Measurement of the ratio $R = \frac{BR(B \to D^0 K)}{BR(B \to D^0 \pi)}$

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The partial widths of $B^- \to D^0_{(CP)} K^-$ decays provide a theoretically clean way of measuring the CKM angle γ.

The ratio $R = \frac{BR(B \to D^0_{flav} K)}{BR(B \to D^0_{flav} \pi)}$ has been measured using the data collected between February 2002 and August 2004 (360 pb^{-1}).

I. INTRODUCTION

There are several methods, based on the partial widths of $B \to D^0 K$ modes, to obtain a measurement of the angle $\gamma = arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$:

- The GLW (Gronau-London-Wyler) method [1][2], that use $D^0_{flav} \to K^-\pi^+$, $D^0_{CP+} \to K^+K^-, \pi^+\pi^-$ and $D^0_{CP-} \to K^0_s\pi^0, K^0_s\Phi, K^0_s\omega$.
- The ADS (Atwood-Dunietz,-Soni) method [3][4], that use $D^0_{flav} \to K^-\pi^+$ and the doubly cabibbo suppressed $D^0_{dcs} \to K^+\pi^-$.
- The Dalitz method [4][5], that use the $D^0_{flav} \to K^0_s\pi^+\pi^-$.

All these methods require no tagging or time-dependent measurements.

Using the GLW method the parameters needed for the γ measurement are:

- $R = \frac{BR(B\to D^0_{flav}K)}{BR(B\to D^0_{flav}\pi)}$
- $R_+ = \frac{BR(B^-\to D^0_{CP+}K^-)+BR(B^+\to D^0_{CP+}K^+)}{BR(B^-\to D^0_{CP+}\pi^-)+BR(B^+\to D^0_{CP+}\pi^+)}$
- $A_{CP+} = \frac{BR(B^-\to D^0_{CP+}K^-)-BR(B^+\to D^0_{CP+}K^+)}{BR(B^-\to D^0_{CP+}K^-)+BR(B^+\to D^0_{CP+}K^+)}$

A_{CP+} and $R_{CP+} = R_+/R$ are related to the angle γ, the magnitude r of the ratio of the amplitudes for the processes $B^- \to \overline{D^0}K^-$ and $B^- \to D^0K^-$, and the relative strong phase δ between these two amplitudes, through the relations $R_{CP+} = 1 + r^2 + 2r\cos\delta\cos\gamma$ and $A_{CP+} = 2r\sin\delta\sin\gamma/R_{CP+}$.

The current analysis measures the ratio $R = \frac{BR(B\to D^0_{flav}K)}{BR(B\to D^0_{flav}\pi)}$ using the small kinematics differences between distinct modes and the particle identification information provided by the specific ionization (dE/dx) in the drift chamber. The dE/dx measurement was calibrated using a sample of kaons and pions from $D^{*+} \to D^0\pi^+ \to [K^-\pi^+]\pi^+$ decays. The resulting K/π separation is $\sim 1.5\sigma$.

II. DATA SAMPLE AND EVENT SELECTION

The analysis is performed on a sample corresponding to an integrated luminosity of $\sim 360\ pb^{-1}$ collected with the CDF II detector [6] between February 2002 and August 2004.

The $B^- \to D^0\pi^-$ with $D^0 \to K^-\pi^+$ (flavour eigenstate) and the $B^- \to D^0_{CP+}\pi^-$ with $D^0_{CP+} \to \pi^+\pi^-$ and $D^0_{CP+} \to K^+K^-$ (CP-even eigenstate) were reconstructed using the cuts summarized in Table I.

The cuts on the tridimensional vertex quality χ^2_{3D} and on the B isolation are very important to suppress the combinatoric background. The B isolation is defined as $\frac{p_T(B)}{p_T(B)+\sum p_T}$, where the sum on the transverse momentum is extended to the tracks contained in a cone with radius = 1 in the η - ϕ space around the B meson flight direction.

The resulting invariant mass distributions, with pion mass assignment to the track from B, were reported in Figs. 1, 2 and 3, respectively the invariant mass distribution of $B^- \to D^0\pi^- \to [K^-\pi^+]\pi^-$, the one of $B^- \to D^0_{CP+}\pi^- \to [K^+K^-]\pi^-$ and the one of $B^- \to D^0_{CP+}\pi^- \to [\pi^+\pi^-]\pi^-$.

