Localization of Electromagnetic waves in two-dimensional disordered systems

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ABSTRACT

We calculate the average transmission for s- and p-polarized electromagnetic (EM) waves and consequently the localization length of two dimensional disordered systems which are periodic on the average; the periodic systems form a square lattice consisting of infinitely long cylinders parallel to each other and embedded in a different dielectric medium. In particular, we study the dependence of the localization length on the frequency, the dielectric function ratio between the scatterer and the background, and the filling ratio of the scatterer. We find that the gaps of the s-polarized waves can sustain a higher amount of disorder than those of the p-polarized waves, due to the fact that the gaps of the s-polarized waves are wider than those of the p-polarized waves. For high frequencies, the gaps of both type of waves are easily disappear, the localization length is constant and it can take very small values. The optimum conditions for obtaining localization of EM waves in 2D systems will be discussed.

I. INTRODUCTION

Recently, there has been growing interest in the propagation of electromagnetic (EM) waves in random media.[1–4] This problem is closely related to the electron transport in disordered solids, although phenomena similar to electron-electron and electron-phonon interactions are not present in the EM case, so, one expect that this is an ideal case to study the variety of complex effects related with the scattering of waves in random media.

For relatively weak disorder, an enhanced backscattering, as a result of the interference between time-reversed waves, has been observed in light scattering experiments[5,6] and it is well understood theoretically.[7,8] By increasing the disorder, the interference of the waves due to the strong scattering creates more and more exponentially localized states and after a certain critical value of the disorder all the states become localized and the diffusion coefficient vanishes.[1–4] It is by now well known that for one- and two-dimensional (1D and 2D) systems, an infinitesimal amount of disorder is enough to make all the states localized (except some special models which do not follow this rule[9]), while for three-dimensional (3D) systems, the states become localized only when the disorder is higher than a finite value.[10,11] However, there is no conclusive experimental evidence for the localization of EM waves in either 2D or 3D systems, although experiments by Genack and collaborators[12] provide some indications that light localization is possible.

The difficulty in localizing EM waves had led to suggestions of alternative pathways to localization. John[13] has proposed that EM localization may be more easily achieved in a weakly disordered system of almost periodically arranged dielectric system in the frequency regime near a band gap. We can very reliably calculate the bands and the gaps of a periodic system. It is very plausible that a connection between the gaps in the periodic system and the regions of localized states in a random system exist,[14,15] at least for weak disorder. Indeed, in that case the regions of localized states (being at the tails inside the gaps) practically coincide with the positions of the gaps.

Recently, McGurn et al.[16] studied the propagation of light of frequency, ω emitted radially from a line source in an infinite 2D disorder dielectric medium by numerically integrating Maxwell's equations in space and time. Their system consisted of N×N identical rods of square cross section and dielectric constant ϵ, that fill a supercell without chinks or overlaps. They introduced disorder by removing half of these rods randomly and replacing them by vacuum (the filling ratio of that system is 0.5). They found that high frequencies and high dielectric contrasts favor localization with small values of the localization length. However, their estimations of the localization lengths should be regarded only as indications of their order of magnitudes.[16]

In this paper, we study the optimum conditions for the appearance of the gaps in 2D periodic dielectric systems and how those conditions are related with the scattering cross section of a single scatterer (section 2). The 2D systems consist of infinitely long cylinders parallel to the z-axis embedded in a different dielectric medium; the EM waves propagates in the x,y plane. Then, disorder is introduced in the 2D systems described previously by either randomly changing the radius of the cylinders, or randomly moving the cylinders away from their periodic positions, or randomly changing the dielectric constant of the cylinders. Although, we expect that all the states become localized with the smallest amount of disorder, there is still the question of what are the optimum conditions for the appearance of the smallest localization length. An answer to that question is very important since that will help the experimental observation of the localization of EM waves in 2D systems. In section 3, we answer that question by considering the different factors which affect the frequency dependent localization length such as the amount of disorder, the filling ratio and the ratio of the dielectric constant between the two media. We find that s-polarized waves give the smallest localization lengths for cylinders with a high dielectric constant, when the filling ratio is around 25%. The gaps of the p-polarized waves are destroyed easily by the disorder.

