Old Dominion University Research Foundation

DEVELOPMENT OF RESPONSE SURFACE MODELS FOR RAPID ANALYSIS & MULTIDISCIPLINARY OPTIMIZATION OF LAUNCH VEHICLE DESIGN CONCEPTS

Final Report

NASA Grant No: NAG-1-2157
ODURF No: 192121

Submitted by:

Dr. Resit Unal
Engineering Management Department
Old Dominion University

Submitted to:

Dr. Mary Kae Lockwood and Roger A. Lepsch

ASCAC/Vehicle Analysis Branch
National Aeronautics and Space Administration
Langley Research Center, Mail Stop 365
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FINAL REPORT

Resit Unal
Old Dominion University

1. BACKGROUND

Multidisciplinary design optimization (MDO) is an important step in the design and evaluation of launch vehicles, since it has a significant impact on performance and lifecycle cost. The objective in MDO is to search the design space to determine the values of design parameters that optimize the performance characteristics subject to system constraints.

Vehicle Analysis Branch (VAB) at NASA Langley Research Center has computerized analysis tools in many of the disciplines required for the design and analysis of launch vehicles. Vehicle performance characteristics can be determined by the use of these computerized analysis tools. The next step is to optimize the system performance characteristics subject to multidisciplinary constraints. However, most of the complex sizing and performance evaluation codes used for launch vehicle design are stand-alone tools, operated by disciplinary experts. They are, in general, difficult to integrate and use directly for MDO.

An alternative has been to utilize response surface methodology (RSM) to obtain polynomial models that approximate the functional relationships between performance characteristics and design variables. These approximation models, called response surface models, are then used to integrate the disciplines using mathematical programming methods for efficient system level design analysis, MDO and fast sensitivity simulations. A second-order response surface model of the form given below (1) has been commonly used in RSM since in many cases it can provide an adequate approximation especially if the region of interest is sufficiently limited.

$$y = b_0 + \sum b_i x_i + \sum b_{ii} x_i^2 + \sum \sum b_{ij} x_i x_j \quad (1)$$

In (1), the x_i terms are the input variables that influence the response (performance characteristic such as weight) y, and b_0, b_i, and b_{ij} are estimated model coefficients. The cross terms represent two-parameter interactions, and the square terms represent second-order non-linearity.

Over the last five years, various design-of-experiments (DOE) based response surface methods have been utilized for efficiently constructing the second-order model (1). These were Taguchi methods [1], central composite designs [2,3,4] and minimum point D-Optimal designs [2,5,12]. These RSM methods were applied successfully to many launch vehicle multidisciplinary design optimization problems at VAB [6,7,8,9,10,11,14]. Current research and applications at VAB on

RSM indicates that "Augmented D-Optimal Designs" may be a good approach for response surface model building using computerized analysis codes [13,15].

The results of the application of RSM have been faster design times, rapid multidisciplinary design optimization and integration of many of the disciplinary analysis codes. Most of this research and applications of RSM have focused on weights and sizing, propulsion and structures disciplines.

2. TECHNICAL OBJECTIVE

A major advantage of using RSM is that it enables the integration of disciplines for rapid MDO using mathematical programming methods. In the applications at VAB, the number of design parameters studied ranged from four to seven. The disciplines involved were weights & sizing, aerodynamics, propulsion and geometry modeling, with the objective performance characteristics usually being dry weight. In these applications, the fitted second-order model (1) predicted the analysis results with good accuracy within the region studied, especially in weights & sizing.

However, in a number of the applications, the prediction accuracy of the aerodynamics related response surface models have been barely adequate, leading to problems in estimating optimum conditions. This indicated that the aerodynamics response surface is more complex or more nonlinear than can be adequately represented by the second order approximation model (1). The objective of this study was to conduct research in an effort to improve the accuracy of the aerodynamics approximation models integration and MDO.

3. METHODOLOGY AND RESULTS

3.1. Aerodynamics Data for a Generic Hypersonic Vehicle Design

The data used in this study involved 3160 wind tunnel data points at subsonic, transonic, supersonic and hypersonic speeds for a generic hypersonic vehicle design. There were 351 data points at each Mach speed of 0.8, 0.9, 0.95, 1.05, 1.50, 2.48, 3.94, 5.94, and 9.93. The data was provided for lift (CL), drag (CD) and pitching moment (Cm) coefficients in terms of angle-of-attack (Alpha) and elevon-deflection (Delev). Alpha ranged from -10 to 16, and Delev ranged from -10 to 20 in a full factorial form. The data was provided in Excel® spreadsheet format. The data was then exported to a statistical analysis software package, JMP® and initial screening analysis was conducted.

3.2. Approximation Model forms

3.2.1. Second Order Model Form: Initial approximation (response surface) model form tried with the data was the second order RSM model (1). CL, CD and Cm data was regressed against Mach speed (Mach), elevon deflection (delev) and angle-of-attack (Alpha) using multivariate least squares regression analysis. The resulting model fits were poor with adjusted R Square values around 0.32, which indicated little correlation.

A study by Scott and Olds [17] has addressed approximation models for vehicle aerodynamic data sets. In this study, they presented a method of transforming aerodynamic data sets generated by

APAS into approximating models [17]. The authors noted that APAS is a very useful tool for conceptual level vehicle aerodynamic design, however, APAS is difficult to integrate into a MDO framework. Hence the need for approximation models [17]. Their research showed that aerodynamic data sets generated in APAS for a given vehicle might be successfully reduced to a set of approximating functions through methods of linear regression. The resulting accuracy of the parametric equations was very good for data set transformations involving force coefficients as a function of Mach number only [17]. However, the accuracy of the equations generated by fitting force coefficients as a function of Mach number and a geometric parameter (wing aspect ratio in this case) was less accurate [17]. They concluded that regression analysis might not applicable in the latter case, "at least not using a regression model of the form used". Their conclusion was that further research would be required to determine a more suitable model, perhaps using additional predictor variables, in order to obtain parametric equations whose accuracy is within an acceptable error range [17].

Utilizing this information, the regression analyses were repeated using the second order model form in terms of Alpha and Delev only, for each Mach number (keeping Mach number fixed). This has significantly improved model fit with adjusted R Square values improving to 0.97 to 0.99. This indicated that better fitting approximation models may be obtained if Mach number was fixed (or within the same Mach number).

3.2.2. Higher Order Polynomial (Taylor Series) Form: Even though, adjusted R Square values of up to 0.99 was obtained, experience indicated that prediction accuracy was still low in many cases and models with better prediction sum of squares (PRESS) was needed. In an effort to improve model accuracy, higher order terms (third, fourth and fifth order) for main effects (parameters) and interactions (cross terms) were included in the models constructed using the data. For the data set in hand, it was possible to do that because there were enough data points and the data was available at many levels (values) ranging from -10 to 16 for Alpha and from -10 to 20 for Delev. In general, one will need data with at least k levels to obtain a model to the k-1 degree. As an example, data with at least three levels (k=3) is needed to obtain a second order model.

With the higher order models, in general, the Adjusted R Square values were much improved (except in transonic speeds) ranging from 0.998 to 0.999, also improving PRESS and root mean square (RMS) errors. These results indicated that the aerodynamic model forms are more complex than a second order model can capture alone.

Appendices one, two and three display the results of the analyses with JMP®, together with the 3-D response surface plots for CL, CD, and Cm as a function of Alpha and Delev for Mach 0.8, 0.95, 2.48 and 5.94.

As can be seen from the 3D plots in Appendix-1, for CL, CD and Cm for the transonic speeds (Mach 0.8 and 0.95), the surface is very complex. It will be hard to capture this surface accurately with a polynomial approximation model within the range studied. The plots for the Predicted CL, CD and Cm are much smoother than the actual surface. When discussed with the VAB engineers, it was mentioned that this behavior can be expected in transonic speeds.

Appendix-2 displays the results for Mach 2.48. At this supersonic speed, the approximation model fits are very good for all coefficients, with Adjusted R Square values ranging from 0.999 to 0.9999. This can also be seen from the well matching plots for the actual data and plots for predicted values using the approximation models. The approximation model form and model coefficients are displayed in the JMP® "Screening Fit" output on the box identified as "Parameter Estimates". As can be seen, model forms are slightly different but in general similar, with higher order terms of, 3rd order, 4th order and 5th order.

Appendix-3 displays the results for Mach 5.94. At this hypersonic speed, the approximation model fits are also very good for all coefficients, with Adjusted R Square values ranging from 0.998 to 0.999. This can also be seen from the well matching plots for the actual data and plots for predicted values using the approximation models. The approximation model form and model coefficients are displayed in the JMP® "Screening Fit" output on the box identified as "Parameter Estimates". As can be seen again, model forms are slightly different but in general similar, with higher order terms of, 3rd order, 4th order and 5th order.

In summary, it appears that approximation model accuracy can be improved (at least for the data in hand) by including higher order terms in the model over the second-order RSM model (1). Discussions with the aerodynamics experts in VAB also suggested that the inclusion of higher order terms in the model is appropriate.

3.3. Improving the Approximation Model

Many studies in the literature suggest the use of transformations for improving model fit and accuracy and give details of the transformations that can be utilized [2, 3, 4, 5]. However, most of these transformations (e.g.
log Alpha) were not applicable in this case since the aerodynamic data included negative values for Alpha and Delev.

Roux, Stander and Haftka [18] note that "in choosing an approximating function one should consider the functional form of the response under consideration, since there might be an analytical relationship that may be utilized. The function form should be chosen using engineering knowledge of the true functional form of the response. An example of using previous engineering results is provided by Vanderplaats"[18,19].

Other useful findings and suggestions from the literature are;

- The use of more experimental points may not improve model accuracy if the model form is not appropriate.

- Approximation model accuracy is largely dependent on the choice of the model and on the region studied [18].

- The selection of sampling points from the design space or the choice of the experimental design has an important influence on the accuracy and the cost of constructing the response surface [18].

- Multiplicative, exponential and power functions can also be used [18].

- The response surface should in general be used only to approximate the part of the response for which the true functional relationship is not available, too difficult to calculate or integrate [18].

3.4. Orthogonal Array Based Latin Hyper cube Designs

The results suggest that the choice of an approximation model's functional form should utilize engineering knowledge of the true form of the response. The results also suggest that approximation model accuracy can be improved by including higher order terms (up to 5^{th} order) in the model over the second-order RSM model (1).

In prior MDO studies using RSM at VAB, central composite designs (CCD) [2,3,4] and D-Optimal designs [2,5,12] were utilized to sample the design space for constructing second-order approximation models for aerodynamics using APAS. The CCDs used were mostly "face centered" [2,3] designs, generating experimental designs at three levels (values). So were the D-optimal designs, sampling the design space at three levels.

One will need data with at least six levels to obtain a model to the 5^{th} degree, and the experimental designs. However, constructing CCD and D-Optimal designs at six or more levels would increase the number of data points or APAS runs required in orders of magnitude and would be prohibitive in most all VAB applications. The question then is how to construct experimental designs that can sample the design space efficiently (without increasing the number of data points or APAS runs required) at six or more levels in order to build approximation models with up to 5^{th} order terms in it.

One way to construct multilevel experimental designs is to utilize the computer programs given by Owen [24, 25]. Owen [25] lists a set of randomized orthogonal arrays (OA) for computer experiments. The Statlib computer programs (http://lib.stat.cmu.edu/designs/) to generate these multilevel orthogonal arrays are also listed by Koehler and Owen [24]. However, these OA may require more experiments than central composite designs at six or more levels.

Tang [20] presents an approach to construct experimental designs efficiently at multiple levels called "Orthogonal Array based Latin Hyper cubes." He [20] notes that experimental designs developed for physical experiments (such as CCDs) may not be appropriate for deterministic computer experiments (such as using APAS).

Using OA based Latin hypercube designs (LHD), one can construct experimental designs at nine levels utilizing a three level OA *without* increasing the number of points required. As an example, Table-1a displays a three level OA for two parameters (X1 and X2). This OA has 9 rows, indicating that 9 design points (e.g. APAS runs) are necessary to construct a second order approximation model. Using Tang's algorithm [20], this OA was converted to an OA based LHD (Table-1b). As can be seen from the Table, there are still nine rows, however, the number of levels have increased to 9, enabling the construction of a higher (i.e. fourth or above) order model efficiently.

Table-1: 3-Level and 9-Level Orthogonal Arrays

	X1	X2
1	1	1
2	2	1
3	3	1
4	1	2
5	2	2
6	3	2
7	1	3
8	2	3
9	3	3

a) Three level OA

	X1	X2
1	1	3
2	2	6
3	3	9
4	4	2
5	5	5
6	6	8
7	7	1
8	8	4
9	9	7

b) Nine level OA based LHD

OA based Latin hypercube experimental designs were utilized by Booker [21] in a Helicopter Rotor optimization study. Booker [21], notes that, OA based LHD for computer experiments, have an appealing "space filling" property which enable a more thorough sampling of the design space as compared with traditional experimental designs such as central composite designs. With a face centered CCD, most of the sampling is done at the outer edges of the parameter design range. Therefore, these experimental designs appear to be a very good choice and better suited for conducting experimentation and for approximation model building.

3.5. Constructing OA based Latin Hypercube Designs

OA based Latin hypercube designs can be constructed using the algorithm given by Tang [20]. Tang's algorithm as given in [20] generates "random" OA based LHD's. The non-uniqueness of OA based Latin Hypercube designs poses a problem of choosing a desirable design. Tang discusses this problem and proposes "correlation" and "distance" criteria [22, 23]. Thus one can generate several designs for given number of variables, and then choose one that has largest "distance" [22, 23].

For selecting a LHD using the correlation criteria, Tang [22] introduces a polynomial canonical correlation of two vectors and suggest that a design which has a small polynomial canonical correlation for each pair of its columns is preferred. He provides an algorithm for reducing polynomial canonical correlations of a Latin hypercube. Tang [23] also uses the Maximin distance criteria for selecting an OA based Latin hypecube. He notes that it is commonly recognized that uniformity of design points is a favored property of a design in cases of little knowledge of the underlying model [23]. Therefore, Tang [23] argues, any criterion oriented toward uniformity can be used for the selection of OA based LHDs. He provides a theorem for this purpose [23].

4. CONCLUSIONS

The models presented in the Appendices and accompanying results are only valid for the data set in hand and the parameters studied. However, some general conclusions may also be drawn as follows:

The results suggest that the choice of an approximation model's functional form should utilize engineering knowledge of the true form of the response. The results also suggest that approximation model accuracy can be improved by including higher order (more than three) terms in the model over the second-order RSM model (1).

Using OA based Latin hypercube designs multiple levels experimental designs can be constructed without increasing the number of points required (in reference to the base OA used), enabling the building of fifth order approximation models efficiently. As a result, OA based Latin hypercube designs appear to be a very good choice for conducting wind tunnel experiments and for experimentation using analysis codes for approximation model building.

5. FUTURE WORK

There is a lot of further research needed in modeling and capturing vehicle aerodynamics. This study has been limited in focusing on the data available, and in the number of parameters included. Also, we were unable to conduct an applied design study using APAS as anticipated. Nevertheless, a contribution was made by the literature findings. A practical approach was added to the RSM toolkit at VAB for generating multiple level experimental designs that can be utilized for approximation model building for vehicle aerodynamic and for MDO studies.

