REPORT No. 178

RELATIVE EFFICIENCY OF DIRECT AND GEARED DRIVE PROPELLERS

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SUMMARY.

This report is an extension of the National Advisory Committee for Aeronautics Technical Report No. 168 and has been prepared for the National Advisory Committee for Aeronautics to show the relative values of various direct and geared drives. It has been assumed that the speed V and the crankshaft revolutions are held constant at each value of $\left(\frac{V}{ND}\right)_2$, corresponding to the maximum efficiency for a two-bladed, direct-drive propeller, so that the corresponding $\left(\frac{V}{ND}\right)$ and maximum efficiency for any other propeller arrangement depends only on N and D, which are easily calculated. The net efficiencies are obtained by allowing 98 per cent for the gears and 95 per cent for the efficiency of a four-bladed propeller relative to a two-bladed propeller.

The net efficiencies so found are given in terms of the efficiency for the two-bladed, direct-drive case, and plotted against $\left(\frac{V}{ND}\right)_2$, so that having given the $\left(\frac{V}{ND}\right)$ corresponding to maximum efficiency for a two-bladed, direct-drive propeller, the relative gain or loss due to any ordinary arrangement may be readily estimated. The conclusion is reached that when $\left(\frac{V}{ND}\right)_2$ is greater than 0.70, gearing is not advisable.

INTRODUCTION.

It is well known that in general a geared-down propeller has higher efficiency than a direct-drive propeller, but the literature on this subject does not present the data in such form that the aeronautical engineer can readily visualize the effect of gearing. This report has been prepared to show the actual net gain or loss in maximum efficiency due to the use of various modifications of the conventional two-bladed, direct-drive propeller.

It was shown in the National Advisory Committee for Aeronautics Technical Report No. 168 that there exists a definite relation between the maximum efficiency and the $\left(\frac{V}{ND}\right)$ at which it occurs. This relation is expressed by the empirical curve of maximum efficiency against $\left(\frac{V}{ND}\right)$, which is reproduced in Fig. 1 in this report. As pointed out in Report No. 168 this curve may be used to study the effects of reduction gearing. However, in order to apply it to four-bladed propellers, the relation between the diameter and efficiency of four-bladed and two-bladed propellers must be determined. These relations have been determined in this report using British test data from R. & M. No. 316.

In order to differentiate between the various conditions studied, the characteristics for the two-bladed and four-bladed propellers with direct drive are denoted by the subscripts 2 and 4, respectively. For the geared drives, 5:4 and 5:3, additional subscripts a and b are used. Thus η_{2a} is the efficiency for a two-bladed propeller geared 5:4 and η_{4b} is the efficiency for a four-bladed propeller, geared 5:3.

RELATION BETWEEN DIAMETERS OF TWO-BLADED AND FOUR-BLADED PROPELLERS.

The relation between the characteristics of two-bladed and four-bladed propellers may be obtained from R. & M. No. 316 of the British Advisory Committee for Aeronautics. Tests were made on two two-bladed propellers of different aspect ratio (5 and 7.5) and on the corresponding four-bladed propellers formed from two similar two-bladed propellers. The essential data applying to this study are given herewith in Tables I and II. It will be noted that the torque coefficients for the four-bladed propellers are 81 per cent greater than for the two-bladed propellers and that the variation in the value of the ratio is quite small. Since the torque varies as V^2D^3 or as VND^4 it will vary as D^4 when V and N are constant. Consequently

$$1.81\ (D_4)^4 = (D_2)^4 \tag{1}$$

where D_2 is the diameter of a two-bladed propeller and D_4 the diameter of a similar four-bladed propeller having the same torque. Therefore

$$D_4 = \frac{D_2}{(1.81)^{1/4}} = \frac{D_2}{1.16} = 0.863\ D_2 \tag{2}$$

The diameter of a propeller may be obtained from the expression for power

$$P \propto N^3D^5 \tag{3}$$

Dividing the right-hand side of (3) by the nondimensional factor $\frac{V}{ND}$ and substituting HP for P gives:

$$HP \propto VN^2D^4$$

or

$$HP = KVND^4$$

Solving for D

$$D = K\sqrt[4]{\frac{HP}{N^2V}} \tag{4}$$

In this equation K is found to vary from 275 to 325 for two-bladed propellers with an average value of about 300, when N is in R. P. M. and V in M. P. H. The equation is more easily solved in the form

$$D_2 = \sqrt[4]{\left(\frac{K}{N}\right)^2 \frac{HP}{V}} \tag{5}$$

K now varying from 75000 to 105000 with an average value of 90000. This variation may be considered unduly large for practical use, although it must be remembered that the variation includes many factors, such as blade form and width, blade section, camber ratios, etc. For geometrically similar propellers K should be substantially constant for reasonable variations in HP, N, and V.