Periodic systems similar with the ones that we study in the present work have already been studied both experimentally and theoretically.[17–21] Disorder can be introduced easily in those systems by either moving apart the cylinders or by changing their radius, so, one can verify the present results experimentally. Also, the effect of the disorder in the gaps is very important especially in applications where we want to construct materials in smaller dimensions (of the order of microns).[1–4] In those cases, the introduction of a disorder similar with those studied in the present work is almost inevitable and one wants to know what is the highest amount of disorder for which the gap disappear. This question is clearly answered in the present work.

II. PERIODIC

For EM waves propagating in the x,y plane, the s (**E** field parallel to the z-axis) and p (**E** field perpendicular to the z-axis) polarized waves can be described by two decoupled wave equations. The equation for the s-polarized wave is:

$$\nabla^2 E + \frac{\omega^2}{c^2}\epsilon E = 0 \tag{1}$$

where $E = E_z$; $\epsilon = \epsilon(r)$ is the dielectric constant, ω is the frequency, and c is the speed of light in the vacuum. Equation (1) is identical with the scalar wave equation. The equation for the p-polarized wave has the form:

$$\nabla(\frac{\nabla H}{\epsilon}) + \frac{\omega^2}{c^2}H = 0 \tag{2}$$

where $H = H_z$. Using the plane-wave (PW) expansion method, we can solve the previous two equations for a periodic lattice[17–21] and therefore find the band structure, $\omega(\vec{k})$.

We have made a systematic effort in order to find the optimal values of the filling ratio, f, and the ratio of the dielectric constant μ for the creation of a gap in 2D square lattices, for both the s- and p-polarization. The minimum value of the relative dielectric constant, μ, for creating a gap is obtained for both dielectric and air cylinders. In Fig. 1, the calculated size of the forbidden gap over the midgap frequency $\delta\omega/\omega_g$ versus the filling ratio of the cylinders, f, for a constant dielectric ratio, μ=10 is shown. For s-polarized waves, a maximum $\delta\omega/\omega_g$ of 33% (30%) appears when f= 0.20 (0.80) for high (low) dielectric constant cylinders, respectively. For the p-polarized waves and low dielectric cylinders, $\delta\omega/\omega_g$ can only reach 8% at f=0.50. However, for the p-polarized waves and high dielectric cylinders with μ=10, there is no gap for any f.

Another interesting information is how easily a band gap can be formed. In particular, for certain value of f, what is the lowest dielectric constant that forms a gap? We plot in Fig. 2 the threshold value of the dielectric constant ratio, μ, for which the first gap just opens, versus the filling ratio of the cylinders, f. The results for s-polarized waves (Fig. 2a) are the following: for high dielectric cylinders, the lowest gap appear when μ=3.0 and f=25%, while for the opposite case of low dielectric cylinders in a high dielectric medium, the threshold value is μ=7.9 and f=70%. For the p-polarized case (Fig. 2b) of high dielectric cylinders, the lowest gap appears when μ=15 and f=40%, while in the opposite case of low dielectric cylinders,μ=4.4 and f=55%.

¿From the present and previous results,[19] one concludes that for the s-polarized waves the optimum conditions for the appearance of the gaps are obtained when f is around 0.25, for the case of high dielectric cylinders in a low dielectric medium. On the other hand, for the p-polarized waves,the optimum conditions are obtained when f is around 0.50, for the case of low dielectric cylinders in high dielectric medium; however, the size of the gaps are small even with that optimum conditions.

Can we interpret the behavior of the band gaps presented in Fig. 1 and 2, by the Mie resonant scattering from a single dielectric cylinder? The behavior of the scattering cross section, σ, for an isolated cylinder of high dielectric material (ϵ=10) in air is shown in Fig. 3a. Notice that the first three resonances of the s-polarization at $d/\lambda_i = 0.3$, 0.7 and 1.25 are very sharp (dashed line in Fig. 3a); in contrast, for the p-polarization (solid line in Fig. 3a) the corresponding three first resonances are weaker and they appear at higher frequencies. We have chosen to present the results for the total scattering cross section versus frequency, as d/λ_i, where d is the diameter of the cylinder and $\lambda_i = 2\pi c/\omega\sqrt{\epsilon_i}$ is the wavelength inside the cylinder. In this 2D case too, the positions of the gaps in periodic 2D dielectric structures can be associated with the Mie resonance frequencies of the isolated cylinder. For an isolated air cylinder in a dielectric medium with $\epsilon = 10$, the scattering cross section (Fig. 3b) is very similar for both polarizations and it exhibits an oscillatory behavior; the differences are more prominent for the first two Mie resonances where they are stronger for the p-polarization. Comparing the results for the s-polarization (dashed lines in Fig. 3a,b), we conclude that cylinders of high dielectric material can create more scattering than the opposite case and this obviously explain why the gaps in the corresponding periodic case appear easier for the s-polarization and they are wider. On the other hand, comparing the results for the p-polarization (solid lines in Fig. 3a,b), we conclude that the strength of the resonances is nearly the same for cylinders of either the low or the high dielectric material, so, we expect that the gaps will appear equally difficult in both cases, as it is really the case.

