Physics 539 | Spring Semester 1997 | Chris Quigg

March 13, 1997

Recommended Reading:

Gauge Theories, §6.5 – 7.2.
Aitchison and Hey, §14-1 – 14-5.
On parity violation in atomic physics: E. D. Commins, "Atomic Parity Violation Experiments," *Physica Scripta* **36,** 468 (1987); M.-A. Bouchiat, "Neutral Currents in Atoms, Experiments in Cesium and Implications," in *Neutral Currents: Twenty Years Later,* edited by U. Nguyen-Khac and A. M. Lutz (World Scientific, Singapore, 1994), p. 247.
E. D. Commins, "Resource Letter ETDSTS-1: Experimental Tests of the Discrete Space-Time Symmetries," *Am. J. Phys.* **61**, 778 (1993).

Cultural Reading:

D. B. Cline, C. Rubbia, and S. van der Meer, "The Search for Intermediate Vector Bosons," *Sci. Am.* **246**, (March, 1982), p. 48.
S. van der Meer, "Stochastic cooling and the accumulation of antiprotons," *Rev. Mod. Phys.* **57**, 689 (1985).
C. Rubbia, "Experimental observation of the intermediate vector bosons W^+, W^-, and Z^0," *Rev. Mod. Phys.* **57**, 699 (1985).
M.-A. Bouchiat and L. Pottier, "An Atomic Preference between Left and Right," *Sci. Am.* **250**, (June, 1984), p. 100.

Deep Background:

M. J. Musolf, *et al.*, "Intermediate-energy semileptonic probes of the hadronic neutral current," *Phys. Rep.* **239**, 1 (1994), gives the nuclear-physicist's perspective on characterizing the neutral current.

Problems (due March 27, 1997)

26. At very low momentum transfers, as in atomic physics applications, the nucleon appears elementary. If the effective Lagrangian for nucleon β-decay can be written in the limit of zero momentum transfer as

$$\mathcal{L}_\beta = -\frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu\bar{p}\gamma^\lambda(1-g_A\gamma_5)n,$$

where $g_A = 1.2601 \pm 0.0025$ is the axial charge of the nucleon (renormalized from unity by the strong interactions), show that the eN neutral-current interactions may be represented by

$$\mathcal{L}_{ep} = \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\bar{p}\gamma^\lambda(1-4x_W-g_A\gamma_5)p$$

and

$$\mathcal{L}_{en} = -\frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\bar{n}\gamma^\lambda(1-g_A\gamma_5)n.$$

Regard the nucleus as a noninteracting collection of Z protons and N neutrons. Now perform the nonrelativistic reduction of the implied nuclear matrix elements. Show that, for a heavy nucleus, the dominant parity-violating contribution to the electron–nucleus amplitude will be of the form

$$\mathcal{M}_{\text{p.v.}} = -\frac{iG_F}{2\sqrt{2}}Q^W\bar{e}\rho_N(\mathbf{r})\gamma_5 e,$$

where ρ_N is the nucleon density as a function of the electron coordinate $\mathbf{r}$, and the weak charge is

$$Q^W = Z(1 - 4x_W) - N.$$

27. The weak charge of ${}^{205}_{81}\mathrm{Tl}$ has been measured by a group at the University of Washington [P. A. Vetter, D. M. Meekhof, P. K. Majumder, S. K. Lamoreaux, and E. N. Fortson, *Phys. Rev. Lett.* **74**, 2658 (1995)] as

$$Q^W = -114.2 \pm 1.3 \pm 3.4 \ ,$$

where the first error is experimental and the second refers to the uncertainty in atomic theory. Using the first-order expression derived in Problem 26, deduce the weak mixing parameter $\sin^2\theta_W$ and its uncertainties.

