).

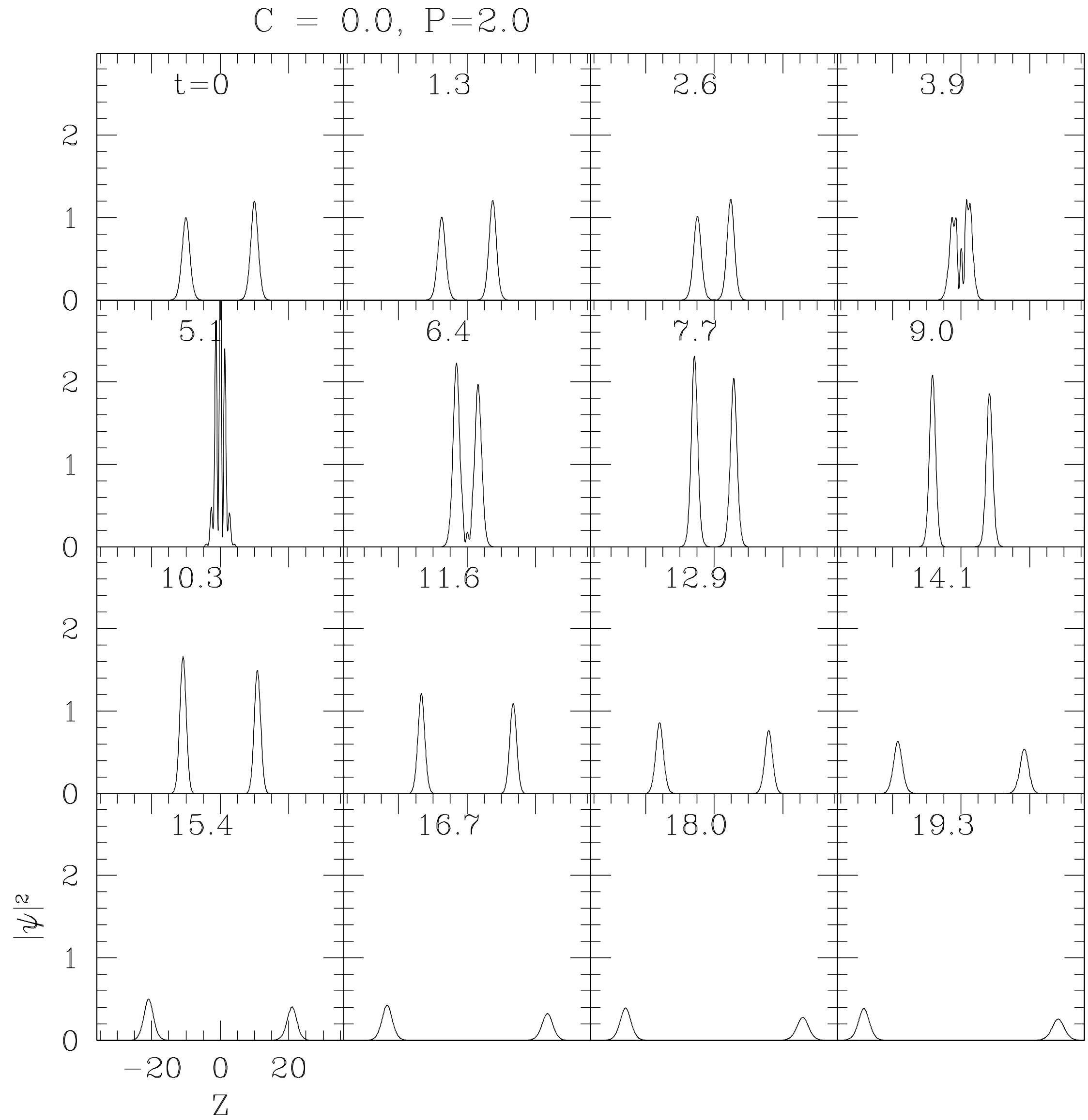

FIG. 2: Time evolution of ϕ^2 on the axis ($\rho = 0$) of two non-equal mass Boson stars. One can see the clear indication of solitonic behavior. It is characterized by shape changing collision.

[4] A. Brandt, Math. Comput. **31**, 333 (1977).