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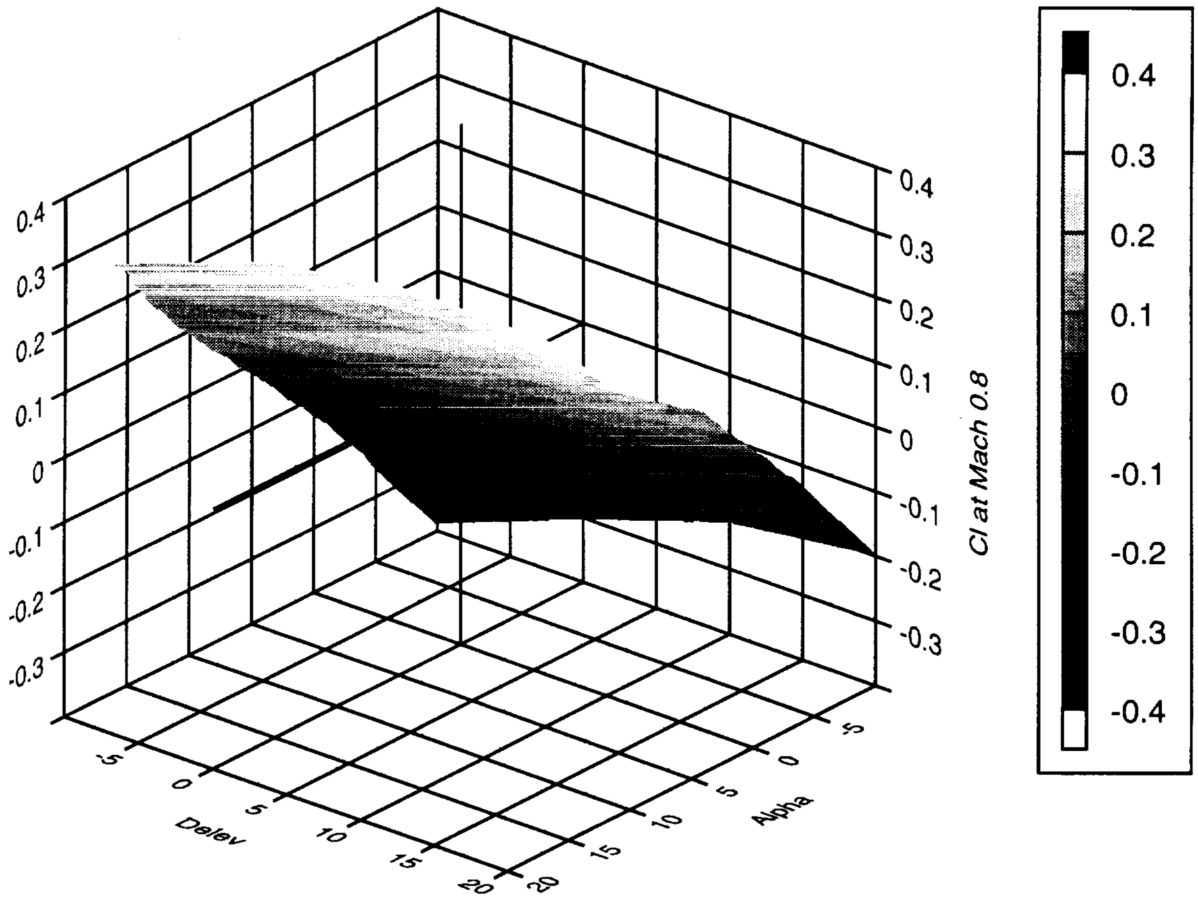
Appendix-1a

Results for Mach 0.8



Cl at Mach 0.8
Delev
Alpha
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4

Screening Fit

CL

Mach 0.8

Summary of Fit

RSquare	0.998831
RSquare Adj	0.998793
Root Mean Square Error	0.006574
Mean of Response	0.030451
Observations (or Sum Wgts)	352

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	12.552150	1.14110	26401.8
Error	340	0.014695	0.00004	**Prob>F**
C Total	351	12.566845		0.0000

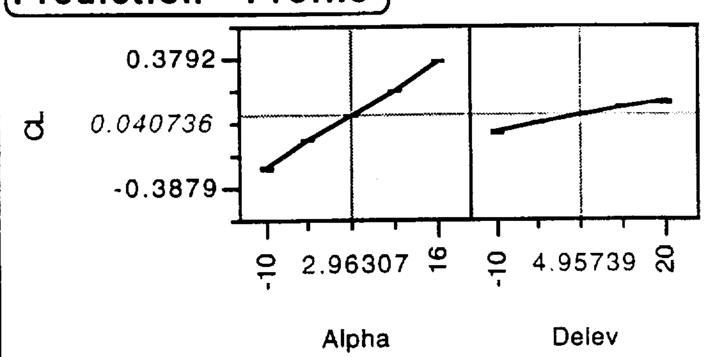
Lack of Fit

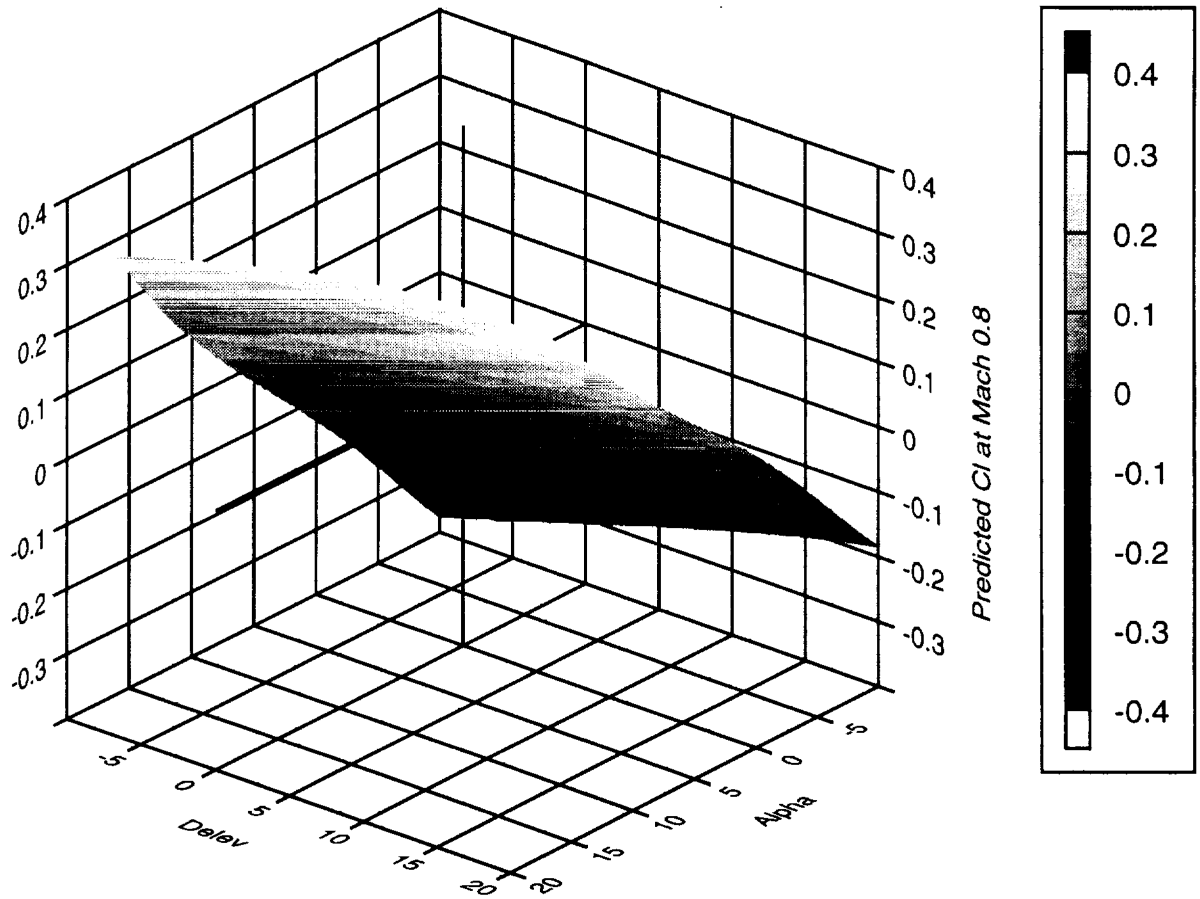
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.057709	0.000919	-62.78	<.0001
Alpha	0.0226172	0.000172	131.32	<.0001
Delev	0.0071657	0.000088	81.51	<.0001
Alpha*Alpha	-0.000315	0.000038	-8.39	<.0001
Delev*Delev	-0.000045	0.00001	-4.67	<.0001
Alpha*Alpha*Alpha	0.0000163	0.000003	5.33	<.0001
Delev*Delev*Delev	-0.000002	5.624e-7	-3.96	<.0001
Alpha*Alpha*Delev	-0.000004	0.000001	-3.38	0.0008
Delev*Delev*Alpha	-0.000004	5.038e-7	-8.53	<.0001
Alpha*Alpha*Alpha*Alpha	0.0000018	3.458e-7	5.08	<.0001
Alpha*Alpha*Alpha*Delev	-2.445e-7	6.994e-8	-3.50	0.0005
Alpha*Alpha*Alpha*Alpha*Alpha	-6.611e-8	2.114e-8	-3.13	0.0019

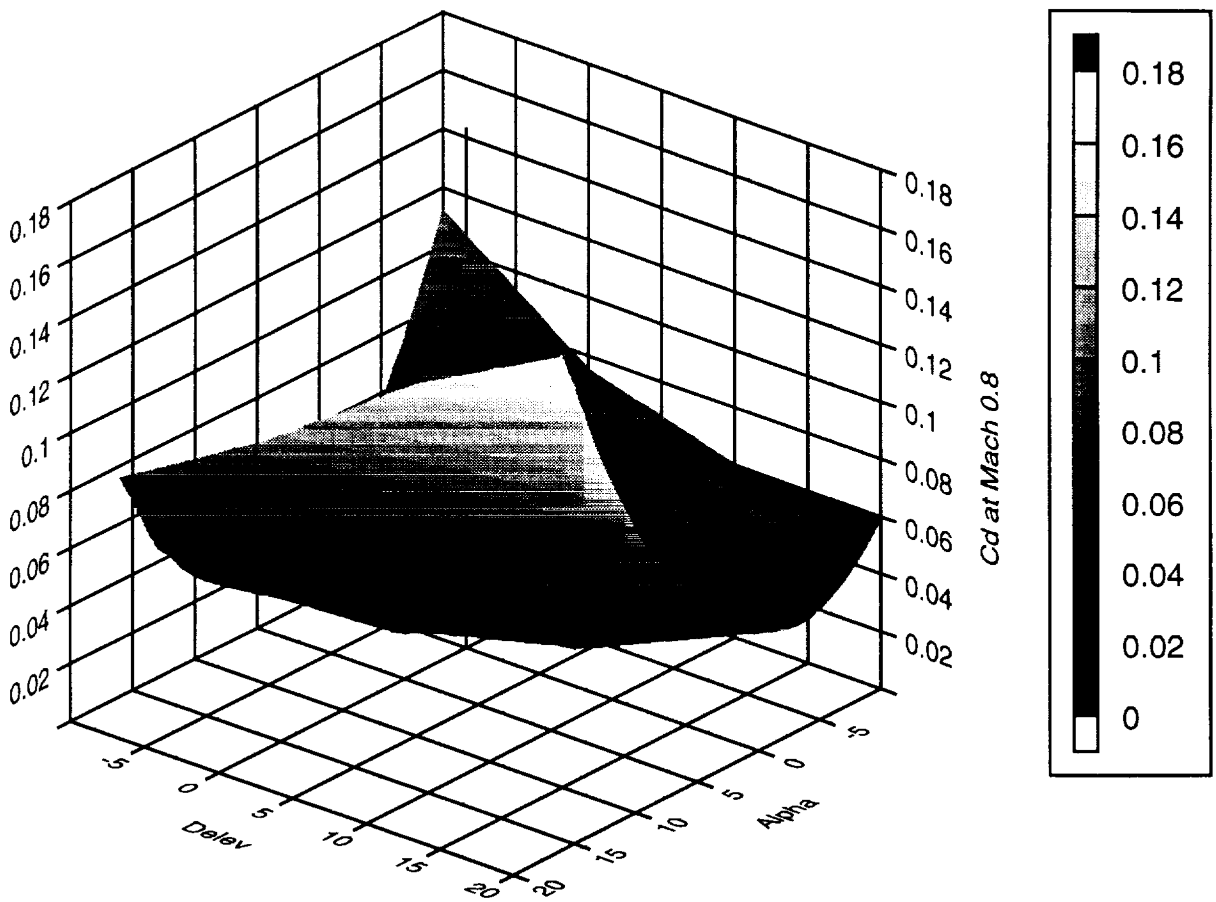
Effect Test

Prediction Profile





0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Predicted Cl at Mach 0.8
Delev
Alpha
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4


0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Cd at Mach 0.8
Delev
-5
0
5
10
15
20
Alpha

Screening Fit

CD

Mach 0.8

Summary of Fit

RSquare	0.99746
RSquare Adj	0.997385
Root Mean Square Error	0.001723
Mean of Response	0.05153
Observations (or Sum Wgts)	352

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	10	0.39739163	0.039739	13390.24
Error	341	0.00101201	0.000003	**Prob>F**
C Total	351	0.39840364		0.0000

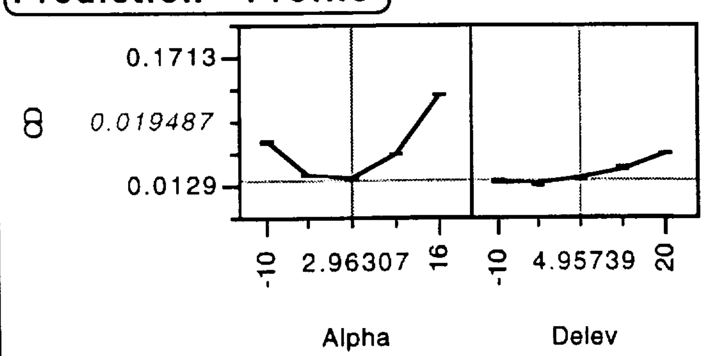
Lack of Fit

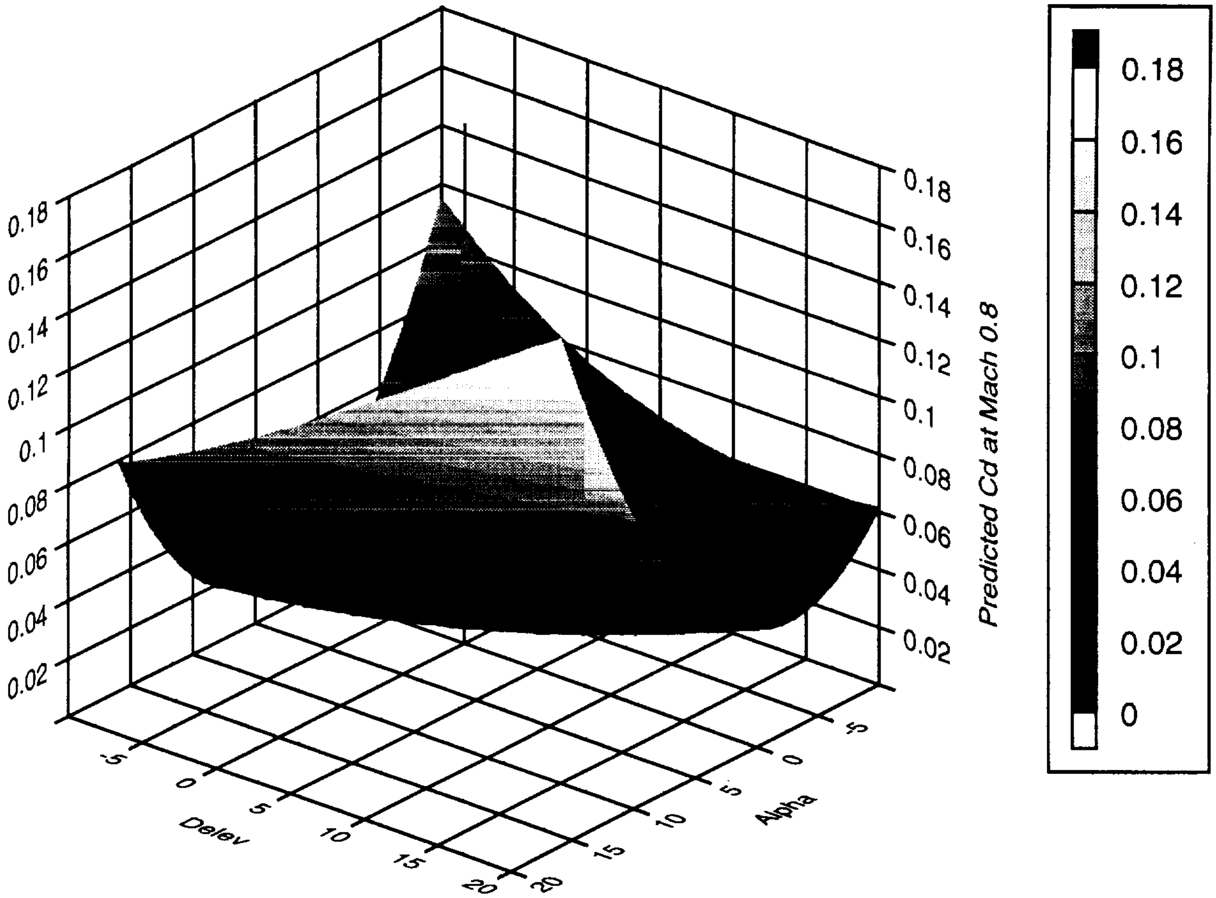
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0155442	0.000218	71.36	<.0001
Alpha	-0.001655	0.00002	-81.80	<.0001
Delev	-0.000049	0.000024	-2.07	0.0388
Alpha*Alpha	0.0004328	0.000005	95.40	<.0001
Delev*Delev	0.0000814	0.000003	32.17	<.0001
Delev*Delev*Delev	-8.224e-7	1.474e-7	-5.58	<.0001
Alpha*Alpha*Delev	-0.000002	2.889e-7	-6.28	<.0001
Delev*Delev*Alpha	-0.000003	1.505e-7	-20.95	<.0001
Alpha*Alpha*Alpha*Alpha	0.0000001	1.982e-8	5.34	<.0001
Alpha*Alpha*Alpha*Delev	-8.741e-8	2.56e-8	-3.41	0.0007
Alpha*Delev	0.000252	0.000003	81.80	<.0001

