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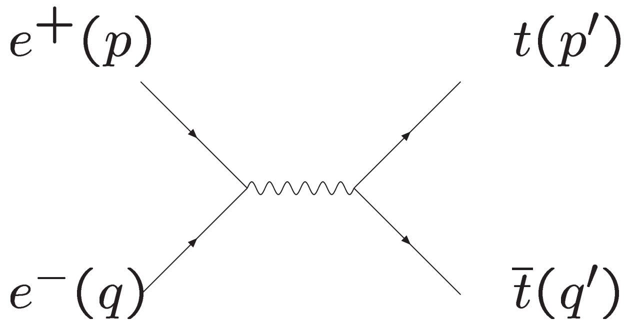
Top Physics, #2:

Top as QCD Laboratory

- $e^+e^- \to t\bar{t}$
  - QCD NLO corrections
  - Threshold behavior

- $e^+e^-, pp \to t\bar{t}h$

$e^+e^- \to t\bar{t}$



Consider only γ exchange:

$$\mathcal{A}_{LO}^{\gamma} = \frac{e^2 Q_e Q_t}{s} \overline{v}(q)\gamma^{\mu} u(p) \overline{u}(p')\gamma_{\mu} v(q')$$

Center-of-mass frame:

$$\begin{aligned} p &= \frac{\sqrt{s}}{2}(1,0,0,1) \\ q &= \frac{\sqrt{s}}{2}(1,0,0,-1) \\ p' &= \frac{\sqrt{s}}{2}(1,0,\beta\sin\theta,\beta\cos\theta) \\ q' &= \frac{\sqrt{s}}{2}(1,0,-\beta\sin\theta,-\beta\cos\theta) \end{aligned}$$

- $\beta = \sqrt{1-\frac{4m_t^2}{s}}$; $\beta = 0$ at threshold

Tree level amplitude-squared:

$$|\mathcal{A}_{LO}|^2 = 64\pi^2\alpha^2 Q_t^2[2+\beta^2(\cos\theta^2-1)]$$

Total cross section:

$$\sigma_{LO} = \frac{1}{2s}|\mathcal{A}_{LO}|^2\,(dPS)_2$$

Phase space in $N = 4-2\epsilon$:

$$(dPS)_2 = \frac{2^{2\epsilon}}{16\pi}\left(\frac{4\pi}{s}\right)^{\epsilon}\beta^{1-2\epsilon}\frac{1}{\Gamma(1-\epsilon)}\int_0^{\pi}\sin^{1-2\epsilon}\theta d\theta$$

$$\begin{aligned}\sigma_{LO} &= \frac{\beta}{32\pi s}\int d\cos\theta\,|\mathcal{A}|^2 \\ &= \frac{2\pi\alpha^2 Q_t^2}{3s}\beta(3-\beta^2)\end{aligned}$$

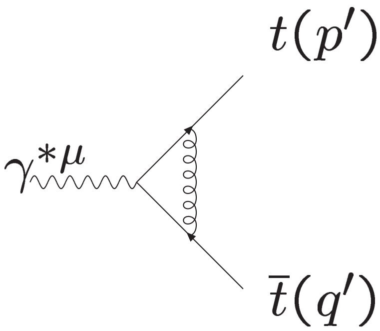
- σ vanishes at threshold, $\sigma \sim \frac{1}{s}$

$\mathcal{O}(\alpha_s)$ corrections to $e^+e^- \to t\bar{t}$

Virtual + real contributions at $\mathcal{O}(\alpha_s)$

$$\sigma_{NLO} = \sigma_{LO} + \sigma_{virtual} + \sigma_{real}$$

- Vertex correction:



$$\begin{aligned}\Gamma_\mu &= (-ieQ_t)\delta_{ij}g_s^2 C_F[N] \\ &\cdot \overline{u}(p')[A\frac{(p'+q')_\mu}{2m_t} + B\gamma_\mu]u(q') \\ N &= \frac{1}{16\pi^2}(\frac{4\pi\mu^2}{M_t^2})^\epsilon \Gamma(1+\epsilon)\end{aligned}$$

