THE STRUCTURE OF PRODUCTION TECHNOLOGY PRODUCTIVITY AND AGGREGATION EFFECTS

By

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CES 91-5 August 1991

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Abstract

This is a sequel to an earlier paper by the author, Dhrymes (1990). Using the LRD sample, that paper examined the adequacy of the functional form specifications commonly employed in the literature of US Manufacturing producing relations. The "universe" of the investigation was the three digit product group; the basic unit of observation was the plant; the sample consisted of **all** "large" plants, defined by the criterion that they employ 250 or more workers. The study encompassed three digit product groups in industries 35, 36 and 38, over the period 1972-1986, and reached one major conclusion: if one were to judge the adequacy of a given specification by the **parametric compatibility of the estimates of the same parameters**, as derived from the **various implications of each specification**, then the three most popular (production function) specifications, Cobb-Douglas, CES and Translog **all fell very wide on the mark.**

The current paper focuses the investigation on **two digit industries** (but retains the plant as the basic unit of observation), i.e., our sample consists of **all** "large" manufacturing plants, in each of Industry 35, 36 and 38, over the period 1972-1986. It first replicates the approach of the earlier paper; the results are basically of the same genre, and for that reason are not reported herein. Second, it examines the extent to which increasing returns to scale characterize production at the two digit level; it is established that returns to scale at the mean, in the case of the translog production function are almost identical to those obtained with the Cobb-Douglas function.[1] Finally, it examines the robustness and characteristics of measures of productivity, obtained in the context of an econometric formulation and those obtained by the method of what may be thought of as the "Solow Residual" and generally designated as Total Factor Productivity (TRP). The major finding here is that while there are some differences in productivity behavior as established by these two procedures, by far more important is the aggregation sensitivity of productivity measures. Thus, in the context of a pooled sample, introduction of time effects (generally thought to refer to productivity shifts) are of **very marginal consequence**. On the other hand, the introduction of **four digit industry effects is of appreciable consequence**, and this phenomenon is **universal**, i.e., it is present in industry 35, 36 as well as 38. The suggestion that aggregate productivity behavior may be largely, or partly, an aggregation phenomenon is certainly not a part of the established literature. Another persistent phenomenon uncovered is the extent to which productivity measures for individual plants are volatile,

[1] The CES function has not been examined in this context; in the previous paper it was found to be slightly inferior to the other two specifications, and this, combined with associated computational complexities has led us to pass it over.

while two digit aggregate measures appear to be stable. These findings clearly call for further investigation.

* The research on which this paper is based was carried out, in part, during the author's tenure of an ASA/NSF/Census Senior Research Fellowship. The views expressed herein are solely the author's and do not necessarily reflect those of the Department of Commerce or the other sponsoring organizations. I would like to express my appreciation to Dr. Eric Bartelsman who carried out most computations; to Dr. Robert McGuckin for general support and encouragement, and to Carol Jones and Linda Moeller for able assistance in carrying out this research.

Contents

1.1 INTRODUCTION AND SUMMARY

The production structure of US Manufacturing has been studied intensively in the sixties and early seventies. Surveys of empirical findings and theoretical developments may be found in Walters (1963), and Nerlove (1967); see also Nerlove (1965), for a specific discussion of empirical findings relative to the Cobb-Douglas function. Other surveys are by Griliches (1967), Jorgenson (1974), (1986) among others. The theoretical underpinnings of production theory have been well established in microeconomic theory for almost a century now. Duality theory has been an interesting and helpful addition to the formulation and interpretation of empirical studies since it was introduced by Samuelson (1954) and Shephard (1953). In Fuss and McFadden (1978), we have an extensive review of modern production theory.

Thus, a review of the literature would be completely redundant on our part.

A number of issues are routinely examined in the literature on the basis of rather limited samples. Such issues are whether production relations are to be considered form the **value added** or the **gross output** points of view; whether the translog function is an appreciable improvement over the Cobb-Douglas specification and, if appropriate whether symmetry and separability (or the associated cost function) prevail. In addition, many authors estimate production functions on the basis of **time series observations** on two digit industry aggregates. This practice invites the question, particularly on the issue of increasing or decreasing returns to scale, of whether the composition of output is responsible for the results and, if so, to what extent. Another issue that merits consideration is whether the measure of productivity, currently favored in the literature, is robust relative to the specification of the underlying production function.

An interesting finding that has substantial bearing on a number of time series studies using **two digit industry data** is that while "time effects" do not make appreciable difference in the interpretation of results, "four digit industry effects" are quite significant, econometrically, and quite appreciate in terms of orders of magnitude. Thus, to the extent that the four digit composition of the output of two digit industries varies over time, phenomena that may resemble productivity movements are generated, so that productivity measurements at the two digit level may simply reflect shifts in the compositional effect.

Spurred by this finding we have examined the dynamic behavior of the "residual" from the fitted production relations, both with and without "time" and "(four digit) industry effects". Two basic results stand out; first, if we classify plants according to the **magnitude** of their "total factor productivity" (residual) **each** year, the (geometric) mean TRP of the i^{th} decile is more or less flat over the 15 year period, except possibly for that of the tenth decile; second, if we classify plants according to the **magnitude of their TFP during 1972 only**, the behavior of the **(geometric) mean**

TFP of nearly all (1972 rank based) deciles is rather erratic. This suggests that the relatively steady behavior of "productivity" at the higher levels of aggregation hides a great deal of movement at more basic levels of production, thus suggesting a new frontier for research.

As in the previous paper, we deal with fifteen cross sections, from 1972 to 1986, and the unit of observation is the plant. We deal with the pooled sample, but we allow for "year effects" and for "four digit industry" effects. We had considered, but rejected, the possibility of arranging our data in the form of a panel. We rejected this alternative since to have worked with a panel (of plants) would have entailed eliminating a very substantial number of observations. Invariably, this must invite considerations of selectivity bias.

