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ABSTRACT

A $k - \epsilon$ model is proposed for turbulent wall bounded flows. In this model, the turbulent velocity scale and turbulent time scale are used to define the eddy viscosity. The time scale is shown to be bounded from below by the Kolmogorov time scale. The dissipation equation is reformulated using this time scale, removing the need to introduce the pseudo-dissipation. A damping function is chosen such that the shear stress satisfies the near wall asymptotic behavior. The model constants used are the same as the model constants in the commonly used high turbulent Reynolds number k-ϵ model. Thus, when it is far away from the wall, the proposed model reduces to the standard k-ϵ model. Fully developed turbulent channel flows and turbulent boundary layer flows over a flat plate at various Reynolds numbers are used to validate the model. The model predictions are found to be in good agreement with the DNS data.

INTRODUCTION

Because of a wide range of scales involved in a turbulent flow, DNS (direct numerical simulation) is limited to flows of moderate Reynolds number and simple geometry. Turbulence modeling is the only viable approach for the calculation of turbulent flows of engineering interest. In turbulence modeling, k-ϵ model is the most used model in engineering calculations and model constants have been computationally optimized, giving what is called the Standard Model (e.g., Spalding and Launder [1], Rodi [2]). The Standard Model is devised for high turbulent Reynolds number flows and is traditionally used in conjunction with a wall function when it is applied to wall bounded turbulent flows. Since universal wall functions do not exist in complicated flow situations, it is necessary to develop a form of the $k - \epsilon$ model such that the equations can be integrated directly down to the wall.

Jones and Launder [3] made the first proposal for the low Reynolds number $k - \epsilon$ model and the equations were successfully integrated to the wall. A number of models have been proposed since then. A critical evaluation of the pre-1985 models was made in the paper by

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Patel et al. [4]. They pointed out that it is important for the damping function in the eddy viscosity to satisfy the near wall asymptotics. More recent models could be found in Shih [5]. Of the models proposed, a pseudo-dissipation is introduced near the wall. However, the definition of the pseudo-dissipation is quite arbitrary, except for the constrains on the wall and far away from the wall. In addition, the model constants used are different from the Standard Model, making the near wall models less capable of handling flows containing both high Reynolds number turbulence and near wall turbulence.

In the present paper, we try to provide remedies for these two shortcomings stated above. In section 2, we list the general form of the low Reynolds number $k - \epsilon$ models. We then propose in section 3 a new model. The proposed model is validated in section 4 for fully developed turbulent channel flows and turbulent boundary layer flows over a flat plate at different Reynolds numbers. Section 5 presents conclusions and possible future works.

EXISTING NEAR WALL $k - \epsilon$ MODELS

In turbulence modeling, the instantaneous quantities of an incompressible flow are decomposed into the mean and the fluctuations. i.e. $\tilde{u}_i = U_i + u_i$, $\tilde{p} = P + p$. The mean field U_i satisfies the following continuity equation and the Reynolds averaged Navier-Stokes equation:

$$U_{i,i} = 0 \tag{1}$$

$$\dot{U}_i + U_j U_{i,j} = -\frac{1}{\rho}P_i + \nu U_{i,jj} - <u_i u_j>_{,j} \tag{2}$$

where the Reynolds stress term, $- <u_i u_j>$, has to be modeled.

In an eddy viscosity model, the Reynolds stress is related to the mean field by

$$- <u_i u_j> = \nu_T(U_{i,j} + U_{j,i}) - \frac{2}{3}k\delta_{ij}, \tag{3}$$

where ν_T is the eddy viscosity and k is the turbulent kinetic energy. From a dimensional reasoning, the eddy viscosity is given by

$$\nu_T \sim u_t l_t,$$

where u_t and l_t are the turbulent velocity scale and turbulent length scale respectively.

In the framework of the $k - \epsilon$ model, $l_t \sim k^{3/2}/\epsilon$ and $u_t \sim k^{1/2}$. In the case of near wall turbulence, the above is modified by introducing a damping function f_μ and a pseudo-dissipations $\tilde{\epsilon}$. The general forms for the eddy viscosity and the transport equations for k and ϵ are (Patel et al. [4]),

$$\nu_T = C_\mu f_\mu \frac{k^2}{\tilde{\epsilon}} \tag{4}$$

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$$\dot{k} + U_j k_{,j} = [(\nu + \frac{\nu_T}{\sigma_k})k_{,j}]_{,j} - <u_i u_j> U_{i,j} - \epsilon + D \tag{5}$$

$$\dot{\epsilon} + U_j \epsilon_{,j} = [(\nu + \frac{\nu_T}{\sigma_\epsilon})\epsilon_{,j}]_{,j} + C_1 f_1 \frac{\epsilon}{k} \nu_T U_{i,j} U_{i,j} - C_2 f_2 \frac{\epsilon \bar{\epsilon}}{k} + E \tag{6}$$

where

$$\bar{\epsilon} = \epsilon - D$$

Various models use different sets of model constants c_μ, σ_k, σ_ϵ, c_1, c_2 and different damping functions f_μ, f_1, f_2 as well as different forms of D and E. Far away from the wall, the damping functions f_μ, f_1, f_2 approach to 1 and the near wall terms D, E are negligibly small so that the model equations become the high Reynolds number forms of $k - \epsilon$ model.

