Nuclear Structure & Dynamics: From Light Nuclei to Neutron Stars

Outline

- Basic Picture
- Computational Aspects
- Light Nuclei:
  e^- scattering, BBN, solar ν's, hadronic PV, ...
- toward Larger Nuclei and Neutron Stars

Collaborators

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V. R. Pandharipande (Urbana) , ...

Ingredients:

- Interactions - (Largely Known)

$$H = \sum_i \frac{-\hbar^2}{2m}\nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

- Currents - (mostly known)

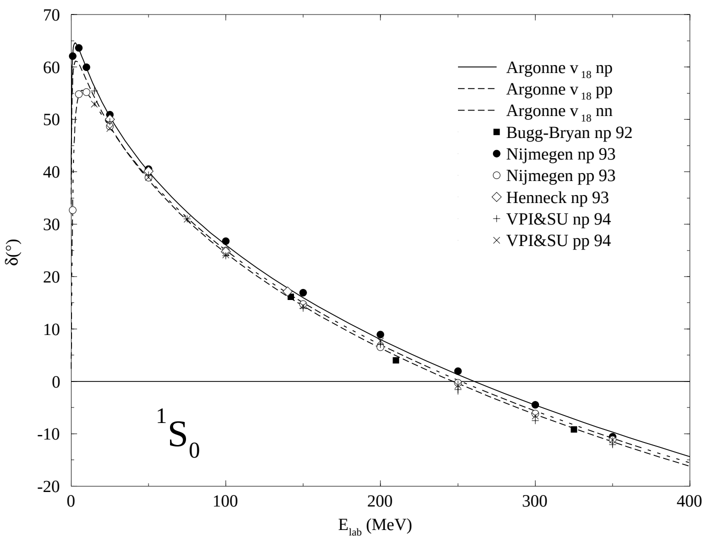
$$\rho = \sum_i \rho_i^1 + \sum_{i<j} \rho_{ij}^2 + \ldots$$

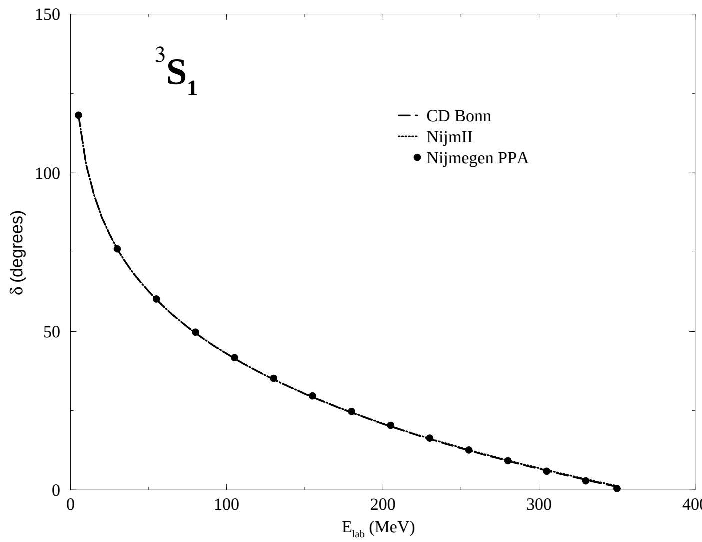
$$\mathbf{j} = \sum_i \mathbf{j}_i^1 + \sum_{i<j} \mathbf{j}_{ij}^2 + \ldots$$

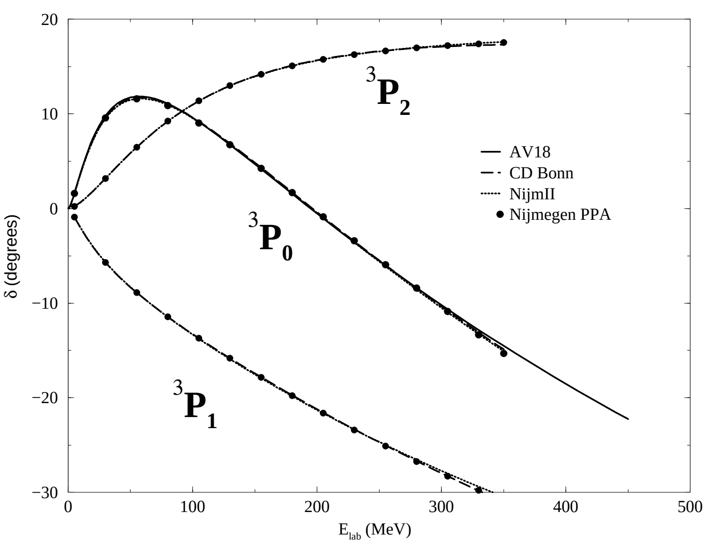
- Non-Perturbative Solutions - (some things known)

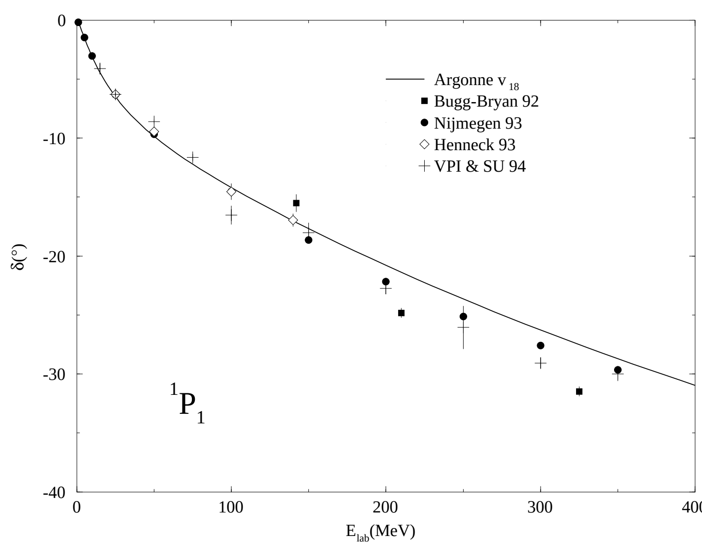
Interaction:

$$H = \sum_i \frac{-\hbar^2}{2m} \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \qquad (1)$$





