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Transmittance measurements in the integrating sphere

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The finite-difference equation method is used to derive expressions for the directional–hemispherical transmittance as measured in the integrating sphere; and for the correction factor necessary with a one-port sphere to account for changes in the enclosure caused by substituting the open port (the standard device) for the sample. Expressions are also derived for the hemispherical reflectance, since this quantity is necessary to compute the correction factor. It is also shown that no correction factor is necessary with a two-port (comparative) sphere.

I. Introduction

Researchers[1,2] are using one- and two-port integrating spheres to measure the optical transmittance of samples placed over a port in the sphere wall. The finite difference equation method has been applied to lighting problems in rooms[3–5] and to derive expressions for the directional-hemispherical reflectance of samples measured in the integrating sphere.[6,7] This paper extends the method to transmittance measurements made with the integrating sphere. The formalism is used to derive an expression for the sample's directional-hemispherical transmittance and for the correction factor necessary with a one-port sphere to account for changes in the enclosure caused by substituting the open port (the standard device) for the sample. Expressions are also derived from the hemispherical reflectance, since this quantity is necessary to compute the correction factor. It is also shown that no correction factor is necessary with a two-port (comparison) sphere.

II. General Equation

Following Hisdal's derivation,[6] if A_j is part of an enclosure comprised of N perfectly diffuse reflecting surfaces $\{A_k\}$, the luminous exitance (lumens/unit area) of A_j is

$$L_j = r_jL_1F_{j1} + r_jL_2F_{j2} \ldots + r_jL_NF_{jN} + r_j'E_j^{\text{ex}}, \tag{1}$$

where F_{jk} is the form factor from A_j to A_k, and E_j^{ex} is the excitation illuminance on A_j arising from an external monochromatic light source. The quantity r_j is the spectral hemispherical reflectance of A_j for diffuse incident radiation from the other surfaces of the enclosure, while r_j' is the spectral directional-hemispherical reflectance of A_j for the excitation flux. The entire set of exitances can be found by solving the system of N linear equatons:

$$\begin{bmatrix} r_1F_{11}-1 & r_1F_{12} & & r_1F_{1N} \\ r_2F_{21} & r_2F_{22}-1 & \ldots & r_2F_{2N} \\ r_3F_{31} & r_3F_{32} & \ldots & r_3F_{3N} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ r_NF_{N1} & r_NF_{N2} & \ldots & r_NF_{NN}-1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \cdot \\ \cdot \\ \cdot \\ L_N \end{bmatrix} = \begin{bmatrix} -r_1'E_1^{\text{ex}} \\ -r_2'E_2^{\text{ex}} \\ -r_3'E_3^{\text{ex}} \\ \cdot \\ \cdot \\ \cdot \\ -r_N'E_N^{\text{ex}} \end{bmatrix}. \tag{2}$$

III. One-Port Sphere

This section will derive an expression for the directional-hemispherical transmittance of an externally illuminated sample mounted over a port in the wall of the sphere. Next, a method for measuring the sample's backreflectance is described, since this quantity is used in one of the factors expressing the transmittance. Finally, it is shown that in a two-port sphere, which operates in a comparison rather than a substitution mode, no correction factor appears in the expression for the transmittance.

A. Measurement of Transmittance

The transmittance of a sample will be determined as the ratio of two illuminances measured in the integrating sphere, one produced when the sample is placed over a port in the sphere and illuminated by an external source, the other resulting from external illumination of the port with sample removed. This ratio will be modified to account for fluctuations of the source and for changes in the enclosure's reflectance resulting from substitution of the sample for the open port (which has zero reflectance back into the enclosure).