III. LIKELIHOOD FIT

Our goal is to measure the relative yield of $B^- \to D^0K^-$ to $B^- \to D^0\pi^-$. We used an unbinned maximum likelihood fit which combines kinematics and dE/dx information.

Other channels contribute to background as we can observe from Fig. 4 where the invariant mass distribution, obtained with a generic MC of B^+ requiring only $D^0 \to K^-\pi^+$, is shown. However, the principal background contribute to the mass region of the DK signal is the $B^+ \to \overline{D^{0*}}\pi^+$. We choose as fit window the range of mass between 5.17 and 5.60 GeV/c; in this region the only significant physics backgrounds are $B^+ \to \overline{D^0}\pi^+$ and $B^+ \to \overline{D^{0*}}\pi^+$.

The variables used in the fit are:

- $M_{D^0\pi}$ which is the invariant mass assigning the pion mass at the track from B.

- The momentum imbalance α defined as:
  - if $p_{tr} < p_{D^0}$ $\quad \alpha = 1 - p_{tr}/p_{D^0} > 0$
  - if $p_{tr} \geq p_{D^0}$ $\quad \alpha = -(1 - p_{D^0}/p_{tr}) \leq 0$

  These two variables provide some separation between $B^+ \to \overline{D^0}\pi^+$ and $B^+ \to \overline{D^0}K^+$ as we can see from Fig. 5 were the mass vs α is shown for the two channels.
- $p_{tot} = p_t + p_{D^0}$.
- ID which contain the dE/dx information for the track from B.

The Likelihood function is $L = \prod_{i=1}^{nev} L_i$. The index i runs over the events and the Likelihood of the i^{th} event is:

$$L_i = b \cdot L_{back} + (1-b) \cdot L_{sign}. \tag{1}$$

The index *sign* (*back*) labels the likelihood term of signal (background), b is the background fraction.

The expression of the signal Likelihood is:

$$L_{sign}(\overrightarrow{\theta}) = \sum_j f_j \cdot pdf_j^{mass}(M_{D^0\pi}|\alpha, p_{tot}; \overrightarrow{\theta}) \cdot pdf_j^{PID}(ID|\alpha, p_{tot}; \overrightarrow{\theta}) \cdot pdf_j^{momentum}(\alpha, p_{tot}; \overrightarrow{\theta}) \tag{2}$$

The index j runs over the two modes $B^- \to D^0\pi^-$ and $B^- \to D^0K^-$, the parameters f_j are their relative fractions determined by the fit.

The mass term is:

$$pdf_j^{mass}(M_{D^0\pi}|\alpha, p_{tot}; \sigma, \overrightarrow{\theta}) = \sum_{i=1}^{3} \frac{A_i}{\sqrt{2\pi}\sigma_i} \cdot \exp\Big(-\frac{1}{2} \cdot \Big(\frac{M_{D^0\pi} - M(\alpha, p_{tot})}{\sigma_i}\Big)^2\Big) \tag{3}$$

The pdf of the $M_{D^0\pi}$ variable is the sum of three gaussians. The resolution σ_i and the relative fraction A_i are obtained by fitting the "correct" invariant mass distribution from MC samples of $B^+ \to \overline{D^0}\pi^+$ and $B^+ \to \overline{D^0}K^+$. We have defined the "correct mass" as $M_{corr} = M_{D^0\pi} - \delta(\alpha, Ptot)$, where $\delta(\alpha, p_{tot}) = M(\alpha, p_{tot}) - M_{Bu}$ is the mass shift due to the incorrect mass assignment ($\delta(\alpha, p_{tot}) = 0$ for $B^+ \to \overline{D^0}\pi^+$). In order to avoid relying on the absolute resolution predicted from the MC, we introduced a free scale parameter in our likelihood fit, multiplying the width of the three gaussians. This parameter is fitted together with all "physics parameters".