COMPARATIVE EFFICIENCIES OF TWO-BLADED AND FOUR-BLADED PROPELLERS—DIRECT DRIVE.

The method of comparison adopted for this study assumes that the $\left(\frac{V}{ND}\right)$ for maximum efficiency of a two-bladed propeller is known and that V and N are to remain constant. Consequently the ratio $\left(\frac{V}{ND}\right)_4$ to $\left(\frac{V}{ND}\right)_2$ will be determined by the diameters only. That is

$$\left(\frac{V}{ND}\right)_4 = \frac{D_2}{D_4}\left(\frac{V}{ND}\right)_2 = 1.16\ \left(\frac{V}{ND}\right)_2 \tag{6}$$

From Tables I and II, it is seen that

$$\eta_4 = 0.95\eta_2 \qquad (7)$$

Therefore we may assume any value of $\left(\frac{V}{ND}\right)_2$ and find the corresponding η_2 from Figure 1. $\left(\frac{V}{ND}\right)_4$ is given by (6) and the corresponding η_4 is read from the curve, representing Equation (7), on Figure 1.

The values of η_2 and η_4 thus obtained are plotted against $\left(\frac{V}{ND}\right)_2$ in Figure 2 so that we may obtain a direct comparison of the efficiencies. That is, under conditions which are represented by $\left(\frac{V}{ND}\right)_2$ a two-bladed propeller would have the efficiency η_2 and a four-bladed propeller to absorb the same power at the same speed and R. P. M. would have the efficiency η_4.

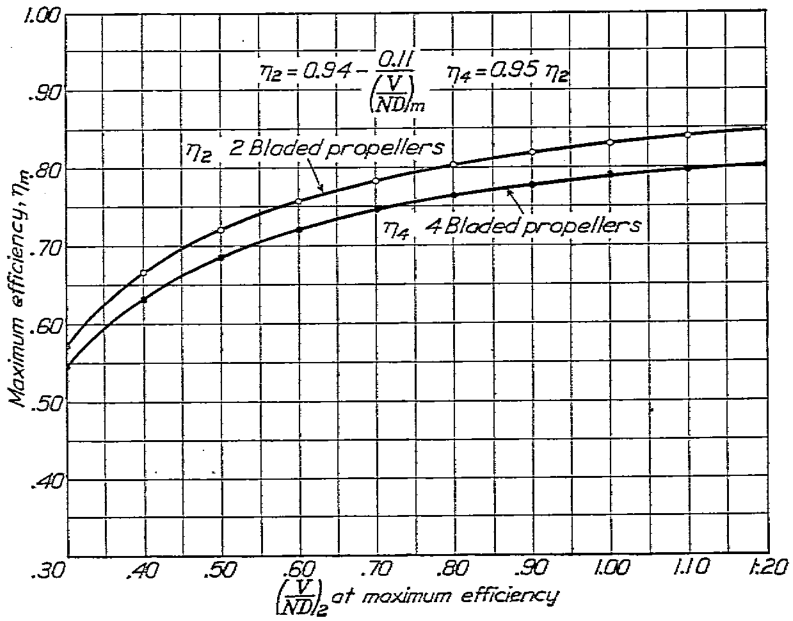

FIG. 1. Maximum efficiency for 2 bladed and 4 bladed propellers. From Durand's tests (see N. A. C. A. Technical Report No. 168).

At this time it is desired to call attention to the fact that the curve of η_2 *vs* $\left(\frac{V}{ND}\right)_2$ given in Figure 1 may be closely approximated by the equation

$$\eta_2 = 0.94 - \frac{0.11}{\left(\frac{V}{ND}\right)_2} \qquad (8)$$

This relation is very convenient in enabling an accurate estimate of the maximum efficiency of a two-bladed propeller to be made without reference to the curves.

COMPARATIVE EFFICIENCIES OF TWO-BLADED AND FOUR-BLADED PROPELLERS, GEARED DRIVE.