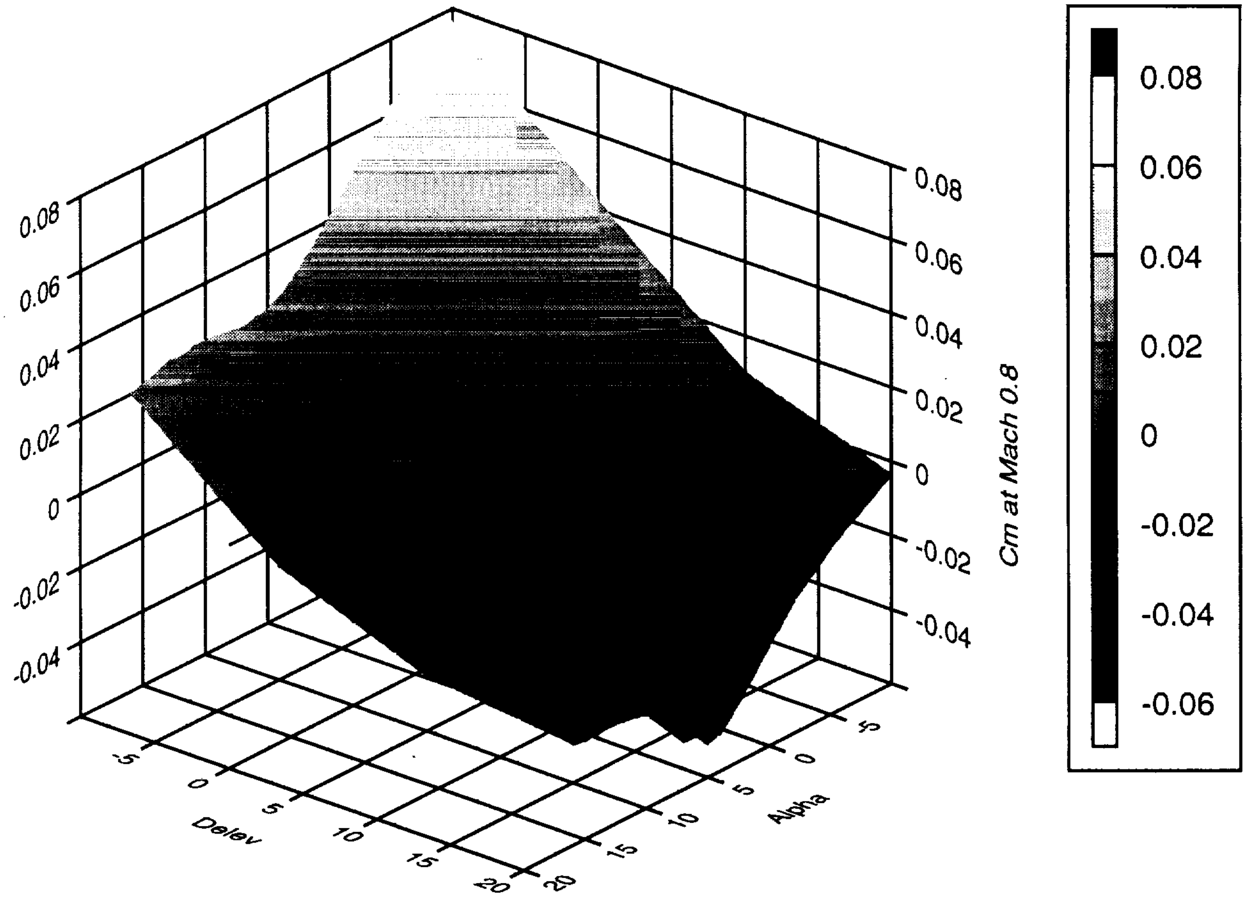
Effect Test

Prediction Profile





0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
Predicted Cd at Mach 0.8
Delev
Alpha
-5
0
5
10
15
20
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0


0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Cm at Mach 0.8
Delev
Alpha
-5
0
5
10
15
20

Screening Fit

Cm

Mach 0.8

Summary of Fit

RSquare	0.991311
RSquare Adj	0.991082
Root Mean Square Error	0.00264
Mean of Response	0.000722
Observations (or Sum Wgts)	352

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	9	0.27194428	0.030216	4335.333
Error	342	0.00238364	0.000007	**Prob>F**
C Total	351	0.27432792		0.0000

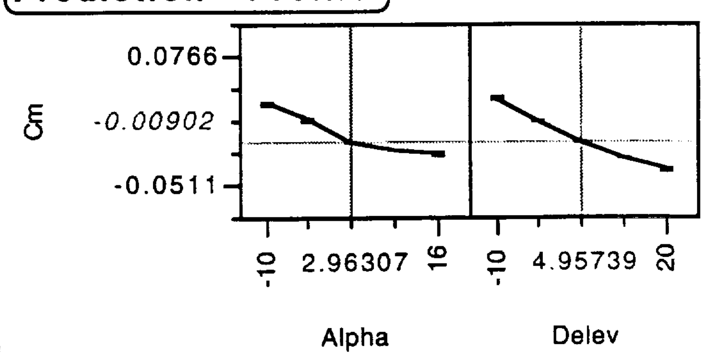
Lack of Fit

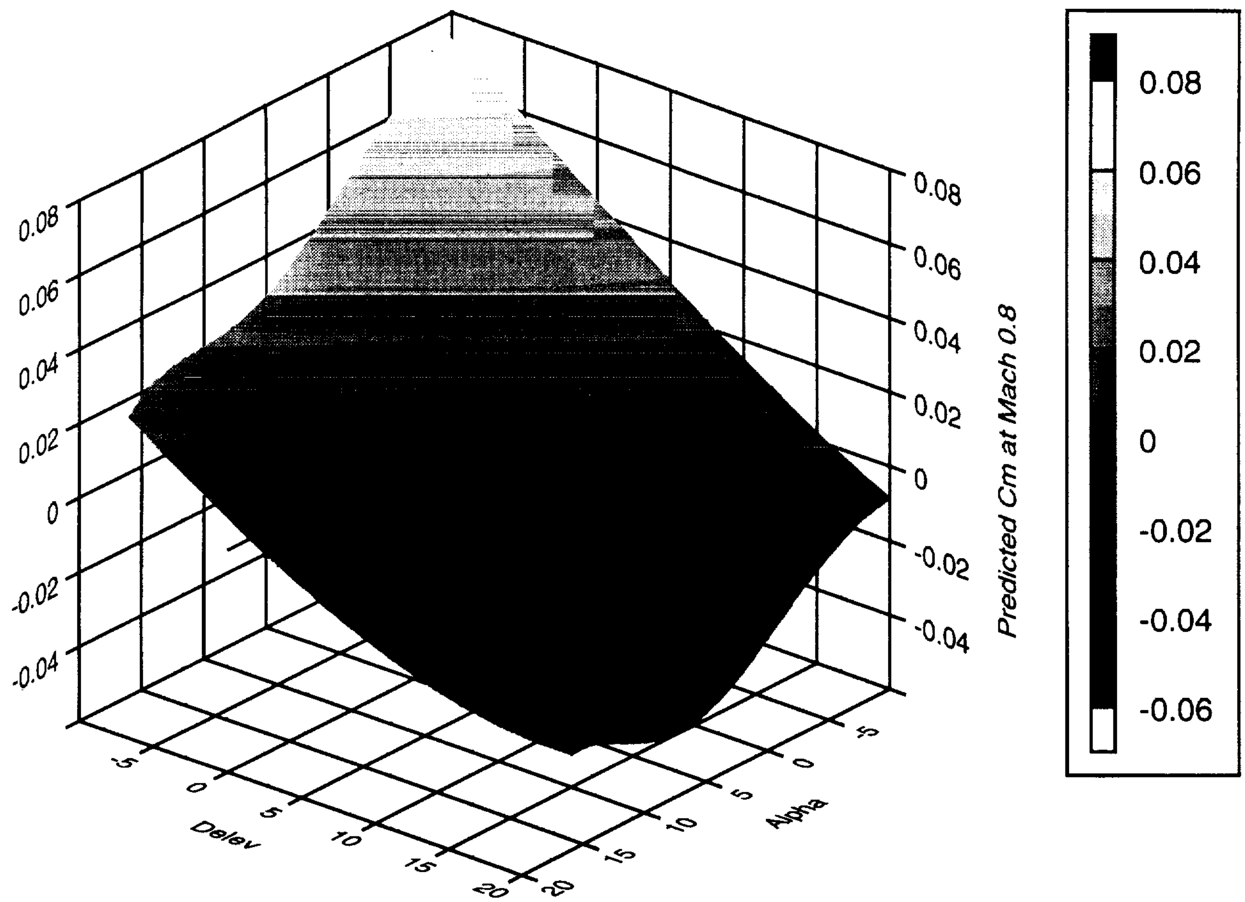
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0141343	0.000311	45.47	<.0001
Alpha	-0.00356	0.000069	-51.57	<.0001
Delev	-0.002833	0.000031	-90.53	<.0001
Alpha*Alpha	0.0000491	0.000004	11.50	<.0001
Delev*Delev	0.0000165	0.000004	4.27	<.0001
Delev*Delev*Delev	0.0000008	2.258e-7	3.47	0.0006
Delev*Delev*Alpha	0.0000014	1.991e-7	7.00	<.0001
Alpha*Alpha*Alpha*Delev	0.0000001	1.638e-8	8.53	<.0001
Alpha*Alpha*Alpha	0.000015	0.000001	13.98	<.0001
Alpha*Alpha*Alpha*Alpha*Alpha	-4.016e-8	3.458e-9	-11.61	<.0001

Effect Test

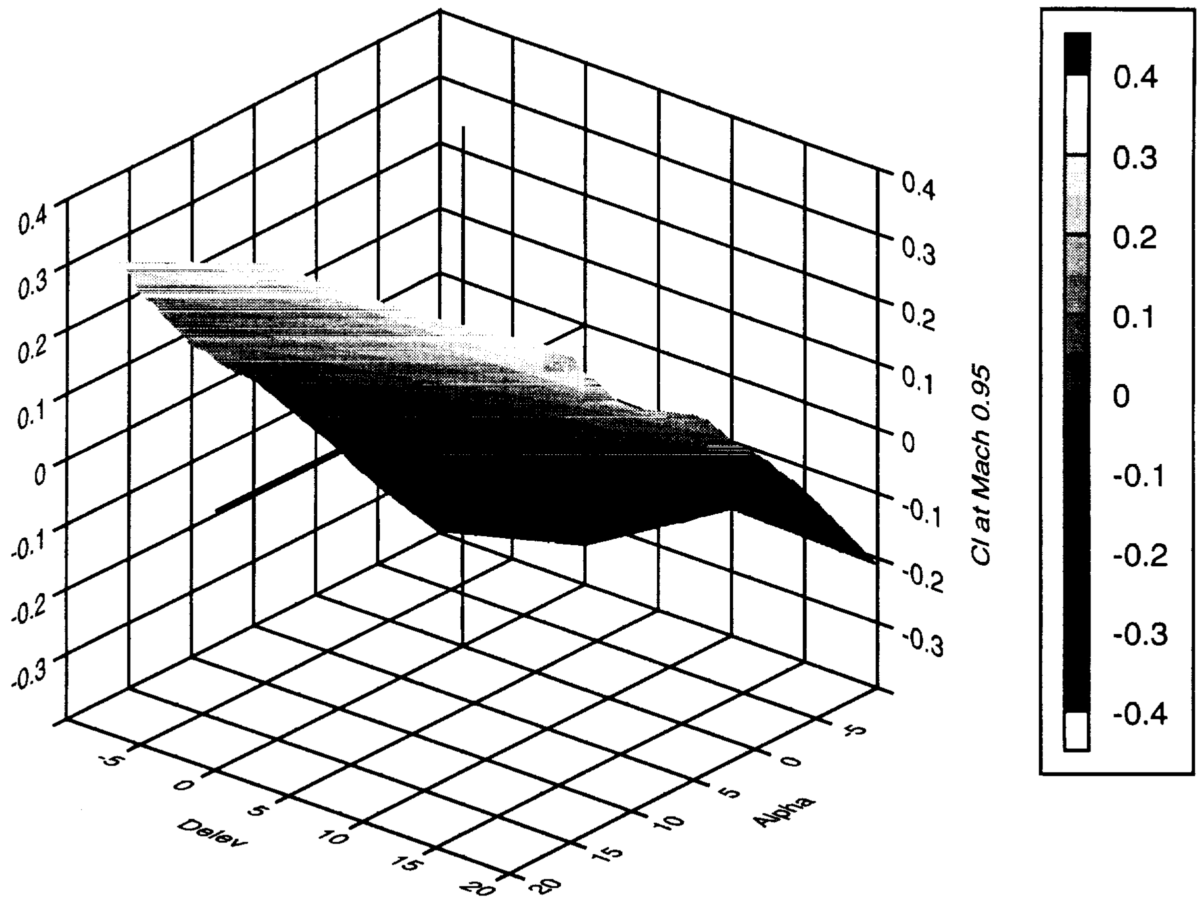
Prediction Profile




0.08
0.06
0.04
0.02
0
-0.02
-0.04
-5
0
5
10
15
20
Delev
Alpha
Predicted Cm at Mach 0.8
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06

Appendix-1b

Results for Mach 0.95



Cl at Mach 0.95
Delev
Alpha
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4

Screening Fit

CL

0.95

Summary of Fit

RSquare	0.998189
RSquare Adj	0.99813
Root Mean Square Error	0.008222
Mean of Response	0.021629
Observations (or Sum Wgts)	351

Analysis of Variance

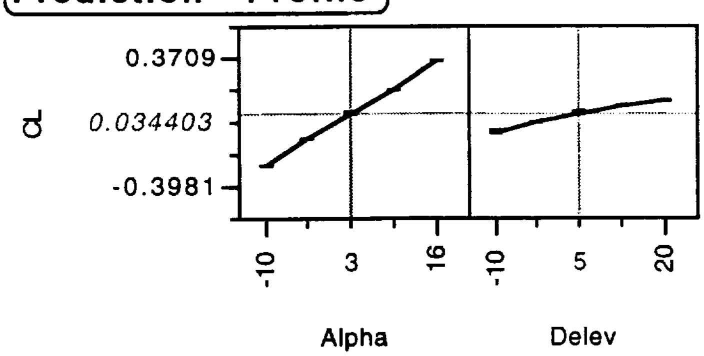
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	12.631651	1.14833	16987.15
Error	339	0.022916	0.00007	Prob>F
C Total	350	12.654568		0.0000