III. DISORDER

In this section, we systematically study the behavior of the frequency dependent transmission coefficient as a function of the strength of the disorder and how the gaps of the periodic case are affected by the introduction of the different types of disorder. The transfer-matrix method (TMM), introduced recently by Pendry and MacKinnon,[22] has been used for the calculation of the transmission. In that method, the space is divided in small cells and the fields in each cell are coupled with those in the neighboring cells; then the transfer matrix can be defined by relating the fields on one side of the material's slab with those on the other side. So, the transmission and reflection coefficients for EM waves of various frequencies incident on a finite thickness material can be obtained; the system assumed periodic in the directions parallel to the interfaces. The method has been used in two-dimensional[23] and three-dimensional[24] structures as well as in cases where the dielectric constant is complex and frequency dependent.[25,26] In all the cases, the agreement between theory and experiment is very good.

The disorder can be introduced by three different ways. First, by randomly changing the radius of each cylinder while they are keeping their periodic positions; in that case the radius of the i-th cylinder is given by $r_i = (1+\gamma)r$ where r is the radius of the cylinder used in the periodic lattice and γ is a random variable uniformly distributed over the interval $[-d_r, d_r]$, so, d_r measures the strength of the disorder. Second, by moving the cylinders from their ideal 2D periodic positions, so the x and y components of the position of the i-th cylinder in the random system differ from those of the periodic case by γ_xa and γ_ya, respectively; γ_x, γ_y are random variables uniformly distributed over the interval $[-d, d]$ and a is the lattice constant of the periodic system. Third, by changing the dielectric constant of each cylinder; in that case, the dielectric constant of the i-th cylinder is $\epsilon_i = (1+\gamma)\epsilon$ where ϵ is the dielectric constant of the cylinders in the corresponding periodic case and γ is a random variable uniformly distributed over the interval $[-d_\epsilon, d_\epsilon]$. For the first two types of disorder, some of the cylinders may overlap when the amount of disorder is higher than a certain value which depends on the filling ratio. The second type of the disorder, the random displacement of the dielectric cylinders, is the easiest to experimentally implement.

As we have pointed in the introduction, for 2D systems, all the states will become localized even with the smallest amount of disorder and the localization length, ℓ, is given by:[10,11]

$$\ell = -\frac{2L}{< ln(T) >} \tag{3}$$

where the $< ln(T) >$ is the logarithmic average of the transmission over the different configurations. In all the following cases, the average lattice is a square with cylinders of circular cross section and filling ratio, f, and the waves propagate along the y-axis while the axis of the cylinders is along the z-axis. The thickness of the system (along the y-axis) is L, its width (along the x-axis) is W, and the lattice constant of the corresponding periodic system is a=1.28 cm (the results can be scaled in any lattice constant). A supercell which include three cylinders along the x-axis (W=3a) is used with periodic boundary conditions at the edges of the supercell. We studied the effect of W in a system similar to the one described in Fig. 4 with $d = 0.4$ and we found that there is a convergence of the $< ln(T) >$ for almost all the frequencies as we increase the W/a from three to four. According to the scaling hypothesis for localization,[10,11] this indicates that the waves are localized. Due to computer time constrains, $< ln(T) >$ is calculated from an average over 10 configurations which gives less than 20% error in the value of $< ln(T) >$. Finally, each unit cell is divided into 10×10 points; calculations with higher divisions per unit cell have less than 5% difference with the present results.