The comparison of efficiencies may be extended from direct drives to geared drives by use of Equation (4). From this equation it is seen that if *HP.* and V remain constant, D varies inversely as $\sqrt{N}$. Consequently for a two-bladed propeller geared down 5:4

$$D_{2a}=\sqrt{1.25}\ D_2=1.118\ D_2 \qquad (9)$$

and

$$\left(\frac{V}{ND}\right)_{2a}=\frac{N_2}{N_{2a}}\cdot\frac{D_2}{D_{2a}}\left(\frac{V}{ND}\right)_2=\frac{5}{4}\cdot\frac{1}{1.118}\left(\frac{V}{ND}\right)_2=1.118\left(\frac{V}{ND}\right)_2 \qquad (10)$$

Similarly, for a two-bladed propeller geared down 5:3

$$D_{2b}=\sqrt{1.667}\ D_2=1.291\ D_2 \qquad (11)$$

$$\left(\frac{V}{ND}\right)_{2b}=\frac{5}{3}\ \frac{1}{1.291}\left(\frac{V}{ND}\right)_2=1.291\left(\frac{V}{ND}\right)_2 \qquad (12)$$

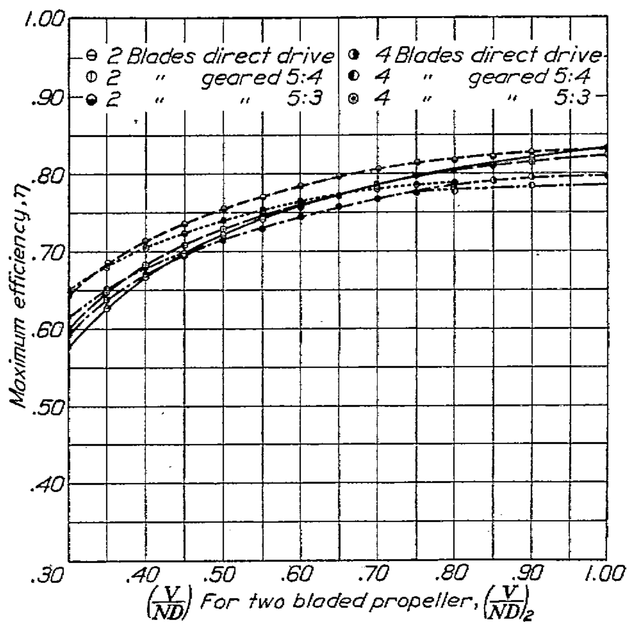

FIG. 2. Comparative net maximum efficiencies for 2 and 4 bladed propellers.

From Equation (2)

$$D_4=\frac{D_2}{1.16} \qquad (2)$$

the characteristics for the corresponding four-bladed propeller may be obtained:

$$D_{4a}=\frac{1.118}{1.16}D_2=.964\ D_2 \qquad (13)$$

$$\left(\frac{V}{ND}\right)_{4a}=\frac{1.25}{0.964}\left(\frac{V}{ND}\right)_2=1.296\left(\frac{V}{ND}\right)_2 \qquad (14)$$

and

$$D_{4b}=\frac{1.291}{1.16}D_2=1.112\ D_2 \qquad (15)$$

$$\left(\frac{V}{ND}\right)_{4b}=\frac{1.667}{1.112}\left(\frac{V}{ND}\right)_2=1.497\left(\frac{V}{ND}\right)_2 \qquad (16)$$

The values of η_{2a}, η_{2b}, η_{4a}, and η_{4b}, corresponding to these values of $\left(\frac{V}{ND}\right)$ may be read from the curves on Figure 1. These efficiencies are gross values and must be corrected for the efficiency of the gearing, which is here taken at 98 per cent, although a slightly higher figure may be obtained by careful design. The net efficiencies η'_{2a}, η'_{2b}, η'_{4a}, and η'_{4b} so obtained by the calculations in Table IV are then plotted on Figure 2 against $\left(\frac{V}{ND}\right)_2$ for direct comparison as previously explained.

Figure 2 now supplies sufficient data for an analysis of the comparative efficiencies of all conventional arrangements in terms of the efficiency for the normal case of two blades—direct drive.

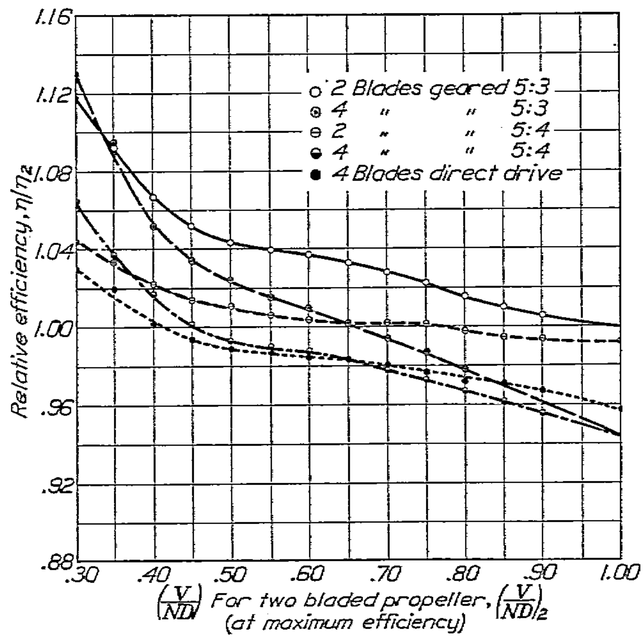

FIG. 3. Relative net maximum efficiencies for 2 and 4 bladed propellers.