THE PROPOSED MODEL

The turbulent length scale is characterized by the size of the energy containing eddies. Near the wall, these eddies would have a size of $O(y)$. Following Patel et al [4], the turbulent velocity field has the following expansions near the wall:

$$\begin{aligned} u' &= u_1 y + u_2 y^2 + ... \\ v' &= v_2 y^2 + ... \\ w' &= w_1 y + w_2 y^2 + ... \end{aligned} \tag{7}$$

where u_1, v_2, w_1 are non-zero in general. Thus, as the wall is approached, both the turbulent length scale and the turbulent velocity scale approach zero. However, the turbulent time scale, which is given by the ratio of the scale of the energy containing eddies to the turbulent velocity scale, approaches to a non-zero value. We expect this time scale to be the Kolmogorov time scale because viscous dissipation is dominating near the wall. We thus have, for the turbulent time scale,

$$T_t = \frac{k}{\epsilon} + T_k \tag{8}$$

where

$$T_k = c_k (\frac{\nu}{\epsilon})^{1/2} \tag{9}$$

is the Kolmogorov time scale and c_k is a constant of order one.

Using this turbulent time scale and $k^{1/2}$ as turbulent velocity scale, the turbulent Reynolds number is

$$R_t = \frac{l_t u_t}{\nu} = \frac{k T_t}{\nu} \tag{10}$$

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and the eddy viscosity is

$$\nu_T = c_\mu f_\mu k T_t \tag{11}$$

The transport equation for k remains the same as eq (5) (with $D = 0$, since we are solving the real dissipation.) The transport equation for ϵ is

$$\dot\epsilon + U_j \epsilon_{,j} = [(\nu + \frac{\nu_T}{\sigma_\epsilon})\epsilon_{,j}]_{,j} + (C_1 \nu_T U_{i,j} U_{i,j} - C_2 f_2 \epsilon)\frac{1}{T_t} + E \tag{12}$$

where

$$f_2 = 1.0 - 0.22 \exp(-\frac{R_t^2}{36}) \tag{13}$$

$$E = \nu \nu_T U_{i,jk} U_{i,jk} \tag{14}$$

Since T_t is always a positive nonzero quantity, the above formulation removes the singularity of the high Reynolds number $k - \epsilon$ model when the wall is approached. f_2 and E are of the form used by Jones and Launder [3] for the low Reynolds number model, in which f_2 is used to capture the final decay of the homogeneous turbulence and E is used to model turbulence in the buffer layer.

Near the wall, the shear stress $- < uv >$ should behave as $O(y^3)$. Thus we would require a damping function which has a near wall behavior of $O(y)$. The damping function is assumed to be of the following form

$$f_\mu = 1.0 - \exp(a_1 y^+ + a_2 (y^+)^2 + a_3 (y^+)^3 + a_4 (y^+)^4) \tag{15}$$

with the constants are determined by calibrating with the DNS data for 2D channel flow at $Re_\tau = 180$. The calibration gives

$$a_1 = 4\text{x}10^{-3}, a_2 = 5\text{x}10^{-5}, a_3 = -2\text{x}10^{-6}, a_4 = 8\text{x}10^{-8}$$

The model constants used are the same as those in the Standard Model. i.e.

$$c_\mu = 0.09, C_1 = 1.44, C_2 = 1.92, \sigma_k = 1.0, \sigma_\epsilon = 1.3$$

Thus, far away from the wall, f_2, f_μ become 1, the Kolmogorov time scale is much smaller than k/ϵ, and E is much smaller than the other terms on the right hand of ϵ equation, our model would reduce to the standard $k - \epsilon$ model, ensuring the good performance of our model in the high turbulent Reynolds number limit.

MODEL VALIDATION

We use 2D fully developed channel flows and turbulent boundary layer flows to validate the proposed model. These flows are attractive for model testing, because they have self-similar solutions so that the initial conditions do not have to be accurately specified. These

flows are very simple, and solutions can be found very efficiently; yet, the effects of the wall on turbulent shear flow are still present. In addition, DNS data providing detailed flow information is available for these flows. We will compare the model predictions with the DNS data.