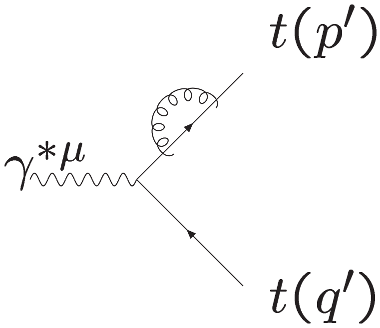
$$
\begin{aligned}
A &= (\beta - \frac{1}{\beta})\Lambda_\beta \\
\Lambda_\beta &= \log(\frac{1-\beta}{1+\beta}) \\
B &= \frac{1}{\epsilon}[1 - (\frac{1}{\beta} + \beta)\Lambda_\beta] - 3\beta\Lambda_\beta \\
&\quad + (\frac{1}{\beta} + \beta)(-\frac{1}{2}\Lambda_\beta^2 + 2\Lambda_\beta \log(\frac{2\beta}{1+\beta}) \\
&\quad + 2Li_2(\frac{1-\beta}{1+\beta}) + \frac{2\pi^2}{3}) \\
&\equiv \frac{B_\epsilon}{\epsilon} + B_{fin}
\end{aligned}
$$

Virtual cross section is interference with lowest order:

$$
\begin{aligned}
\sigma_{vertex} &= 2\int (dPS)_2 \mathcal{A}_{LO}\mathcal{A}^*_{vertex} \\
&= \frac{\alpha_s}{2\pi}(\frac{4\pi\mu^2}{m_t^2})^\epsilon C_F \sigma_{LO}[\frac{B_\epsilon}{\epsilon} + A + B_{fin}]
\end{aligned}
$$

Add wavefunction renormalization for top:

$\delta Z_{tt} = \frac{\partial \Pi(p^2)}{\partial p^2} \mid p^2 = m_t^2$



$$\delta Z_{tt} = -\frac{\alpha_s}{4\pi}\left(\frac{4\pi\mu^2}{m_t^2}\right)^\epsilon C_F \Gamma(1+\epsilon)\left(\frac{3}{\epsilon}+4\right)$$

This contributes $\sigma_Z = 2\delta Z_{tt}\sigma_{LO}$

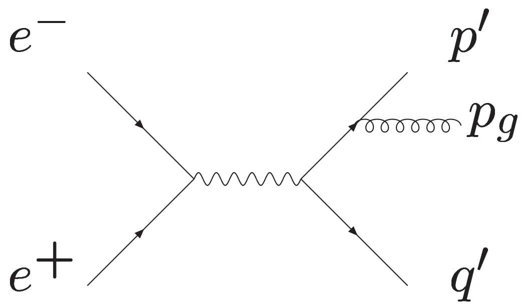
Total virtual contribution is :

$$\begin{aligned}\sigma_{VIRT} &= \sigma_{vertex} + \sigma_Z \\ &= \sigma_{LO}\frac{\alpha_s}{2\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu^2}{M_t^2}\right)^\epsilon [(-3+B_\epsilon)\frac{1}{\epsilon} \\ &\quad -4+A+B_{fin}]\end{aligned}$$

[Jersak, Laermann, Zerwas, PRD25 (1982) 1218]

- Virtual singularities cancelled by real gluon emission:

$$e^+e^- \to t\bar{t}g$$



$$\frac{1}{(p'+p_g)^2 - m_t^2} = \frac{1}{2p' \cdot p_g} = \frac{1}{2E'E_g(1-\beta\cos\widehat{\theta})}$$

- m_t regulates collinear singularity

- Soft singularity, $E_g \to 0$

Phase Space Slicing

Divide gluon phase space into soft+ hard regions: $\sigma_{real} = \sigma_{soft} + \sigma_{hard}$

Soft region:

$$E_g < \delta_s \frac{\sqrt{s}}{2}$$

Hard region:

$$E_g > \delta_s \frac{\sqrt{s}}{2}$$

σ_{hard} finite and evaluated numerically

[MADGRAPH, for example]

σ_{soft} and σ_{hard} depend on cutoff, δ_s

Independence of sum is check on calculation [Baer, PRD40 (1989) 2844]

Three body phase space in soft limit:

Work in rest frame of incoming e^+e^-

$$\begin{aligned} p+q &= p'+q'+p_g \\ E_g &= (\frac{s-s_{p'q'}}{2\sqrt{s}}) \\ s_{p'q'} &= (p'+q')^2 \end{aligned}$$

Separate 3-body phase space into 2 body phase space times soft phase space:

$$\begin{aligned} (dPS)_3 &= \int \frac{d^{n-1}p'}{2E_{p'}(2\pi)^{n-1}} \int \frac{d^{n-1}q'}{2E_{q'}(2\pi)^{n-1}} \\ &\cdot (2\pi)^n \delta^n(p+q-p'-q'-p_g) \\ &\cdot \int \frac{d^{n-1}p_g}{2E_g(2\pi)^{n-1}} \end{aligned}$$