The expression of the B mass in the kaon hypothesis as function of α and p_{tot} is:

- if $\alpha > 0$

$$\begin{aligned} M^2(\alpha, p_{tot}) = {} & M_{Bu}^2 + m_\pi^2 - m_K^2 + 2 \cdot \sqrt{m_{D^0}^2 + (p_{tot}/(2-\alpha))^2} \cdot \\ & \cdot \Big(\sqrt{(m_\pi^2 + (p_{tot}(1-\alpha)/(2-\alpha))^2} - \sqrt{m_K^2 + (p_{tot}(1-\alpha)/(2-\alpha))^2}\Big) \end{aligned} \tag{4}$$

- if $\alpha \leq 0$

$$\begin{aligned} M^2(\alpha, p_{tot}) = {} & M_{Bu}^2 + m_\pi^2 - m_K^2 + 2 \cdot \sqrt{m_{D^0}^2 + (p_{tot}(1+\alpha)/(2+\alpha))^2} \cdot \\ & \cdot \Big(\sqrt{(m_\pi^2 + (p_{tot}/(2+\alpha))^2} - \sqrt{m_K^2 + (p_{tot}/(2+\alpha))^2}\Big). \end{aligned} \tag{5}$$

The PID pdf was extracted using the $D^0 \to K\pi$ sample. We applied the particle identification at the track from B. In this way we can distinguish between BDK and BDπ mode. The ID variable is defined as:

$$ID = \frac{\frac{dE}{dx}_{meas} - \frac{dE}{dx}_{exp}(\pi)}{\frac{dE}{dx}_{exp}(K) - \frac{dE}{dx}_{exp}(\pi)}. \tag{6}$$

In particular ID = 0 if the track is a pion and ID = 1 if the track is a kaon.

The kinematics term is:

$$pdf(\alpha, p_{tot}) = \sum_{i=0}^{6} a_i U_i(p_{tot}) \cdot \exp(a_7 p_{tot}) \cdot \sum_{j=0}^{6} b_j(p_{tot})\alpha_j \tag{7}$$

in which the a_i and b_j modes are different for each decay mode. U_i are the Chebyshev polynomial of second kind.

The expression of the background Likelihood is:

$$L_{back}(\overrightarrow{\theta}) = f_D \cdot pdf_D^{mass}(M_{D^0\pi}|\alpha, p_{tot}; \overrightarrow{\theta}) \cdot pdf_D^{PID}(ID|\alpha, p_{tot}; \overrightarrow{\theta}) \cdot pdf_D^{momentum}(\alpha, p_{tot}; \overrightarrow{\theta})$$
$$+ (1 - f_D) \cdot pdf_{comb}^{mass}(M_{D^0\pi}|\alpha, p_{tot}; \overrightarrow{\theta}) \cdot pdf_{comb}^{PID}(ID|\alpha, p_{tot}; \overrightarrow{\theta}) \cdot pdf_{comb}^{momentum}(\alpha, p_{tot}; \overrightarrow{\theta}) \quad (8)$$

where f_D is the fraction of the physics background ($B^+ \rightarrow \overline{D^{*0}}\pi^+$) and $1 - f_D$ is the fraction of combinatoric background.

The mass term of the physics background is parameterized with three gaussians plus an exponential from MC.

To describe the mass of the combinatorial background we used an exponential, in the central fit all the parameters are left free to vary.

The PID pdf for the combinatorial background was obtained assuming that it's composed only of pions and kaons:

$$a \cdot pdf_\pi(ID) + (1 - a) \cdot pdf_K(ID) \quad (9)$$

where a is the fraction of pions in the combinatorial background and is left free to float in the central fit.

The bidimensional $pdf(\alpha, p_{tot})$ distributions are parameterized with the following functions:

- $pdf(\alpha, p_{tot})$ for $B^+ \rightarrow \overline{D^{*0}}\pi^+$

$$\sum_{i=0}^{5} a_i p_{tot}^i \cdot \exp(a_6 p_{tot}) \cdot \sum_{j=0}^{6} b_j(p_{tot})\alpha_j \quad (10)$$

- $pdf(\alpha, p_{tot})$ for combinatorial background

$$[1 + \left(\frac{p_{tot} - \lambda}{a}\right)^2]^{-m} \cdot \exp[-\nu \cdot \arctan\left(\frac{p_{tot} - \lambda}{a}\right)] \cdot \sum_{j=0}^{6} b_j(p_{tot})\alpha_j \quad (11)$$