The boundary condition for ϵ on the wall is determined by applying k equation to the wall, which gives

$$\epsilon_w = \nu k_{,yy}. \tag{16}$$

In this study, the following boundary condition for ϵ, which is mathematically equivalent to the above but computationally much more robust, is used.

$$\epsilon_w = 2\nu(\frac{dk^{1/2}}{dy})^2 \tag{17}$$

An implicit finite difference scheme is used to solve the momentum equation and the transport equations for k and ϵ. The coefficients for the convective terms are lagged one step in the marching direction, and the source terms in the k and ϵ equations are linearized in such a way that numerical stability is ensured.

A variable grid spacing is used to resolved the sharp gradient near the wall. The grid distribution is controlled by $\delta y_i/\delta y_{i-1} = \alpha$. Both α and the total number of the grid, N, are varied to ensure the grid independence of the numerical results. The marching step size, δx, is also varied to ensure accuracy. It is found that $N = 150$, $\alpha = 1.03$, and $\delta x = 0.05$ are sufficient to give a solution with a less than 0.5% error.

Computations are carried out for 2D fully developed turbulent channel flows at $Re_\tau = 180$ and $Re_\tau = 395$. Fig. 1 shows the mean velocity profile for $Re_\tau = 180$, together with the DNS data of Kim et al [6]. In this figure, and in the following figures, both the dependent and the independent variables are normalized by u_τ and ν. The agreement between model prediction and DNS data is excellent. This may not be unexpected since the constants in the damping function are calibrated for this case. We then show the results for 2D channel flow at a different Reynolds number, $Re_\tau = 395$. Fig. 2 to Fig. 5 shows the profiles of the mean velocity, shear stress, turbulent kinetic energy and the dissipation, respectively. The predictions of Jones and Launder [3] and Chien [7] are also shown. These predictions are compared with the DNS statistics for this case (Mansour [8]). Overall, the proposed model gives a better prediction.

Predictions of the mean velocity, shear stress, turbulent kinetic energy and dissipation rate for turbulent boundary layer over a flat plate at $Re_\theta = 1410$ are shown in Fig. 6 to Fig. 9. The model predictions are compared with the DNS data of Spalart [9]. Similar computations are made for $Re_\theta = 670$, where the DNS data is also available (Spalart

[9]). The results for $Re_\theta = 670$ are not shown here, due to space limitation. The model predictions for the skin friction are shown in Fig. 10, together with the DNS data of Spalart [9] at $Re_\theta = 670$ and $Re_\theta = 1410$.

In the above computations, $c_k = 1.0$ is used. The constant c_k was varied in the range of 0.5 - 3.0 and the solutions were found to be quite insensitive to c_k in this range. As more flow situations are tested, the value of c_k could be optimized by fine tuning.

CONCLUSION

We have presented a $k - \epsilon$ model for wall bounded flows. We have shown that near the wall, the time scale is characterized by the Kolmogorov time scale rather than zero. By using this time scale and a turbulent velocity scale, the singularity in the standard dissipation equation is removed as the wall is approached and the equation can be integrated down to the wall. The use of this time scale also renders pseudo-dissipation unnecessary, which contains an arbitrary element in its definition.

The proposed model uses the same set of model constants as that used in the Standard Model, and when it is far away from the wall, the proposed model reduces to the Standard Model. Thus, the proposed model would be applicable in both the near wall turbulence and the high Reynolds number turbulence.

The damping function is chosen in such a way that the near wall asymptotics for the shear stress is satisfied, a feature which is very important in predicting the near wall properties (Patel et al [4]). The form of the damping function is assumed to be a function of y^+ with the constants calibrated with the DNS data for a particular case. This y^+ dependence of the damping function must be viewed as a drawback of the model, for y^+ is not clearly defined, or even does not exist for complex flows (corner flows, flows with separation, for example.) In addition, y^+ makes the modeling equation not invariant in coordinate transformations. Currently, effort is being made to remove these deficiencies.