It is necessary to shield the detector from the sample port to avoid error when the sample's transmitted flux directly excites the detector, but that of the open port

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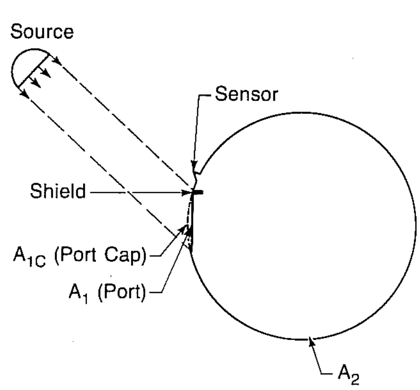
does not (or *vice versa*). Otherwise, the ratio of the illuminances with and without the test device in place will not accurately indicate the true ratio of the (diffuse) illuminances in the sphere. Shielding the sensor from direct view of the planar port also ensures that the measured illuminance does not depend on the sensor's location in the spherical portion of the enclosure, because, as is well known, the form factor from any portion of the sphere to a sensor (embedded in the sphere) depends only on the areas of the sensor and sphere.

To use the theory for a diffuse enclosure, it is necessary that each surface of the enclosure be uniformly irradiated.[5,8,9] Any two portions of the spherical surface uniformly irradiate one another. (This follows from the property of form factors mentioned above.) However, a planar sample mounted over the port does not share this desirable property. Although, as the area of the port decreases, the planar sample approaches the spherical surface, and the necessary conditions for enclosure theory are obtained.

The integrating sphere consists of a spherical shell A_s from which a spherical cap A_{1c} is removed. Figure 1 shows the sphere consisting of a spherical surface A_2 and a planar port A_1. Denote the area of a surface A_j by the symbol a_j. Surface A_2 will not have uniform exitance, because it is typically excited by a nonuniform spatial distribution of transmitted flux. Moreover, the planar surface A_1, unlike the spherical cap A_{1c}, will not reflect flux uniformly over A_2. We will show that for a port-shielded sensor embedded anywhere in A_2, the measured illuminance depends only on the total excitation flux over A_2, irrespective of its spatial distribution. This allows us to then derive the expression for the transmittance by applying the matrix formalism to the simplified enclosure illustrated in Fig. 1.



A_S (Sphere) = $A_{1C} \cup A_2$

Fig. 1. One-port (substitution) sphere with a nondispersive sample over port.

Use of a shield on the sensor allows use of the enclosure formalism because it ensures that the sensor can view only the spherical portion of the enclosure. All flux reflected from the planar device will, after reflection from the spherical surface, be uniformly distributed over the spherical surface. The nonuniform illumination of the device by reflected flux from an arbitrary excitation of the spherical surface will not matter with the assumptions of diffusely reflecting surfaces, a port-shielded sensor, and a reflectance r_1 that is constant over the sample.

The previously discussed property of the form factor between any two portions of a sphere allows us to treat the spherical portion of the enclosure as a single surface with constant excitation. Suppose that for an arbitrary nonconstant excitation of A_2, A_2 was divided into subsurfaces A_{2j}. Each spherical subsurface A_{2j} reflects its excitation flux uniformly over the spherical surface A_2. Thus the distribution of this initially reflected flux over A_2 does not depend on the spatial distribution over A_2 of the excitation flux transmitted by the sample. The surface A_2 can, therefore, be treated as a single uniformly excited surface.

The matrix formalism will now be applied to the enclosure comprising two surfaces, as shown in Fig. 1. The transmittance of a planar test device is determined with respect to that of the open-port standard. As shown in Fig. 1, assume that monochromatic beam radiation is incident on a test device mounted over the sphere port A_1 (a circle of radius r) from a direction characterized by altitude angle θ and azimuth angle ϕ (relative to a coordinate system on the sphere port). No background incident radiation is assumed. The discussion at the end of Sec. III shows that there is no loss of generality in restricting the discussion to a monochromatic source. If Φ_{inc} is the flux uniformly incident on the sample port (at the instant we measure the illuminance due to the flux transmitted through the sample), the excitation flux transmitted into the sphere is $T(\theta,\phi)\Phi_{\text{inc}}$, where $T(\theta,\phi)$ is defined to be the spectral directional-hemispherical transmittance of the device. Note that this definition allows T to have values greater than unity, as it would, for example, if the sample incorporated reflectors which increase the effective area of the port. For a given source position, temporal fluctuations in Φ_{inc} may be determined by monitoring the source with a sensor external to the sphere.