IV. FIT RESULT

The raw results of the maximum Likelihood fit are reported in table II. The raw result has to be corrected for detector effects and analysis efficiencies. The corrected result is:

$$R = \frac{BR(B^- \rightarrow D^0 K^-)}{BR(B^+ \rightarrow D^0 \pi^+)} = 0.065 \pm 0.007. \quad (12)$$

V. SYSTEMATICS UNCERTAINTIES

- *Mass resolution tails.* In the MC mass distribution there are some tails that we can parameterize adding a constant to the three gaussians. The resulting systematics is 0.001.
- *Input mass.* The B_u mass is an input parameter to the fit through the analytic expression of M(α,p_{tot}). We repeat the fit varying the input mass measured by CDF [7] of $\pm 1\sigma$ statistical uncertainty and the resulting systematics is 0.001.
- *dE/dx induced systematics.* To evaluate the dE/dx systematics we have repeated the fit varying the likelihood function parameters randomly, in a 1σ-radius multidimensional sphere, in the space of the parameters of dE/dx calibrations. The resulting systematics is 0.0015.
- *Combinatorial background systematics.*
  - *Mass model.* Our central fit assumes a mass-shape of the combinatorial background events distribuited as an exponential function. To evaluate the systematics we have changed the mass shape. We repeated the fit using as mass shape different polynomial functions of increasing degree, up to the third. The resulting systematics is 0.001.

  - *Kinematical parameterization.* To evaluate the systematics associated with the kinematical parameterization of the combinatorial background we repeat the fit using the $P(\alpha, p_{tot})$ distributions of $D^0\pi$ and of D^0K for the background. The resulting systematics is 0.001.
  - *Particle Identification.* In the main fit we assumed that the fraction of pions in combinatorial background is not mass dependent. To verify that, we fitted two distinct pions fractions, one in the window [5.17-5.4] and one in the window [5.4,5.6] . We obtained:
    * fraction of pions in [5.17-5.4] $= 0.76 \pm 0.05$
    * fraction of pions in [5.4-5.6] $= 0.72 \pm 0.05$

    in agreement within the error.

    Another assumption is relative to the composition of the combinatorial background. We assumed a background of only pions and kaons. To evaluate the systematics associated with ignoring protons and electrons we made 100 toy MC with also protons and electrons in the background. The background composition of each toy is 70% of pions, 10% of protons, 2% of electrons and 18% of kaons. We fitted the toy MC with the main fit and we obtained a residual distribution of the ratio $\frac{N(B\to D^0K)}{N(B\to D^0\pi)}$ centered in zero. The fit is not sensitive to the electrons and protons contribution.

- *$D^{*0}\pi$ mass model.* To evaluate this systematics we left free in the main fit some parameters of the $D^{*0}\pi$ mass parameterization. We parameterized the $D^{*0}\pi$ mass with three gaussians plus an exponential between 5.06 and 5.6. The idea is to see how the result (the ratio $\frac{N(B\to D^0K)}{N(B\to D^0\pi)}$) change letting free in the main fit the parameters of the exponential or of the gaussian that falls in the fit window [5.17,5.6]. We made some test:
  - We left free in the main fit the slope of the exponential and we obtained $\frac{N(B\to D^0K)}{N(B\to D^0\pi)} = 0.069 \pm 0.007$.
  - We left free the mean of the gaussian obtaining $\frac{N(B\to D^0K)}{N(B\to D^0\pi)} = 0.066 \pm 0.007$
  - We left free simultaneously the slope of the exponential and the mean of the gaussian obtaining $\frac{N(B\to D^0K)}{N(B\to D^0\pi)} = 0.066 \pm 0.007$.

  The resulting systematics is 0.003.

  We also make the fit with a different parameterization for the $D^{*0}\pi$ mass (two gaussians plus an exponential). The resulting systematics is 0.001.

- *Monte Carlo statistics.* This is the uncertainty deriving from the limited statistics of the Monte Carlo samples used to estimate the relative efficiencies and from the uncertainty on the XFT efficiency. The resulting systematics is 0.002.

VI. RESULTS

We measured the ratio

$$R = \frac{BR(B \to D^0_{flav}K)}{BR(B \to D^0_{flav}\pi)} = 0.065 \pm 0.007(stat) \pm 0.004(sys). \quad (13)$$

The world average for this ratio is 0.0830 ± 0.0035. This result comes out from the combined measurement of Belle ($0.077 \pm 0.005 \pm 0.006$ [8]), Babar ($0.0831 \pm 0.0035 \pm 0.002$ [9]) and Cleo ($0.099^{+0.014+0.007}_{-0.012-0.006}$ [10]).