1.2 MODEL SPECIFICATIONS AND IMPLICATIONS

1.2.1 Duality and Production Theory

The typical (static) model of production theory, and many dynamic models, require (for equilibrium) that certain optimality conditions hold for every time t. Such models entail, typically, the assumption of perfect competition in the product and factor markets and represent the economic agent as a profit maximizer. If

$$\Pi(p, x) = p_0 Q - \sum_{i=1}^{n} p_i^* x_i, \tag{1.1}$$

$$p_0 \frac{\partial F}{\partial x_i} = p_i^*, \quad or \quad \frac{\partial F}{\partial X_i} = p_i, \quad p_i \frac{P_i^*}{P_0}$$

is the profit function, where p_0 is the price of output, Q is output obtained through a production function *f(x)*, with inputs, *x*, then under perfect competition the economic agent operates according to the rule

(1.2)

In the preceding, we have taken output, Q, and the *numeraire* so that all prices are stated relative to the price of output; we shall follow this practice in the remainder of the paper unless otherwise indicated.

A solution to the system in Eq. (2) expresses **the demand** for the factors of production in terms o input prices, i.e., we have a solution, $x_i = x_i(p)$, and the representation $Q = F[x(p)] - G(P)$, is said to be the **indirect production function.** While it is possible to derive from the preceding demand relations as functions of p and Q, this is not generally done in the literature. Rather, the representation of demand as a function of factor prices **and** output is obtained in the context of the mathematical dual of the profit maximization problem posed above. This is the **(cost) minimization problem subject to an expected output constraint,**

$$\min_{x} \sum_{j=1}^{n} p_i x_i, \quad \text{subject to } \bar{Q} = F(x),$$

whose first order conditions are,

$$\lambda \frac{\partial F}{\partial X_i} = p_i, \quad i = 1, 2, \ldots, n, \quad \bar{Q} = F(x), \tag{1.3}$$

where 8 is the Lagrange multiplier. Denote the solution to this problem by *x (p, Q*), 8 *(p, Q)*, and consider the cost function[2]

$$C(p, Q) = \sum_{j=1}^{n} p_j x_j (p, Q) . \tag{1.4}$$

Thus,

$$\frac{\partial C}{\partial p_j} = x_j (p, Q) + \sum_{i=1}^{N} p_i \frac{\partial x_i}{\partial p_j} . \tag{1.5}$$

From the second (constraint) equation of Eq. (3) we find

$$0 = \sum_{i=1}^{N} \frac{\partial F}{\partial x_i} \frac{\partial x_i}{\partial p_j} = 0 \tag{1.6}$$

Substituting in Eq. (5), we find, in view of the fact that 8 *(p, Q)* … *0*,

[2] For east of notation we have eliminated the overbar on the output symbol *Q*.

$$\frac{\partial C}{\partial p_j} = x_j(P, Q), \tag{1.7}$$

i.e., the equilibrium employment of the j^{th} factor is representable as the partial derivative of the cost function with respect of the j^{th} factor price.

Eq. (7) is a crucial relationship, and establishes the link between alternative representations of econometrically useful relations. Notice, in particular, that if we proceed from the first order conditions of the profit maximization problem, we obtain relations between the share of output accruing to the various factors of production and **factor inputs**, while if we proceed from the cost function derivation, we establish a similar relationship between the shares of cost and **input prices**. This is a particularly prominent feature of the translog specification.

The duality between the cost and production function representation of technology has led to many studies of the characteristics of manufacturing technology through the cost function, but to relatively few such characterizations through the production function. Since a translog production function does not, generally, have a translog cost function for its dual, one is led to wonder whether similar conclusions are obtained from these two venues. A mention of this problem seems to have appeared in Burgess (1975), but to have received little, if any, attention since.

1.2.2 Cobb-Douglas Production Functions

In the Cobb-Douglas (Cobb-Douglas) case the basic model is

$$Q_t = A\prod_{j=1}^{n} x_{t_i}^{\alpha_i} e^{u_t}, \tag{1.8}$$

where Qt represents the t^{th} observation on real output, x_{ti} the t_{th} observation on the i^{th} input, u_t is the t^{th} observation on a zero mean i.i.d. random variable with finite variance, and the remaining symbols represent parameters to be estimated. In nearly all applications in the literature, it is assumed that the markets for inputs as well as products are purely competitive, and that the economic agents proceed on the basis of either cost minimization or profit maximization. This, almost invariably, leads to the additional condition of homogeneity of degree one. It is shown in Dhrymes (1962) and Drèze, Kmenta, and Zellner (1966), that under profit maximization the input quantities are independent of the structural error in the production function. The same may be shown when one assumes cost minimization, subject to an expected output constraint.

The implications of these assumptions are several. First, under expected profit maximization, we can estimate the unknown parameters of the production process through the General Linear Model (GLM),

$$\ln Q_t = \ln A + \sum_{j=1}^{n} \alpha_j \ln x_{tj} + u_t. \tag{1.9}$$

Second, we can estimate all the parameters above, with the exception of ln A, through the relations

$$s_{ti} = \alpha_i e^{v_{ti}}, \quad i = 1,2,\ldots,n, \tag{1.10}$$

where s_{ti} is the observed share of output accruing to the i^{th} factor. Thus, we can test whether the translog production function is appropriate, relative to the Cobb-Douglas function, either through the production specification directly, or through the share equations. The cost functions corresponding to the Cobb-Douglas production function is given by

$$C(\boldsymbol{p}, \boldsymbol{Q}) = f(\theta)\, \boldsymbol{g}(\boldsymbol{p})\, h(\boldsymbol{Q}),$$

(1.11)

where

$$\theta = (\boldsymbol{A}, \alpha_1, \alpha_2, \ldots, \alpha_2)', \quad f(\theta) = \alpha \boldsymbol{A}^{-\frac{1}{\alpha}} \left(\prod_{j=1}^{n} \alpha_j^{-\frac{\alpha_j}{\alpha}} \right)$$

$$\boldsymbol{g}(\boldsymbol{p}) = \left(\prod_{j=1}^{n} \boldsymbol{p}_j^{\frac{\alpha_j}{\alpha}} \right), \quad h(\boldsymbol{Q}) = \boldsymbol{Q}^{\frac{1}{\alpha}}$$

We note that the cost function is **separable** in factor prices and output and, moreover, if " = 1, we have, for given factor prices, **constant marginal costs**; if " < 1, we have increasing marginal costs and if " > 1 we have decreasing marginal costs. The standard comprehensive model of production **requires the condition that** " =