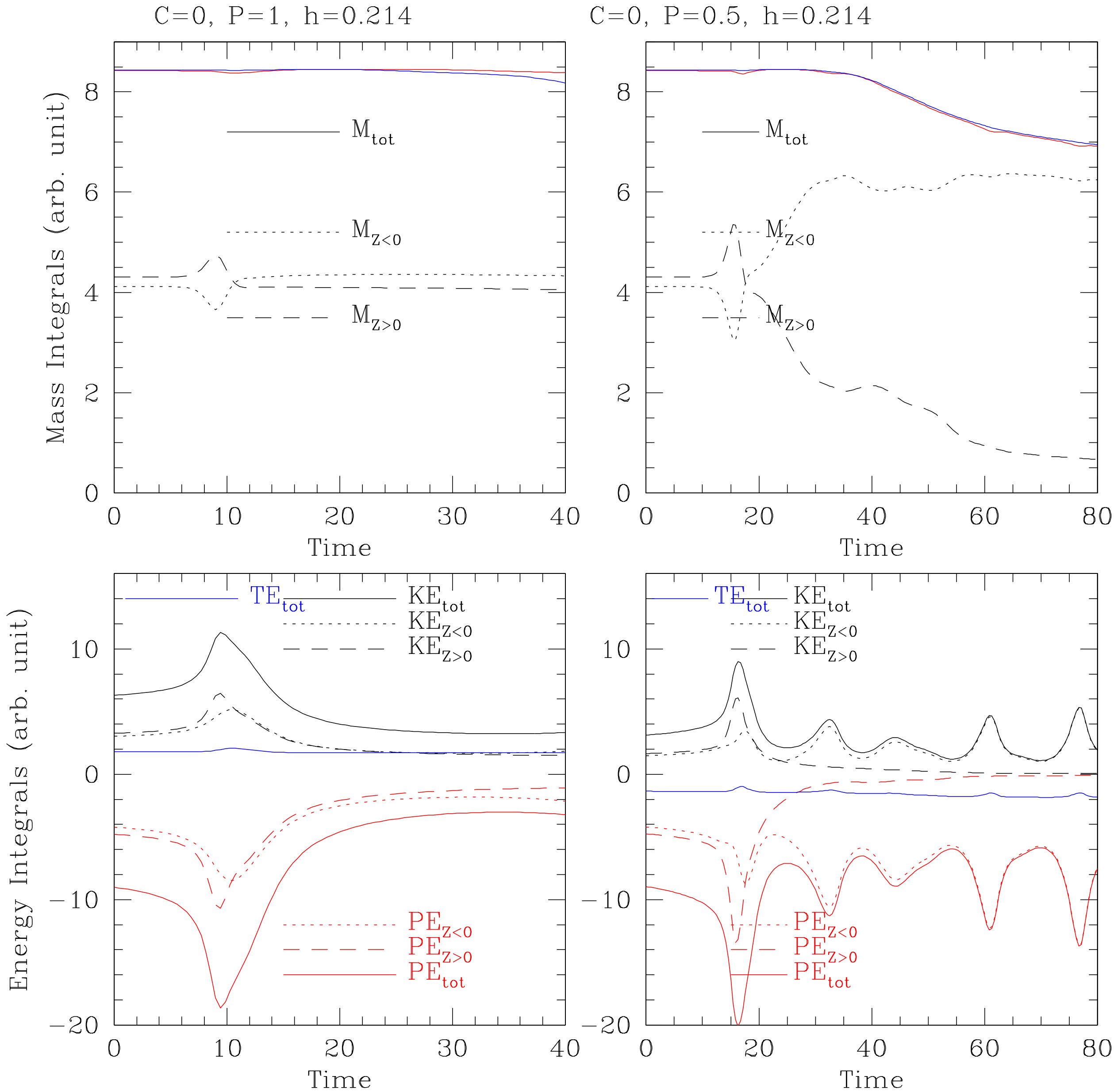

FIG. 3: total mass and total energy conservation for the head-on collision of two non-equal mass Boson stars. Initial momentum is given by $P = \pm 1$(left column), ± 0.5(right column). and initial position is given by $Z_0 = \pm 10$.