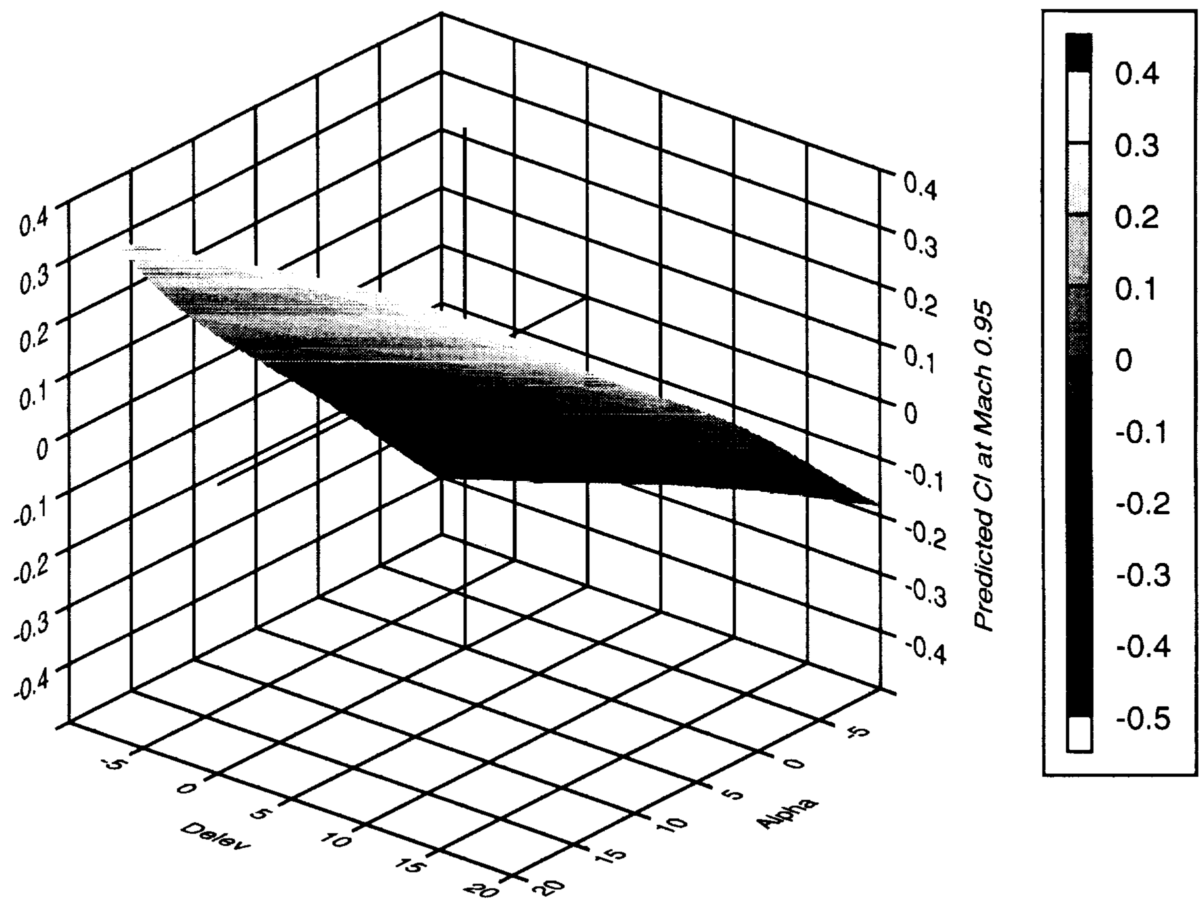
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.07296	0.000953	-76.60	<.0001
Alpha	0.0238654	0.000097	246.04	0.0000
Delev	0.0079703	0.000188	42.46	<.0001
Alpha*Delev	-0.000063	0.000015	-4.25	<.0001
Alpha*Alpha	-0.000212	0.000022	-9.66	<.0001
Alpha*Alpha*Delev	-0.000006	0.000001	-4.13	<.0001
Delev*Delev*Alpha	-0.000003	7.253e-7	-3.77	0.0002
Delev*Delev*Delev	-0.000005	0.000002	-2.74	0.0066
Alpha*Alpha*Alpha*Alpha	0.0000012	9.499e-8	12.16	<.0001
Delev*Delev*Delev*Delev	-3.244e-7	1.058e-7	-3.07	0.0023
Alpha*Alpha*Alpha*Delev	-3.39e-7	1.232e-7	-2.75	0.0063
Delev*Delev*Delev*Delev*Delev	1.7269e-8	4.327e-9	3.99	<.0001

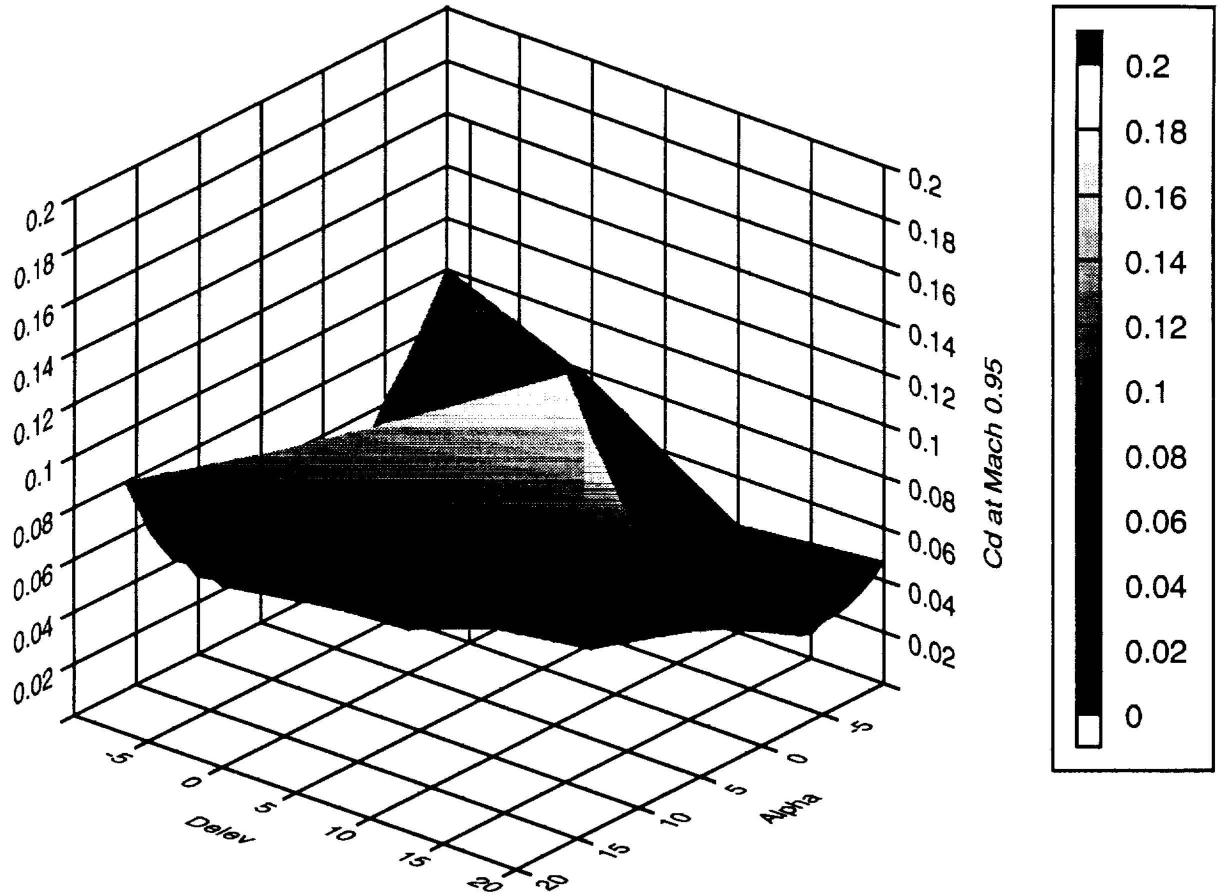
Effect Test

Prediction Profile





Predicted Cl at Mach 0.95
Delev
Alpha
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5



0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
Cd at Mach 0.95
-5
0
5
10
15
20
Delev
Alpha
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0

Screening Fit

CD

0.95

Summary of Fit

RSquare	0.99491
RSquare Adj	0.994745
Root Mean Square Error	0.002482
Mean of Response	0.052775
Observations (or Sum Wgts)	351

Analysis of Variance

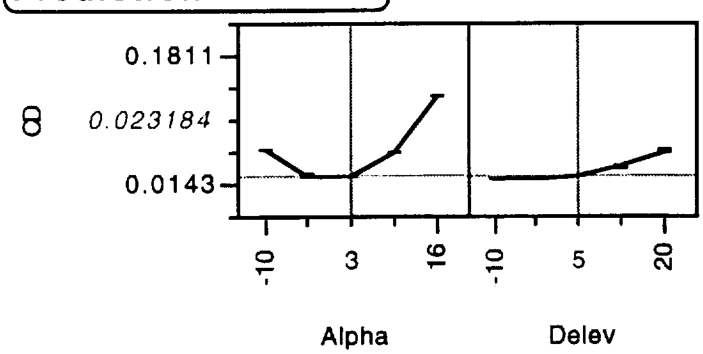
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	11	0.40826307	0.037115	6023.508
Error	339	0.00208880	0.000006	**Prob>F**
C Total	350	0.41035187		0.0000

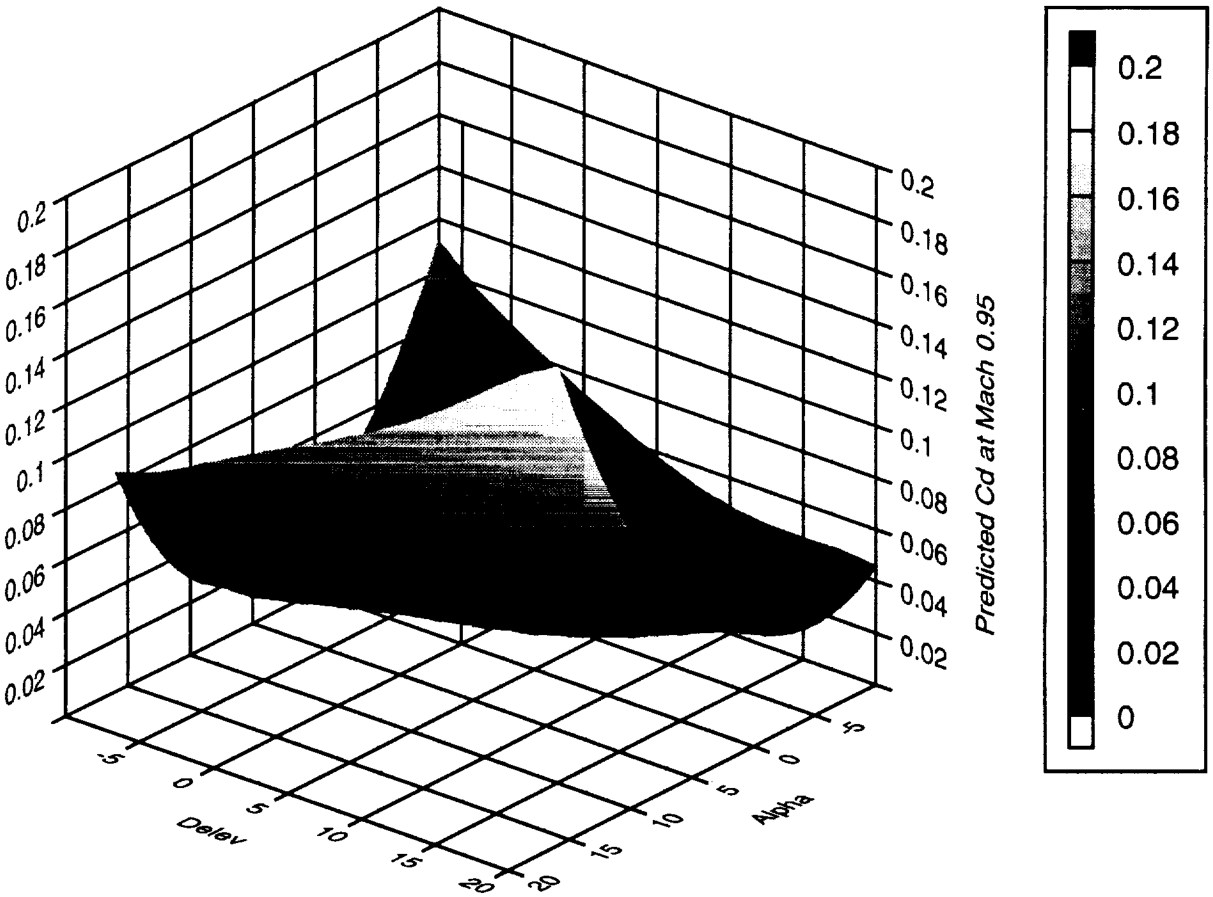
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.020622	0.000288	71.71	<.0001
Alpha	-0.001311	0.000029	-44.77	<.0001
Delev	-0.00019	0.000057	-3.35	0.0009
Alpha*Delev	0.0002659	0.000004	59.65	<.0001
Alpha*Alpha	0.0003619	0.000007	54.73	<.0001
Alpha*Alpha*Delev	-0.000003	4.248e-7	-7.67	<.0001
Delev*Delev*Alpha	-0.000002	2.19e-7	-9.87	<.0001
Delev*Delev*Delev	0.0000019	5.932e-7	3.12	0.0019
Alpha*Alpha*Alpha*Alpha	0.0000003	2.868e-8	10.65	<.0001
Delev*Delev*Delev*Delev	0.0000005	3.195e-8	16.51	<.0001
Alpha*Alpha*Alpha*Delev	-5.594e-8	3.72e-8	-1.50	0.1335
Delev*Delev*Delev*Delev*Delev	-2.269e-8	1.306e-9	-17.37	<.0001

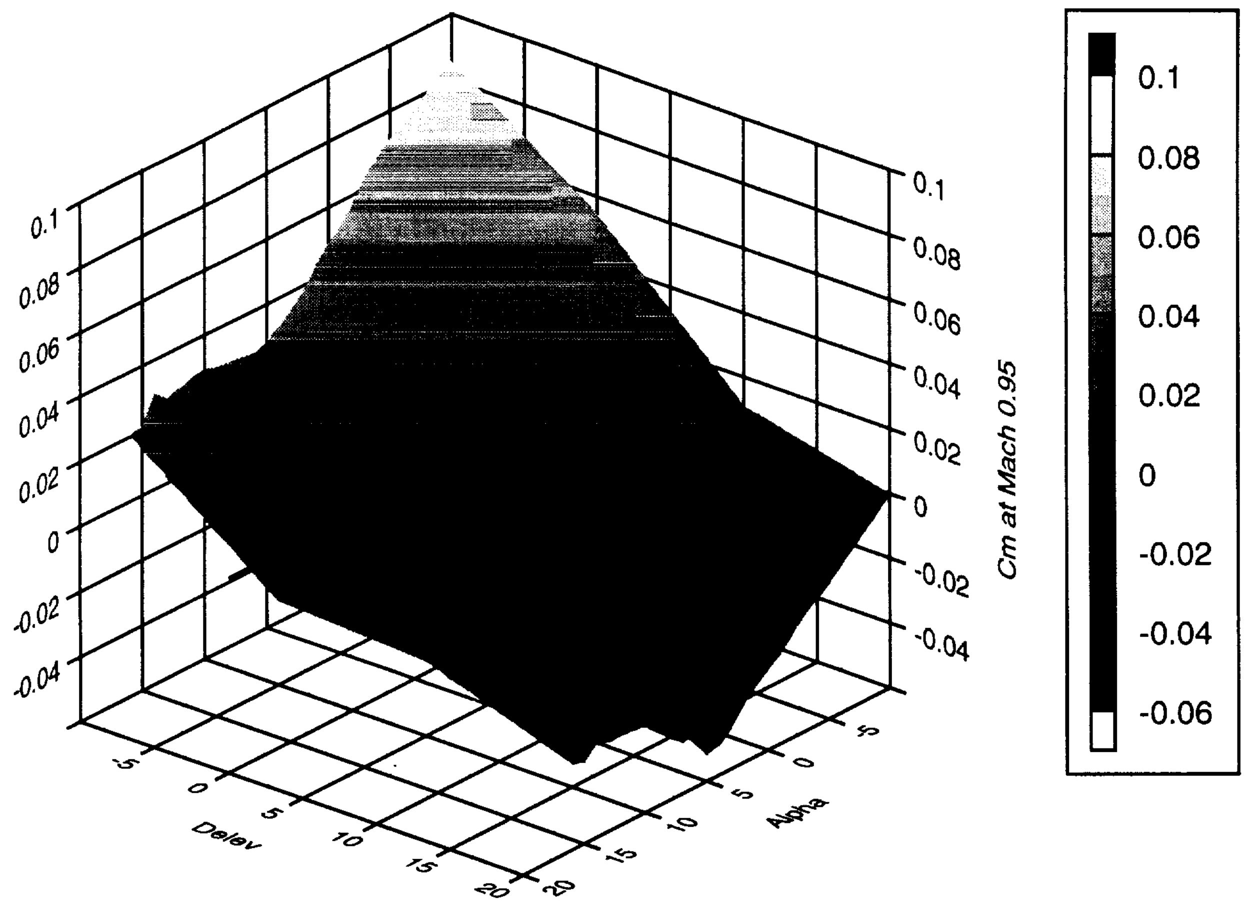
Effect Test

Prediction Profile





0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
Predicted Cd at Mach 0.95
-5
0
5
10
15
20
Delev
Alpha
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0