Note that to accurately measure transmittance, either the incident flux must be uniform over the port or the test sample must have a relatively constant transmittance over its surface (for a given source position). This follows from the observation that the open port standard has a constant transmittance of unity over the port. A shading device like an overhang, however, has a transmittance of zero over the shaded portion of the port and unity over the remainder. This will unequally weight a nonuniform incident flux distribution and thereby lead to inaccurate determination of transmittance. Devices composed of repetitive elements (e.g., slats) may be accurately tested with nonuniform incident flux if their variation in transmittance occurs on a scale, that is small relative to the size of the port.

The following properties of form factors are used in the derivation:

$F_{jk} = a_j/a_s$ for A_j, A_k in A_s;
$F_{jj} = 0$ for planar A_j;
$F_{jk} = (a_{kc}/a_s)$ for A_j in A_s and A_k planar;
$F_{jk} = a_k/(a_s - a_{jc}) = (a_{jc}/a_j)(a_k/a_s)$ for A_j planar and A_k in A_s;[6]
$F_{jk} = (a_{jc}/a_j)(a_{kc}/a_s)$ for A_j, A_k planar.

These properties can be applied to the areas shown in Fig. 1. Writing f_c for a_{1c}/a_s, r_1 for port reflectance, r_2' for the sphere wall (directional) reflectance for the excitation flux, and r_2 for sphere wall reflectance, Eqs. (2) become

$$\begin{bmatrix} -1 & r_1 \\ r_2 f_c & r_2(a_2/a_s) - 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -r_2'(T\Phi_{inc})/a_2 \end{bmatrix}, \tag{3}$$

We made use of the facts that F_{12} = unity, since all radiation reflected from A_1 strikes A_2, and that $(a_2/a_s) = (a_s - a_{1c})/(a_s)\ 1 - f_c$. Note that L_1 does not include the flux $T\Phi_{inc}$, which can be considered to leave an imaginary source inside the sphere and to excite only the surface A_2, where it is transformed into part of L_2, the diffuse luminous exitance from A_2. The equations can be solved by Cramer's method, where we first calculate the determinant

$$\Delta = 1 - r_2(1 - f_c) - r_1 r_2 f_c. \tag{4}$$

Next we solve for the unknowns L_1, L_2:

$$L_1 = \left(\frac{1}{\Delta}\right) \begin{vmatrix} 0 & r_1 \\ -r_2' T\Phi_{inc}/a_2 & r_2(1 - f_c) - 1 \end{vmatrix}$$

$$= \left(\frac{1}{\Delta}\right)(r_1 r_2' T\Phi_{inc})(1/a_2); \tag{5}$$

$$L_2 = (1/\Delta)(-r_2' T\Phi_{inc})(1/a_2). \tag{6}$$

Following Hisdal,[6] we can write the illuminance measured by a detector shielded from the port as $E_d = L_2 F_{d-2} = L_2(a_2/a_s) \equiv E$. Thus

$$E = (1/\Delta)(-r_2' T\Phi_{inc})(1/a_2)(a_2/a_s)$$

$$= \frac{r_2' T\Phi_{inc}/a_s}{1 - r_2(1 - f_c) - r_1 r_2 f_c}. \tag{7}$$

A similar expression for the standard illuminance (detected by a port-shielded sensor when the open port is used as reference transmitter) can be found by substituting zero for r_1 and unity for T yielding

$$E_0 = (1/\Delta_0)(r'_2 \Phi_{inc,0}/a_s), \tag{8}$$

where Δ_0 is the value of the determinant (4) obtained by substituting zero for r_1, and $\Phi_{inc,0}$ is the flux incident on the sample port at the instant of the standard measurement (sample dismounted). The ratio $\Phi_{inc,0}/\Phi_{inc}$ accounts for temporal variation of the source.