Fig. 4a shows the $< ln(T) >$ vs the frequency for a system consisting of cylinders with dielectric constant $\epsilon_i = 10$ and f= 0.28 embedded in air; the thickness is L=25a, the E-field of the wave is parallel with the axis of the cylinders (s-polarized). For the periodic case (solid line in Fig. 4a), there are two sharp drops in the transmission which correspond to the first two gaps and some smaller drops which correspond to the higher gaps. Comparing the position of the gaps with the Mie resonances (dotted line in Fig. 3a), we conclude that the gaps appear between the Mie resonance frequencies. In particular, the first three Mie resonances appear at 4.5, 9.1, and 14.5 GHz, while the first two gaps appear at around 6 and 11.5 GHz. With the introduction of the first type of disorder where the radius of the cylinders changes randomly, the higher gaps disappear even with a small amount of disorder ($d_r = 0.2$, dotted line in Fig. 4a) while the first two gaps survive; especially the first one survives for d_r as high as 0.4. For the p-polarized waves (Fig. 4b), there are three large drops in the transmission of the periodic case (see solid line in Fig. 4b), but they are not correspond to full gaps since, as we have mentioned in the previous section, there are not full gaps for that polarization and for ϵ=10. For that reason, those drops disappear easily with the introduction of the disorder; for d_r =0.2 (dotted line in Fig. 4b), only the first drop survives, although much smaller, while, for d_r =0.4 (dashed line in Fig. 4b), there are no drops.

There are some common characteristics of the average transmission for both polarizations. For relatively high frequencies (ω higher than about 20 GHz in our case, or, in general, ω higher than about c/a), $< ln(T) >$ is almost independent of the frequency; we shall refer to that value as $< ln(T) >_s$, the saturated value of $< ln(T) >$. Also, this value is almost independent on the amount of the disorder within the accuracy of our statistical average. In particular, for p-polarized waves, $< ln(T) >_s$ is about -6 (Fig. 4b) for both d_r = 0.2 and 0.4 while for the s-polarized waves, $< ln(T) >_s$ is about -6.5 (Fig. 4a). Similar conclusions have been found for one-dimensional random bilayer systems[27] and they have been explained as follows: At high frequencies (small wavelengths) the phase coherence between the scattering at different interfaces is lost and the dominant factor is the the scattering from each interface. But, the transmission and reflection coefficients at a sharp interface are frequency independent, therefore the $< ln(T) >$ and ℓ are also frequency independent at high frequencies.[27] In the present 2D case, for high frequencies, one expect that the geometric limit to be reached and the dominant scattering factor to be from the interface, so, $< ln(T) >$ will be frequency independent. On the other hand, at low frequencies (high wavelengths), the wave is not affected by the inhomogeneity of the system, but it essentially "sees" a uniform medium with an effective dielectric constant, so, we expect that $< ln(T) >$ tends to zero as the frequency approaches zero and it is independent of the thickness of the system and the amount of the disorder. This is clearly shown for the p-polarization in Fig. 4b; $< ln(T) >$ is almost the same as in the perfect case for frequencies less than 6.5 GHz and 5 GHz for the d_r = 0.2 and 0.4 cases respectively. For the s-polarization, this happens for frequencies smaller than 4 GHz which is the lowest frequency in our calculations.

At intermediate frequencies, the drops of the $< ln(T) >$ are actually the reminiscent of the corresponding gaps of the periodic case. Our calculations show that the wider the gap of the periodic case is, the larger the amount of disorder needed to close it. At frequencies inside the gaps, the $< ln(T) >$ and the localization length increases by increasing the disorder as a result of the fact that the density of states (DOS) become higher due to the creation of localized states inside the gap.[27–30] In order to find how the different types of disorder affect the gaps we plot (Fig. 5) the localization length of s-polarized waves at 6.5 GHz (this is in the middle of the first gap; see Fig. 4a) as a function of the rms error of the dielectric constant, $\sqrt{< \epsilon^2 >}$. This is defined as:

$$< \epsilon^2 >=< \frac{1}{N}\sum_{i=1}^{N}(\epsilon_i^d - \epsilon_i^p)^2 > \qquad (4)$$

where ϵ_i^d, ϵ_i^p are the dielectric constants at site i in the disorder and the periodic case respectively; N is the number of sites in the system and $< ... >$ means an average over different configurations. Notice that ℓ increases, initially, as the disorder increases, against the usual expectations. The localization length is around 1.5a for $\sqrt{< \epsilon^2 >}$ less than about one and then increases. As we explained previously, this is due to the fact that the DOS increases in that particular range of frequencies. However, we expect that for high amount of disorder ($\sqrt{< \epsilon^2 >}$ higher than the values that we have used in our calculations), the localization length will start decreasing because the DOS will eventually reach its maximum value and the localization effects will become dominant. The behavior is very similar for disorder in the dielectric constant as well as in the radius of the cylinders (compare dashed and solid lines in Fig. 5). For the case with randomness in the position of the cylinders, $\sqrt{< \epsilon^2 >}$ reaches a constant value when the strength of the disorder, d, is higher than about 0.5 and that cause difficulties in getting any conclusions.