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Parameter	Value
$\frac{L_{xy}(B)}{\sigma_{L_{xy}(B)}}$	> 8
$L_{xy}(D)$ wrt B	> -0.0015 cm
$\lvert d_0(B)\rvert$	< 0.008
p_t (track from B)	> 2 GeV/c
$\Delta R_{\pi,D^0} = \sqrt{\Delta\phi^2 + \Delta\eta^2}$	< 2

TABLE I: Selection cuts

Parameter	Raw fraction	Raw yield
$B \to D^0\pi$	0.936 ± 0.006	3265 ± 38
$B \to D^0 K$	0.064 ± 0.006	224 ± 22
$\frac{N(B\to D^0 K)}{N(B\to D^0\pi)}$	0.0069 ± 0.007	

TABLE II: Raw results from the maximum likelihood fit.

Source	Shift wrt central fit
Mass resolution tails	0.0006
Input mass	0.001
dE/dx	0.0015
combinatorial background model	0.001
$D^{*0}\pi$ left free in the fit	0.003
changing $D^{*0}\pi$ mass model	0.001
MC statistics/XFT eff	0.002
Total(sum in quadrature)	0.004

TABLE III: Systematics summary.

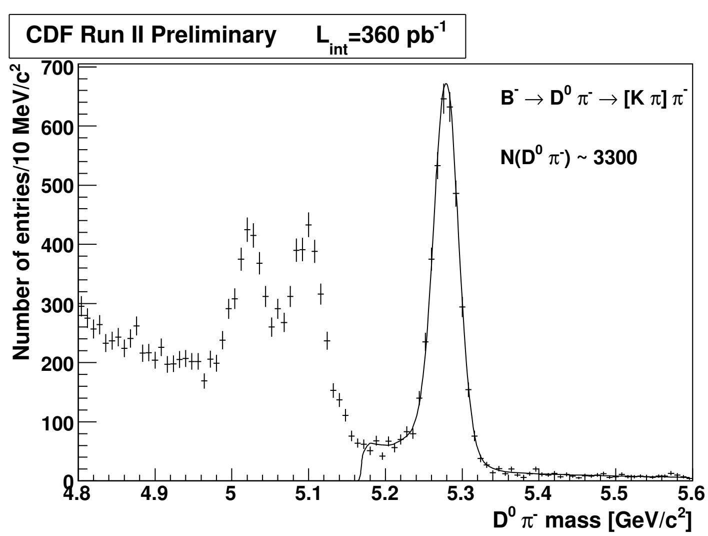


FIG. 1: Invariant mass distribution of $B^- \to D^0\pi^-$ with $D^0 \to K^-\pi^+$ (flavour eigenstate)

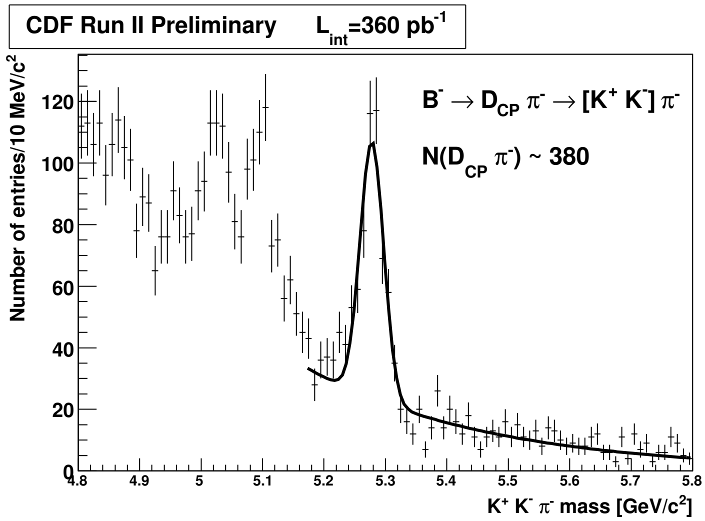


FIG. 2: Invariant mass distribution of $B^- \rightarrow D^0_{CP+}\pi^-$ with $D^0_{CP+} \rightarrow K^+K^-$ (CP-even eigenstate)