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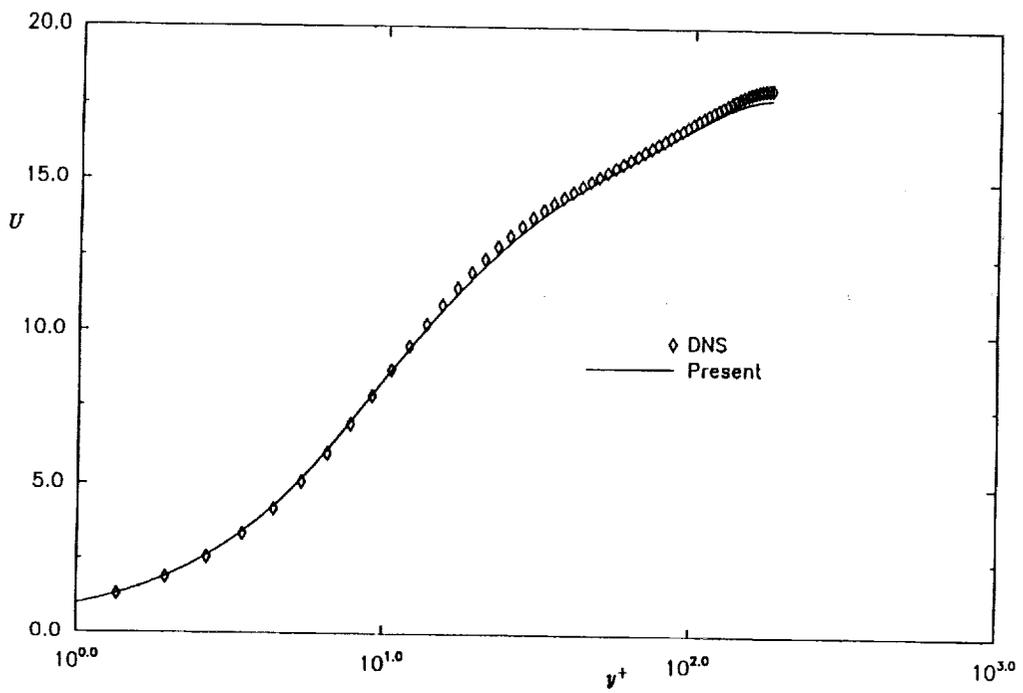
Figure 1. Mean velocity profile for channel flow, $Re_\tau = 180$.
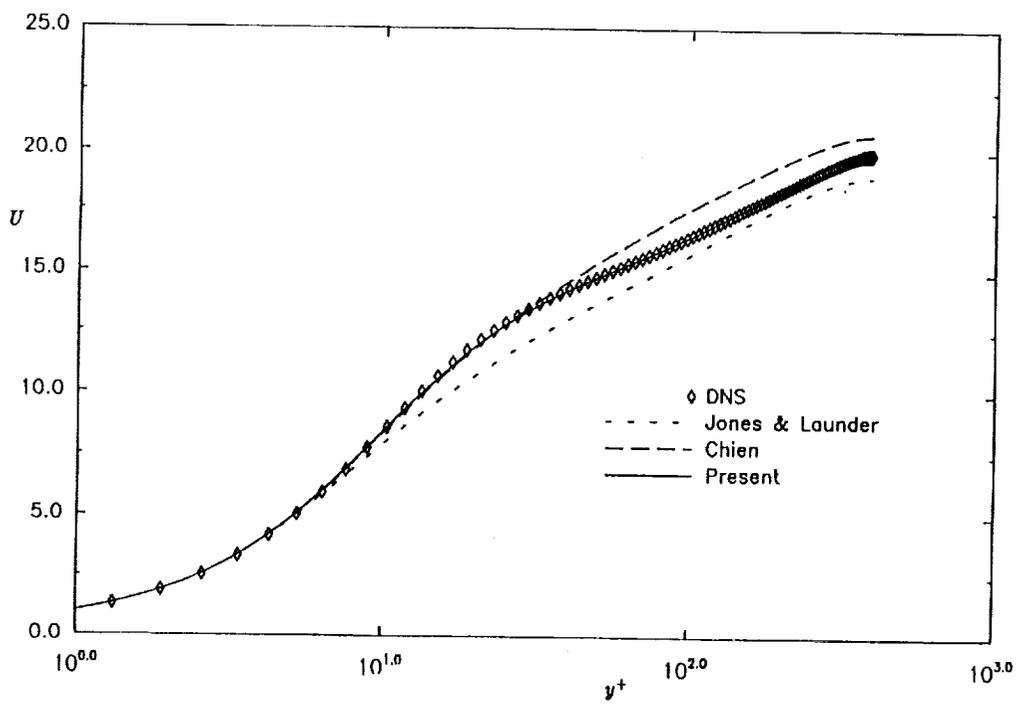
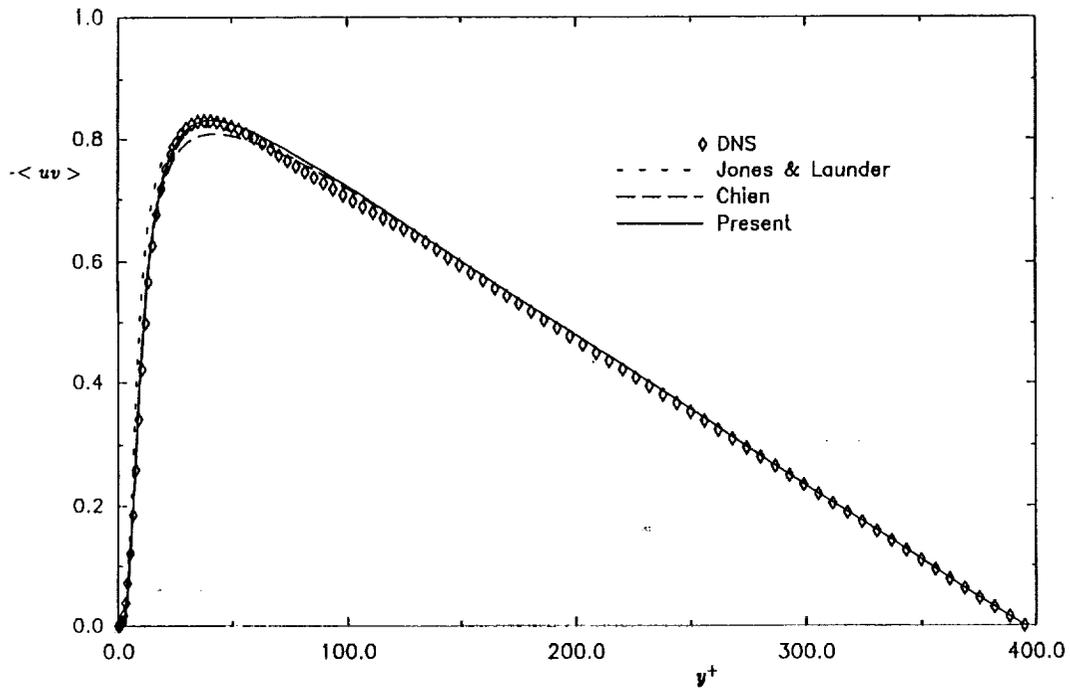Figure 2. Mean velocity profile for channel flow, $Re_\tau = 395$.