1, **i.e., that the production function is homogeneous of degree one.** This is so since **if, as asserted, factor and product markets are perfectly competitive**, returns to the factors of production are governed by the marginal productivity conditions; thus, what accrues to them (factors of production) is given by

$$\sum_{j=1}^{n} p_j x_j = \sum_{j=1}^{n} \frac{\partial F}{\partial x_j} x_j = hF$$

for functions homogeneous of degree *h*. Since the Cobb-Douglas function we have employed is homogeneous of degree ", anything different from unity raises the issue of over- or under exhaustion of output. Thus, we have an incomplete and potentially contradictory theory. Besides, in the typical empirical practice, (and in the national income accounts), **it is assumed that the shares sum to unity**, by attributing to capital what is left over, after compensation of all other factors of production[3] This practice is perfectly admissible but, if we take it up as part of our framework then we **cannot, at the same time, employ the relations implied by the marginal productivity conditions for capital!** This was pointed out in Dhrymes (1965), but the practice of implementing estimation procedures with increasing or decreasing

[3] Other procedures, such as for example Hall (1989), which independently attribute a return to capital, generally **do exhibit over- or under-exhaustion of output**. One is then left to explain, how in an equilibrium context we can have, systematically, such over- and under-exhaustions.

returns to scale **and** the competitive first order (marginal productivity) conditions still persists to this day!

1.2.3 Translog Production Functions

The term translog production function is really a misnomer, in that the translog function is not a proper production function over the nonnegative orthant, as is commonly the case with other specifications. Rather, it has the customary properties of production functions only over a restricted subset of the admissible input space. As such, it is not generally viewed as a production function in its own right, but as an "approximation" to a more general, but unspecified functional form. Noting that, if

$$Q = F(x) \tag{1.12}$$

is an unspecified general function serving as a production function, we may expand it around ln x_j^0, where x_j^0 = 1, for all j, by Taylor's series, retaining only linear and quadratic terms.[4] This yields

$$\ln Q = \ln F(x^0) + \frac{\partial F}{\partial x}(x^0)(\ln x) \tag{1.13}$$

[4] In connection with this development, note that the Cobb-Douglas function can always be thought of as a Taylor series approximation, retaining only linear terms, to an arbitrary underlying production function. The difference is that this approximation is a production function in its own right, while the quadratic approximation is not!

$$+\frac{1}{2}(\ln x)' \begin{bmatrix} \frac{\partial^2 F}{\partial \ln x_1 \partial \ln x_1} & \frac{\partial^2 F}{\partial \ln x_1 \partial \ln x_2} & \cdots & \frac{\partial^2 F}{\partial \ln x_1 \partial \ln x_n} \\ \frac{\partial^2 F}{\partial \ln x_2 \partial \ln x_1} & \frac{\partial^2 F}{\partial \ln x_2 \partial \ln x_2} & \cdots & \frac{\partial^2 F}{\partial \ln x_2 \partial \ln x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 F}{\partial \ln x_n \partial \ln x_1} & \frac{\partial^2 F}{\partial \ln x_n \partial \ln x_2} & \cdots & \frac{\partial^2 F}{\partial \ln x_n \partial \ln x_n} \end{bmatrix} (\ln x) .$$

If **we replaced, by parameters, all derivatives evaluated at the point** $x = x^0 = e$, where e is a vector of unities, then we have the standard translog function

$$\ln Q = \alpha_0 + \alpha' \ln x + \frac{1}{2}(\ln x)' B (\ln x) \tag{1.14}$$

Under perfect competition in the product and factor markets, as well as profit maximization, we obtain the share equations,

$$s_{ti} = \alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln x_{tj} + v_{ti}, \quad i = 1, 2, ..., n. \tag{1.15}$$

This is easily verified from Eq. (14), if we note that the right member of Eq. (15) is simply the derivative,

$$\frac{\partial \ln Q}{\partial \ln x_i} = \frac{x_i p_i}{Q}, \quad \textit{owing to the fact that} \quad \frac{\partial Q}{\partial x_i} = p_i .$$

Several remarks are in order, regarding Eq. (15). First, all the parameters of the translog function, with the exception of the scale parameter, "$_0$, may be estimated from the *n* share equations.[5] Second, as indicated in the footnote, we are forced to adopt the conditions,

$$\sum_{i=1}^{n} \alpha_i = 1, \quad e'\mathrm{B} = 0, \quad \textit{and, since } \mathrm{B} \textit{ is symmetric}, \quad \mathrm{B}'e = 0.$$

Collectively, these conditions imply that the approximating (translog) function is homogeneous of degree one. Alternatively, we may estimate the relevant parameters, without any restrictions, or assumptions regarding the nature of product and factor markets. This may be done by simply regressing the logarithm of output on

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[5] The reader should note that since, in Eq. (15), all share equations contain the **same variables**, and since the data is such that all shares add up to unity, **least squares applied to the share equations** produces estimates that obey the conditions,

$$\sum_{i=1}^{n} \hat{a}_i = 1, \quad \sum_{i=1}^{n} \hat{\beta}_{ij} = 0, \quad \textit{for all } j.$$

The proof of this is straightforward. Let *S, X* be the data matrices, i.e., the matrices containing the observations of the *n* shares and inputs respectively. The least squares estimates of the parameters are

$$(X'X)^{-1}X'S = \begin{pmatrix} \hat{\alpha}' \\ \hat{\mathrm{B}}' \end{pmatrix} \quad \textit{hence}, \ (X'X)^{-1}X'Se = (X'X)^{-1}Xe = e_{\cdot 1},$$

where e is a vector of unities, and $e_{\cdot i}$ is a vector all of whose elements are zero, except the i_{th}, which is unity. But this imposes on use the assumption that the function is homogeneous of degree one!

the inputs. The resulting parameter estimates, may then serve as the test statistics for testing the null

$$H_0: \ \boldsymbol{e}'\boldsymbol{\alpha} = 1, \quad \textit{and} \ \boldsymbol{e}'\mathrm{B} = \mathbf{0},$$

as against the alternative that the parameters in question are unrestricted.