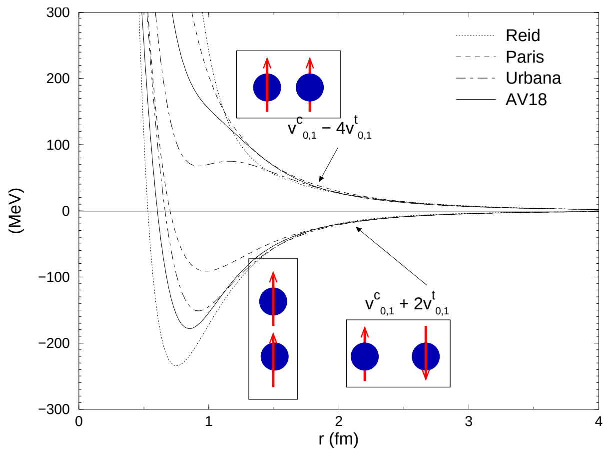


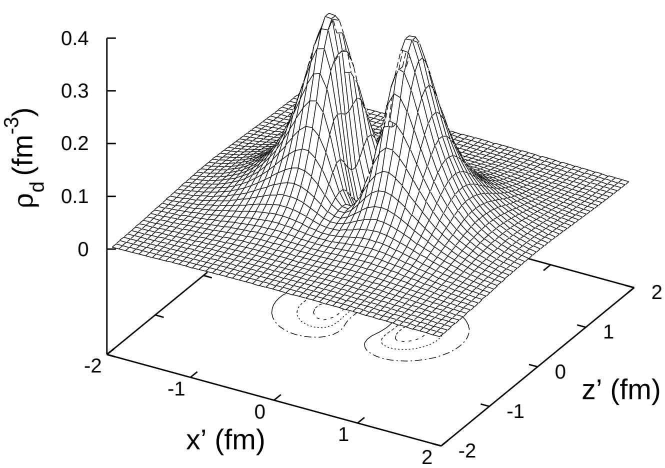


NN Interactions

Interaction of order 1 GeV, 0.1 fm

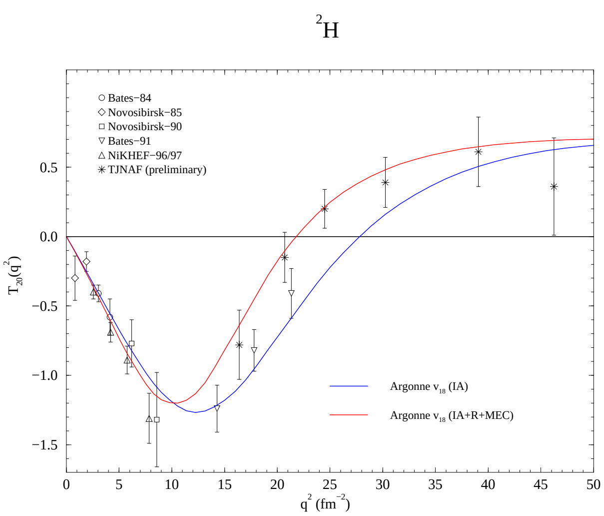
Physics of order 1 MeV, up to 10 fm

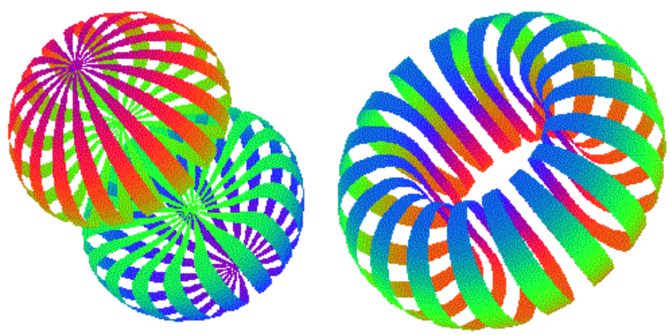




Short-distance repulsion

Spin Dependence





Forest, et al. (96)

Scaling with Particle Number:

$$\Psi = \sum_{i=1}^{2^A} \sum_{j=1}^{\frac{A!}{Z!(A-Z)!}} \psi(\mathbf{R}\chi_\sigma(i)\chi_\tau(j))$$

Simple Methods (Diagonalization):

$$\#\text{ of components} \approx \left(\frac{5\text{ fm}}{0.1\text{ fm}}\right)^{3(A-1)} 2^A \frac{A!}{(Z)!(A-Z)!} \tag{2}$$

Nucleus:	^{2}H	^{4}He	^{8}He	^{8}Be	^{16}O
$\log_{10}$(# components):	6	17	39	40	85

Alternative: Monte Carlo Integration over spatial coordinates:

Variational Monte Carlo:

$$|\Psi_T\rangle = \mathcal{S} \prod_{i<j} F_{ij} \, |\Phi(J,T)\rangle$$

Green's function Monte Carlo:

$$|\Psi_0\rangle = \prod \exp[-[H - E_0]\Delta\tau]|\Psi_T\rangle$$

Scaling with Particle Number:

Sparse linear algebra within each node (large by A=10)

Branching random walks across parallel nodes

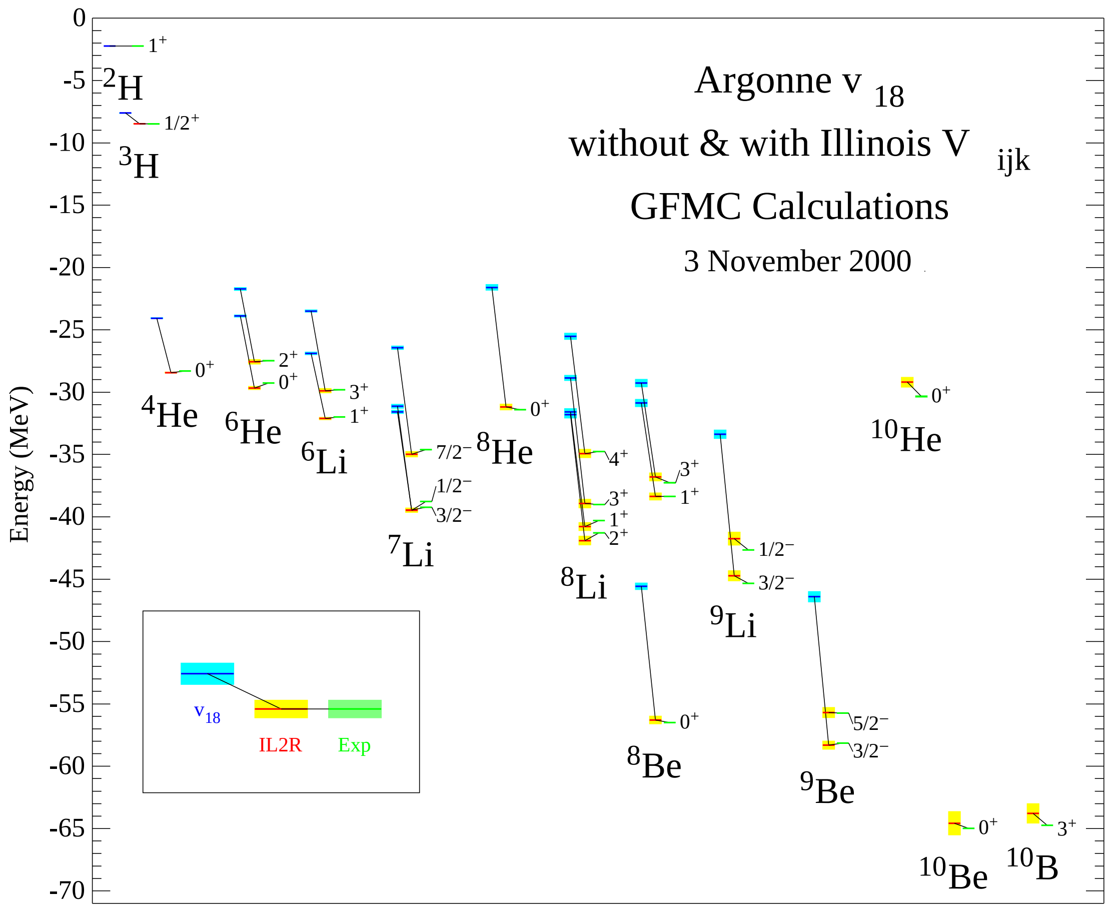
Measured 95 % parallel efficiency up to 512 processors

	Samples (1000's)	Statistical Error (MeV)	Processor hours*
^{6}Li	180	0.08	40
^{7}Li	105	0.25	340
^{8}Be	240	0.2	300
^{8}Li	190	0.2	600
^{9}Be	43	0.5	3,400
^{9}Li	45	0.5	8,300
^{10}B	75	0.6	11,000
^{10}Be	65	0.6	22,500

$^{*}A = 6 - 8$: IBM SP3 or SGI at MCS

^{9}Be: NERSC IBM SP (Phase I)

^{9}Li, ^{10}Be, ^{10}B: 500 MHz Pentium-III (MCS Chiba City)



Three Nucleon Interactions must be included :