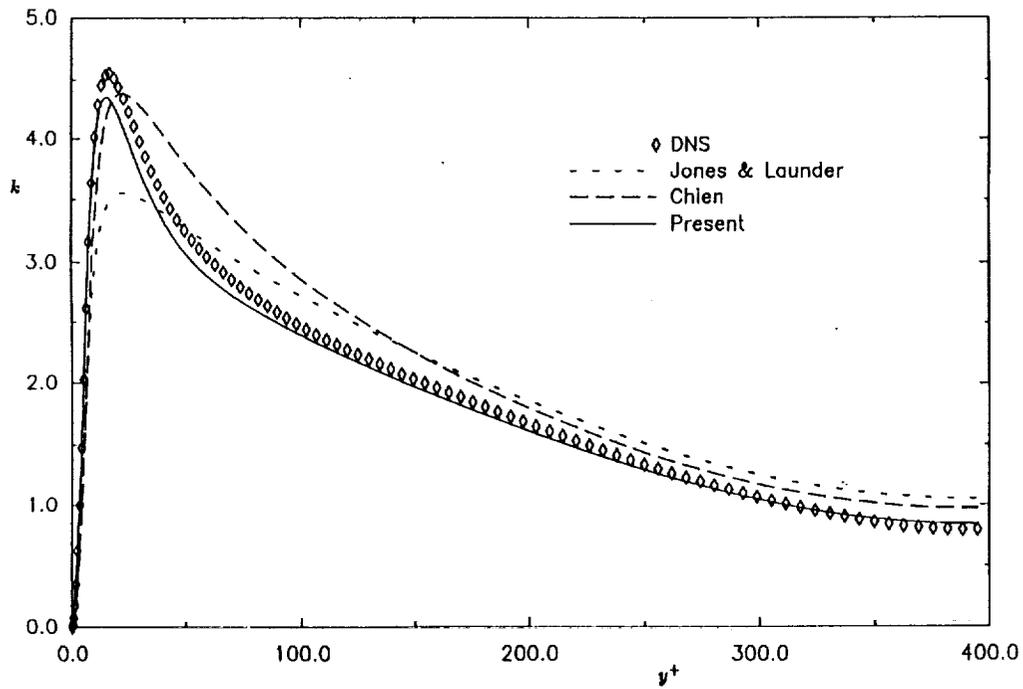Figure 3. Turbulent shear stress for channel flow, $Re_\tau = 395$.