0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Cm at Mach 0.95
Delev
Alpha
-5
0
5
10
15
20

Screening Fit

Cm

0.95

Summary of Fit

RSquare	0.982486
RSquare Adj	0.982128
Root Mean Square Error	0.003996
Mean of Response	0.0034
Observations (or Sum Wgts)	351

Analysis of Variance

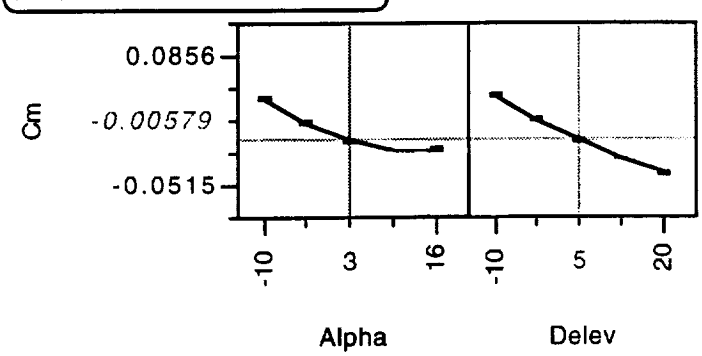
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	0.30724512	0.043892	2748.693
Error	343	0.00547715	0.000016	**Prob>F**
C Total	350	0.31272227		<.0001

Parameter Estimates

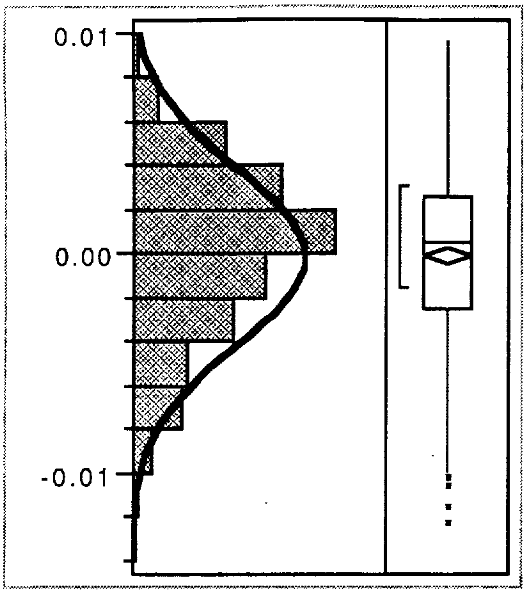
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0169912	0.000383	44.42	<.0001
Alpha	-0.002994	0.000046	-65.39	<.0001
Delev	-0.00298	0.000036	-83.15	<.0001
Alpha*Alpha	0.0000932	0.000004	22.79	<.0001
Delev*Delev*Alpha	0.0000015	2.99e-7	5.04	<.0001
Delev*Delev*Delev*Delev	0.0000002	4.122e-8	5.24	<.0001
Alpha*Alpha*Alpha*Delev	0.0000003	2.445e-8	12.71	<.0001
Delev*Delev*Delev*Delev*Delev	-7.748e-9	2.05e-9	-3.78	0.0002

Effect Test

Prediction Profile



Resid Cm

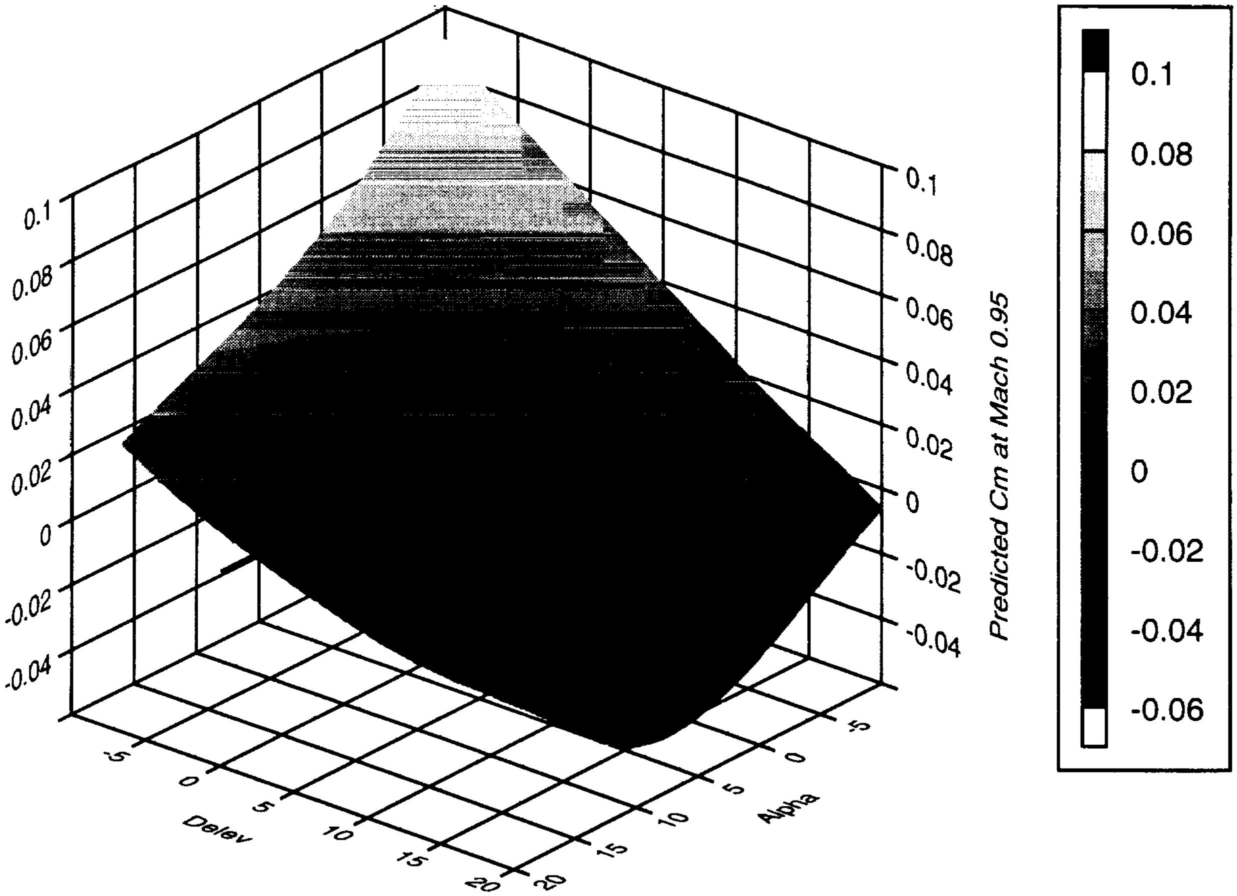


Quantiles

maximum	100.0%	0.00965
	99.5%	0.00911
	97.5%	0.00707
	90.0%	0.00467
quartile	75.0%	0.00266
median	50.0%	0.00059
quartile	25.0%	-0.0024
	10.0%	-0.0059
	2.5%	-0.0092
	0.5%	-0.0116
minimum	0.0%	-0.0121

Moments

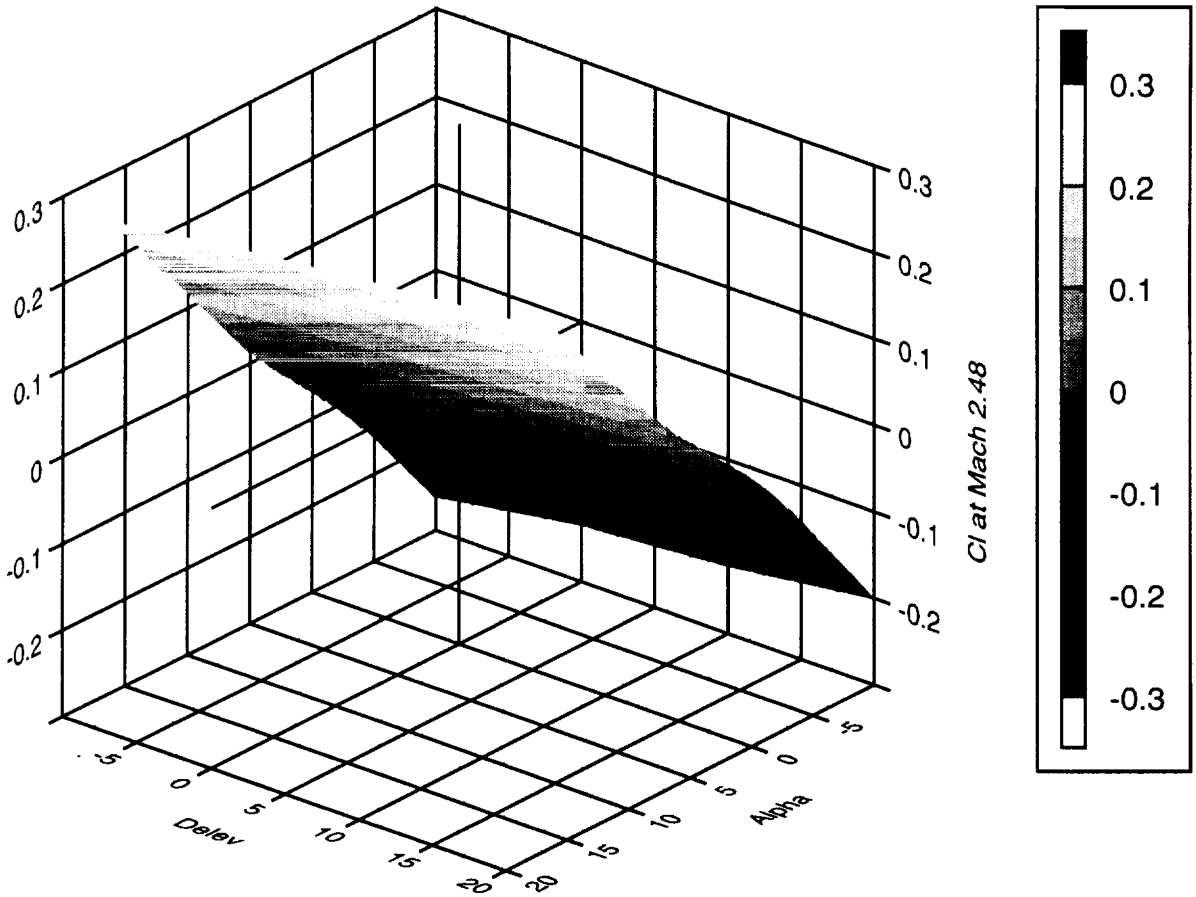
Mean	-0.0000
Std Dev	0.0040
Std Error Mean	0.0002
Upper 95% Mean	0.0004
Lower 95% Mean	-0.0004
N	351.0000
Sum Weights	351.0000



0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Predicted Cm at Mach 0.95
Delev
Alpha
-5
0
5
10
15
20

Appendix-2

Results for Mach 2.48



0.3
0.2
0.1
0
-0.1
-0.2
-5
0
5
10
15
20
Delev
Alpha
Cl at Mach 2.48
-0.3

Screening Fit

CL

Summary of Fit

RSquare	0.999903
RSquare Adj	0.9999
Root Mean Square Error	0.001472
Mean of Response	0.021387
Observations (or Sum Wgts)	351

Analysis of Variance

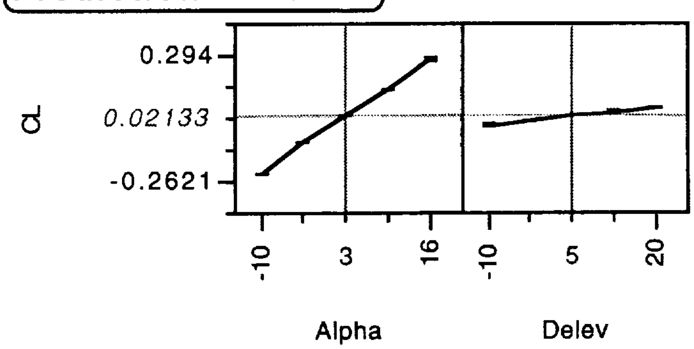
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	10	7.5676005	0.756760	349461.9
Error	340	0.0007363	0.000002	**Prob>F**
C Total	350	7.5683367		0.0000

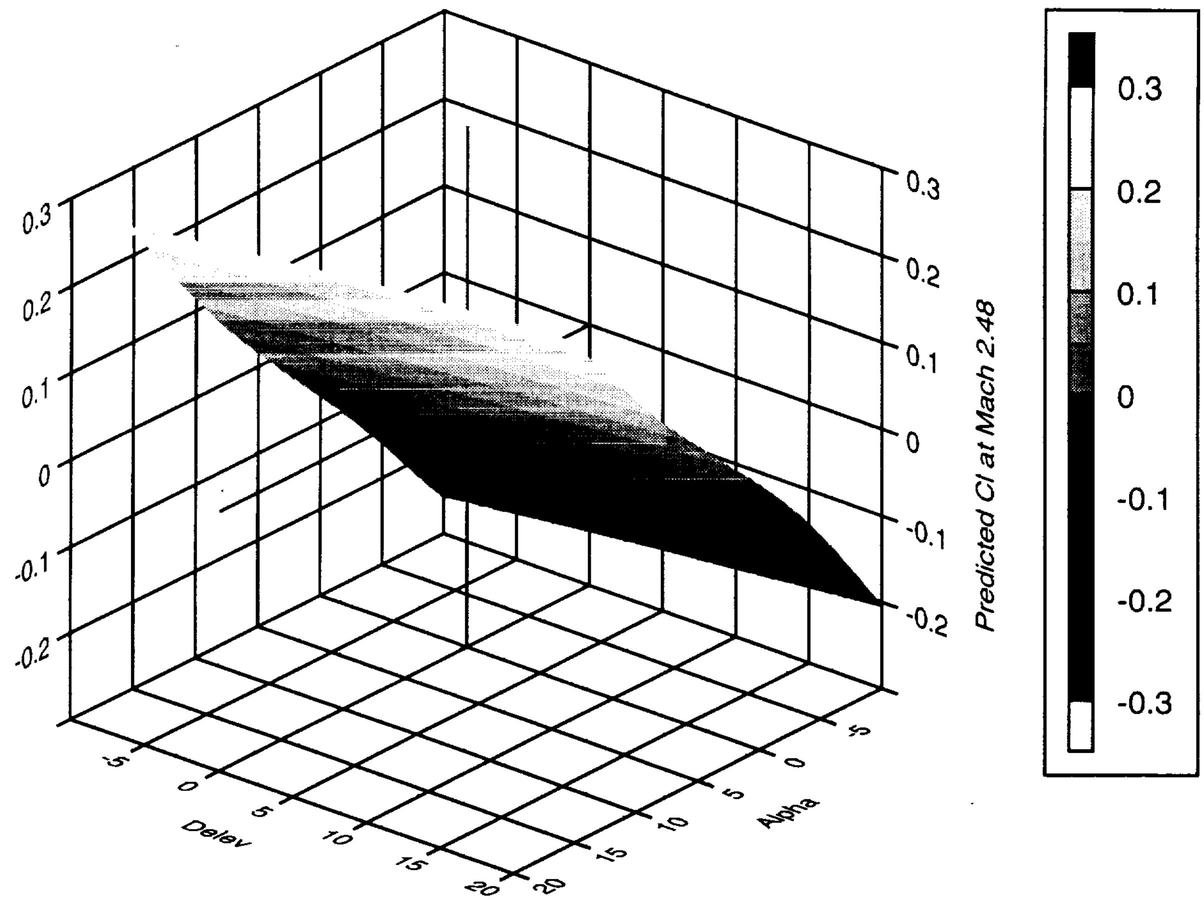
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.041273	0.000189	-218.6	0.0000
Alpha	0.0169315	0.000038	451.13	0.0000
Delev	0.0026815	0.000017	160.07	0.0000
Alpha*Alpha	-0.000201	0.000008	-23.91	<.0001
Delev*Delev	-0.000014	0.000001	-13.55	<.0001
Alpha*Alpha*Alpha	0.0000291	6.884e-7	42.27	<.0001
Alpha*Alpha*Delev	-0.000007	2.438e-7	-27.59	<.0001
Alpha*Alpha*Alpha*Delev	0.0000002	1.753e-8	12.57	<.0001
Alpha*Alpha*Alpha*Alpha	0.0000011	7.802e-8	14.58	<.0001
Delev*Delev*Delev*Alpha	-3.411e-8	6.045e-9	-5.64	<.0001
Alpha*Alpha*Alpha*Alpha*Alpha	-1.043e-7	4.753e-9	-21.94	<.0001