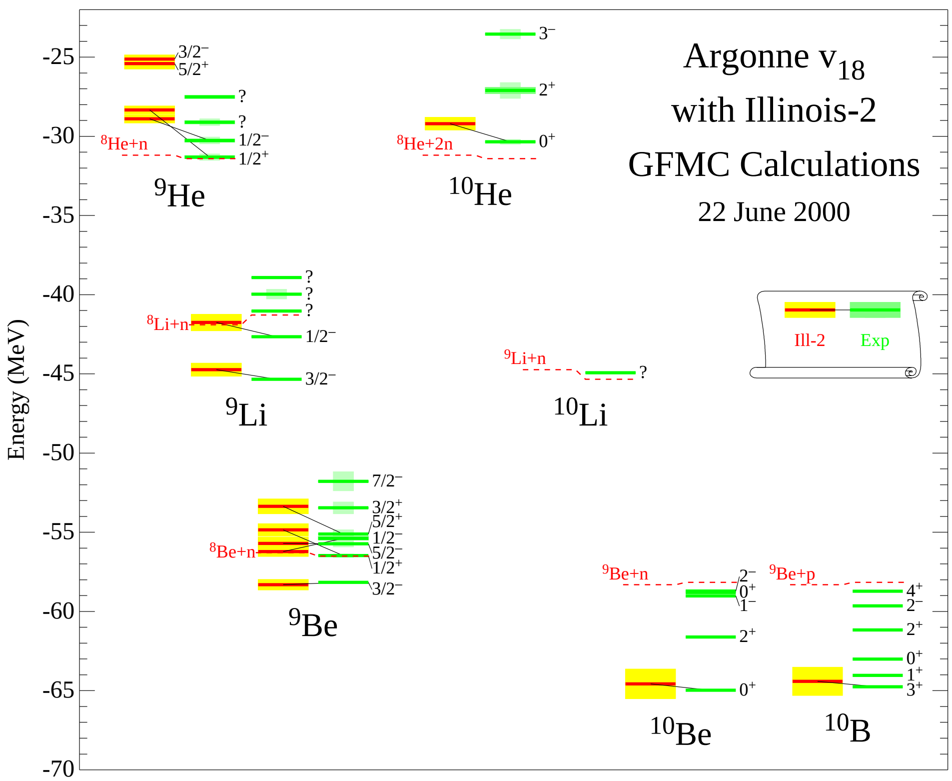
light p-shell nuclei (nearly) unbound

wrong ordering of states in some cases

To Be Addressed:

- Chiral Two-Pion Exchange in NN interaction Models
- Alternative Three-Nucleon Interactions
- To what extent do these affect physical observables?

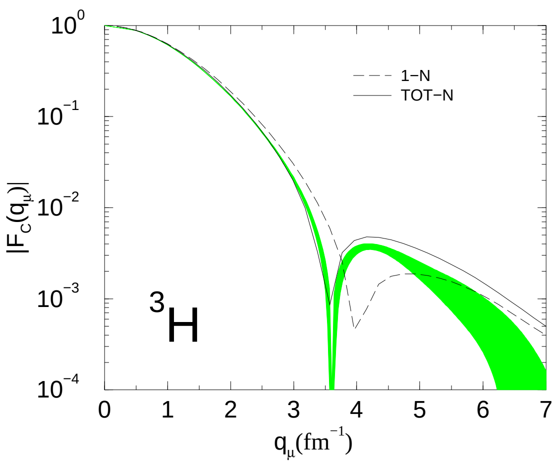
Interaction: Mass 9-10 Spectra from Illinois Vijk

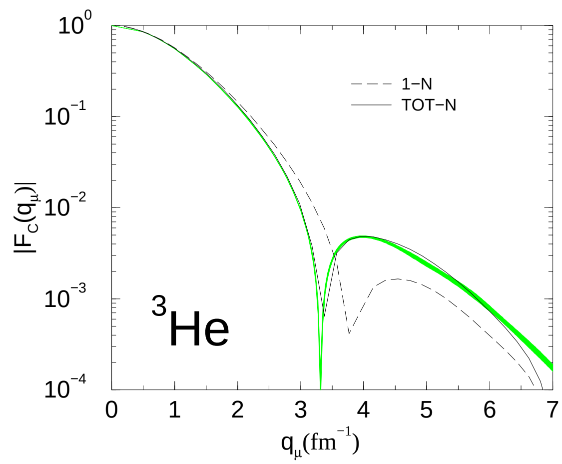


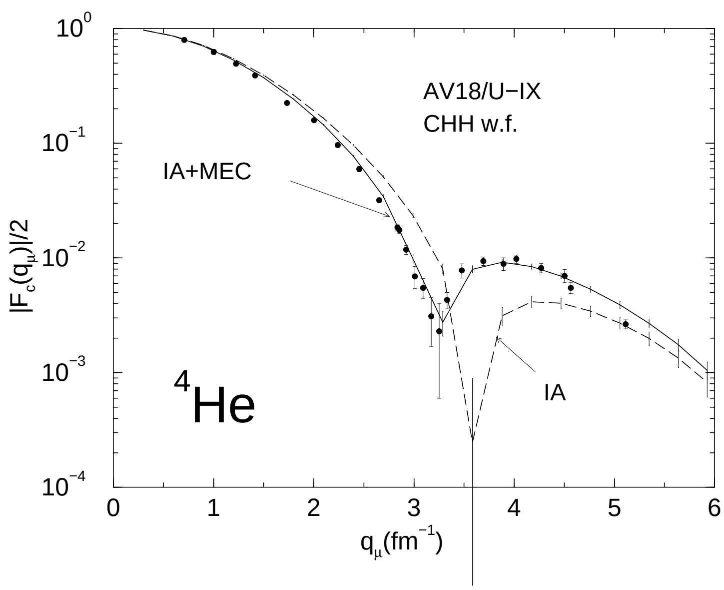
Electroweak Transitions: Charge Operators

$$\rho(\mathbf{q}) = \sum_i \rho_i^1 + \sum_{i<j>} \rho_{ij}^2 + \ldots$$

$$\rho_{i,\mathrm{NR}}^{(1)}(\mathbf{q}) = \epsilon_i \, e^{\mathbf{i}\mathbf{q}\cdot\mathbf{r}_i}$$