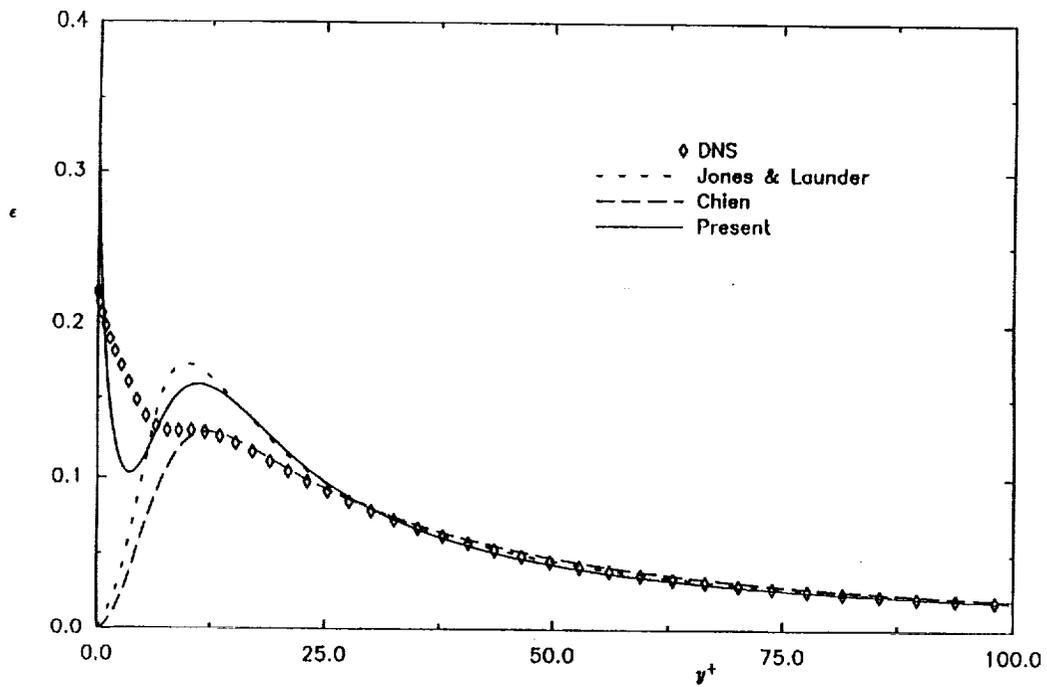Figure 4. Turbulent kinetic energy for channel flow, $Re_\tau = 395$.

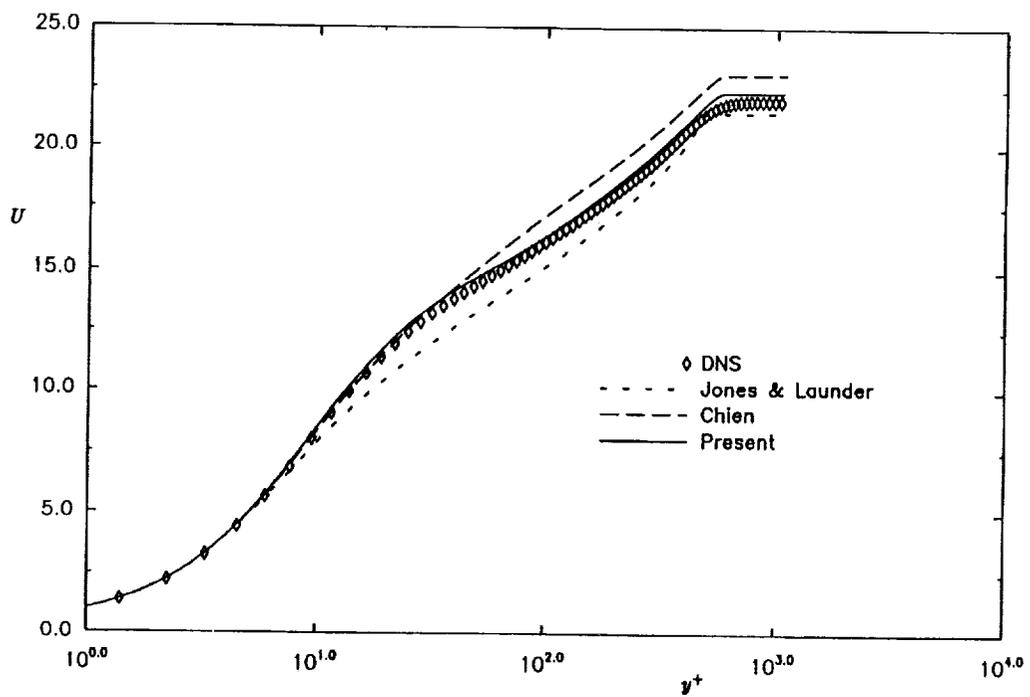Figure 5. Dissipation rate for channel flow, $Re_\tau = 395$.