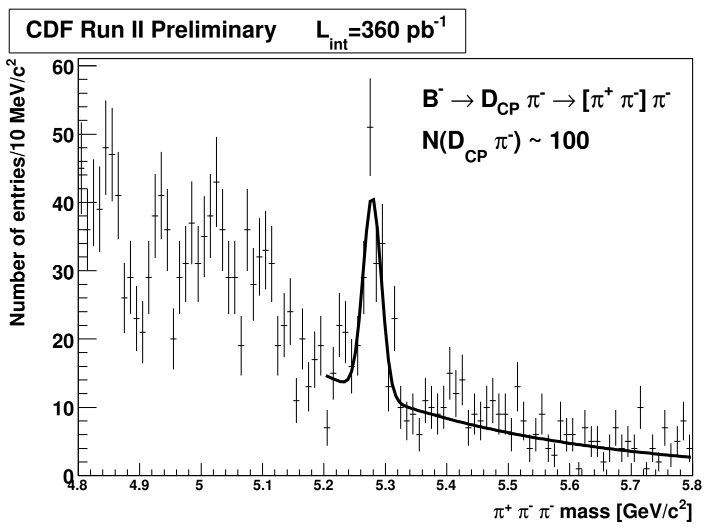
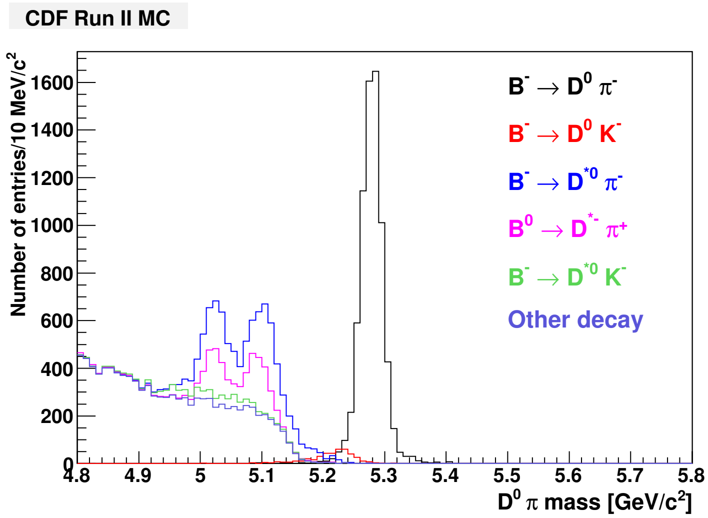


FIG. 3: Invariant mass distribution of $B^- \rightarrow D^0_{CP+}\pi^-$ with $D^0_{CP+} \rightarrow \pi^+\pi^-$ (CP-even eigenstate)



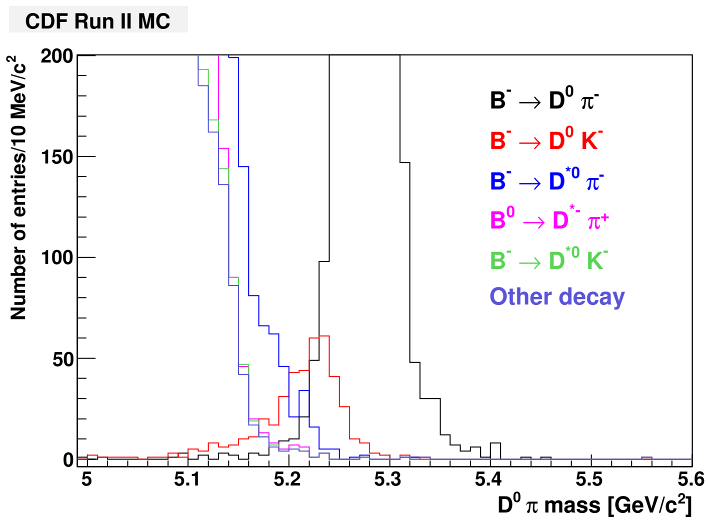


FIG. 4: Generic MC of B^+. Black: $B^+ \rightarrow \overline{D^0}\pi^+$. Red: $B^+ \rightarrow \overline{D^0}K^+$. Blue: $B^+ \rightarrow \overline{D^{0*}}\pi^+$. Magenta: $B^+ \rightarrow D^{*-}\pi^+$. Green: $B^+ \rightarrow \overline{D^{0*}}K^+$ and in violet other decay.