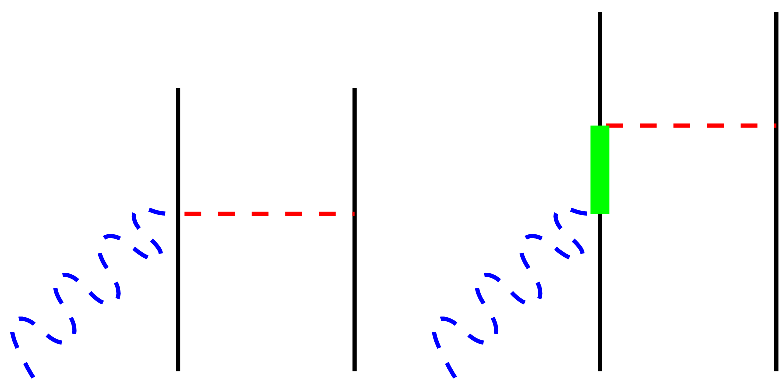


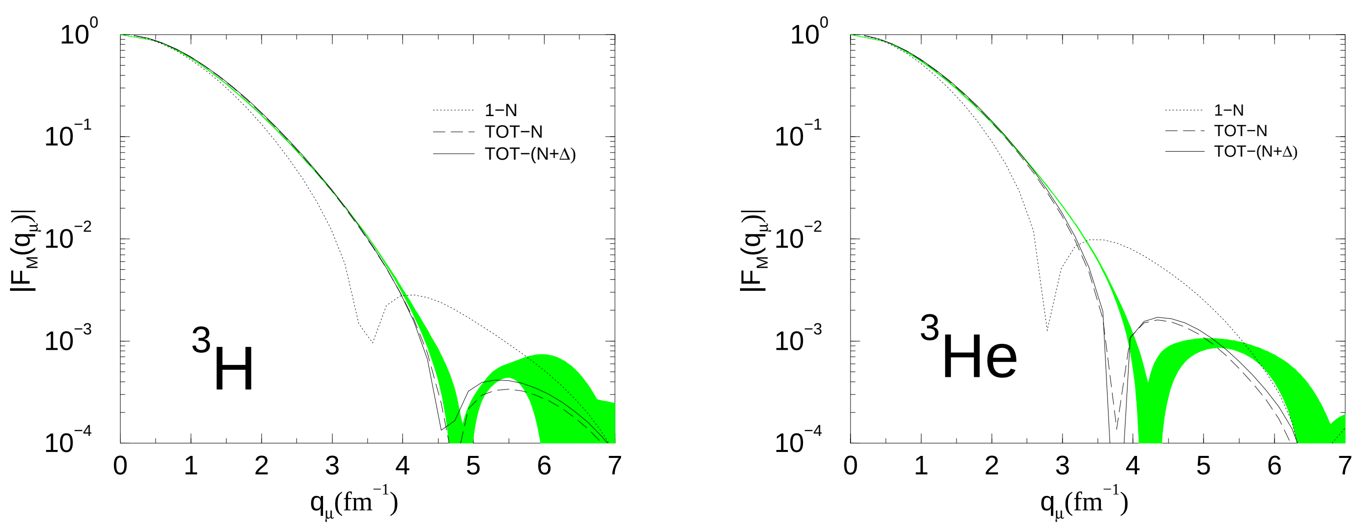


EM Currents

$$\mathbf{j}_i^{(1)}(\mathbf{q}) = \frac{1}{2m}\epsilon_i \left\{\mathbf{p}_i \, , \, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}\right\} - \frac{\mathrm{i}}{2m}\mu_i \, \mathbf{q} \times \sigma_i \, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}$$

$$\mathbf{j}_{ij,\pi}^{(2)}(\mathbf{k}_i, \mathbf{k}_j) = 3\mathrm{i}(\tau_i \times \tau_j)_z \, G_E^V(Q^2) \left[v_\pi(k_j)\sigma_i(\sigma_j \cdot \mathbf{k}_j) - v_\pi(k_i)\sigma_j(\sigma_i \cdot \mathbf{k}_i)\right]$$





A=6 Form Factors

Fig.2 Wiringa & Schiavilla

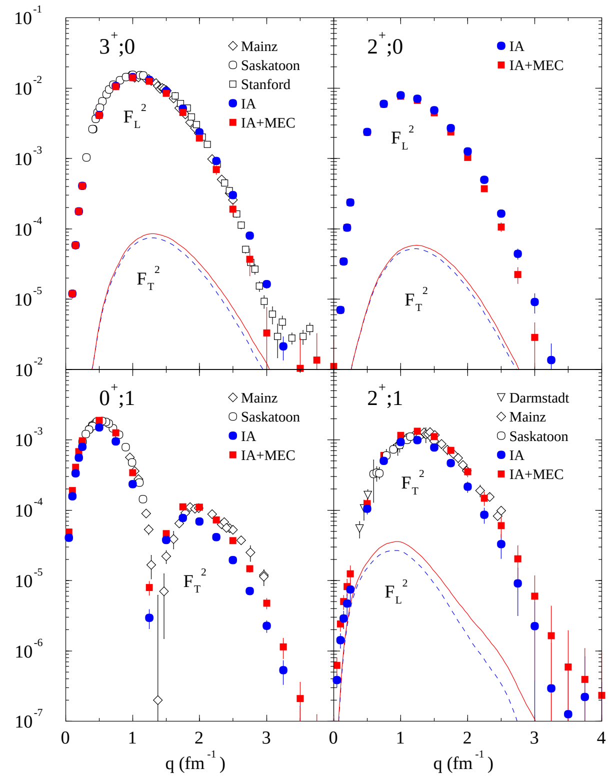
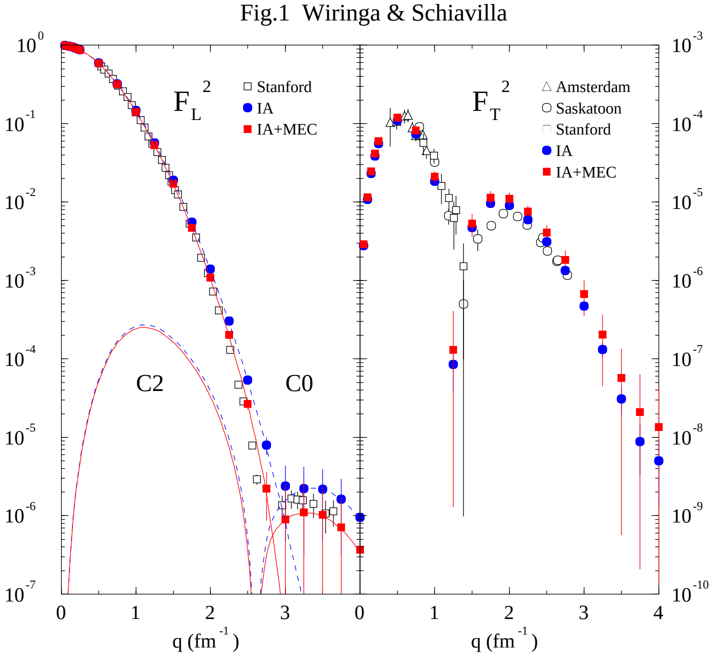


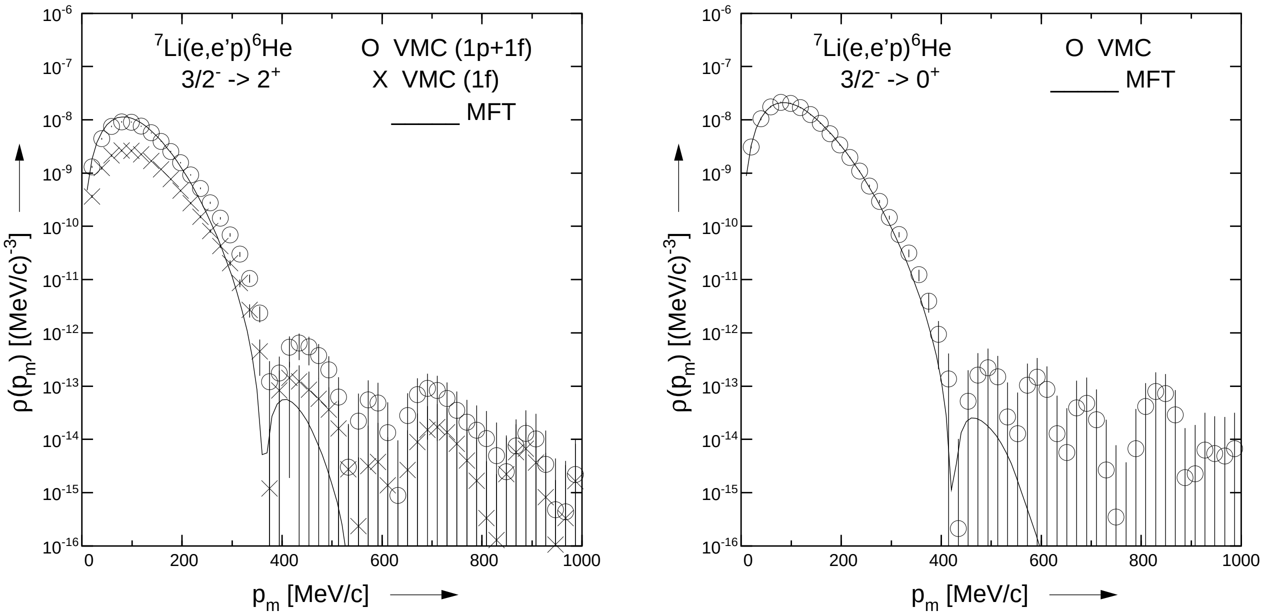
Fig.1 Wiringa & Schiavilla



Caveats

- Relativistic Treatment consistent with charge operator
- Experimental constraints on currents beyond Pion (brehmstrahlung)
- Weak Currents...