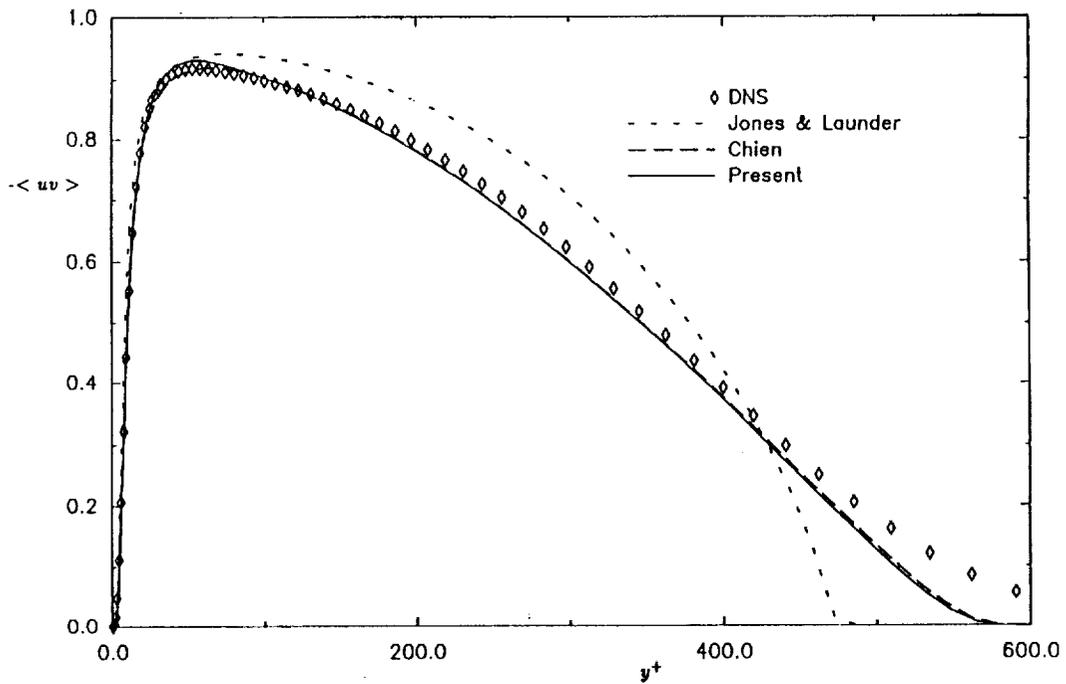Figure 6. Mean velocity profile for flat plate boundary layer, $Re_\theta = 1410$.

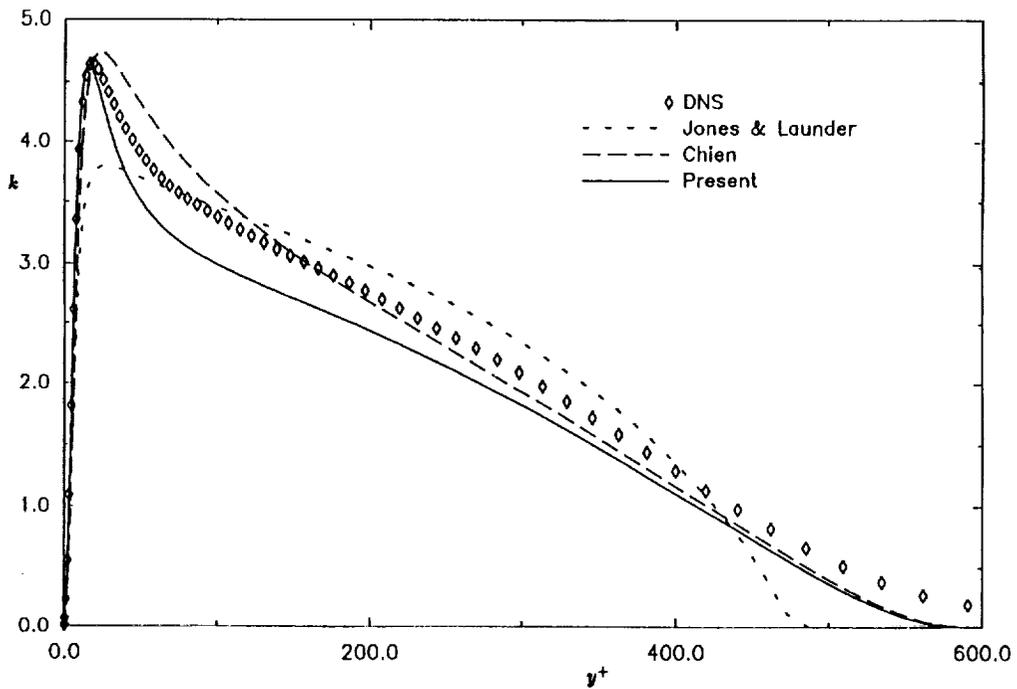Figure 7. Turbulent shear stress for flat plate boundary layer, $Re_\theta = 1410$.