CONCLUSIONS.

In Table V there are given the actual values of the efficiencies previously calculated, together with the relative values referred to in the case of two blades, direct drive. These relative values are plotted against $\left(\frac{V}{ND}\right)_2$ in Figure 3 which show directly the gain *or loss* in maximum efficiency due to gearing under any ordinary conditions. Remembering that $\left(\frac{V}{ND}\right)_2$ is the value of $\left(\frac{V}{ND}\right)$ corresponding to the maximum efficiency η_2 for a direct drive two-bladed propeller, the following conclusions may be drawn from Figure 3:

1. For values of $\left(\frac{V}{ND}\right)_2$ below 0.415 the efficiency of a four-bladed direct drive propeller is greater than that of a two-bladed direct drive propeller and vice versa.

2. For values of $\left(\frac{V}{ND}\right)_2$ greater than 0.40, gearing to reasonable ratios does not result in any great increase in efficiency when four-bladed propellers are used.

3. For values of $\left(\frac{V}{ND}\right)_2$ greater than 0.70 gearing is not advisable, even for two-bladed propellers since a geared drive must show a definite improvement, say 3%, before its use is justified.

It should be noted that at low speeds a geared propeller gives greater thrust than the corresponding direct drive propeller and this feature is of considerable importance in enabling an otherwise overloaded seaplane to take off in a calm.

The foregoing conclusions have been based on calculations which assume the ratio of $\frac{\eta_4}{\eta_2}$ to be substantially constant at all values of $\frac{V}{ND}$ within the usual working range. Recent test data, not available for use at this time, seem to indicate that the ratio of efficiencies is not constant. The conclusions must therefore be modified when our knowledge of the variation of $\frac{N_4}{N_2}$ with $\frac{V}{ND}$ is more definite, but the method of comparison will be unchanged.

TABLE I.—*Comparison of two and four bladed propellers.*

PROPELLER "A"—ASPECT RATIO 5.0.

[Data from Br. A. C. A., R. & M. 316.]

$\frac{V}{ND}$	Two blades—A.		Four blades.		T_{c4}/T_{c2}	Q_{c4}/Q_{c2}	$\frac{\eta_4}{\eta_2}$
	T_{c2}	η_2	T_{c4}	η_4			
0.54	0.337	0.655	0.580	0.618	1.720	1.825	0.943
.58	.270	.675	.463	.635	1.715	1.820	.940
.62	.216	.687	.375	.660	1.730	1.810	.958
.66	.175	.702	.307	.683	1.750	1.800	.972
.70	.142	.716	.250	.699	1.760	1.803	.975
.74	.113	.735	.200	.705	1.770	1.845	.960
.78	.088	.723	.156	.702	1.770	1.824	.971
.82	.068	.710	.118	.678	1.740	1.824	.955
Average..	....	....	....	....	1.744	1.819	.959

TABLE II.—*Comparison of two and four bladed propellers.*

PROPELLER "B"—ASPECT RATIO 7.5.

[Data from Br. A. C. A., R. & M. 316.]

$\frac{V}{ND}$	Two blades.		Four blades.		T_{c4}/T_{c2}	Q_{c4}/Q_{c2}	$\frac{\eta_4}{\eta_2}$
	T_{c2}	η_2	T_{c4}	η_4			
0.44	0.410	0.655	0.695	0.605	1.70	1.835	0.925
.48	.318	.672	.535	.630	1.68	1.795	.936
.52	.252	.687	.424	.647	1.68	1.783	.943
.56	.198	.704	.337	.664	1.70	1.804	.943
.60	.155	.715	.268	.678	1.73	1.825	.948
.64	.121	.720	.210	.680	1.73	1.830	.945
.68	.093	.715	.158	.680	1.70	1.790	.950
.72	.065	.685	.112	.658	1.72	1.790	.960
Average..	....	....	....	....	1.705	1.806	.944

TABLE III.—*Comparison of two and four bladed propellers.*

DIRECT DRIVE.