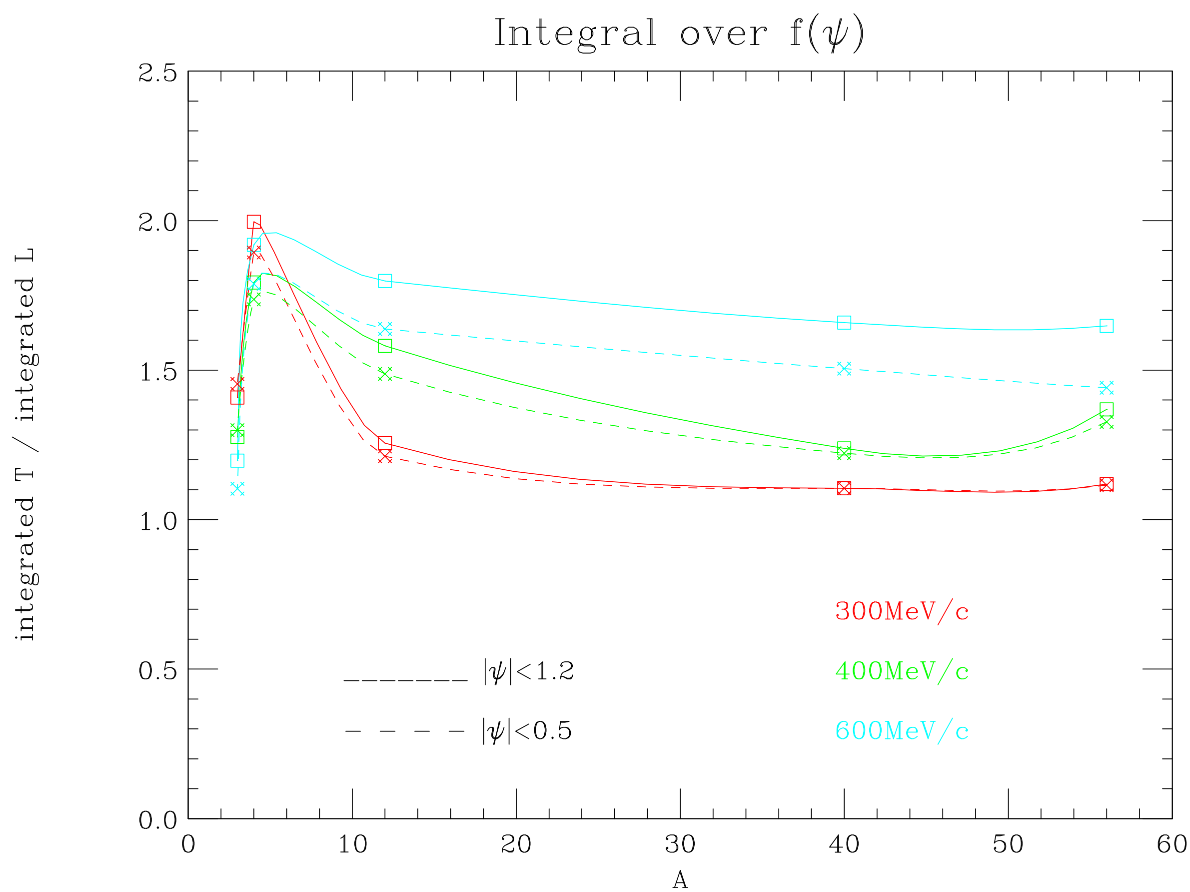
Electron Scattering: (e,e'p)



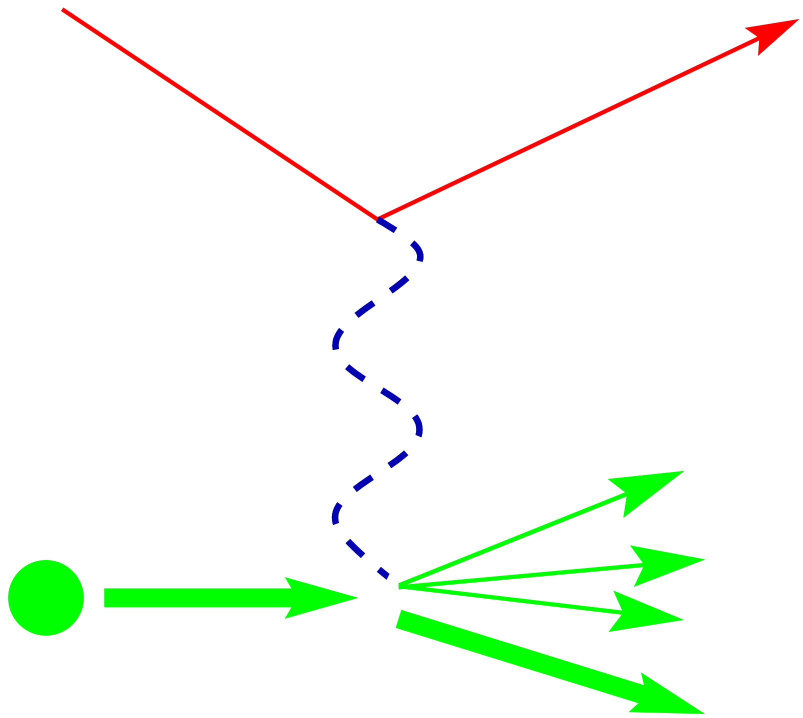
Calculation:

- Microscopic calculations of nuclear wvfns, overlaps
- Optical Potential Treatment
- Challenges: Microscopic determination of exclusive scattering, studies of energy dependence, etc.