Complimentary and additional information can be obtained by examining the effect of the disorder on the density of states (DOS) of the system. The density of states is obtained by an order(N) spectral method, which is discussed in detail elsewhere.[31] Briefly, the method integrates the Maxwell's Equations numerically in the time-domain, and the spatial derivatives (the "curls") are determined by finite differences. The E and H fields are stored as time series, and when a sufficient number of time steps have been accumulated (which governs the resolution in the frequency domain), the field intensities are Laplace-transformed from the time-domain to the frequency domain to obtain the spectral intensities. If the initial fields corresponds to random numbers, the spectral intensities would correspond to the density of states of the system under consideration. The following results are obtained with a supercell containing 64 unit cells (8x8), and each cell is discritized by a 32x32 grid. Periodic boundary condition is imposed. For 2D systems, the Maxwell Equations decouples to E-polarized modes and H-polarized modes, so that we can define a DOS for each polarization.

The DOS of the E-polarized modes (s-polarization) for the case of no disorder and disorder in radius with $d_r = 0.2$ and $d_r = 0.4$ are compared in Fig.6. We note from Fig.6a, which shows the DOS for the perfect structure, that there are spectral gaps for this polarization centered at about 6.5 GHz, 11.5 GHz, and the 16 GHz and they will be referred to as the first, second and the third gap respectively. These spectral gaps give rise to the corresponding dips in the transmission coefficients in Fig. 4a. The transmission results in Fig. 4a seems to indicate that the first gap is bigger than the second gap. In fact, the second gap is bigger, as can be seen in the DOS results. The apparent difference is due to the fact that transmission results correspond to one direction of propagation, while DOS results correspond to those of all directions. The transmission at the midgap of the second gap in the transmission results in Fig.4a is smaller than that of the first gap, consistent with the fact that the second gap is actually bigger. When a disorder in radius corresponding to d_r=0.2 is introduced (Fig.6b), the third gap is already closed completely. The second gap is almost closed, and has become a psuedo-gap with a strong depletion in DOS. The first gap survives, the disorder in radius have only made it smaller. These observations can be used to interpret the transmission results in Fig.4a: The dip corresponding to the third gap disappeared, the transmission in the second gap increased substantially and becomes very narrow, corresponding to the imminent closing of the gap, while the dip corresponding to the first gap is just reduced in width. With a disorder d_r=0.4 is introduced (Fig.6c), all the gaps are destroyed, leaving behind a dip in the DOS corresponding to what used to be the first gap in the perfect structure. This again correlates well with the transmission results in Fig.4a. For the case of d_r=0.2, we have results for 5 different realizations of the randomness (by using different seeds in the random-number-generator); and the results are basically the same : the third gap is closed entirely, the first gap survived but is reduced in size, and the second gap is just on the verge of being closed.

We also have DOS for the H-polarization (p-polarization), and they are shown in Fig. 7(a-c), for d_r=0,0.2 and 0.4 respectively. We note that there are no spectral gaps for this polarization, so that the very strong dips in the transmission data (Fig.4b) just represent a gap in the Γ-X direction in k-space. Once disorder is introduced, the sense of direction is destroyed and the dips in the transmission are destroyed very quickly, consistent with the fact that there is no full gap for this polarization.