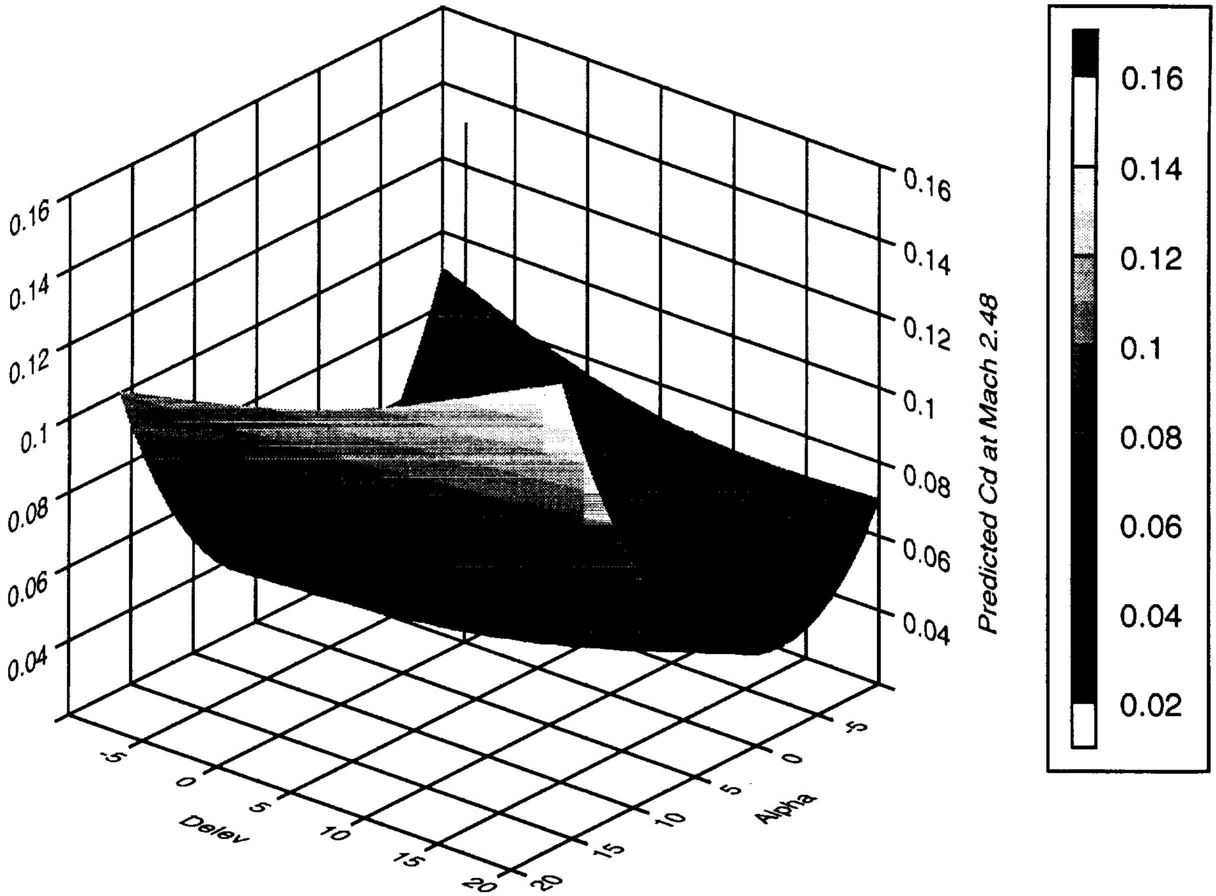
Effect Test

Prediction Profile





0.3
0.2
0.1
0
-0.1
-0.2
Predicted Cl at Mach 2.48
-5
0
5
10
15
20
Delev
Alpha
0.3
0.2
0.1
0
-0.1
-0.2
-0.3


0.16
0.14
0.12
0.1
0.08
0.06
0.04
-5
0
5
10
15
20
Delev
Alpha
Predicted Cd at Mach 2.48
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02

Screening Fit

CD

Summary of Fit

RSquare	0.999329
RSquare Adj	0.999309
Root Mean Square Error	0.000655
Mean of Response	0.056921
Observations (or Sum Wgts)	351

Analysis of Variance

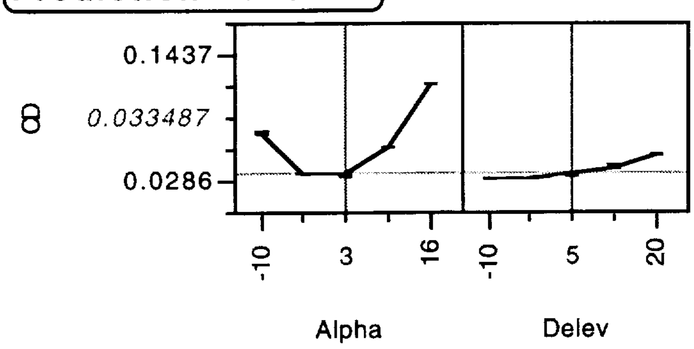
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	10	0.21682298	0.021682	50607.32
Error	340	0.00014567	0.000000	**Prob>F**
C Total	350	0.21696865		0.0000

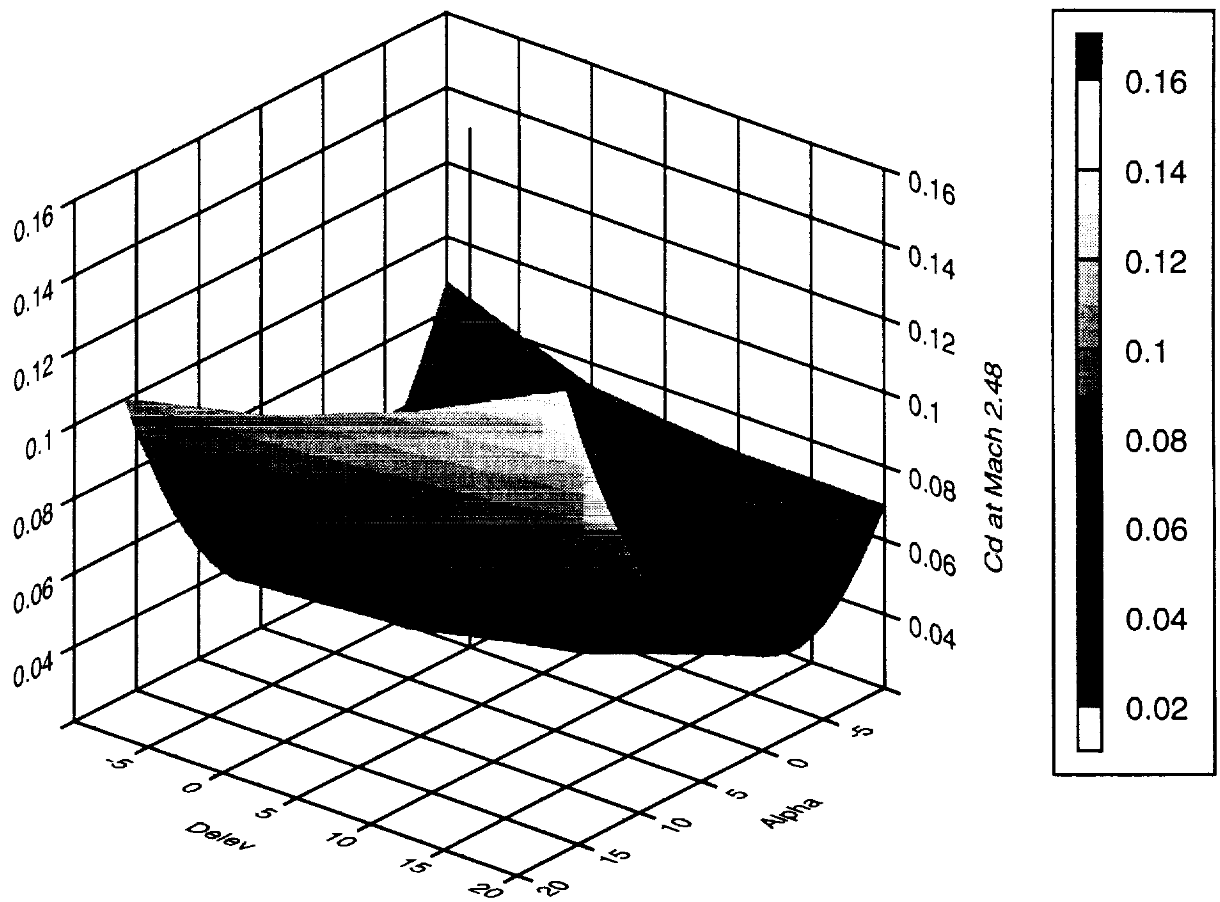
Parameter Estimates

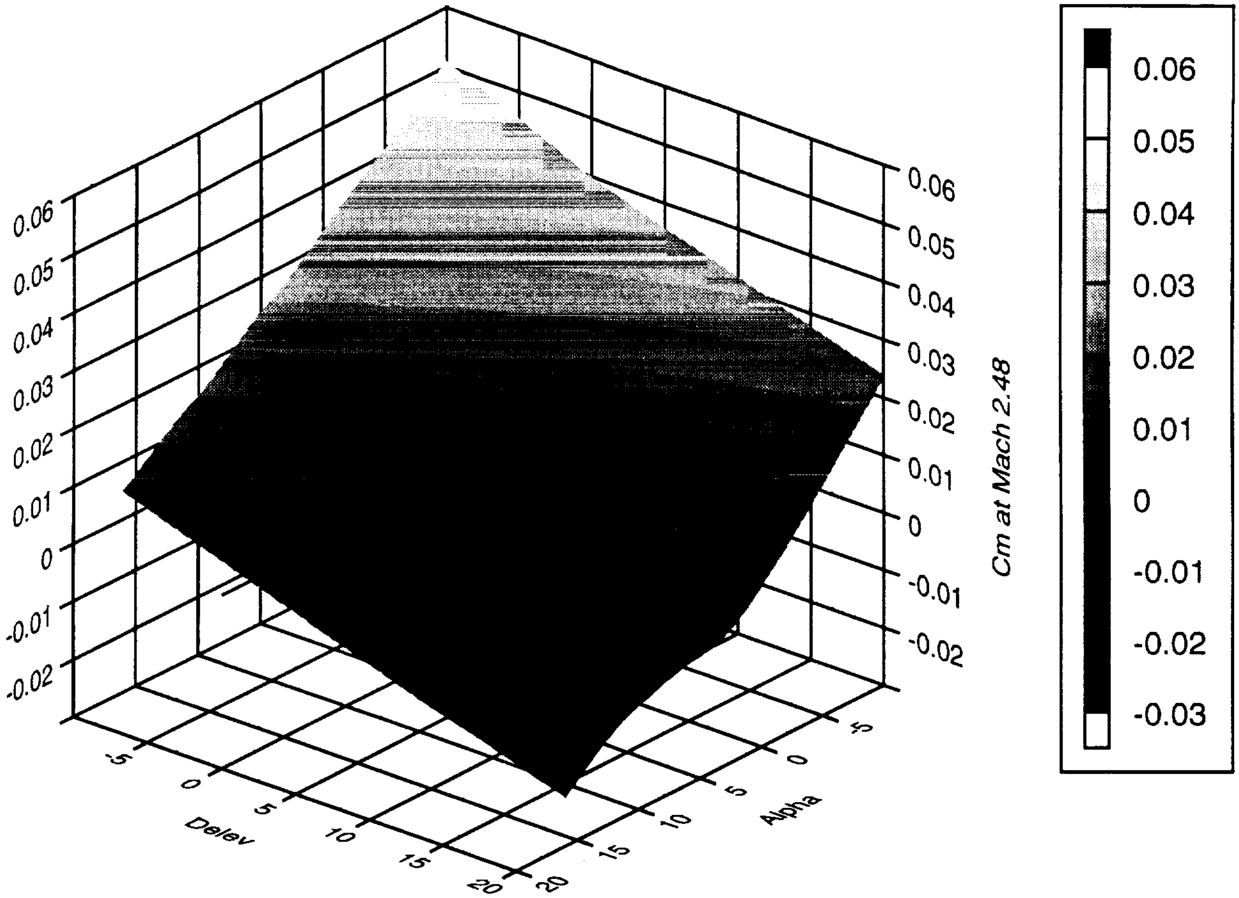
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0298352	0.000075	398.88	0.0000
Alpha	-0.000619	0.000016	-38.49	<.0001
Delev	0.0000522	0.000008	6.71	<.0001
Alpha*Alpha	0.0003262	0.000002	174.09	0.0000
Delev*Delev	0.0000343	4.749e-7	72.24	<.0001
Alpha*Alpha*Alpha	-0.000004	2.004e-7	-18.70	<.0001
Alpha*Alpha*Delev	-8.91e-7	1.144e-7	-7.79	<.0001
Alpha*Alpha*Alpha*Delev	-9.373e-8	1.014e-8	-9.24	<.0001
Alpha*Alpha*Alpha*Alpha	0.0000003	1.41e-8	23.77	<.0001
Delev*Delev*Delev*Alpha	-2.46e-8	3.346e-9	-7.35	<.0001
Alpha*Delev	0.0001061	0.000001	81.56	<.0001

Effect Test

Prediction Profile




0.16
0.14
0.12
0.1
0.08
0.06
0.04
Cd at Mach 2.48
-5
0
5
10
15
20
Delev
Alpha
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02



0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-5
0
5
10
15
20
Delev
Alpha
Cm at Mach 2.48
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03

Screening Fit

Cm

Summary of Fit

RSquare	0.999395
RSquare Adj	0.999377
Root Mean Square Error	0.000401
Mean of Response	0.01163
Observations (or Sum Wgts)	351

Analysis of Variance

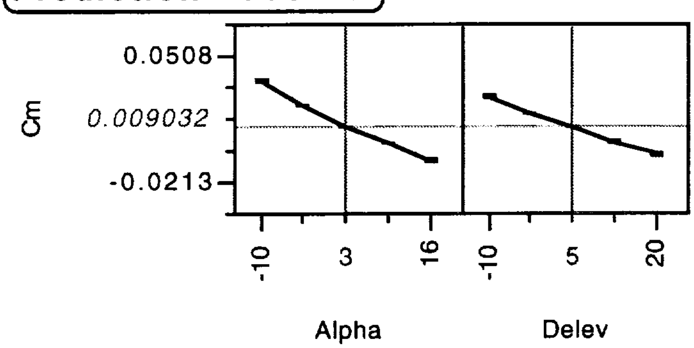
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	10	0.09021670	0.009022	56184.29
Error	340	0.00005459	0.000000	**Prob>F**
C Total	350	0.09027130		0.0000