Inclusive Response: Integrated Transverse / Longitudinal Strength



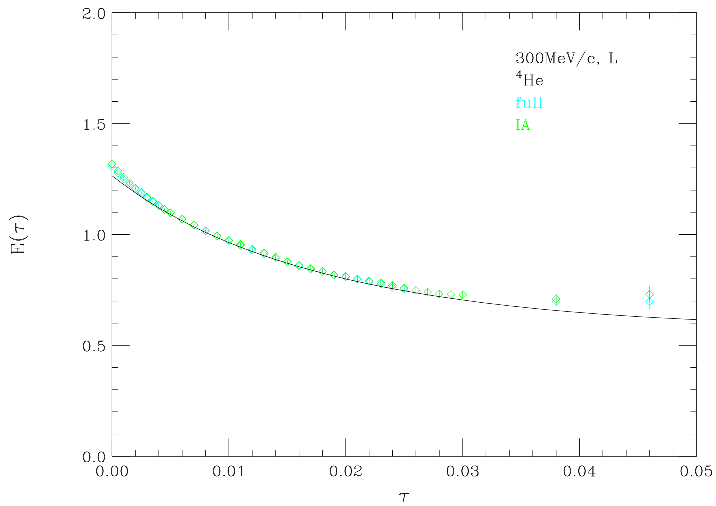
Inclusive Response

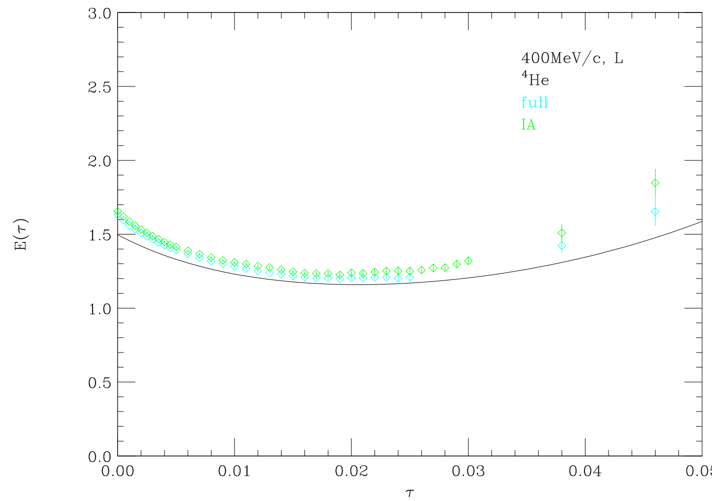


Euclidean Response

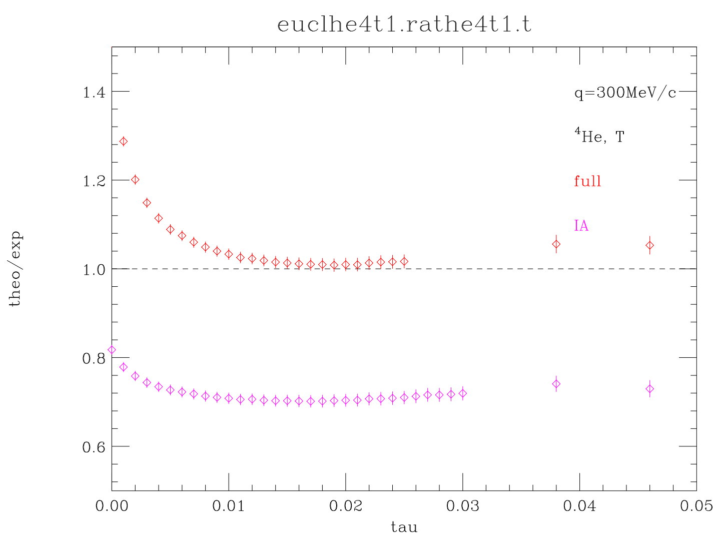
$$
\begin{aligned}
E_L(q) &= \langle 0|\rho^\dagger(\mathbf{q})\exp[-(H - E_0 - q^2/(2m))\tau]\rho(\mathbf{q})|0 > \\
&\quad -|\langle 0_{\mathbf{q}}|\rho(\mathbf{q})|0\rangle|^2 \exp[-E_r\tau] \\
&= \int d\omega\ S_L(q,\omega)\ \exp(-\omega\tau) \\
E_T(q) &= \langle 0|\mathbf{J}^\dagger(\mathbf{q})\exp[-(H - E_0 - q^2/(2m))\tau]\mathbf{J}(\mathbf{q})|0 > \\
&\quad -|\langle 0_{\mathbf{q}}|\mathbf{J}(\mathbf{q})|0\rangle|^2 \exp[-(E_r - q^2/(2m)\tau] \\
&= \int d\omega\ S_T(q,\omega)\ \exp(-(\omega - q^2/(2m)\tau)
\end{aligned}
$$

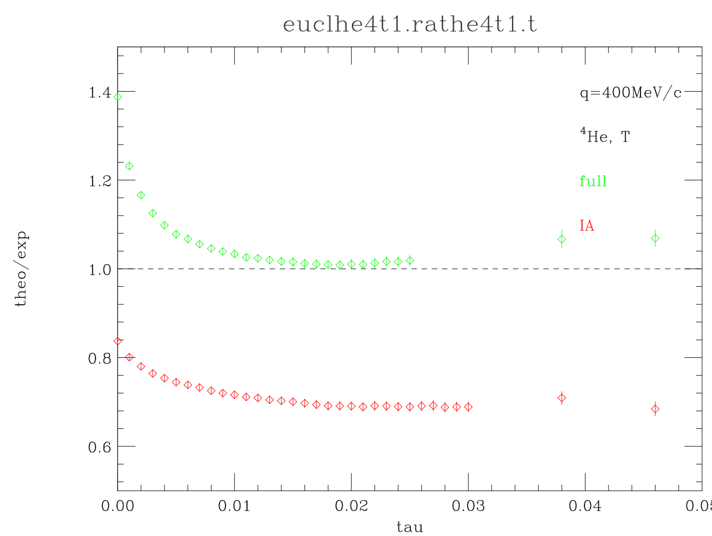
^{4}He Longitudinal Theory vs. Exp





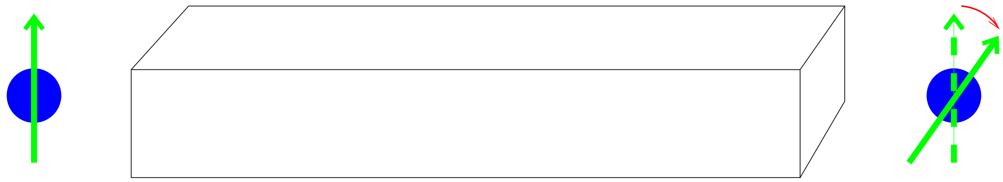
^{4}He Transverse Theory / Exp



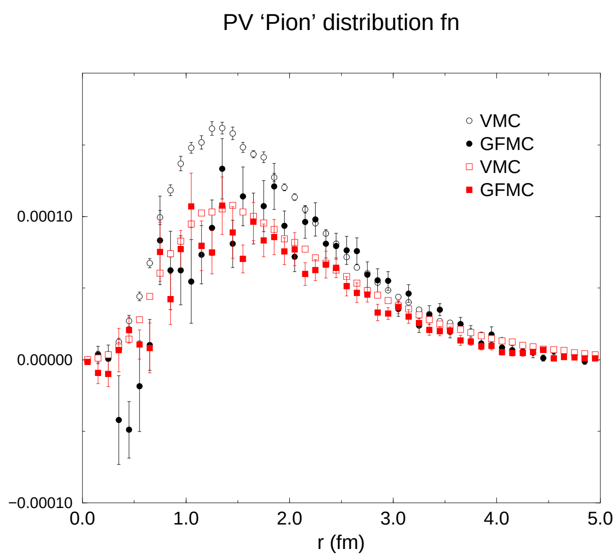


Low-Energy Scattering

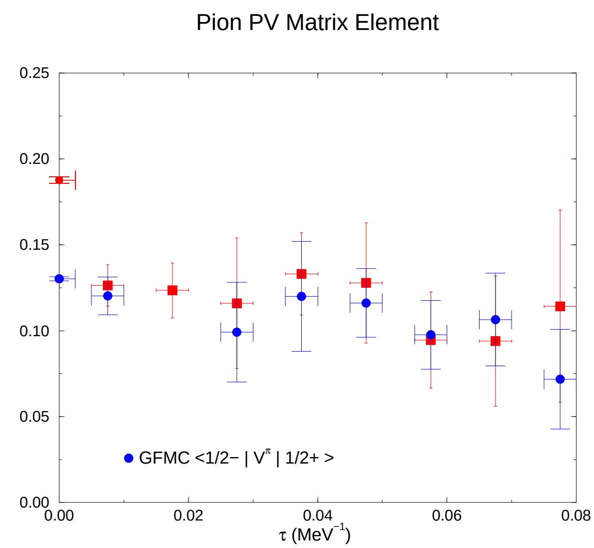
- Solar neutrino reactions (very low-energies)
- BBN reactions (100's of keV)
- Hadronic PV interactions: $\vec{n} - \alpha$