28. LEP measurements on the Z^0 were summarized by Joachim Mnich, *Phys. Rep.* **271**, 181 (1996), as follows:

M_Z	91.1884 ± 0.0022 GeV/c^2
Γ_Z	2.4963 ± 0.0032 GeV
$\Gamma(Z \to \text{hadrons})$	1744.8 ± 3.0 MeV
$\sigma_{\text{had}}(\text{pole})$	41.488 ± 0.078 nb
$\Gamma(Z \to e^+e^-)$	83.92 ± 0.17 MeV
$\Gamma(Z \to \mu^+\mu^-)$	83.92 ± 0.23 MeV
$\Gamma(Z \to \tau^+\tau^-)$	83.85 ± 0.29 MeV
$\Gamma_{\text{leptonic}} = \Gamma(Z \to \ell^+\ell^-)$	83.93 ± 0.14 MeV
$\Gamma_{\text{invisible}}$	499.9 ± 2.5 MeV
a_e	-0.5011 ± 0.0004
v_e	-0.0380 ± 0.0007

Let us confront these results with the lowest-order predictions of the Weinberg–Salam model.

(a) Use the value of the Z^0 mass to determine $x_W \equiv \sin^2\theta_W$.

(b) Using your value of x_W, predict the $W^\pm$ mass and compare it with the current world average: $M_W = 80.38 \pm 0.09$ GeV/c^2.

(c) Use the measured value of the Z^0 mass to predict the partial width $\Gamma(Z \to \nu\bar{\nu})$. By comparing with $\Gamma_{\text{invisible}}$ measured at LEP, determine the number of light neutrino species.

(d) Use your value of the weak mixing parameter x_W to predict $\Gamma_{\text{leptonic}} = \Gamma(Z \to \ell^+\ell^-)$, $\Gamma(Z \to \text{hadrons})$, a_e, and v_e. Neglect the masses of all fermions but the top quark.

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Supplement to references on inverse muon decay, p. 95 of *Gauge Theories*:
¶ S. R. Mishra, *et al.*, *Phys. Lett.* **252B**, 170 (1990).

$$\sigma(\nu_\mu e \to \mu^- \nu_e)/\sigma_{\text{th}} = 0.981 \pm 0.049 \text{ (stat.)} \pm 0.030 \text{ (syst.)}$$

for neutrino energies up to 600 GeV.
¶ P. Vilain, *et al.* (CHARM II Collaboration), *Phys. Lett.* **364B**, 121 (1995).

$$\sigma(\nu_\mu e \to \mu^- \nu_e)/\sigma_{\text{th}} = 0.929 \pm 0.019 \pm 0.048$$

in the SPS wideband beam ($\langle E_\nu \rangle = 23.8$ GeV, neutrino energies up to 150 GeV).
¶ J. Dorenbosch, *et al.* (CHARM Collaboration), *Z. Phys.* C**41**, 567 (1989).

$$\sigma(\nu_\mu e \to \mu^- \nu_e)/\sigma_{\text{th}} = 1.10 \pm 0.08 \text{ (stat.)} \pm 0.13 \text{ (syst.)}$$

in the SPS wideband beam.

Additional references on $\nu_\mu e$ neutral current reactions:
¶ N. J. Baker, *et al.*, *Phys. Rev.* D**40**, 2753 (1989), *ibid.* **41**, 1715E (1990).
¶ L. A. Ahrens, *et al.*, *Phys. Rev.* D**41**, 3297 (1990), Brookhaven experiment E-734.
¶ P. Vilain, *et al.* (CHARM II Collaboration), *Phys. Lett.* **335B**, 246 (1994).
¶ J. Dorenbosch, *et al.* (CHARM Collaboration), *Z. Phys.* C**41**, 567 (1989).

Detection of $\bar{\nu}_e e$ scattering:
F. Reines, H. S. Gurr, and H. W. Sobel, *Phys. Rev. Lett.* **37**, 315 (1976).

Detection of $\nu_e e$ scattering:
R. C. Allen, *et al.*, *Phys. Rev.* D**47**, 11 (1993).

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First class after spring break: Monday, March 24.

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to http://lutece.fnal.gov/Princeton/.