For soft limit, set $p_g = 0$ in delta function:

$$(dPS)_3(soft) = (dPS)_2 \int \frac{d^{n-1}p_g}{2E_g(2\pi)^{n-1}}$$

In $p+q$ rest frame:

$$p_g = E_g(1, ..., \sin\theta_1 \sin\theta_2, \sin\theta_1 \cos\theta_2, \cos\theta_1)$$

$$\begin{aligned} d^{n-1}p_g &= d \mid \vec{p}_g \mid\mid \vec{p}_g \mid^{n-2} d\Omega_{n-2} \\ &= dE_g E_g^{n-2} \sin^{n-3}\theta_1 d\theta_1 \\ &\quad \cdot \sin^{n-4}\theta_2 d\theta_2 d\Omega_{n-4} \end{aligned}$$

Three body phase space in soft limit:

$$\begin{aligned} \Omega_{n-4} &= \frac{2\pi^{(n-3)/2}}{\Gamma(\frac{n-3}{2})} \\ (dPS)_3(soft) &= (dPS)_2(\frac{4\pi}{s})^\epsilon \frac{\Gamma(1-\epsilon)}{8\pi^2\Gamma(1-2\epsilon)} dPS_{soft} \\ dPS_{soft} &= \frac{1}{\pi}(\frac{s}{4})^\epsilon \int_0^{\delta_s \frac{\sqrt{s}}{2}} dE_g E_g^{1-2\epsilon} \\ &\quad \cdot \sin^{1-2\epsilon}\theta_1 d\theta_1 \sin^{-2\epsilon}\theta_2 d\theta_2 \end{aligned}$$

[Sterman, Introduction to QFT]

$e^+e^- \to t\bar{t}g$ in $p_g \to 0$ limit:

$$
\begin{aligned}
i\mathcal{A}_{soft} &= \overline{u}(p')(-ig_sT^A\gamma^\sigma)\frac{i(p' + p_g + m_t)}{(p'+p_g)^2 - m_t^2}\ldots\ldots \\
&= g_sT^A[\frac{p'^\sigma}{p'\cdot p_g} - \frac{q'^\sigma}{q'\cdot p_g}]\mathcal{A}_{LO}
\end{aligned}
$$

- Eikonal (double pole) Approximation
- Soft limit always works this way

$$
\begin{aligned}
\mid \mathcal{A}_{soft} \mid^2 &= g_s^2 Tr(T^AT^A) \mid \mathcal{A}_{LO} \mid^2 \\
&\quad [-\frac{m_t^2}{(p'\cdot p_g)^2} - \frac{m_t^2}{(q'\cdot p_g)^2} + \frac{s - 2m_t^2}{p'\cdot p_g q'\cdot p_g}]
\end{aligned}
$$

Problem reduces to evaluating soft integrals:

Typical Integral:

Choose frame

$$
\begin{aligned}
p_g &= E_g(1, ... \sin\theta_1 \sin\theta_2, \sin\theta_1 \cos\theta_2, \cos\theta_1) \\
p' &= \frac{\sqrt{s}}{2}(1, 0, 0, \beta)
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{1}{(p' \cdot p_g)^2} dPS_{soft} &= \frac{1}{\pi}\left(\frac{s}{4}\right)^\epsilon \int_0^{\delta_s \frac{\sqrt{s}}{2}} dE_g E_g^{1-2\epsilon} \\
&\quad \cdot \sin^{1-2\epsilon}\theta_1 d\theta_1 \sin^{-2\epsilon} d\theta_2 \\
&\quad \cdot \left(\frac{4}{sE_g^2}\right) \frac{1}{(1-\beta\cos\theta_1)^2} \\
&= \frac{2}{m_t^2}\left(-\frac{1}{2\epsilon} + \log(\delta_s) + \frac{1}{2\beta}\Lambda_\beta\right)
\end{aligned}
$$

Drop terms $\mathcal{O}(\delta_s)$

[See Reina, Dawson, Wackeroth, hep-ph/0109066]

$$\sigma_{soft} = \sigma_{LO}\frac{\alpha_s}{2\pi}C_F(\frac{4\pi\mu^2}{s})^\epsilon[\frac{C_\epsilon}{\epsilon}+C_{fin}]$$

- $B_\epsilon = -C_\epsilon$ so total answer is finite
- C_{fin} depends on δ_s

$$\begin{aligned} C_{fin} &= -4\log(\delta_s) - \frac{2}{\beta}\Lambda_\beta - 2(\frac{1}{\beta}+\beta) \\ &\quad \cdot[Li_2(\frac{2\beta}{1+\beta}) + \frac{\Lambda_\beta^2}{4} + \log(\delta_s)\Lambda_\beta] \end{aligned}$$

$$\begin{aligned} \sigma_{vertex}+\sigma_{soft} &= \frac{\alpha_s}{2\pi}C_F\sigma_{LO}[(\frac{4\pi\mu^2}{m_t^2})^\epsilon\frac{B_\epsilon}{\epsilon} \\ &\quad +(\frac{4\pi\mu^2}{s})^\epsilon\frac{C_\epsilon}{\epsilon}+\ldots] \end{aligned}$$