The $< ln(T) >$ and consequently the localization length also depends on the filling ratio. At intermediate frequencies, there may be some drops in the $< ln(T) >$ (depending on the amount of the disorder and the filling ratio) and their position can be predicted from the gaps of the corresponding periodic structures. At low frequencies, $< ln(T) >$ tends to zero. So, the most natural frequency region to work is at high frequencies where $< ln(T) >$ is almost constant. Making an average of $< ln(T) >$ over the frequency region between 20 GHz and 23.5 GHz, we find the saturated value of the localization length at high frequencies, ℓ_s. Fig. 8 shows the ℓ_s as a function of the filling ratio, f. For all the cases we have examined, ℓ_s has a minimum at intermediate f and ℓ_s tends to infinity when f tends to either zero or one because the system becomes homogeneous at those two limiting cases. This minimum value of ℓ_s is slightly smaller for s- rather than p-polarized waves. Also, for both s- and p-polarized waves (Fig. 8a,b), the minimum value of ℓ_s is smaller in the case of dielectric cylinders than in the case of air cylinders. For disorder in the radius (d_r=0.4) and for dielectric cylinders, the minimum ℓ_s is 6.8 (7.2) at f around 0.45 (0.30) in the case of s- (p-) polarized waves; for air cylinders, the minimum ℓ_s is 8.3 and 11.1 at f around 0.75 in the case of s- and p-polarized waves respectively. For randomness in the dielectric constant of each cylinder (d_ϵ=0.4), the minimum ℓ_s is 5.2 and 7.1 at f around 0.45 and 0.30 in the case of s- and p-polarized waves. These results also show that relatively small localization lengths can be achieved not only at the edges of a gap but also at the high frequency regions; note that the smallest value of ℓ_s can be 5.2a (see Fig. 8a).

Using the average value of $< ln(T) >$ at high frequencies, we calculate the saturated value of the localization length, ℓ_s, as a function of the dielectric constant, ϵ (Fig. 9). The system consists of air cylinders with filling ratio 0.5 in a dielectric medium with dielectric constant ϵ and disorder is introduced in the position of the cylinders (d=0.2). In general, ℓ_s of the s-polarized waves is smaller than the ℓ_s of the p-polarized waves. Also, the ℓ_s tends to infinity as ϵ goes to one (since, in that case, the system becomes homogeneous), while, at high values of ϵ (greater than about 20), ℓ_s is almost constant; this high ϵ value of ℓ_s is 3.5 and 3.8 for s- and p-polarized waves. In general, higher dielectric constant ratios favor the localization of the waves in accordance with the results of Ref. 15.

The absorption is another factor that may change the physical picture described previously. Fig. 10 shows the $< ln(T) >$ vs the frequency for the same system as the one described in Fig. 4 except that the dielectric constant of the cylinders is 10 + 0.5 i instead of 10. In general, the transmission will be reduced for all frequencies in the presence of the absorption.[23] However, the reduction become larger as the frequency increases, although, there are some exceptions from this general rule with most notable the frequency regions inside the gaps of the periodic case; in particular, for the first two gaps of the s-polarization, the transmission, T, is almost the same with that of the non-absorbing case (compare solid lines in Figs 4a and 10a at around 6 GHz and 11.5 GHz). This is because the wave penetrates only the first few layers at frequencies inside the gaps, so, it is not affected that much from the absorption. But, even outside of the gap, there are some exceptions from the general rule. The transmission of p-polarized waves at 14 GHz and 16 GHz are an order of magnitude smaller than the transmission at 20 GHz (see solid line in Fig. 10b). This is because the wave at 20 GHz has its maxima mostly in non-absorbing regions, while, at 14 GHz and 16 GHz, the waves have their maxima in absorbing regions (e.g., inside the cylinders). These differences are inherited also in the disordered cases, especially, when the amount of disorder is small. Note that the transmission, T, of the s-polarization is almost unaffected from the absorption for d_r= 0.2 at around 6.5 GHz (where the first gap was), but as the disorder increases, more and more localized states are created inside the gap, so, T becomes more affected from the absorption. It has been suggested that the combined effects of disorder and absorption can be described by the following formula:[29]

$$\frac{1}{\ell_T} = \frac{1}{\ell} + \frac{1}{\ell_a} \tag{5}$$

where ℓ_T is the total decay length in the presence of both absorption and disorder, ℓ is the localization length in the absence of absorption and ℓ_a is the decay length due to the absorption and in the absence of disorder. ℓ_a is actually given by the formula: $\ell_a = -c/v\epsilon^{''}$ where v is the frequency of the EM wave and $\epsilon^{''}$ is the imaginary part of the dielectric constant.[32] According to Eqs. (3) and (5), by subtracting the $< ln(T) >$ of an absorbing case from the corresponding non-absorbing one, the result must be equal to $(-2L/\ell_a)$ which must be independent of the disorder. Although, it has been shown that this is true in disorder cases where the average system is homogeneous[32], it is not true in the present disorder cases where the average system is a periodic lattice due to the reasons mentioned earlier in this paragraph. This conclusion regarding the effect of the absorption in disorder systems which are periodic on the average, is the same with the conclusion reached recently by studing elastic waves propagating in layered systems which are periodic on the average.[33]