Third, this functional form is almost never employed in the literature. Instead, what is employed are share equations **derived from duality theory**, which means that one operates with the associated cost function. Now, **if the cost function is, actually, of the translog type**, i.e.,

$$\ln C = \boldsymbol{a}_0 + \boldsymbol{a}' \ln \boldsymbol{p} + \frac{1}{2}(\ln \boldsymbol{p})' \boldsymbol{B} (\ln \boldsymbol{p}) + \boldsymbol{g}(\ln \boldsymbol{Q}) + \boldsymbol{error}, \tag{1.16}$$

we obtain the relations

$$\frac{\partial \ln C}{\partial \ln \boldsymbol{p}_i} = \frac{\boldsymbol{p}_{ti} x_{ti}}{C_t} = s_{ti} = \boldsymbol{a}_i + \sum_{j=1}^{n} \boldsymbol{b}_{ij} \ln \boldsymbol{p}_{tj} + v_{tj}, \quad \boldsymbol{i} = 1, 2, \ldots, n. \tag{1.17}$$

The representation above is valid, provided the cost function is **separable**, as in the cases of the Cobb-Douglas and the CES based cost functions.

If separability is denied, the cost function should be rendered as

$$\ln C = \boldsymbol{a}_0 + \boldsymbol{a}' \ln \boldsymbol{p} + \alpha \ln \boldsymbol{Q} + \frac{1}{2} \begin{pmatrix} \ln \boldsymbol{p} \\ \ln \boldsymbol{Q} \end{pmatrix}' \begin{bmatrix} \boldsymbol{B} & c \\ c' & \gamma \end{bmatrix} \begin{pmatrix} \ln \boldsymbol{p} \\ \ln \boldsymbol{Q} \end{pmatrix} + \boldsymbol{error}. \tag{1.18}$$

In this context, the share equations above become

$$\frac{\boldsymbol{p}_{t\boldsymbol{i}} x_{t\boldsymbol{i}}}{C_t} = s_{t\boldsymbol{i}} \, \boldsymbol{a}_{\boldsymbol{i}} + \sum_{\boldsymbol{j}=1}^{n} \boldsymbol{b}_{\boldsymbol{ij}} \ln \boldsymbol{p}_{t\boldsymbol{j}} + c_{\boldsymbol{i}} \ln \boldsymbol{Q} + v_{t\boldsymbol{j}}, \quad \boldsymbol{i} = 1, 2, \ldots, n, \tag{1.19}$$

and a test of decomposability, or separability, could be carried out in the form of the hypothesis test

H_0: $c = 0$,

as against the alternative

H_1: $c \neq 0$.

Consequently, a test of separability, may be carried out by estimating the parameters of Eq. (19), and testing the hypothesis

H_0: $c = 0$,

as against the alternative

H_1: $c \neq 0$,

while a test of homogeneity of degree one (constant returns to scale), given separability, may be carried out through the hypothesis test

H_0: $\gamma = 0.$ $\alpha = 1.$

Since duality implies that the production process exhibits increasing returns to scale if the cost function has the property that an increase in output, by a factor λ, leads to an increase in cost, by a factor less than λ, and conversely for decreasing returns to scale, a simple calculation shows that the cost function in Eq. (18) allows, in principle, for ranges (of output) corresponding to decreasing, constant and increasing returns to scale. The change in logarithm of cost, following a change in output by factor 8, is given by

$$[\alpha + c'\ln\boldsymbol{p} + \gamma(\ln\boldsymbol{Q} + \frac{1}{2}\ln\lambda)]\ln\lambda.$$

Thus, for 8 > 1,

$$\alpha + c'\ln\boldsymbol{p} + \gamma(\ln\boldsymbol{Q} + \frac{1}{2}\ln\lambda) \leq 1,$$

implies **non decreasing returns to scale**, while

$$\alpha + c'\ln\boldsymbol{p} + \gamma(\ln\boldsymbol{Q} + \frac{1}{2}\ln\lambda) > 1,$$

implies decreasing returns to scale.

1.2.4 Productivity Measurement

The most widely used measure of productivity, Total Factor Productivity (TFP), derives from the early work of Solow (1957) and we shall refer to it as the "Solow Residual"; the initial formulation assumed a production function

$$\boldsymbol{Q}_t = \boldsymbol{A}(t)F(\boldsymbol{x}), \tag{1.20}$$

with "Hicks neutral technical change" function *A(t)* and a production component *F*. Taking logarithmic derivatives, we find

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + \sum_{j=1}^{n} \frac{x_i}{F} \frac{\partial F}{\partial x_i} \frac{\dot{x}_i}{x_i}. \tag{1.21}$$

Noting that

$$\frac{x_i}{F} \frac{\partial F}{\partial x_i} = \frac{x_i}{Q} \frac{\partial Q}{\partial x_i} = s_i,$$

we may interpret the relation above as designating the **observed share of output accruing as income to the** i^{th} factor of production. This becomes possible by the interpretation of the partial derivative *(MQ/Mx$_i$)* as the "wage" of the i^{th} input in units of the output, which is here taken to be the *numeraire*. This, of course, immediately necessitates the assumption that there is perfect competition in the factor markets and that the production function is homogeneous of degree one, otherwise there will be over- or under- exhaustion of output. Notice, further, that Eq. (21) may also be rendered as

$$\frac{d}{dt} lnQ \approx \frac{d}{dt} lnA + \frac{d}{dt} \sum_{j=1}^{n} s_j lnx_j,$$

on the assumption that the s_j are nearly constant. Hence, up to an additive constant, we have, approximately,

$$lnQ \approx lnA + \sum_{j=1}^{n} s_j lnx_j, \tag{1.22}$$

so that we can write

$$\ln \boldsymbol{A} \approx \ln \boldsymbol{Q} - \sum_{j=1}^{n} s_j \ln x_j . \tag{1.23}$$

Initially, Solow used Eq. (21), thus obtaining the relation between the rate of growth of output and the rates of growth of the inputs plus the rate of growth of "technical change", or productivity, giving rise to a literature of "growth accounting".[6] He then "integrated" the rate of growth of productivity function to obtain what we would call today the Total Factor Productivity (TFP). From Eq. (23), we see that if we insert a time subscript, we shall obtain

$$\ln \boldsymbol{A}_t = \ln \boldsymbol{Q}_t - \sum_{j=1}^{n} s_{jt} \ln x_{jt},$$

where the share, s_{jt}, is computed for each observation (time period) in the sample.

