Sensitivity of $K^+ \to l^+\nu e^+e^-$ and $K^+ \to e^+\nu\pi^0$ decays to tensor quark-lepton coupling

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Abstract

The sensitivity of $K \to l\nu e^+e^-$ and $K \to l\nu\pi^0$ decays to the quark-lepton tensor coupling are compared.

Standard Model supposed V-A quark lepton coupling:

$$\frac{G_F V_{us}}{\sqrt{2}}\,\overline{u}\gamma^\mu(1+\gamma^5)s\;\overline{e}\gamma^\mu(1+\gamma^5)\nu \tag{1}$$

If additionally tensor coupling g_T presents

$$\frac{G_F V_{us}}{\sqrt{2}}\,g_T\,\overline{u}\sigma^{\mu\nu}s\;\overline{e}\sigma^{\mu\nu}(1+\gamma^5)\nu, \tag{2}$$

then $K^+ \to l^+\nu\gamma^*$ ($K_{l2\gamma}$) amplitude aquires a term:

$$-i\frac{eG_F V_{us}}{\sqrt{2}}\,F_T\,\varepsilon^\mu q^\nu\,\overline{e}\sigma^{\mu\nu}(1+\gamma^5)\nu, \tag{3}$$

where ε^μ is a photon polarization vector or e^+e^- electromagnetic current $\frac{e}{q^2}\overline{e}_+\gamma^\mu e_-$, and q^μ is photon (e^+e^- pair) 4-momentum.

Tensor term might also appear in $K^+ \to l^+\nu\pi^0$ (K_{l3}) decay amplitude:

$$\frac{G_F V_{us}}{\sqrt{2}}\,\frac{2f_T}{M_K}p_K^\mu p_\pi^\nu\,\overline{l}\sigma^{\mu\nu}(1+\gamma^5)\nu \tag{4}$$

Here p_K and p_π are kaon and pion momentums, respectively.

To be able to compare the constraints on tensor coupling g_T from $K_{l2\gamma}$ and K_{l3} decays, corresponding matrix elements of tensor quark-lepton coupling (2) must be calculated.

In case of $\pi \to e\nu\gamma$ decay form factor F_T was related to the coupling g_T using PCAC technique [1]:

$$F_T = \frac{2}{3}\frac{\chi\langle|\overline{q}q|\rangle}{f_\pi} g_T \approx 0.4 g_T \tag{5}$$

Here, $f_\pi = 131$ MeV is pion decay constant, $\langle|\overline{q}q|\rangle = -(0.24 \text{ GeV})^3$ is vacuum expectation value for the quark condensate and $\chi = -5.7 \pm 0.6 \text{ GeV}^{-2}$ is the quark condensate magnetic susceptibility [2, 3] defined as

$$\langle|\overline{q}\sigma^{\mu\nu}q|\gamma\rangle = e_q\chi\langle|\overline{q}q|\rangle\mathcal{F}^{\mu\nu} \tag{6}$$

where e_q is electric charge of quark and $\mathcal{F}^{\mu\nu}$ is the external electromagnetic field.

The same ratio (5) between decay form factor F_T and quark-lepton coupling g_T is expected for $K^+ \to l^+\nu\gamma^*$ decay [4]. It was claimed in this Ref. [4] that

> ...the decay$K^+ \to \pi^0 e^+ \nu_e$ which has a large branching ratio and could be interesting for detecting the tensorial interaction of eq. (2) suppressed by PCAC. Indeed, using the PCAC approximation and the well-known reduction formula for the matrix element $\langle\pi^0(p_\pi)|B_{\mu\nu}(0)|K^+(p_K)\rangle$, where $B_{\mu\nu}(x) \equiv \overline{s}(x)\sigma^R_{\mu\nu}u(x)$, we obtain
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> $$\langle\pi^0(p_\pi)|B_{\mu\nu}(0)|K^+(p_K)\rangle \simeq \frac{1}{f_\pi}\int d^3x\langle 0|[A^0_0(x), B_{\mu\nu}(0)]_{x_0=0}|K^+(p_K)\rangle = 0 \tag{7}$$
>
> where A^0_μ is the axial weak hadronic neutral current which annihilates the final pion state $\langle\pi^0|$. The result (7) holds because it is not possible to construct an antisymmetric tensor with only one momentum. The matrix element of (2) will be different from zero only at higher orders in the chiral perturbation theory.

This means that $f_T \to 0$ in chiral limit.

Tensor form factor in K_{l3} decay was calculated [5] using quark graph shown on Fig. 1. In fact, the following quark-lepton coupling was considered

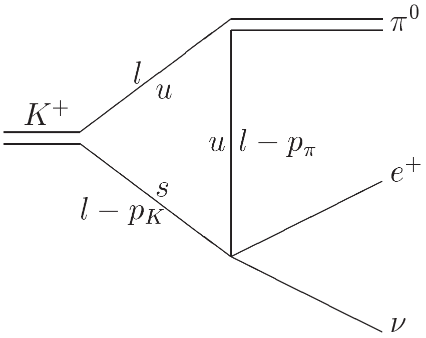

Figure 1: $K^+ \to l^+\nu\pi^0$ quark graph

in this Ref.:

$$\frac{G_F V_{us}}{\sqrt{2}}\, f_t\, \frac{q^\mu q^\lambda}{q^2} \overline{u}\sigma^{\lambda\nu} s\, \overline{e}\sigma^{\mu\nu}(1+\gamma^5)\nu, \tag{8}$$