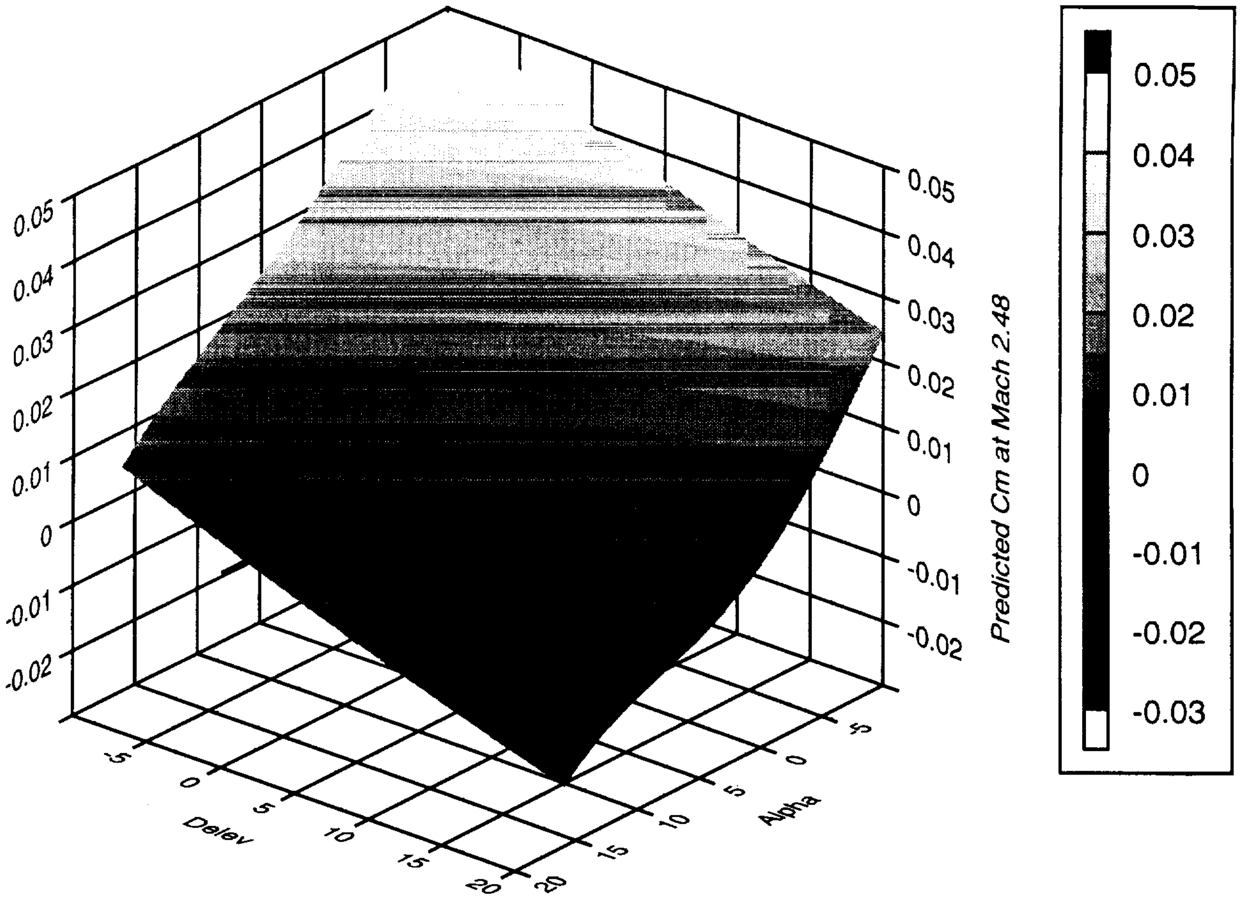
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0197154	0.000051	385.93	0.0000
Alpha	-0.001846	0.00001	-176.7	0.0000
Delev	-0.00118	0.000005	-252.6	0.0000
Alpha*Alpha	0.0000518	0.000002	22.59	<.0001
Delev*Delev	0.0000056	2.758e-7	20.16	<.0001
Alpha*Alpha*Alpha	0.0000006	1.885e-7	3.30	0.0011
Alpha*Alpha*Delev	0.0000024	7.002e-8	34.55	<.0001
Alpha*Alpha*Alpha*Delev	-9.58e-8	6.207e-9	-15.43	<.0001
Alpha*Alpha*Alpha*Alpha	-2.393e-7	2.125e-8	-11.26	<.0001
Alpha*Delev	0.0000036	6.398e-7	5.68	<.0001
Alpha*Alpha*Alpha*Alpha*Alpha	4.509e-9	1.294e-9	3.48	0.0006

Effect Test

Prediction Profile

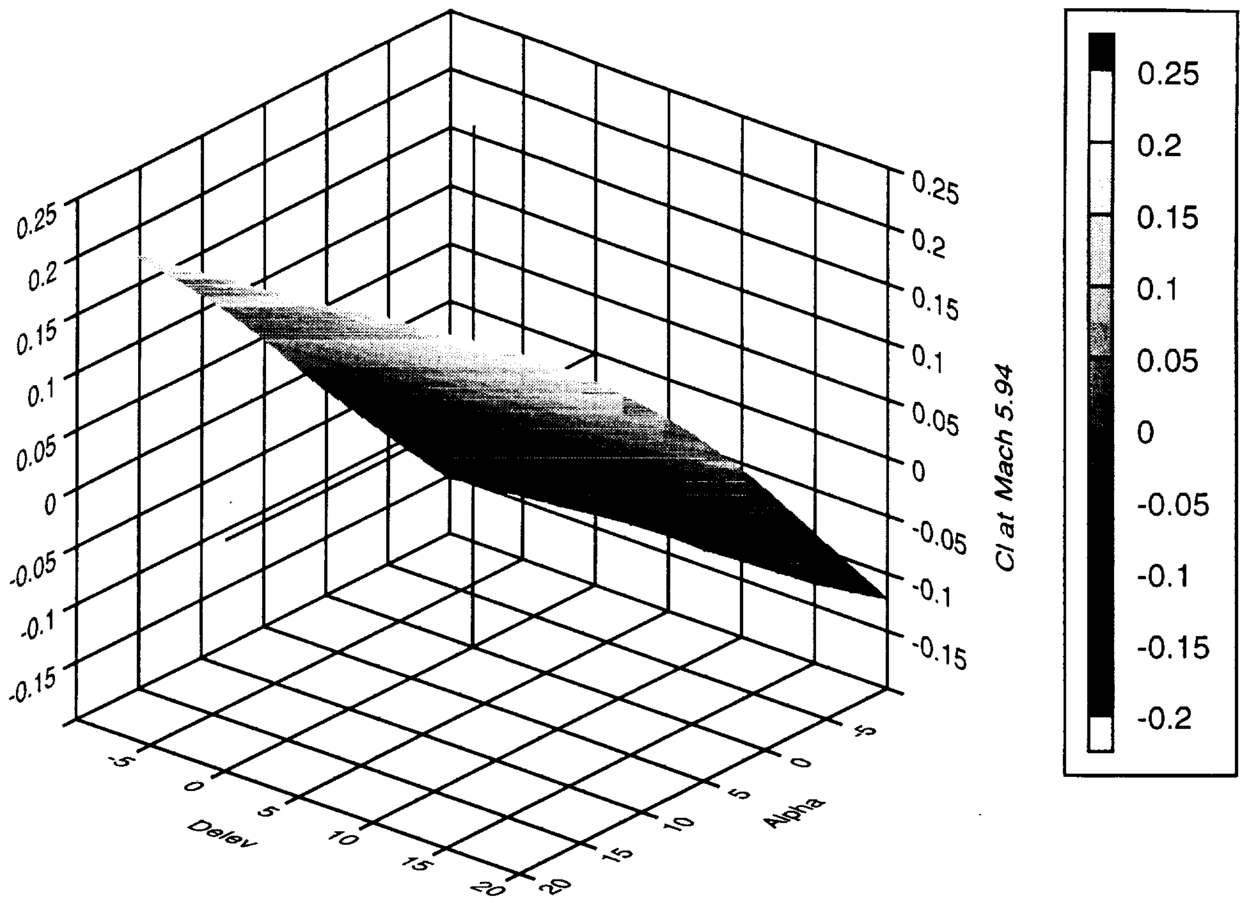




0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
Predicted Cm at Mach 2.48
Delev
-5
0
5
10
15
20
Alpha
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03

Appendix-3

Results for Mach 5.94



0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
Cl at Mach 5.94
Delev
-5
0
5
10
15
20
Alpha
-5
0
5
10
15
20
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2

Screening Fit

CL

Summary of Fit

RSquare	0.999897
RSquare Adj	0.999894
Root Mean Square Error	0.001064
Mean of Response	0.036868
Observations (or Sum Wgts)	351

Analysis of Variance

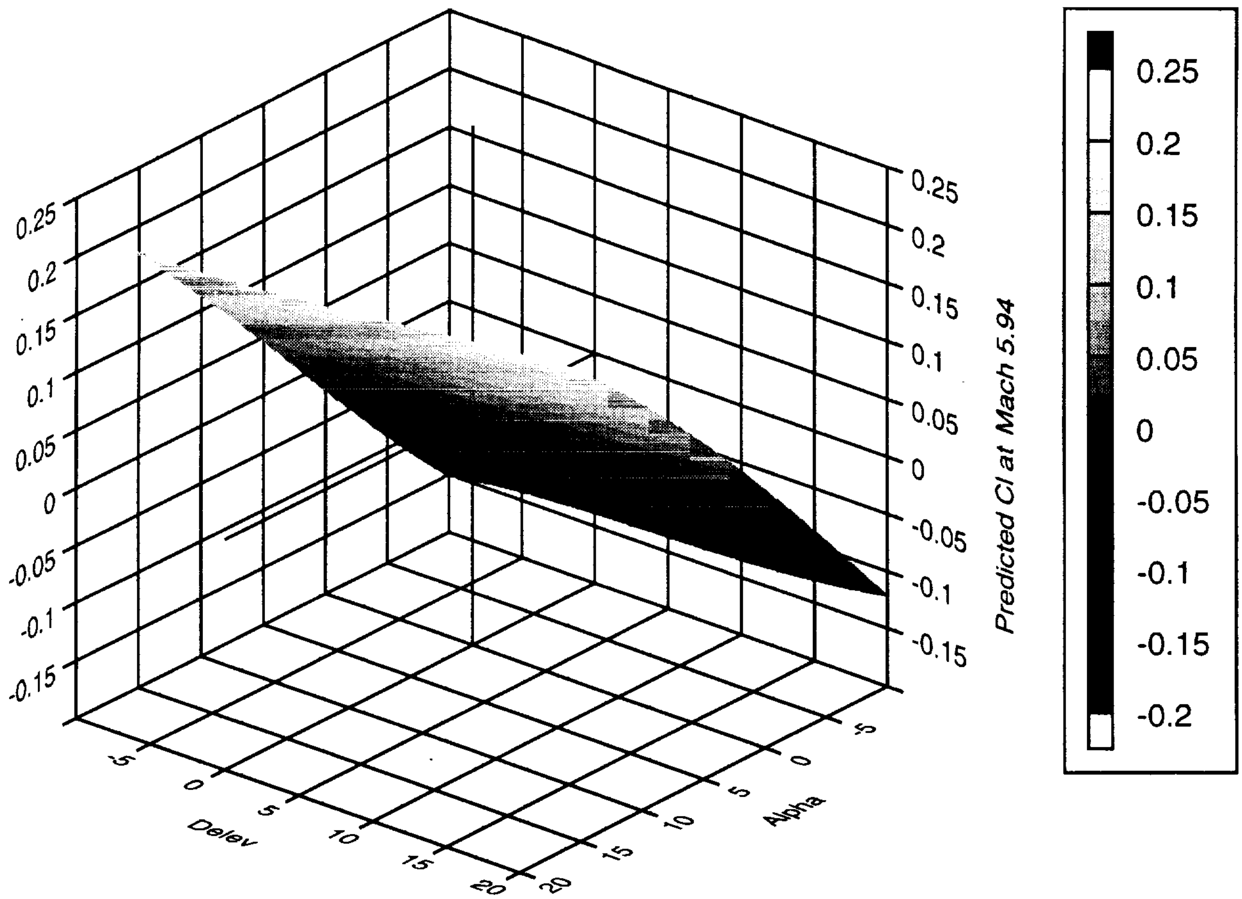
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	12	3.7248770	0.310406	274297.1
Error	338	0.0003825	0.000001	**Prob>F**
C Total	350	3.7252595		0.0000

Parameter Estimates

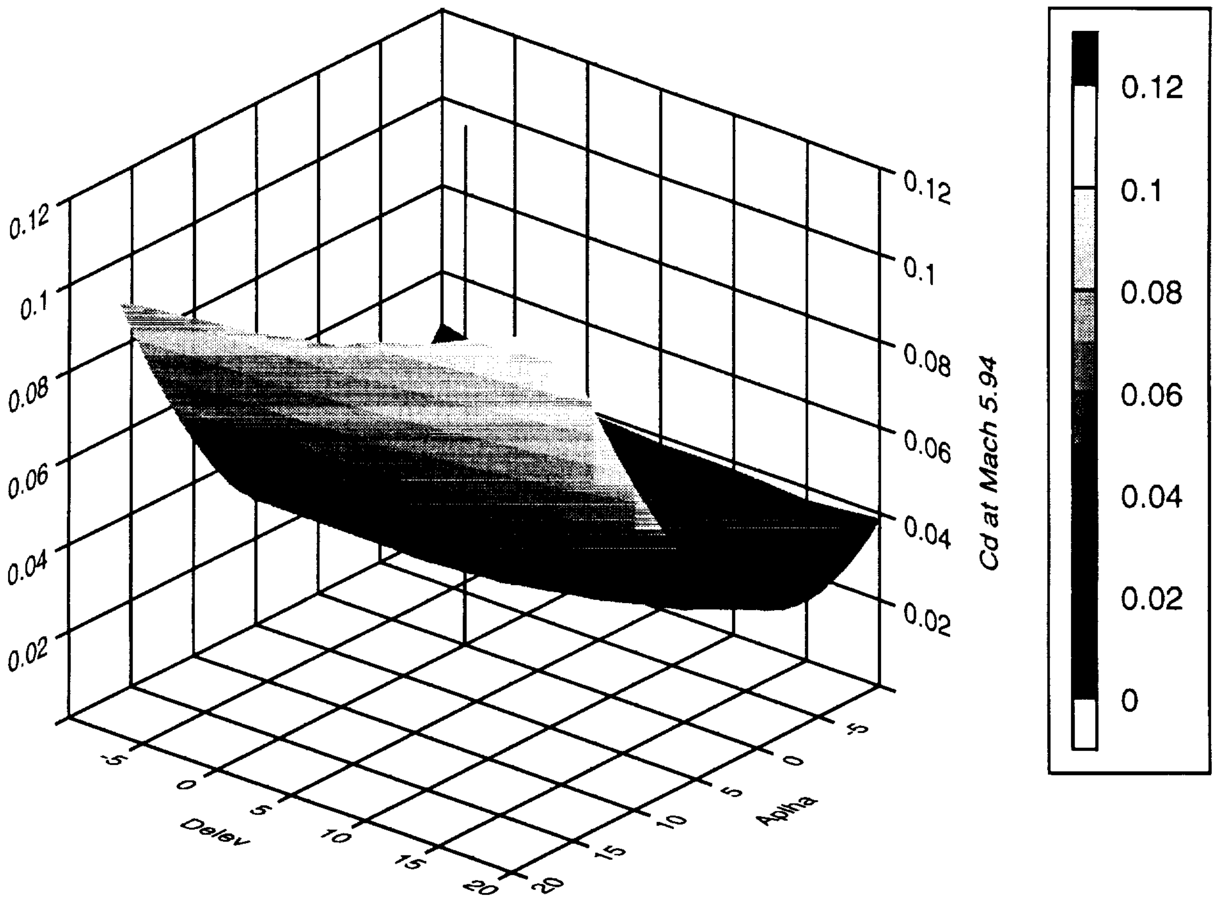
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-0.007874	0.000134	-58.94	<.0001
Alpha	0.0135424	0.000029	461.35	0.0000
Delev	0.0013063	0.000015	87.08	<.0001
Alpha*Delev	0.000028	0.000002	12.93	<.0001
Delev*Delev	-0.000007	0.000002	-4.04	<.0001
Alpha*Alpha	-0.000007	0.000002	-3.77	0.0002
Alpha*Alpha*Alpha	-0.000005	4.415e-7	-11.27	<.0001
Delev*Delev*Delev	0.0000011	9.785e-8	11.62	<.0001
Alpha*Alpha*Delev	-0.000004	1.859e-7	-23.14	<.0001
Delev*Delev*Alpha	0.0000031	1.994e-7	15.42	<.0001
Alpha*Alpha*Alpha*Alpha*Alpha	4.712e-9	1.396e-9	3.38	0.0008
Alpha*Alpha*Alpha*Delev	0.0000001	1.648e-8	6.32	<.0001
Delev*Delev*Delev*Alpha	-1.69e-7	1.172e-8	-14.41	<.0001