$\left(\frac{V}{ND}\right)_2$ (2 blades).	$\left(\frac{V}{ND}\right)_4$ (4 blades).	η_2	η_4
0.30	0.348	0.577	0.594
.35	.407	.627	.610
.40	.464	.668	.670
.45	.522	.698	.694
.50	.580	.722	.714
.55	.638	.741	.732
.60	.696	.756	.745
.65	.754	.771	.757
.70	.812	.784	.768
.75	.870	.795	.777
.80	.930	.806	.784
.85	.987	.814	.790
.90	1.043	.821	.794
1.00	1.160	.832	.796

TABLE IV.—*Comparison of two and four bladed propellers.*

GEARED DRIVES.[1]

$\left(\frac{V}{ND}\right)_2$	Two blades, geared 5:4.			Two blades, geared 5:3.			Four blades, geared 5:4.			Four blades, geared 5:3.		
Two blades, direct drive.	$\left(\frac{V}{ND}\right)$	η_{2a}	η'_{2a}	$\left(\frac{V}{ND}\right)$	η_{2b}	η'_{2b}	$\left(\frac{V}{ND}\right)$	η_{4a}	η'_{4a}	$\left(\frac{V}{ND}\right)$	η_{4b}	η'_{4b}
0.30	0.336	0.614	0.602	0.387	0.658	0.645	0.389	0.627	0.615	0.449	0.662	0.649
.35	.391	.662	.649	.452	.699	.685	.453	.664	.651	.524	.695	.682
.40	.447	.697	.683	.516	.728	.713	.518	.694	.690	.598	.718	.703
.45	.503	.723	.708	.581	.750	.735	.583	.713	.700	.673	.737	.722
.50	.559	.745	.730	.645	.769	.753	.648	.731	.717	.748	.755	.740
.55	.615	.761	.745	.710	.786	.770	.712	.749	.735	.823	.769	.753
.60	.671	.776	.760	.774	.801	.785	.777	.762	.747	.898	.780	.764
.65	.727	.790	.773	.839	.813	.796	.842	.772	.757	.973	.788	.772
.70	.783	.802	.786	.903	.822	.806	.908	.782	.767	1.048	.795	.779
.75	.840	.813	.797	.978	.830	.813	.970	.790	.774	1.123	.800	.784
.80	.895	.820	.803	1.032	.835	.818	1.036	.794	.778	1.196	.803	.787
.85	.952	.827	.810	1.097	.840	.823	1.101	.798	.782	....	....	....
.90	1.006	.833	.816	1.161	.844	.827	1.165	.800	.784	....	....	....
1.00	1.118	.842	.825	....	....	....	....	....	....	....	....	....

[1] The primed values are net efficiencies.

TABLE V.—*Relative efficiency of two and four bladed propellers.*

FROM TABLES III AND IV.

$\left(\frac{V}{ND}\right)_2$	Actual efficiency.						Relative efficiency.				
	η_2	η'_{2a}	η'_{2b}	η_4	η'_{4a}	η'_{4b}	$\frac{\eta'_{2a}}{\eta_2}$	$\frac{\eta'_{2b}}{\eta_2}$	$\frac{\eta_4}{\eta_2}$	$\frac{\eta'_{4a}}{\eta_2}$	$\frac{\eta'_{4b}}{\eta_2}$
0.30	0.577	0.602	0.645	0.594	0.615	0.649	1.044	1.118	1.030	1.065	1.125
.35	.627	.649	.685	.640	.651	.682	1.034	1.092	1.020	1.037	1.087
.40	.668	.683	.713	.670	.680	.703	1.022	1.067	1.002	1.017	1.052
.45	.698	.708	.735	.694	.700	.722	1.014	1.052	.994	1.002	1.034
.50	.722	.730	.753	.714	.717	.740	1.011	1.043	.989	.993	1.025
.55	.741	.745	.770	.732	.733	.753	1.006	1.039	.988	.990	1.015
.60	.756	.760	.785	.745	.747	.764	1.004	1.037	.985	.988	1.010
.65	.771	.773	.796	.757	.757	.772	1.003	1.033	.983	.983	1.002
.70	.784	.786	.806	.768	.767	.779	1.003	1.028	.980	.978	.994
.75	.795	.797	.813	.777	.774	.784	1.002	1.023	.978	.973	.987
.80	.806	.803	.818	.784	.778	.787	.998	1.015	.973	.967	.978
.85	.814	.810	.823	.790	.782	....	.995	1.010	.972	.962	....
.90	.821	.816	.827	.794	.784	....	.994	1.006	.968	.956	....
1.00	.832	.825	....	.796	....	....	.992	....	.958	....	....