Dividing these latter two equations one by the other and rearranging yields the directional-hemispherical transmittance

$$T = (E/E_0)(\Phi_{inc,0}/\Phi_{inc})(\Delta/\Delta_0) \tag{9}$$

or

$$T = (E/E_0(\Phi_{inc,0}/\Phi_{inc}) \left[1 - \frac{r_1 r_2 f_c}{1 - r_2(1 - f_c)}\right]. \tag{10}$$

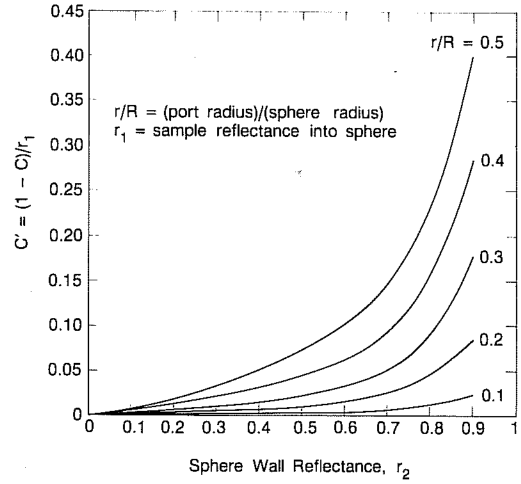


Fig. 2. $C' = (1/r_1)(1 - C)$, where C is the sphere correction factor.

The right-hand term (in brackets) is the correction factor C, which compensates (with respect to the non-reflecting open-port standard) for the sample's nonzero backreflectance r_1. If uncorrected, this difference yields spuriously high values for transmittance, since the illuminance measured with the test device in place is increased relative to that measured when the open port is in place.

The correction factor C can be written as $C = (1 - r_1 C')$, where C' is a function of the sphere radius R, the port radius r, and the sphere wall reflectance r_2. Figure 2 plots C' vs r_2 for various ratios r/R. Since the correction factor depends on the sample hemispherical reflectance r_1, the next section derives an expression for this quantity by means of measurements using a light source within the sphere. Alternatively, the directional-hemispherical reflectance (as measured by the methods of Ref. 6) could be utilized as a proxy for the hemispherical reflectance.

B. Measurement of Reflectance

Figure 3 illustrates the procedure for determining r_1. A light source, emitting a flux Φ, is placed within the sphere. A shield prevents direct illumination of the port or the sensor (located in surface A_2 and shielded from the port). Expressions for r_1 will be derived by comparing the measured illuminance E when a planar sample is in place (surface A_1) to the illuminance E_{st} measured when a standard device (reflectance $r_{1,st}$) is substituted for the sample. Three standards will be considered: planar with $r_{1,st} = 0$; planar with $r_{1,st} = r_2$, and a spherical cap having $r_{1,st} = r_2$, where r_2 is the reflectance of the sphere coating.

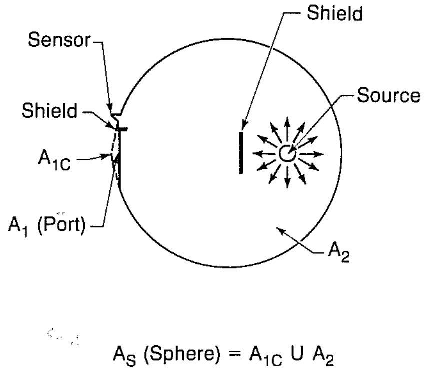


Fig. 3. Substitution method for determining sample backreflectance into the sphere.

For a planar device, Eqs. (2) become

$$\begin{bmatrix} -1 & r_1 \\ r_2f_c & r_2(1-f_c)-1 \end{bmatrix}\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -r'_2\Phi/a_2 \end{bmatrix}, \tag{11}$$

where again $f_c \equiv a_{1c}/a_s \equiv a_c/a_s$. Solving in the previous manner yields

$$\Delta = 1 - r_2(1-f_c) - r_1r_2f_c, \tag{12}$$

$$L_2 = (1/\Delta)\begin{vmatrix} -1 & 0 \\ r_2f_c & -r'_2\Phi/a_2 \end{vmatrix} = (1/\Delta)(r'_2\Phi/a_2), \tag{13}$$

$$E = L_2F_{22} = L_2(a_2/a_s) = (1/\Delta)(r'_2)(\Phi/a_s). \tag{14}$$

Letting the subscript st denote quantities pertaining to a planar standard substituted for the sample, Eq. (14) is used twice to express the ratio of measured illuminances:

$$\frac{E_{st}}{E} = \frac{\Delta}{\Delta_{st}}\frac{\Phi_{st}}{\Phi} = \left[\frac{1-r_2(1-f_c)-r_1r_2f_c}{1-r_2(1-f_c)-r_{1,st}r_2f_c}\right]\frac{\Phi_{st}}{\Phi}, \tag{15}$$

which may be solved for the sample reflectance

$$r_1 = \left(\frac{1}{r_2f_c}\right)\left\{1 - r_2(1-f_c) - \frac{E_{st}\Phi}{E\Phi_{st}}[1-r_2(1-f_c)-r_{1,st}r_2f_c]\right\}. \tag{16}$$

If the open port is used as the reference standard, set $r_{1,st} = 0$ to obtain

$$r_1 = \left[\frac{1-r_2(1-f_c)}{r_2f_c}\right]\left(1 - \frac{E_{st}}{E}\frac{\Phi}{\Phi_{st}}\right). \tag{17}$$

If a planar standard with the same coating as the sphere is used, set $r_{1,st} = r_2$ to obtain

$$r_1 = \left[\frac{1-r_2(1-f_c)}{r_2f_c}\right]\left\{1 - \frac{E_{st}}{E}\frac{\Phi}{\Phi_{st}}\left[1 - \frac{r_2^2f_c}{1-r_2(1-f_c)}\right]\right\}. \tag{18}$$

Finally, consider a standard consisting of the spherical cap (surface A_{1c}) coated with the same material as the sphere. Then $r_{1,st} = r_2$, $F_{11} = f_c$, $F_{12} = 1 - f_c$, and $\Phi = \Phi_{st}$. Equations (2) become

$$\begin{bmatrix} r_2f_c - 1 & r_2(1-f_c) \\ r_2f_c & r_2(1-f_c)-1 \end{bmatrix}\begin{bmatrix} L_{1,st} \\ L_{2,st} \end{bmatrix} = \begin{pmatrix} 0 \\ -r'_2\phi_{st}/a_2 \end{pmatrix}. \tag{19}$$

Solve this to obtain $\Delta_{st} = 1 - r_2$, and

$$L_{2,st} = (1/\Delta)\begin{vmatrix} r_2f_c - 1 & 0 \\ r_2f_c & -r'_2\Phi_{st}/a_2 \end{vmatrix} = (1/\Delta)(1-r_2f_c)(r'_2\Phi_{st}/a_2), \tag{20}$$

$$E_{st} = L_{2,st}F_{22} = L_{2,st}(a_2/a_s) = 1/\Delta_{st}(1-r_2f_c)(r'_2\Phi_{st}/a_s). \tag{21}$$

Divide this expression by Eq. (14) and rearrange to obtain the sample reflectance

$$r_1 = \left(\frac{1}{r_2f_c}\right)\left(1 - r_2(1-f_c) - \frac{E_{st}\Phi}{E\Phi_{st}}\frac{(1-r_2)}{(1-r_2f_c)}\right). \tag{22}$$

This method of determining r_1 relies on discriminating small differences between large numbers, since the ratio E_{st}/E approaches unity as the port area decreases with respect to the area of the sphere. Other methods are described in the literature.[10]

It is now possible to calculate the correction factor using the appropriate expression for r_1 and the previous expression for C'. To experimentally confirm Eq. (10) devices for testing on a 2-m diam sphere were constructed from perforated hardboard coated with either white sphere paint or a flat black enamel. Transmittance at normal incidence was determined as the ratio of open area to total area. The sample reflectance was determined by luminance scanning comparisons with a known standard (both illuminated beneath a hemisphere having uniform luminance), or was measured by the preceding method using alternately the open port and a planar port coated with sphere paint as standards. Using a 2.0-m diam sphere (adaptable to either 0.89- or 0.51-m diam ports) with the port illuminated at normal incidence, a measured value for C was determined from Eq. (10) as $T(E_0/E)(\Phi_{inc,0}/\Phi_{inc})$. This value was compared with the theoretical value derived from Eq. (10). Using both the small and large ports and devices with (T,r_1) = (0.4,0.5), (0.4,0.04), (0.3,0.7), and (0.3,0.05), the error ranged from 1.5 to 4.3%.