μ is arbitrary renormalization scale

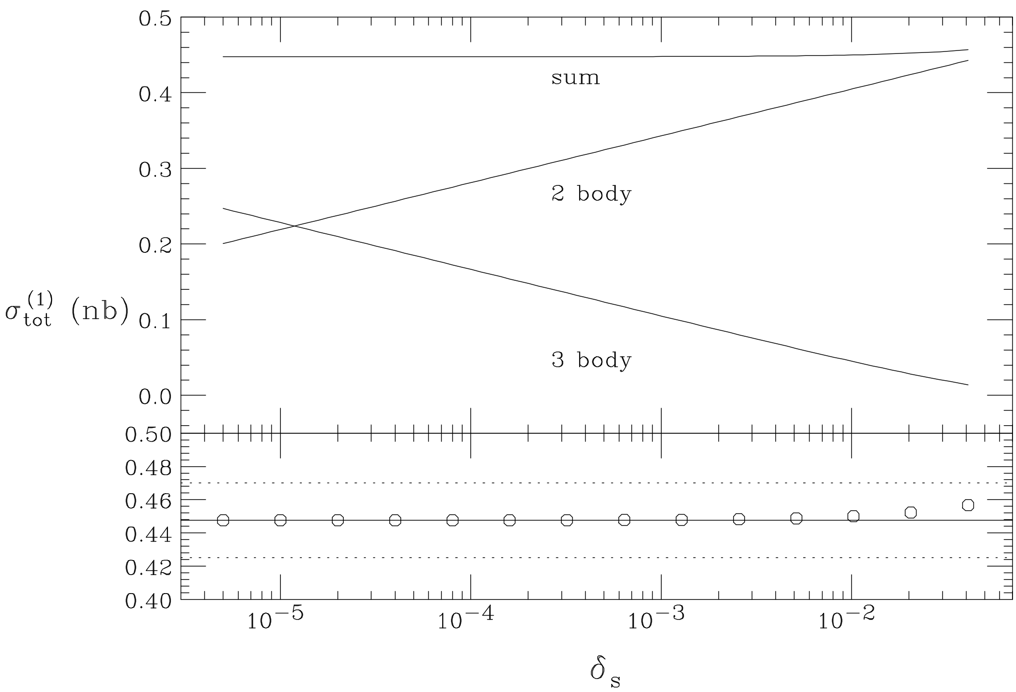
Final answer:

$$\sigma_{NLO} = \sigma_{LO}+\sigma_{vertex}+\sigma_{soft}+\sigma_{hard}$$

- Could have done this example analytically

- Phase space slicing useful for more complicated examples

- PSS useful to construct Monte Carlo distributions



$(\sqrt{s} = 11\ GeV, m = m_b = 5\ GeV)$

[Harris and Owens, hep-ph/0102128]

Threshold Scan in $e^+e^- \to t\bar{t}$

- Location of peak depends on m_t, height of peak on Γ_t

- Problems: Peak moves around at higher orders, large renormalization scale dependence

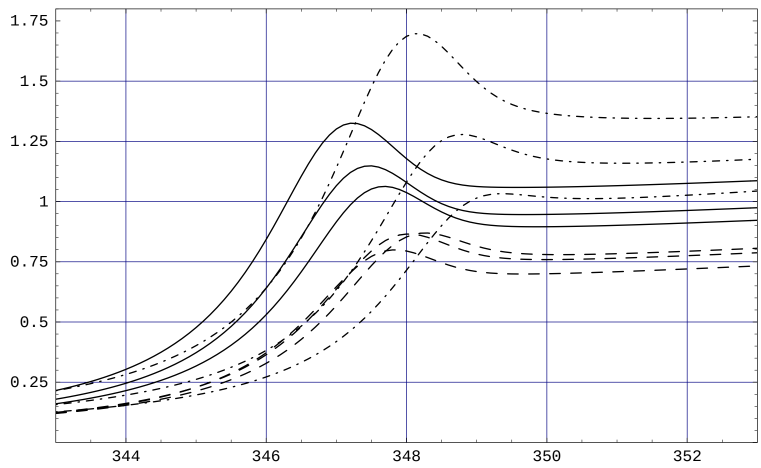
- Two small parameters: α_s, β

$$\sigma(\beta \to 0) = \sigma_{LO}(1 + \frac{4\pi\alpha_s}{9\beta})$$

$\longrightarrow$ Coulomb singularity

- Use non-relativistic quantum mechanics

Pole mass:



(LO: dot-dashed, NLO: dashed, NNLO, solid)

[Groote and Yakovlev, hep-ph/0012234]

Which m_t? Pole mass:

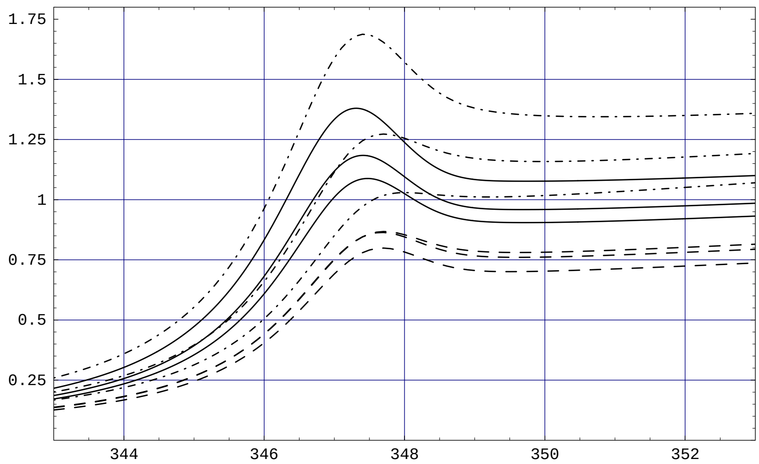
$$D(p) = \frac{i}{p - m_t\,|_{pole} - \Sigma(p)}$$

- $m_t\,|_{pole}$ measured by reconstructing top quark decay products
- $m_t\,|_{pole}$ uncertain by QCD hadronization effects, $\mathcal{O}(\Lambda_{QCD})$
- Define short distance mass:

$$m_{1S} = m_t\,|_{pole}\,(1 - \frac{2\alpha_s^2}{9} ....)$$

- σ peaks at $2m_{1S}$

Use $1S$ mass definition:



(LO: dot-dash, NLO: dash, NNLO,solid)
[Groote and Yakovlev]

NNLO:$\sim$ 20% scale uncertainty

$\longrightarrow \delta m_t \sim 200\ MeV, \quad \mathcal{L} = 50\ fb^{-1}$

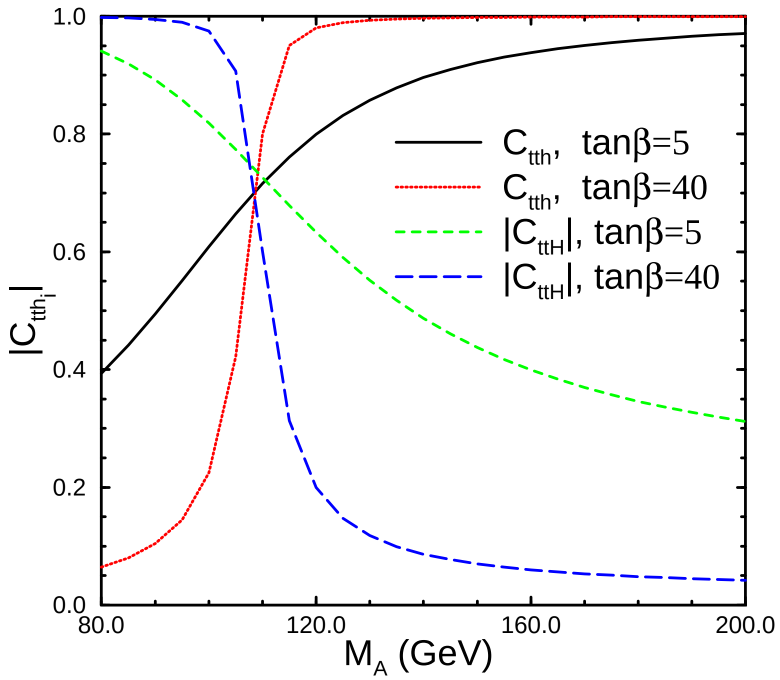
Compare LHC: $\delta m_t \sim 1 - 2\ GeV$

tth production in e^+e^- and pp collisions:

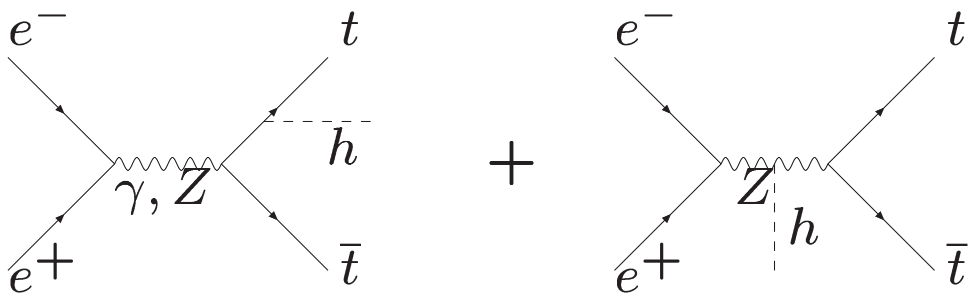
- Standard Model: $g_{tth}^{SM} = -\frac{M_t}{v}$

- g_{tth} can be very different in SUSY models:

$$g_{tth}^{SUSY} = C_{tth} g_{tth}^{SM}$$

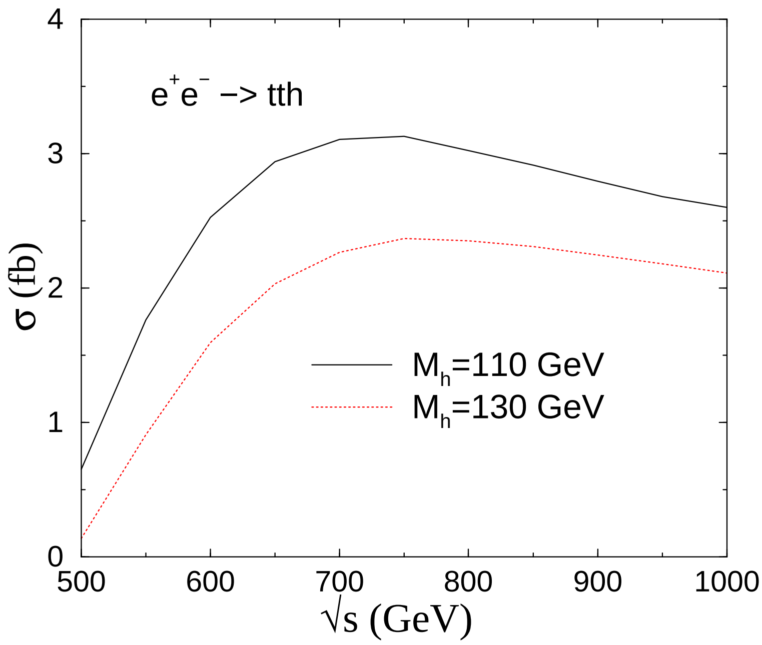


$t\bar{t}h$ at e^+e^- collider:



- Contribution from Z exchange few %

NLO results:



[Dawson and Reina, hep-ph/9808443; Dittmaier et al, hep-ph/9808433]

Top Yukawa coupling in e^+e^- collisions:

Combine hadronic and semi-leptonic W decay channels

- $\sqrt{s} = 800$ GeV , $\mathcal{L} = 1000$ fb^{-1}, $M_h = 120$ GeV

$$\frac{\delta g_{tth}}{g_{tth}} \sim 5.5\%$$

[Juste and Merino, hep-ph/9910301]

- $\sqrt{s} = 500$ GeV, $\mathcal{L} = 1000$ fb^{-1}, $M_h = 120$ GeV

$$\frac{\delta g_{tth}}{g_{tth}} \sim 20\%$$

(Decreased precision due to reduced rate)

[H. Baer, S. Dawson, and L. Reina, hep-ph/9906491]