In Dhrymes (1961), (1963), we have an econometric reformulation of this problem in which it is assumed, explicitly,

$$\boldsymbol{Q}_t = \boldsymbol{A}(t) \prod_{j=1}^{n} x_j^{\alpha_j}; \tag{1.24}$$

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[6] This, of course, was in the early innocent days of applied econometrics, when it was firmly believed that empirical relations, once established, would last for eternity, or at least until the next year!

obtaining appropriate estimates of the exponents, say $\hat{a}_j$, we obtain the total factor productivity as

$$TRP_t = Q_t \prod_{j=1}^{n} x_j^{-\hat{a}_j} \tag{1.25}$$

Notice that the rationale of the Solow approach **almost assumes** the Cobb-Douglas production function, while the approach in Dhrymes (1961) allows for the specification of **any** production function, since the basic scheme may be described as specifying the relation

$$Q_t = A(t) F(x_{\cdot j}), \tag{1.26}$$

estimating the parameters of the function *F* and obtaining

$$TFP_t = \frac{Q_t}{\hat{F}(x_{\cdot t})} \tag{1.27}$$

The only issue remaining here is whether the "productivity" or "technical change" function *A(t)*, should include the scale constant, customary in production function specifications. Evidently, in the Solow residual approach, TFP **includes** the scale constant in question. We shall address this issue when we discuss the empirical results.

1.3 DATA AND EMPIRICAL RESULTS

1.3.1 Data Sources

All data employed in this study are taken from the Census' LRD files. They comprise essentially shipments, inventory, inventory change, production worker compensation, nonproduction worker compensation, production workers' hours of work (as well as number of production workers), number of nonproduction workers, investment in plant and equipment, purchases of materials and energy, as well as the associated prices or implicit price deflators. Data were available on an annual basis, for all plants employing 250 workers or more, for SIC industries 35, 36 and 38. From these, plant specific capital stocks were constructed, utilizing the plant and equipment investment available by plant, and the appropriate deflators. In addition, value added was constructed from shipments plus inventory change minus purchases of energy and materials, divided by the shipments deflator. This value added served as the measure of output in most instances. When gross output was taken to correspond to the theoretical notion of output, it was defined as shipments plus inventory change, deflated by the shipments deflator. The implicit price deflators of production worker and nonproduction worker compensation served as a measure of wages, and the returns to capital, divided by the real capital stock we have constructed, served as a measure of the implicit price (rental) of capital. Thus, we have obtained information on: output, production workers, nonproduction workers and capital (we also had experimented with structure and equipment capital treated separately), annually over the period 1972-1986.

1.4 EMPIRICAL RESULTS: RETURNS TO SCALE AND AGGREGATION

As noted in the introduction, the basic unit of observation is the plant, and the universe investigated is variably industries 35 (Machinery, Except Electrical), 36 (Electrical Machinery and Electronic Equipment) and 38 (Instruments and Related Products). For industry 35 we have 17,724 observations, for industry 36, 17,126 and for industry 38 we have 5,054 observations.[7]

Generally, the same parameters being estimated from different implications of the production model, as discussed in sections 2.1, 2.2 and 2.3 gave rise to very different point estimates, much in the manner documented in Dhrymes (1990). Thus, nothing will be gained by further discussion, except to confirm that the same phenomenon extends through the two digit level. For that reason, in what follows, we shall report extensively only on the other findings, including the dynamic behavior of productivity, as determined by the residuals of econometrically fitted productions, first given in Dhrymes (1961), as well as productivity determined in the standard fashion of today, and first suggested in Solow (1957), with antecedents in Kendrick (?) and others. Finally, we note that, in this study, we confine our attention to the Cobb-Douglas and Translog production functions.

[7] In point of fact we had available to us 24,187, 22,772 and 6,503 observations on individual plants over the period 1972-1986. A number of observations were then eliminated if they had nonpositive value added, zero shipments, or experienced a more than doubling of their labor employment over the previous period.

1.4.1 Value Added versus Gross Shipments

In Tables A1 through A3, in the Appendix, we give the estimation results using **Gross Shipments** as the measure of output. Gross shipments means Shipments plus changes in Inventories. In Tables A4 through A6 we give estimation results using Value Added as the measure of output. Value Added is defined as gross shipments minus purchases from other firms classified as Materials and Energy. In the Gross Shipments version we have four inputs, Capital, designated by K, Production Worker hours, designated by L_1, non-Production Workers hours, designated by L_2 and Materials and Energy, designated by M. Each Table gives the results for the Cobb-Douglas and Translog specifications. The numbers in parentheses, under each coefficient estimate, represent the estimated standard errors. The remainder of the notations of the tables are self evident; thus, KK stands for the coefficient of $\ln K^2$, L_1L_2 stands for the coefficient of $\ln L_1 \ln L_2$ etc.; N.D. stands for the "no dummies" version of the specification, T.D. stands for the "time dummies" only specification and T.D. and I.D. stands for the "time dummies and (four digit) industries dummies" specification. Finally, for the **Cobb-Douglas** specification, Test 1, refers for the test of constant returns to scale (homogeneity of degree 1); Test 2, under the heading T.D., refers to the test of the hypothesis that all time dummies are the same (no time effect, or more precisely, zero time contrasts); under the heading "T.D.

and I.D.", Test 2 refers to the test of the hypothesis that all **industry dummies** are the same (zero four digit industry contrasts).

In The **Translog** specification, Test 1 is the homogeneity of degree 1 test, Test 2 is a test of whether the Translog is significantly different from the Cobb-Douglas specification, i.e., that all the extra parameters of the translog specification are zero. This designation is the same in all three columns, under the Translog heading. Test 3, under the heading T.D., refers to the test of the hypothesis that all time dummies are the same (no time effect, or more precisely, zero time contrasts); under the heading "T.D. and I.D.", Test 3 refers to the test of the hypothesis that all **industry dummies** are the same (zero four digit industry contrasts). The entries in the row corresponding to Tests 1, 2 and 3, give the p-value, i.e., the probability that the test statistic obtained, or a higher value, could have been obtained under the null hypothesis; thus a p-value greater than an appropriate significance level (such as, e.g., .01 or .05 or .1), indicates **acceptance of the null hypothesis**; a value **less than that, indicates rejection**.