One can easily show that factor $q^\mu q^\lambda/q^2$ may be substituted by $g^{\mu\lambda}/2$, *i.e.* to get matrix element of interaction (2) f_t should be substituted by $2g_T$. Also, form factor defined in Ref. [5] differs by factor 2 from (4). After these corrections one derives from Ref. [5]

$$f_T/f_+ = \frac{N_c}{2\pi^2} M_K\, [M_u \tilde{a} + (M_s - M_u)\tilde{b}] g^2 g_T \tag{9}$$

where $f_+ = 1/\sqrt{2}$ is $K^+ \to l^+\nu\pi^0$ vector form factor, $M_u = 340$ MeV and $M_s = 490$ MeV are the *constituent* masses of u and s quarks, $N_c = 3$ is the color factor, $\tilde{a}$ and $\tilde{b}$ are defined by

$$16\pi^2 i \int \frac{d^4 l}{(2\pi)^4} [(l-p_\pi)^2 - M_u^2]^{-1} [l^2 - M_u^2]^{-1} [(l-p_K)^2 - M_s^2]^{-1} = \tilde{a}(q^2) \tag{10}$$

$$16\pi^2 i \int \frac{d^4 l}{(2\pi)^4} l^\mu [(l-p_\pi)^2 - M_u^2]^{-1} [l^2 - M_u^2]^{-1} [(l-p_K)^2 - M_s^2]^{-1} = \tilde{b}(q^2) p_K^\mu + \tilde{c}(q^2) p_\pi^\mu \tag{11}$$

and they depend weakly on q^2. Their average values are $\tilde{a} \approx 4.0$ GeV^{-2} $\tilde{b} \approx 1.2$ GeV^{-2}. With the help of Goldberger-Treiman relation, the constant g may be expressed either through the pion decay constant $f_\pi = 131$ MeV or the kaon decay constant $f_K = 160$ MeV: $g \approx 2M_u/\sqrt{2}f_\pi \approx (M_u + M_s)/\sqrt{2}f_K \approx 3.67$. Numerically

$$f_T/f_+ \approx 1.5\, g_T \approx 3.8\, F_T \tag{12}$$

In the chiral limit (*current* quark masses $m_{u,s} \to 0$) the *constituent* quark masses remain approximately unchanged $M_u \approx M_s \approx 340$ MeV. Since M_K in (9) appeared only due to the definition of form factor (4), it is rather constant than a kaon mass, which depends on quark current masses as $M_K^2 \sim m_u + m_s$. This means that K_{l3} tensor form factor will still be approximately equal to (12) in strong contradiction with the conclusion of Ref. [4].

One can match the result of calculation [5] with chiral limit of Ref. [4] by substituting of *constituent* quark masses in Eq. (9) by *current* quark masses ($m_u \approx 4$ MeV, $m_s \approx 150$ MeV). In this case the value of K_{l3} tensor formfactor will be significantly reduced:

$$f_T/f_+ \approx 0.2\, g_T \approx 0.5\, F_T \tag{13}$$

Certaintly, such "improvement" of (9) **at least is not proved**, nontheless it illustrates that calculatuion of tensor K_{l3} form factor (12) might be unreliable and that significant decreasing of value of f_T can not be excluded.

The expected result [1] of E865 study of $K^+ \to \mu^+(e^+)\nu e^+e^-$ decays is that F_T is compatable with 0 and $\sigma(F_T) \approx 0.01$. This corresponds to $\sigma(f_T/f_+) \approx 0.04$ which may be compared with PDG value $|f_T/f_+| = 0.38 \pm 0.11$ [6] and preliminary result of KTeV (E799) experiment [7], which may be displayed as [2] $|f_T/f_+|^2 = 0.08^2 \pm 0.10^2$

The expected sensitivity of E865 $K \to l\nu ee$ to the tensor quark lepton coupling (2) is expected to be better than the sensitivity of existing $K \to l\nu\pi^0$ experiments. However, this conclusion is based on the result of calculation of matrix element which seems to be unreliable.

References

[1] V.M. Belyaev and Ian I. Kogan, Phys. Lett. **B280**, 238 (1992)

[1] The current (January 8, 2001) result is $F_T = 0.005 \pm 0.009$ if additionally constraints on F_V and F_A from theory and previous $K \to \mu(e)\nu\gamma$ experiments is accounted

[2] f_T^2 is actually measured in $K \to e\nu\pi^0$

[2] B.L. Ioffe and A.V. Smilga, JETP Lett. 37 (1983) 298; Nucl. Phys. B232 (1984) 109

[3] V.M. Belyaev and Ian I. Kogan, Sov. J. Nucl. Phys. 40 (1984) 659

[4] E. Gabrielli, Phys. Lett. B 301 (1993) 409

[5] M.V. Chizhov, Mod. Phys. Lett. **A8** (1993) 2753

[6] D.E. Groom et al., The European Physical Journal **C15** 1 (2000)

[7] Richard J. Tesarek, e-print arXiv: hep-ex/9903069