VII. CONCLUSIONS

The transfer matrix method has been used successfully for the calculation of the properties of 2D disordered systems which are periodic on the average; the corresponding periodic systems consist of cylinders forming a square lattice and embedded in a different dielectric medium. By introducing disorder in these periodic systems, the higher gaps, which are narrow, disappear quickly and the logarithmic average of the transmission, $< ln(T) >$, or the localization length, ℓ, become almost constant at relatively high frequencies (ω higher than about c/a). These high frequency values of the localization length depend on the filling ratio and they can be as small as 5.2 a (a is the lattice constant of the unperturbed periodic system) for the cases that we studied. On the other hand, for low frequencies, $< ln(T) >$ is not affected by the disorder and it is close to zero which correspond to very high localization lengths. At intermediate frequencies, there are large drops in the $< ln(T) >$ which correspond to the lowest two gaps of the periodic cases; the wider the gaps of the periodic cases are, the higher the amount of disorder needed to close those gaps. The gaps of the s-polarized waves are generally wider and survive at high amount of disorder in contrast with the gaps of the p-polarized waves where they destroyed easily by the disorder. A systematic study of the optimum conditions for the appearance of the gaps have shown that those conditions are fulfilled for cylinders of high dielectric material with filling ratio around 0.25 and s-polarized waves; in those cases, the gaps are wider and they survive even for high amount of disorder resulting to localization lengths smaller than 5 a.

Experiments on 2D disordered systems similar with the ones studied in the present work can be performed based on previous experiments in periodic systems.[20,21] As we described previously, the disorder can be introduced by a well controllable way, by either moving apart the cylinders or changing their radius.

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FIGURE CAPTIONS

FIG. 1. Gap over midgap frequency for s-polarized waves vs the filling ratio of the cylinders forming a 2D square lattice embedded into a different medium. Solid and dotted lines are for cases where the cylinders are the high, and low dielectric material. The dielectric constant ratio between the two media is 10.

FIG. 2. The threshold value of the dielectric constant ratio, μ, for which the first gap just opens, vs the filling ratio, f, for s (a) and p (b) polarized waves. Solid and dotted lines correspond to cylinders of high and low dielectric materials, respectively.

FIG. 3. Scattering cross section of s- and p-polarized waves (dashed and solid lines) incident on a cylinder embedded in a different medium vs the frequency. Cylinders of high and low velocity material correspond to panels A and B respectively. The dielectric constant ratio between the two media is 10.

FIG. 4. $< ln(T) >$ vs the frequency for 2D disordered media with d_r=0, 0.2 and 0.4 (solid, dotted and dashed lines respectively) and s- and p-polarized waves (a and b panels). The filling ratio of the cylinders is around 0.29; the dielectric constants are 10 and 1 in the cylinders and in the surrounding medium respectively; the lattice constant is a=1.28 cm and the thickness of the system is L=25 a.

FIG. 5. Localization length, ℓ, at 6.5 GHz for the system described in Fig. 4a vs the rms error of the dielectric constant, $\sqrt{< \epsilon^2 >}$. Results for randomness in the radius (squares) and the dielectric constant (circles) of the cylinders are shown. ℓ is normalized to the lattice constant.

FIG. 6. Total Density of States (DOS) of s-polarized waves propagating in a system similar with the one described in Fig. 4. One configuration has been used with d_r=0, 0.2 and 0.4 (a, b and c panels respectively).

FIG. 7. The same as in the Fig. 6 for p-polarized waves.

FIG. 8. The high frequency value of the localization length, ℓ_s, for s- and p-waves (panels a and b, respectively) vs the filling ratio, f. The systems consisting of dielectric (square) and air (circles) cylinders with randomness in the radius of the cylinders (d_r=0.4). Results for a similar system with dielectric cylinders and randomness in their dielectric constant are shown (triangles). ℓ_s is normalized to the lattice constant.

FIG. 9. The high frequency value of the localization length, ℓ_s, for s- and p-waves (squares and circles, respectively) vs the dielectric constant of the surrounding medium, ϵ_o. The system consisting of air cylinders with filling ratio around 0.5 with randomness in the position of the cylinders (d=0.2) and thickness, L=25 a. ℓ_s is normalized to the lattice constant, a.

FIG. 10. $< ln(T) >$ vs the frequency for systems similar with those in Fig. 4, except that the dielectric constant of the cylinders is 10+0.5 i.