Returns to Scale: Gross Shipments

In point of fact, all tests reported in Tables A1 through A6 result in the rejection of the null hypotheses, since the largest p-value obtained in .02, in the case of "time contrasts" for the Translog function. This would indicate that at the .01 level of significance we would accept the hypothesis that, for all years, the time effect is the same.

Since we reject the degree one homogeneity hypothesis in all cases, it would be desirable to comment on the magnitude of this parameter, as estimated from our data. While for the Cobb-Douglas function the parameter estimate is unambiguous we shall report below the sum of the exponents of the various inputs. For the Translog, we shall evaluate returns to scale **at the mean**. The relevant means are given below in Table 1.

TABLE 1					
Industry	No. OBS.	K	L_1	L_2	M
35	17,824	8.8823	6.3958	5.6621	8.9211
36	17,126	8.8554	6.5742	5.5296	8.8283
38	5,054	8.7247	6.4022	5.7372	8.6229

The returns to scale estimates are given in Table 2, below. In the case of the Translog production function, the returns to scale parameter is evaluated at the sample mean; sample means and other relevant information were given in Table 1, above.

TABLE 2		
Industry	Returns to Scale	
	C.D.	Translog
35	.994	.021
36	1.013	1.034
38	1.020	1.021

These point estimates confirm the results given under Test 1, in Tables A1 through A3, viz., that in either the Cobb Douglas or the Translog specification we cannot reject the hypothesis of nonconstant returns to scale; the magnitude of the scale parameter, however, is rather close to one. We should also note that the results presented above correspond to the specification that includes time and (four digit) industry dummies and that, especially in the Translog specification, we have occasionally point estimates (components of the vector ") which are negative!

The returns to scale parameter is somewhat larger when industry and time dummies are omitted, indicating another important incidence of aggregation effects.

The results presented herein should be tempered by the realization that there has been no correction for possible autocorrelation in plant disturbances, which is a subject that merits further indications. Of course, one might argue, perhaps with equal justification, that autocorrelation correction is irrelevant, since one may view the "error" or "shock" component of the specification as a central limit theorem cumulation of factors, individually infinitesimal and unaccounted for, which, collectively, constitute the productivity phenomenon.

Other Issues: Gross Shipments

Certain other features of the results stand out and we comment on these below, always in the context of the specification that includes both time and (four digit) industry contrasts.

1. In the Cobb-Douglas case, materials (and energy) dominate the production process, i.e., the elasticity of output with respect to materials and energy is generally about twice the elasticity with respect to any other input. In industry 35, this elasticity is of the order of .7, while in Industries 36 and 38 it is of the order of .5;

2. The hypothesis that the additional terms (beyond Cobb-Douglas) have nonnull coefficients is invariably accepted; the

additional terms, however, do not contribute materially to the explanatory power of the relation.

3. Another persistent finding is that the sum of squared errors is reduced only by the order of 9 - 15%, when we move from the Cobb-Douglas to the Translog specification, **but there is a very significant reduction when we introduce, in either the Cobb-Douglas or the Translog specification (four digit) industry effects).** This will be further discussed below.

The hypotheses of no time contrasts and non (four digit) industry contrasts are uniformly rejected, meaning that the scale constants in the specifications, whether Cobb-Douglas or Translog, vary according to the time (year) or four digit industry pertaining to a given plant. But perhaps what is far more significant **is the fact that the introduction of time effects** reduces the sum of squared errors **relatively little**, while the **introduction of four digit industry effects** reduces the sum of squared errors **very considerably**. We display the magnitude of these reductions, due to the introduction of four digit industry effects, in Table 3 below.

TABLE 3		
	Reduction in SSE	
Industry	C.D.	Translog
35	58%	53%
36	34%	31%

38	14%	13%

The preceding represents a "new finding" in the sense that this point has not been made in the literature, and raises a number of issues regarding productivity measurements, the most important of which is whether what is called "Total Factor Productivity", or in earlier times "Technical Change" is, largely or partly, an aggregation phenomenon that has little to do with technical improvements or "productivity" in their very basic meaning.

Returns to Scale: Value Added

We discuss here the same issues as above, for the case where output is defined by Value Added. The pertinent results are given in Tables A4 through A6 in the Appendix. By and large the results are basically those established in the previous case, except that now the returns to scale (point) estimate is somewhat higher, and slightly more uniform across industries, as is clear from the table below.

TABLE 4

Industry	Returns to Scale C.D.	Translog
35	1.042	1.040
36	1.029	1.039
38	1.020	1.020

Generally, the returns to scale parameter estimates here are of about the same magnitude as with Gross Shipments, with the possible exception of Industry 35.

Other Issues: Value Added

As in the previous case, the hypothesis that the "extra" terms of the Translog function have null coefficients is uniformly rejected. Again, the introduction of time contrasts reduces the sum of squared errors (SSE) rather slightly, while the introduction of four digit industry contrasts has a more powerful effect. The reductions in SSE are slightly smaller than in the previous case. The relevant results are given in Table 5, below.

TABLE 5

Industry	Reduction in SSE C.D.	Reduction in SSE Translog
35	54%	48%
36	23%	22%
38	07%	07%

The results in Table 4, above, show that the introduction of (four digit) industry contrasts results in appreciable reduction in the sum of squared residuals for industries 35 and 36. For industry 38, however, the reduction is only slight, 7%. Finally, we note that the basic features of the Table are invariant to the production function specification, i.e., the entries under C.D. and Translog are nearly identical. We had noted a similar result, earlier, when we considered the case where output was defined in terms of Gross Value Added.

1.5 EMPIRICAL RESULTS; PRODUCTIVITY IMPLICATIONS

In the preceding sections we had assumed, in effect, parametric homogeneity across all plants in a given **two digit** industry and progressively relaxed that by allowing "time contrasts" and "(four digit) industry contrasts". Either procedure, allows the scale constant for the production function to be different over time, or across four digit classifications. In the relatively long history of productivity studies, **variation over time** has, more or less, been the basis of productivity comparisons. Often, the departure of observed output from the (estimated or hypothesized) specification of inputs has been termed "technical change"; in many instances the variation of this entity over time has been attributed to research and development expenditures, or other manifestations of the change in the applicable technology, such as the number of patents issued, perhaps in some specified field, over a given time period. In equally as many, or perhaps in even more numerous studies, this entity has been "explained" by time; see, for example Solow (1957) or Dhrymes (1961).