Radial Dependence



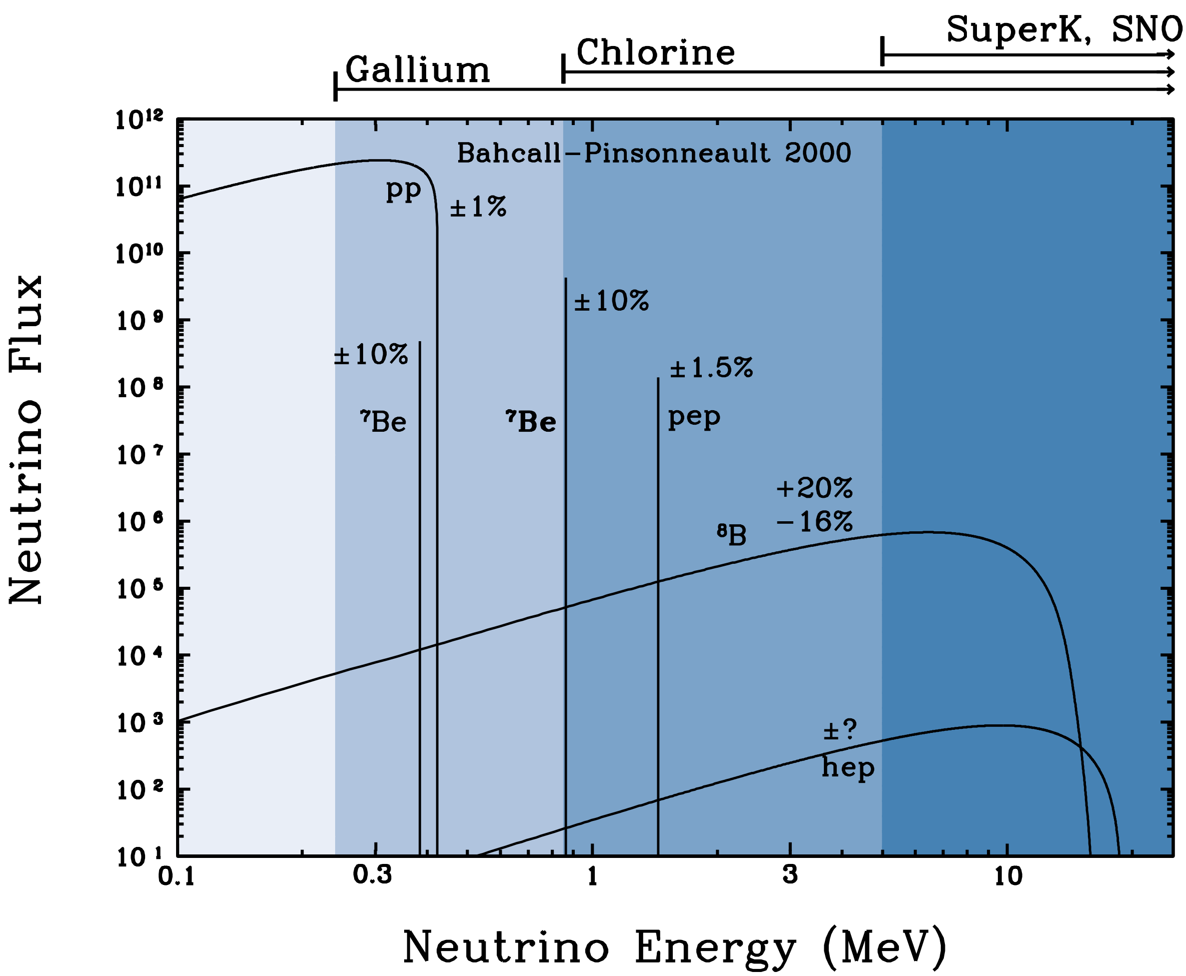
Matrix Element Convergence



Calculation:

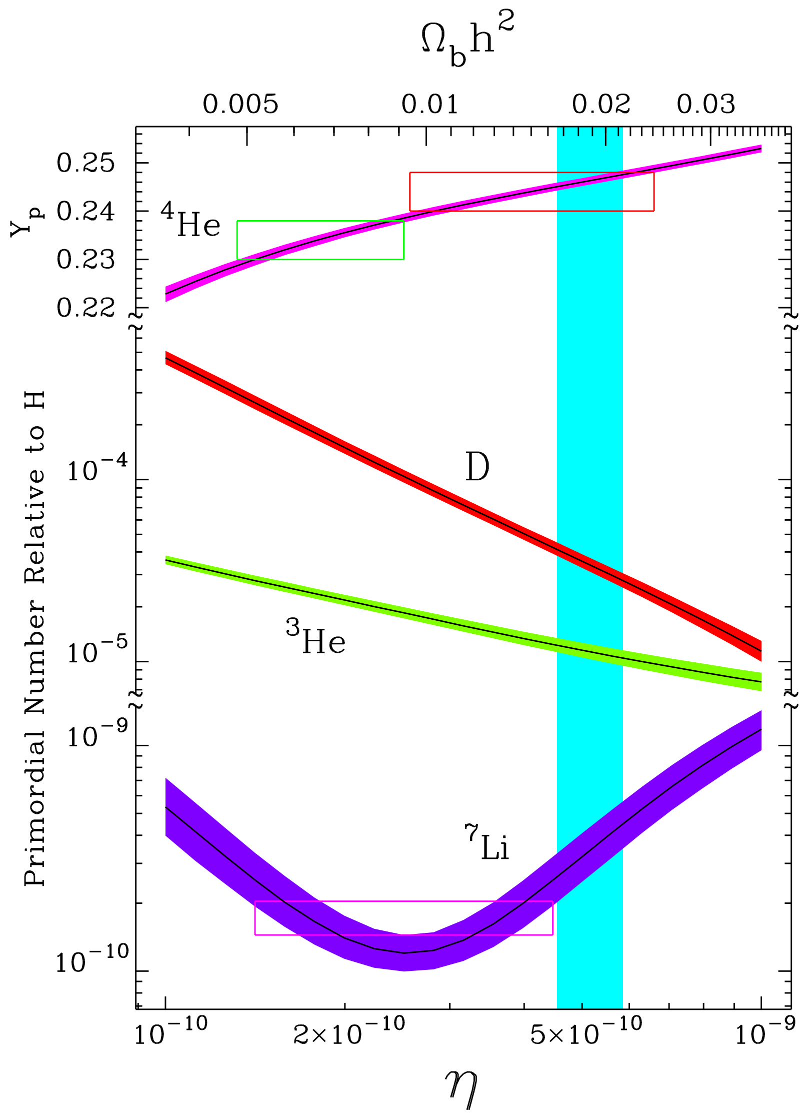
- Extract Pion PV (and other) contributions to $\vec{n} - \alpha$ spin rotation
- Similar Efforts underway in β-decay of p-shell nuclei

Solar neutrinos



- pp capture: Constrained to ≤ 0.2 per cent by interaction, tritium β decay
- hep capture: Extremely suppressed, constrained to approximately 30% (no discrepancy with SuperK)

BBN reactions



Burles, Nollett, Fields, ...

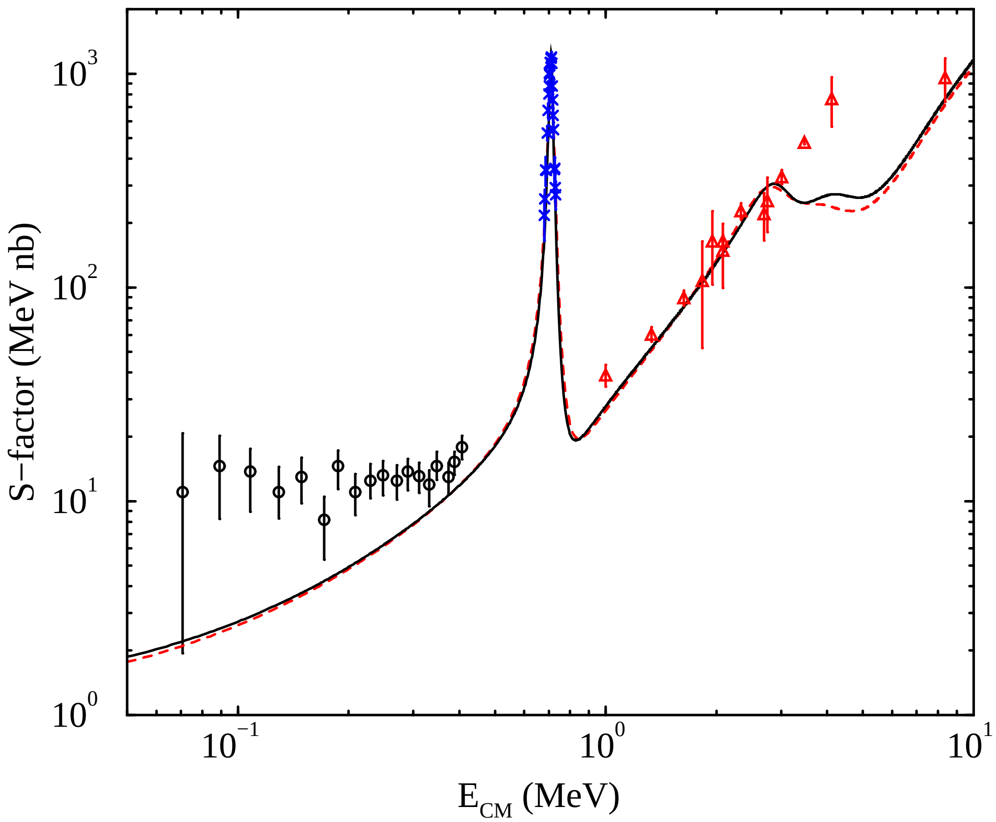
All calculations of $np \to d\gamma$ at relevant energies agree to O(1%).