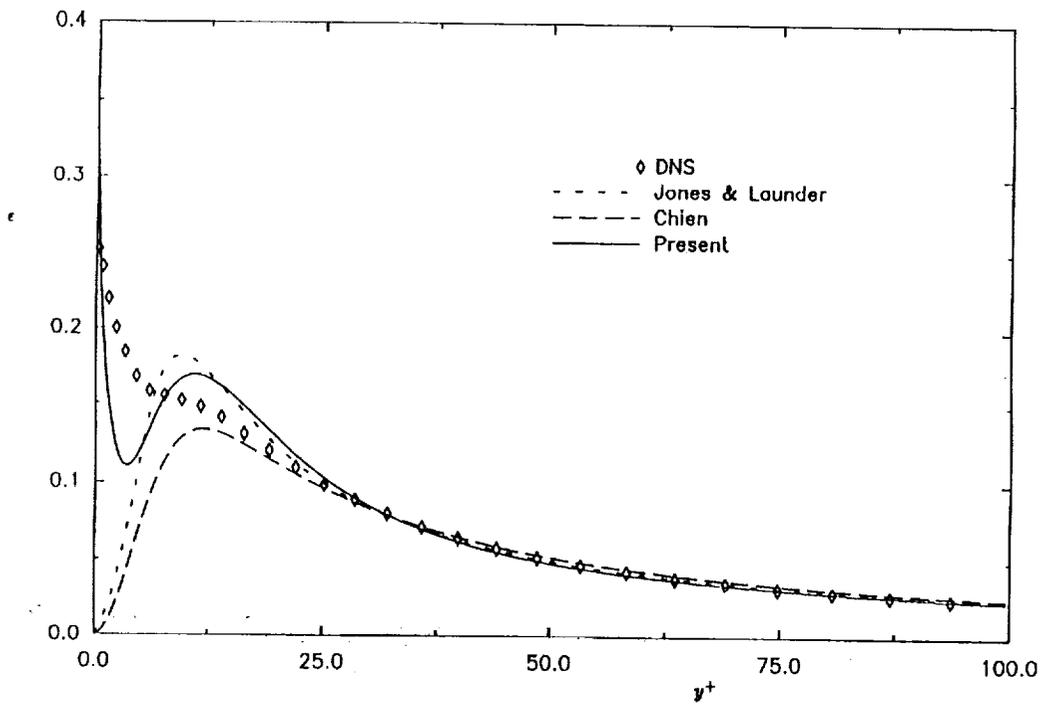Figure 8. Turbulent kinetic energy for flat plate boundary layer, $Re_\theta = 1410$.

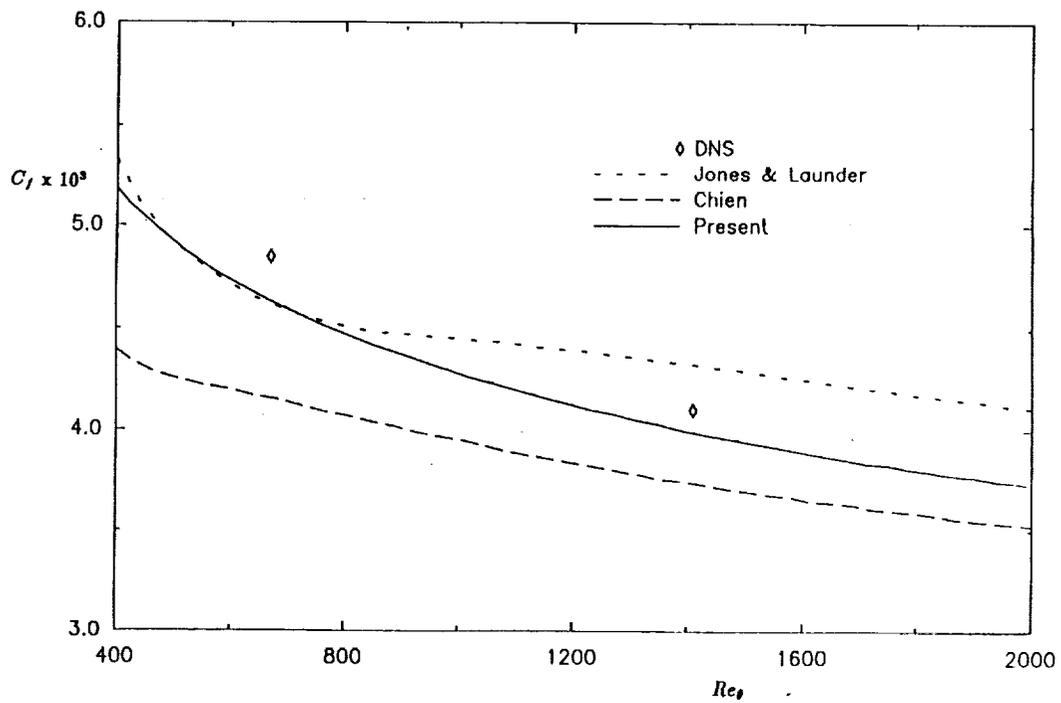Figure 9. Dissipation rate for flat plate boundary layer, $Re_\theta = 1410$.

Figure 10. Skin friction coefficient for flat plate boundary layer.

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13. ABSTRACT (Maximum 200 words)

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