In our study we have a unique opportunity to examine several facets of this problem, owing to the particularly rich data base available to us. We begin the **initial exploration** of this topic herein and reserve the study of several other issues for the subsequent papers.

One important question that is often asked in cross section studies, is whether all entities follow the best industry practice and, if not, whether we can isolate those that are "most efficient", "average" or "least efficient". In this context, this may be translated as: can we find a classification of plants into those that exhibit least TFP, average TFP and those that exhibit most TFP. A corollary question is: is productivity (TFP or residual) growing over time? Finally, how much difference does it make in the **measurement of productivity**, if the approach is completely econometric, as in Dhrymes (1961), or is only **partially econometric** as in Solow (1957), and most of the work currently carried out. The latter approach, which we term "Solow Residual" in the graphs of the Appendix, is the ratio of observed output to a geometric (weighted) mean of the inputs, the weights being the observed shares accruing to the enumerated factors of production. In a variation of this basic approach, the weights are chosen as a Divisia Index of the shares over two periods. In the econometric approach, TFP is defined as the ratio of observed output and $\bar{F}(K, L_1, L_2)$, the latter being the **estimated production function** in terms of the inputs. Generally, we deal with the logarithm of this entity.

As we have observed in an earlier section, in order to make the two "residuals" have the same interpretation, we can either obtain TFP in the econometric procedure from the version that has **neither time nor industry** contrasts, or we can simply "regress" the Solow residual on time and (four digit) industry dummies, and use the residual of that regression as a measure of TFP. The graphs reflect this last approach.

1.5.1 Contemporaneous Rank

Graphs A1 through A6, in the Appendix, contain the course of (logarithmic) mean TFP by decile. More precisely, what is done is as follows: having determined the TFP corresponding to a given plant we rank plants in accordance with the magnitude of their TFP, **in each year**. What is plotted on the graph, then, is the logarithm of the geometric mean of TFP or, equivalently, the mean of the logarithm of the TFP of the plants in a given decile; evidently, the lowest graph corresponds to the first decile; the next corresponds to the second decile and so on. Three remarkable features emerge:

i. the qualitative aspects of productivity behavior are **almost completely independent** of the underlying production function specification, i.e., it makes little difference whether the TFP of plants is determined as a residual from a Cobb-Douglas or a Translog production function.

ii. The time profile of productivity **for deciles three through eight is remarkably flat.** One might interject that, perhaps, this was to be expected since we may well have removed any upward time trend by introducing the **time contrasts** in the estimation of the underlying production relation. This, however, cannot be argued very cogently since, as we had seen earlier, these time contrasts are only marginally significant and reduce the sum of squared errors by relatively small magnitudes.

iii. The first and last two (first, second, ninth and tenth) deciles, vary considerably over the 15-year period. Thus, in industry 35, mean TFP for the first decile rises considerably and that for the second declines somewhat so that the difference between them, which is large at the beginning of the period is considerably reduced by the end of the period. For industries 36 and 38, however, the first decile profile shows a decline, as does the second; the difference between them remains fairly constant or declines somewhat.

     The ninth and tenth deciles exhibit a rising profile, the ninth only slightly, the tenth very appreciably, so that by the end of the period the difference between the two shows a very substantial increase.

From previous results appearing in the literature one would have expected a stationary or slightly declining productivity in the middle to late seventies, and substantial growth following the 1980-81 recession. What we find, by contrast, is the essential absence of relatively significant dynamic shifts in productivity behavior, at least from the three two digit industries under consideration, over the years 1972-1986. The major upward shifts are confined to the upper decile and the major downshifts are confined to the first decile; this is hardly a result that supports the hypothesis of vigorous technical change or productivity growth.

<u>1.5.2 First Year Rank</u>

Since, in the classification scheme of the previous section, the identity of the plants in each decile is constantly changing, we also examined the behavior of mean productivity by deciles, **when the classification of plants is based solely on their rank in their initial year**. To be precise, what is done is to rank plants according to the magnitude of their TFP in 1972; thereafter plants keep this rank, so that, e.g., the entry for the first decile in 1974, is the (logarithmic) mean of TFP, for plants that were ranked in the **first decile in 1972**. These results appear in Graphs A7 through A12. Their salient features are:

i. the results are qualitative quite similar whether derived from the Cobb-Douglas or the Translog residuals;

ii. in industry 35, plants in the first and second decile (as of 19720 exhibit dramatic growth in productivity in subsequent years, and in the 80's they dominate other plants in terms of TFP. This suggests that such plants must have something in common, such as e.g., their SIC four digit classification, or substantial investment in modernization; plants in other deciles tend to become very closely bunched, indicating increasing similarity in their TFP behavior.

iii. For industry 36, plants in the first decile exhibit enormous growth, but also enormous fluctuations in their TFP behavior; to a lesser degree, the same is true for plants in the second decile. Plants in the second decile exhibit less vigorous, **but fairly steady growth**. The remaining plants exhibit the same compression in their TFP growth as those in industry 35, although they generally tend to keep their original ranking. These results are rather intriguing and require further investigation.

iv. In industry 38, we find increased "entropy", in that the time profile of the first and second deciles is similar to what has been observed in industries 35 and 36; the paths of the other deciles, however, cross much more frequently. Thus, what we find is that the relative placidity of productivity behavior is replaced by considerable dynamic movements of plants in their TFP characteristics. In turn, this suggests that there is a potentially interesting research problem in studying the transition of plants into and out of various TFP classifications.