(EFT, Potential Models, R-matrix analysis, ...)

Calculations calibrated to thermal neutron capture.

Low Energy Scattering: α-d capture

K. Nollett and R. B. Wiringa



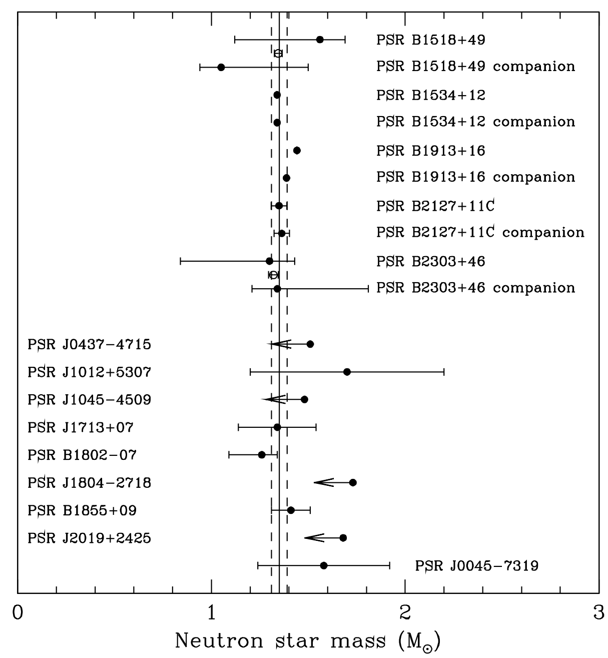
Calculation:

- VMC calculation of α, ^{6}Li wave functions
- Include careful attention to asymptotic boundary conditions
- Optical Model Potential for α-d scattering
- Consistent with Microscopic Interaction ?

Neutron Stars

Maximum Mass/solar tied to nuclear EOS: Expt'l Scenarios

1. ≈ 2.2 suggested by QPO (no exotica)
2. ≈ 1.9 from Vela X-1 ($1.87 + 0.23 - .17$); Cygnus X-2 (1.78 ± 0.23) (modest exotica)
3. ≈ 1.6 from radio pulsars, no obs in 1987A (significant)



Exotica:

- Kaon or Pion condensation
- Hyperons
- CFL state of dense matter (few times NM density)

Assuming calculations are ok...

Neutron Matter: Test of Var. Methods: 14N w/ PBC

- Test of short-distance physics
- Importance of elementary diagrams
- Accuracy of variational wvfn

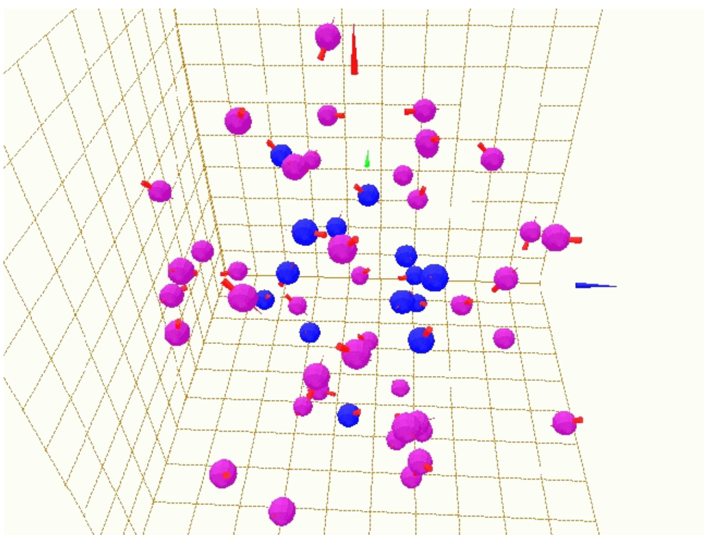
FHNC vs. GFMC (pbc)

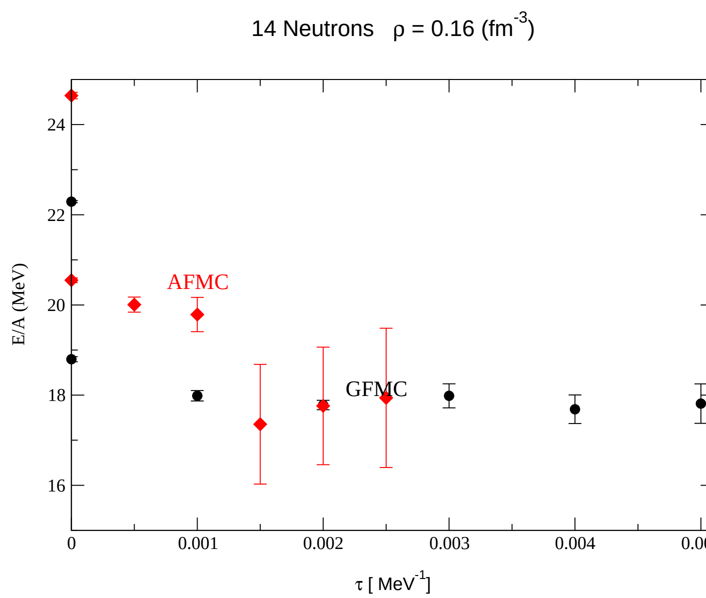
	ρ=0.16			$\rho = 0.24 fm^{-3}$		
H	FHNC	GFMC	Δ(%)	FHNC	GFMC	Δ(%)
V4	21.9	21.2(1)	-3	34.9	34.4(1)	-1.5
V4LS	18.5	19.3(3)	4	28.2	29.8(6)	5
V6	21.2	20.0(2)	-6	34.5	32.1(1)	-7.5
V8'	17.7	17.9(2)	1	27.4	29.1(3)	6

- Errors range up to $\approx$ 10%.
- Larger errors in $< V >$ at $\rho = 0.24$; primarily OPEP - effects on neutrino response?

AFDMC : Schmidt and Fantoni

- Samples path integrals over coordinates AND spin/isospins
- Large systems possible (up to 100)
- Approximate constraint on Path Integral needed





Also possible to treat spin susceptibility, ...

Challenges

- Spectra / Static Properties:
  - Larger Nuclei (GFMC up to 12; AFDMC beyond)
  - Neutron & Nuclear Matter: EOS, Long-Distance Properties
  - β decay of p-shell nuclei (CKM unitary in A=10)
- Low Energy Scattering:
  - Electroweak Capture Reactions (BBN, solar, ...)
  - PV processes
- Response and Beyond
  - Mass Dependence of EM response in nuclei
  - Inclusive Neutrino scattering
  - Polarizabilities, ...