Effect Test

Prediction Profile



0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
Delev
-5
0
5
10
15
20
Alpha
Predicted Cl at Mach 5.94
-0.2



0.12
0.1
0.08
0.06
0.04
0.02
Cd at Mach 5.94
-5
0
5
10
15
20
Delev
Aplha
0.12
0.1
0.08
0.06
0.04
0.02
0

Screening Fit

CD

Summary of Fit

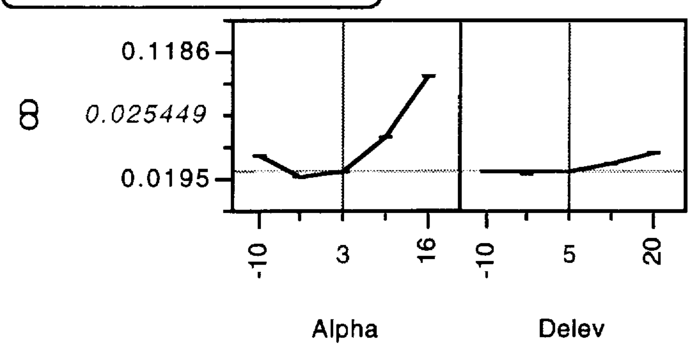
Analysis of Variance

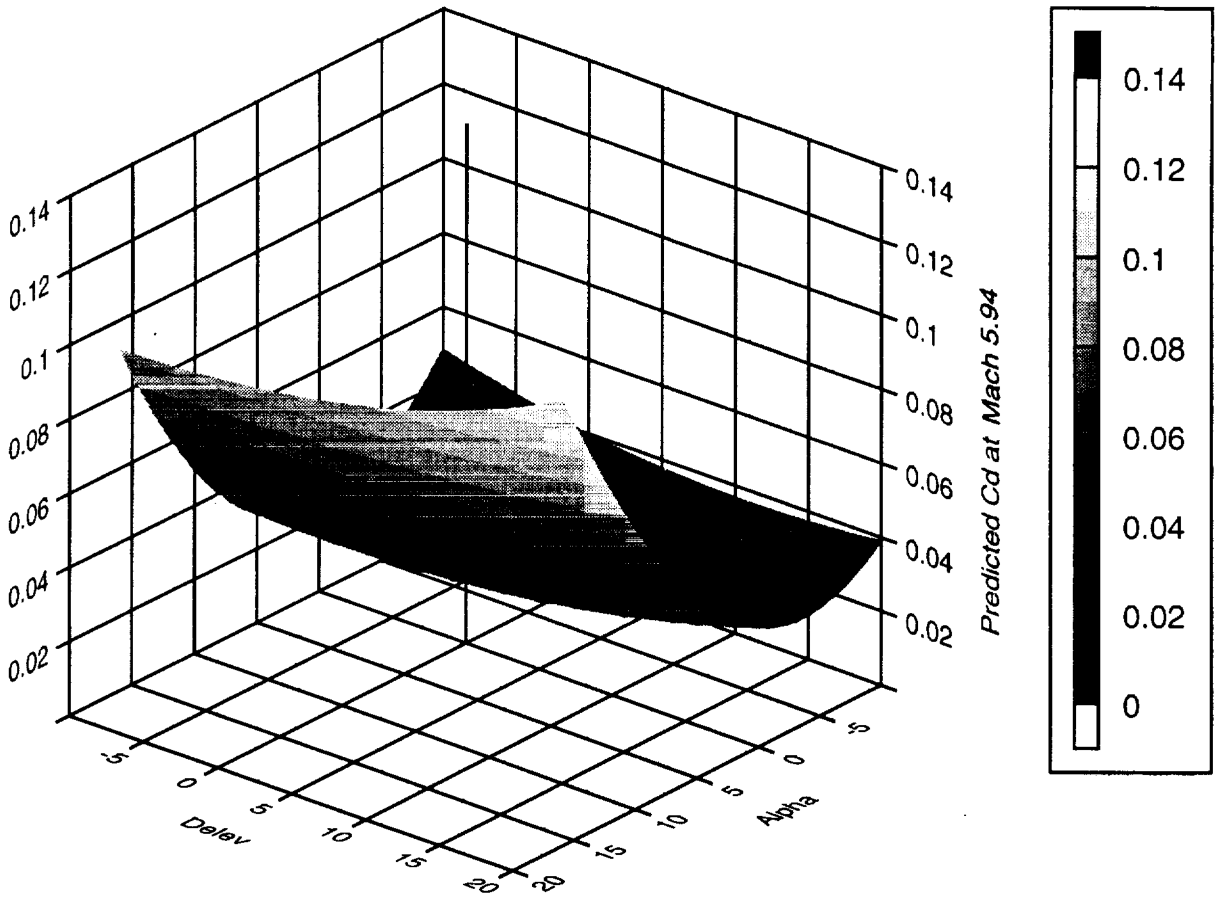
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0200184	0.000063	319.04	0.0000
Alpha	0.0005059	0.000012	41.57	<.0001
Delev	-0.000017	0.000007	-2.43	0.0157
Alpha*Delev	0.0000598	9.898e-7	60.40	<.0001
Delev*Delev	0.0000281	7.232e-7	38.91	<.0001
Alpha*Alpha	0.0002655	0.000001	187.23	0.0000
Alpha*Alpha*Alpha	0.0000008	1.517e-7	5.18	<.0001
Delev*Delev*Delev	0.0000001	4.321e-8	3.21	0.0015
Alpha*Alpha*Delev	-4.956e-7	8.656e-8	-5.72	<.0001
Alpha*Alpha*Alpha*Delev	-7.011e-8	7.673e-9	-9.14	<.0001
Delev*Delev*Delev*Alpha	1.7945e-8	2.573e-9	6.97	<.0001
Alpha*Alpha*Alpha*Alpha	-6.687e-8	1.067e-8	-6.27	<.0001

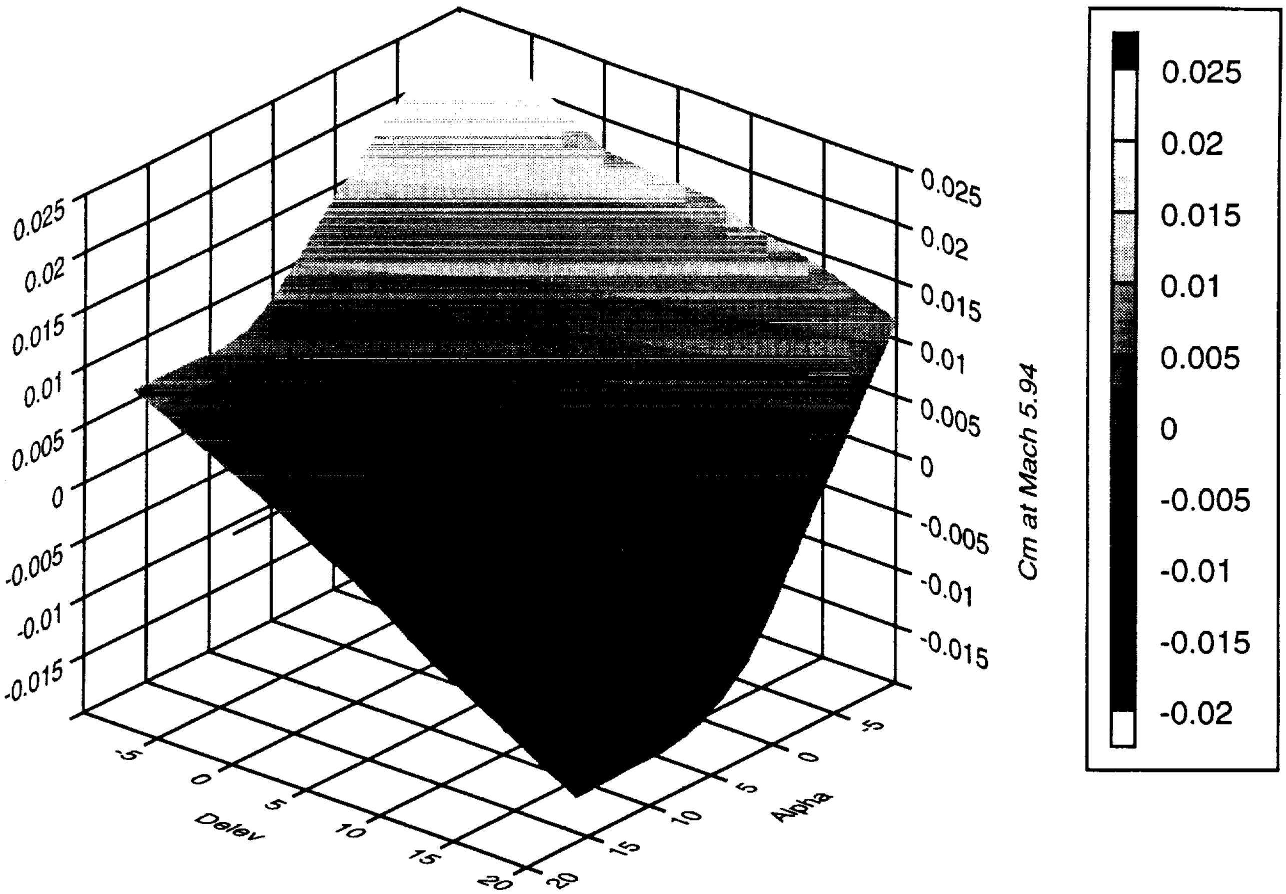
Effect Test

Prediction Profile





0.14
0.12
0.1
0.08
0.06
0.04
0.02
Predicted Cd at Mach 5.94
Delev
-5
0
5
10
15
20
Alpha
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0


0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
Delev
-5
0
5
10
15
20
Alpha
5
0
5
10
15
20
Cm at Mach 5.94
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02

Screening Fit

Cm

Summary of Fit

RSquare	0.998213
RSquare Adj	0.99815
Root Mean Square Error	0.000408
Mean of Response	0.002674
Observations (or Sum Wgts)	351

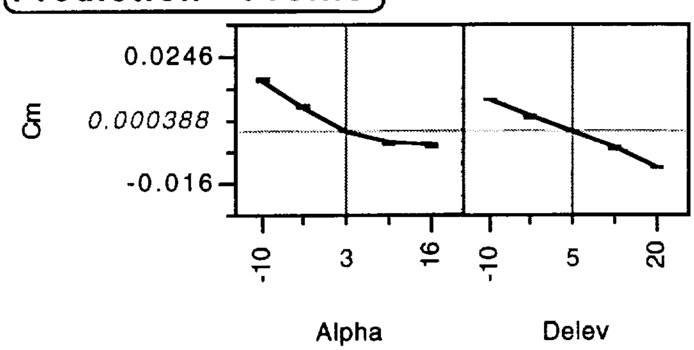
Analysis of Variance

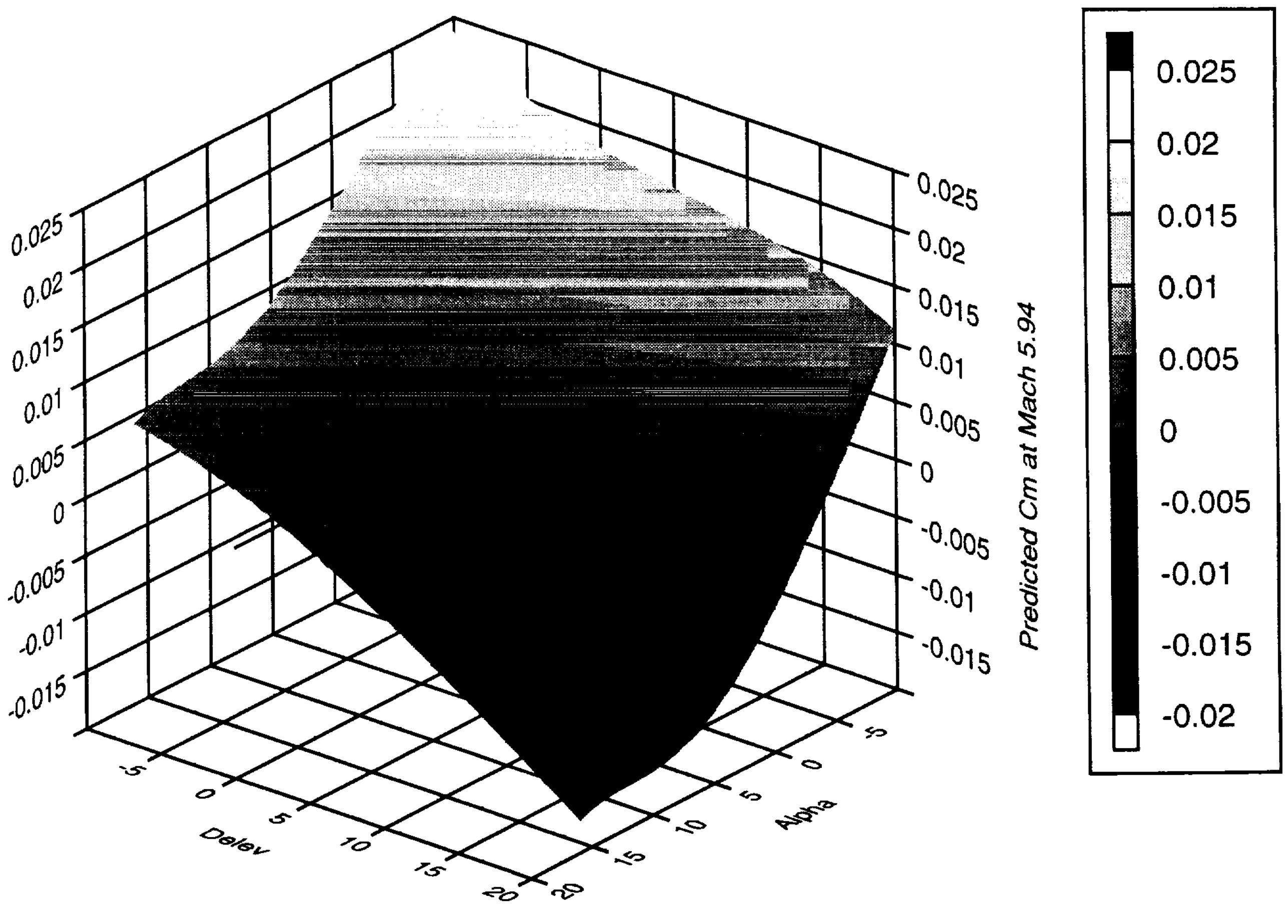
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.006842	0.000051	133.47	<.0001
Alpha	-0.001176	0.000011	-104.4	<.0001
Delev	-0.000613	0.000006	-106.4	<.0001
Alpha*Delev	-0.000013	8.298e-7	-15.95	<.0001
Delev*Delev	0.0000045	6.386e-7	6.98	<.0001
Alpha*Alpha	0.0000226	7.566e-7	29.84	<.0001
Alpha*Alpha*Alpha	0.0000036	1.694e-7	21.54	<.0001
Delev*Delev*Delev	-4.706e-7	3.755e-8	-12.53	<.0001
Alpha*Alpha*Delev	0.0000013	7.133e-8	18.85	<.0001
Delev*Delev*Alpha	-0.000001	7.651e-8	-15.74	<.0001
Alpha*Alpha*Alpha*Alpha*Alpha	-8.767e-9	5.36e-10	-16.37	<.0001
Alpha*Alpha*Alpha*Delev	-3.835e-8	6.323e-9	-6.07	<.0001
Delev*Delev*Delev*Alpha	4.497e-8	4.498e-9	10.00	<.0001

Effect Test

Prediction Profile




Predicted Cm at Mach 5.94
Delev
Alpha
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02