1.5.3 The Solow Residual

Contemporaneous Rank

The time profile of Solow residuals, by contemporaneous rank, is given by deciles, in Graphs A13 through A18, first in the manner usually presented in the literature and thereafter by removing "time effects" and "four digit industry" effects. Precisely, the

Solow residual is regressed on "time dummies" and "four digit industry dummies" and the **residuals from that regression are taken to be the measure of TFP.** This last measure is the one most comparable with the results obtained through the econometric approach. Their graphs are labelled "Solow Regression". The corresponding TFP will be referred to below as SR TFP. Several aspects of these graphs are worth noting.

i. In the graphs labelled "Solow Residual, Sorted", where the time and four digit industry effects are not removed, mean TFP by decile is substantially higher than is the case for the econometrically derived results, where such effects had been removed.

ii. When such effects are removed, the SF TFP profiles are quite similar to those obtained earlier with Cobb-Douglas and Translog production functions, except that SR TFP is **smoother**. This is generally the consequence of using a great deal more parameters in obtaining SR TFP in the sense that, with the econometric approach, we are using a limited number of share parameters; four or three in the case of the Cobb-Douglas, and nine or fourteen in the case of the Translog function. By contrast, in the SR context we may use upwards of 34,000 **independent** share parameters for industries 35 and 36, and upwards of 10,000 parameters in the case of industry 38. Evidently, in the econometric approach we can also increase the number of share parameters by simply allowing different

Cobb Douglas exponents in each year. The question then is: to the extent that the change in the parametric structure leads to different measures of productivity, have we submerged some aspect of "technical change induced TFP" under another category? Or have we attributed to TFP something that is the result of inefficient handling of data?

Since the issue of productivity measures is imbedded in the production technology literature and purports to measure the extend to which "technical change" or other "improvements in technique enhance the productivity" or the factors of production, it is more appropriate to employ the econometrically based approach to productivity measurement. In that context, it may be said that the Solow residual approach gives a **misleadingly smooth** representation to the phenomenon under study.

Initial Rank

In Graphs A19 through A24 we give the time profile of the Solow residual measure of TFP with initial rank designation of plants. The major features are as follows.

i. A general characteristic running through all graphs is that there are fewer crossovers than is the case with econometrically derived TFP. In industry 35, the tenth decile exhibits considerable growth; this growth, however, almost completely disappears when time and four digit industry effects are removed. Since, generally, time effects are quite weak, it would appear that this phenomenon is largely illusory and simply reflects the four digit industry composition of that decile. When the composition effects are removed in Graph A20, we observe the same phenomenon as in the previous section, viz., the strong TFP growth of first decile plants. As we also remarked above, SR TFP is **more smooth** than econometrically derived TFP.

ii. In industry 35, SR TFP shows the consistent decline in the tenth decile and appreciable growth in the first decile, as noted earlier. In fact, the first three deciles exhibit very similar SR TFP in the eighties.

iii. In industry 38, we see the same phenomenon noted above, viz., increased entropy, although the frequency of crossovers is appreciable smaller than in the case of econometrically derived TFP, reflecting the **smoothness** of the Solow residual approach.

1.6 CONCLUSIONS

In this paper we sought to complete the objectives set in Dhrymes (1990) by investigating, at the two digit industry level, the compatibility of estimates of the parametric structure of a given specification. This is attained by **exploiting all implications of that specification**. The results were that none of the popular specifications, such as the Cobb-Douglas, the Translog, or the CES, production functions have a clear advantage over the others. In fact, all of them show great incompatibility. This is vexing and raises grave doubts regarding the theoretical foundations of production studies, either in the specification of technology or in the specification of the institutional milieu in which production is carried out. Since this was extensively documented at the three digit product group level in Dhrymes (1990) we have not reported the results in this paper. Instead, we focused our attention increasingly on issues of returns to scale and productivity measurement. A number of findings stand out.

i. There are mildly increasing returns to scale at the two digit level, at least in the case of industries 35,3 6, and 38.

ii. The translog specification is a slightly preferable specification, in the sense that (some of) the non (log)linear terms (may) have nonzero coefficients. On the other hand, all results of interest such as returns to scale, aggregation effects, or productivity measurements, do not seem to be appreciably affected by the specification of the production process. This would argue, in terms of the principle of simplicity, that Cobb-Douglas should be the production specification of choice, despite the great econometric attraction of the Translog.

iii. Allowing for "time" effects improves the fit very slightly, while allowing for (four digit) "industry" effects improves the fit very substantially.

iv. The results above are valid whether output is defined by Gross Shipments or Value Added.

In taking up issues relative to the measurement of productivity, we have relied exclusively on results obtained from the Value Added formulation. The salient conclusions in the phase of the study are:

i. TFP, interpreted as the (within sample) residual of observed output and the estimated relation, is qualitatively almost identical whether computed on the basis of the Cobb-Douglas or the Translog specification. We have followed the practice of computing TFP on the basis of the production function specification that includes "time" and (four digit) "industry" contrasts. While this particular version may evoke some objections, we note that even when these contrasts are suppressed the results do not change very substantially.

ii. Ranking plants according to TFP, in each year, and graphing the mean TFP by decile, i.e., the mean TFP of plants in the first decile, per year; the second decile, per year, and so on, gives the general impression that aside from the first and tenth decile, the time profile of the other deciles is rather flat. This suggests that it is only at the very bottom and at the very top of the productivity scale that "growth" occurs, and that the growth in question is rather slight.

iii. In the observations made under ii. above, it should be noted that the **identity of the plants** within the various deciles is, at least in principle **constantly shifting**. To gain a different view of the process, we rank plants by their TFP in the **initial year**, 1972, and thereafter follow these same plants in subsequent years. In this framework, quite a different behavior emerges. Dynamic upward growth characterizes some groups of plants, while for others we observe dramatic declines, generally the first decile experiencing the most dynamic movement **upwards** and the tenth decile the most dynamic movement **downwards.**

The findings in this paper lead to a number of questions and suggest a number of topics for further research.

i. what is the nature of the transition process, i.e., the manner in which plants move from one decile (or other classificatory scheme) to another; is it completely random or are there certain commonalities? Are there distinct characteristics for plants that make frequent transitions and those that don't? Indeed, are there stationary plants? Do plants that leave the sample tend to be those with high, medium, or low productivity?

ii. Should productivity include "time" effects and aggregation or "compositional" effects, i.e., should what we wish to call "productivity" consist of the contribution to output of things other than the specified labor and/or capital inputs, or should it be **net** or predictable compositional and/or "time" effects?

iii. Can we model the transition process, econometrically, and determine what factors are most potent in effectuating transition to higher or lower states of TFP, or a stationary status?

iv. Does TFP, as defined by the Solow residual method, given unwarrentedly "smooth" time profiles of the phenomenon?

These issues are reserved for later investigation.

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APPENDIX