The assumption of a monochromatic source does not restrict the generality of the results. The transmittance derived in the preceding may be viewed as a spectral directional-hemispherical transmittance T_λ. The integrated directional-hemispherical transmittance may then be calculated from the expression

$$T = \int_{\lambda=0}^{\infty} T_\lambda(dE_\lambda/d\lambda)d\lambda, \tag{23}$$

where the T_λ are measured as described above, and separate measurements characterize the source's relative spectral irradiance.

The following section discusses modifications to the sphere and experimental procedure which eliminate the correction factor.

IV. Two-Port Sphere

By placing a dedicated open port in the sphere, the test device may be left over the sample port while the open port is illuminated. This comparative mode of operation eliminates the correction factor arising in the above substitutive mode. Figure 4 defines the two-port sphere. With no loss of generality assume

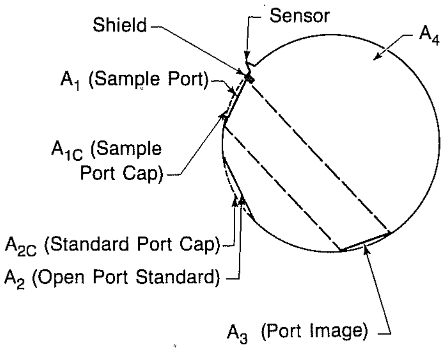


Fig. 4. Two-port (comparison) sphere.

the dedicated open port has the same area as the sample port. Then A_{1c}, A_{2c}, and A_3 are congruent and have area a_c. The system of Eqs. (2) becomes

$$\begin{bmatrix} -1 & r_1(a_c/a_1)f_c & r_1(a_c/a_1)f_c & r_1(a_c/a_1)(1-3f_c) \\ 0 & -1 & 0 & 0 \\ r_3f_c & r_3f_c & r_3f_c - 1 & r_3(1-3f_c) \\ r_3f_c & r_3f_c & r_3f_c & r_3(1-3f_c)-1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -r'_3T\Phi_{\text{inc}}/a_c \\ 0 \end{bmatrix}. \quad (24)$$

The determinant is

$$\Delta = 1 - r_3(1-2f_c) - r_3r_1f_c(1-3f_c)(a_c/a_1) + r_3r_1(a_c/a_1) + r_3r_1(a_c/a_1)(f_c^2). \quad (25)$$

Now the measured illuminance in the two-port sphere can be calculated. For a detector (in A_3 or A_4) shielded from both the standard and sample ports, $E = L_3F_{43} + L_4F_{44}$. Using expressions for L_3 and L_4 calculated from Eq. (24) yields

$$E = (1/\Delta)(r'_3T\Phi_{\text{inc}})(1/a_c)(f_c) = (1/\Delta)(r'_3T\Phi_{\text{inc}})(1/a_s). \quad (26)$$

When the open port standard is illuminated by the source, the preceding expression can be used if T is replaced by unity and Φ_{inc} by $\Phi_{\text{inc},0}$. The measured illuminance due to transmittance through the open port standard is

$$E_0 = (1/\Delta)(r'_3\Phi_{\text{inc},0})(1/a_s), \quad (27)$$

since the substitution leaves Δ unchanged.

Dividing and rearranging the last two equations yields

$$T = (E/E_0)(\Phi_{\text{inc},0}/\Phi_{\text{inc}}). \quad (28)$$

Note the lack of a correction factor, which allows determination of T without the device backreflectance r_1.

V. Applications

Researchers at the Applied Science Division of Lawrence Berkeley Laboratory have constructed a 2-m diam integrating sphere, with a single port or variable diameter (0.25–1.0 m). The sphere is used to measure the directional-hemispherical transmittance of both scale and full-size glazing materials and fenestration components.[11,12] These measured values characterize performance at a realistic range of source/device relative positions, and are necessary to optimize the effects of glazing on building energy management.

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