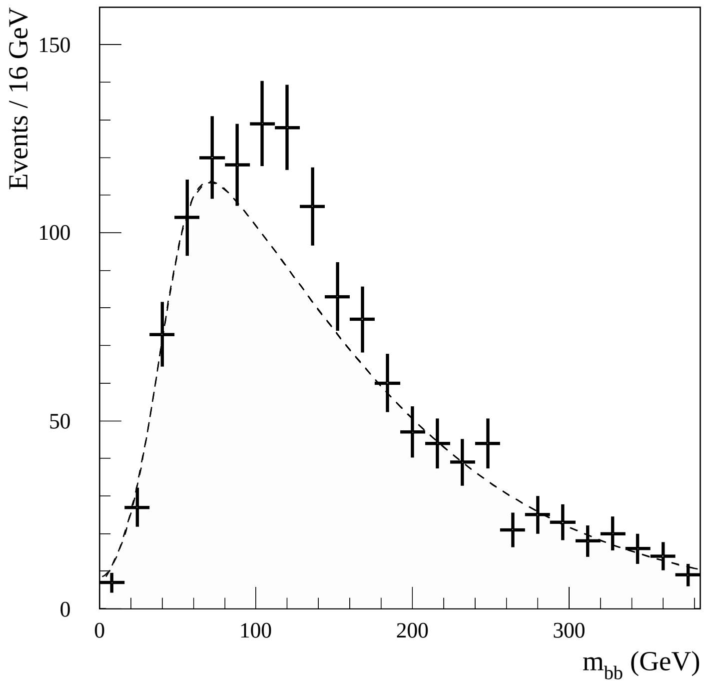
At LHC, $pp \to t\bar{t}h$

- Look for $h \to b\bar{b}$; final state is $W^+W^-b\bar{b}b\bar{b}$

ATLAS with $\mathcal{L} = 100\ fb^{-1}$, $M_h = 120\ GeV$

Background, dashed line; signal, data points

Can measure $\frac{\delta g_{tth}}{g_{tth}} \sim 16\%$



[LHC Top Study, hep-ph/0003033]

QCD corrections to $p\overline{p} \to t\bar{t}h$

At Tevatron, $q\overline{q} \to t\bar{t}h$ is dominant

At LHC, $gg \to t\bar{t}h$ is dominant

Ingredients of NLO calculation:

- $\sigma_{virtual}$:

  Finite/UV singularities evaluated with FF

  IR singularities evaluated analytically

- σ_{soft}:

  Initial and final state soft singularities

- σ_{hard} :

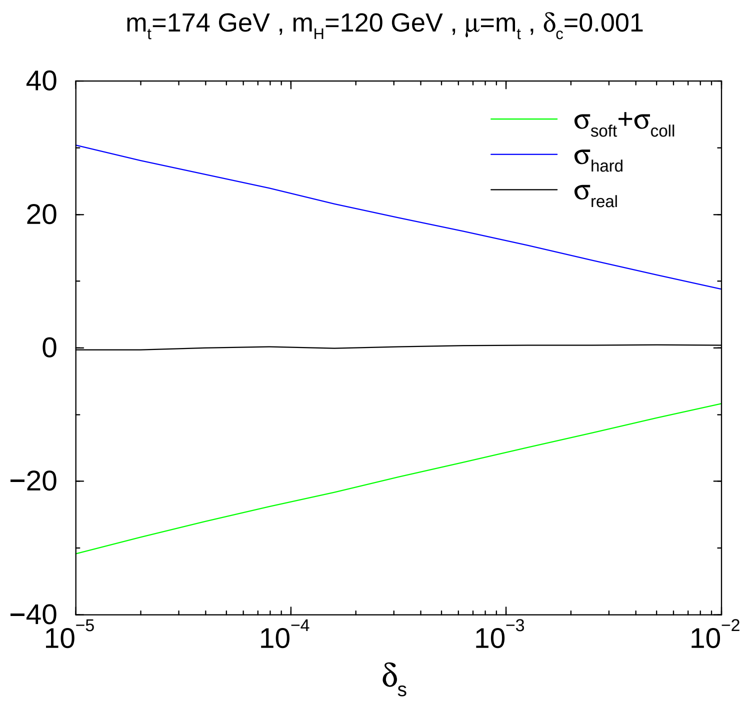
  Collinear singularities for massless partons

Follow the singularities:

$\sigma_{soft} + \sigma_{hard}($ final state collinear$) + \sigma_{virtual}$

$\longrightarrow$ finite

$$p\overline{p} \rightarrow t\bar{t}h \text{ at } \sqrt{s} = 2\ TeV$$



$\sigma_{hard}($ initial state collinear$)$

$\longrightarrow$ singularities absorbed into PDFs

$\sigma_{hard}($not collinear$)$

$\longrightarrow$ finite

Collinear Singularities

Phase Space Slicing

- As before, divide $q\overline{q} \to t\bar{t}hg$ into soft $+$ hard regions

- Further divide σ_{hard}

$$\sigma_{hard} = \sigma_{hard}\,|_{coll} + \sigma_{hard}\,|_{not-coll}$$

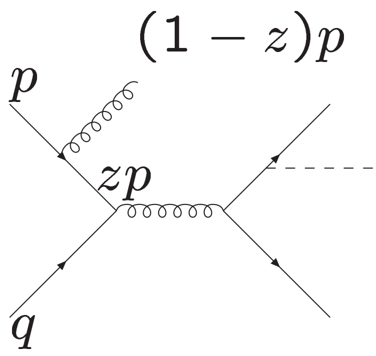
- Separation depends on new cut-off, δ_c

- Hard/not-collinear:

$$\frac{2p \cdot p_g}{E_g\sqrt{s}} > \delta_c, \quad \frac{2q \cdot p_g}{E_g\sqrt{s}} > \delta_c$$

Matrix element in collinear limit factorizes:

Parton $i \longrightarrow$ hard parton i' + collinear g:



$$
\begin{aligned}
p_{i'} &= zp_i \\
p_g &= (1-z)p_i \\
s_{ig} &= 2p_i \cdot p_g
\end{aligned}
$$

Current example, $q \to qg$

Leading pole approximation:

$$
\begin{aligned}
\mid \mathcal{A}(ij \to t\bar{t}h) \mid^2 &\to g_s^2 \mid \mathcal{A}_{LO}(i'j \to t\bar{t}h) \mid^2 \frac{2P_{ii'}(z)}{zs_{ig}} \\
P_{ii'}(z) &= P_{qq}(z) = C_F[\frac{1+z^2}{1-z} - \epsilon(1-z)]
\end{aligned}
$$

Phase space also factorizes in collinear limit:

$$(PS)_4(coll) \to (PS)_3(i'j \to t\bar{t}h)\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\frac{(4\pi)^\epsilon}{16\pi^2}$$
$$\cdot\, z dz ds_{ig}[(1-z)s_{ig}]^{-\epsilon}\theta(\frac{(1-z)}{z}p_{i'}\cdot p_j\delta_c - s_{ig})$$

[Giele and Glover, PRD 46 (1992) 1980]

Hard-collinear cross section:

$$\sigma_{hard/coll} = \frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}(\frac{4\pi\mu^2}{m_t^2})^\epsilon(-\frac{1}{\epsilon})\delta_c^{-\epsilon}$$
$$\cdot\,\{\int_0^{1-\delta_s} dz[\frac{(1-z)^2 s'}{2zm_t^2}]^{-\epsilon}P_{ii'}(z)\sigma_{LO}(i'j \to t\bar{t}h)$$
$$+\,(i \leftrightarrow j)\}$$

- Note δ_c, δ_s dependence
- $1-\delta_s$ limit on z integration excludes soft gluon region

Mass Factorization

The idea:

Lowest order hadronic cross section:

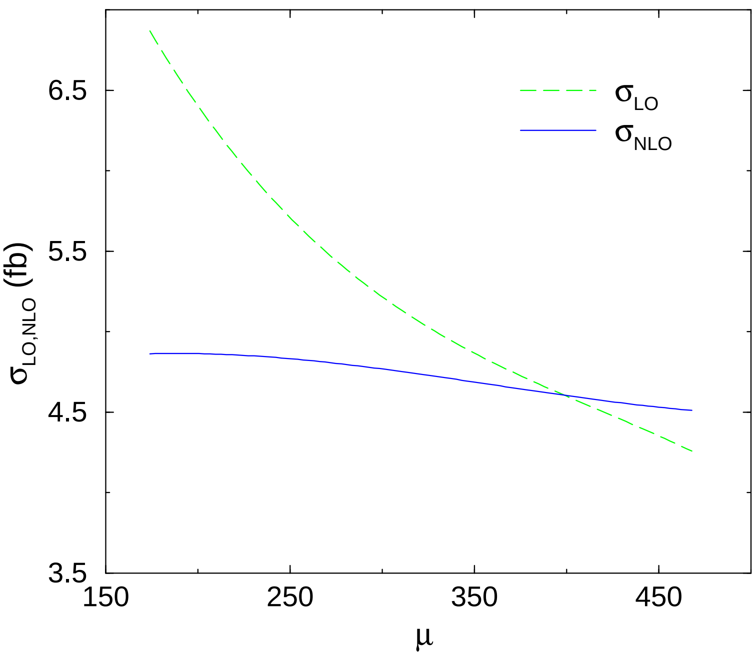
$$\sigma_H(S) = \Sigma_{ij} \int f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}(x_1 x_2 S)$$

- Define scale dependent PDFs to absorb hard/collinear singularity:

$$\begin{aligned} f_i(x,\mu) \equiv\ & f_i(x) + (-\frac{1}{\epsilon})\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}(\frac{4\pi\mu^2}{\mu_f^2})^{\epsilon} \\ & \cdot\{\int_z^1 \frac{dz}{z} P_{ii'}(z) f_{i'}(\frac{x}{z}) + C_F(2\log(\delta_s) + \frac{3}{2})\} \end{aligned}$$

This definition of PDFs exactly absorbs singularities from initial state collinear gluons

$p\bar{p} \to t\bar{t}h$ at Tevatron:



- Note reduced μ dependence of NLO rate

- $K = \frac{\sigma_{NLO}}{\sigma_{LO}}$ has no physical significance

[Reina, Dawson, hep-ph/0107101; Beenakker etal, hep-ph/0107081]

Conclusions:

Well defined procedure for QCD corrections

Virtual + Real + Mass factorization

$\longrightarrow$ finite result

New Physics/QCD effects often similar

Must understand QCD!