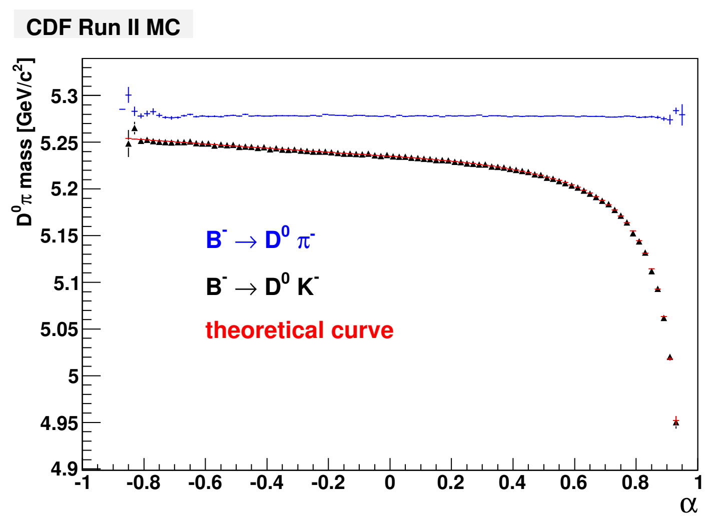


FIG. 5: Mass vs α using MC. Blue: $M_{D^0\pi}$. Black: M_{D^0K}. Red: theoretical curve obtained using the analytical formula

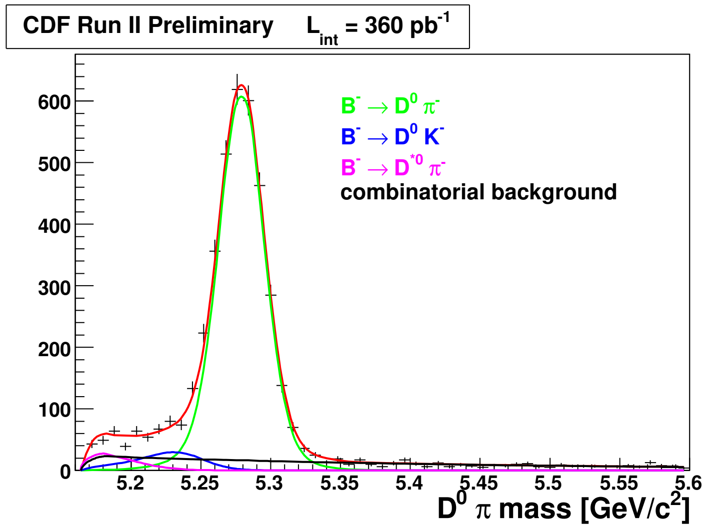


FIG. 6: Mass Projection

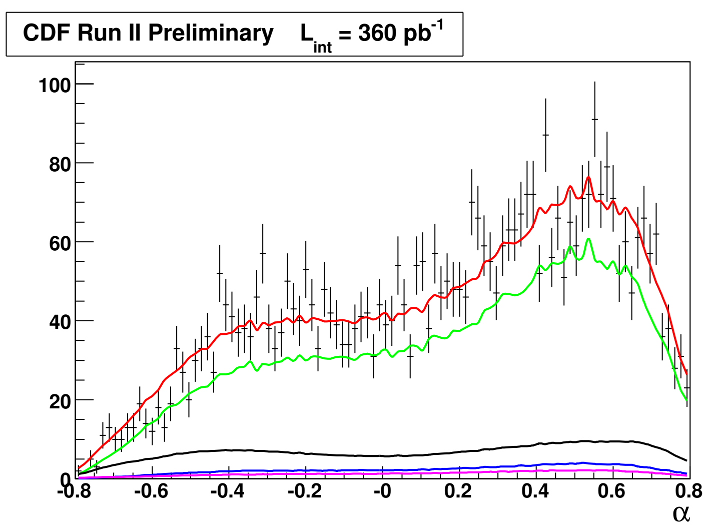


FIG. 7: α Projection

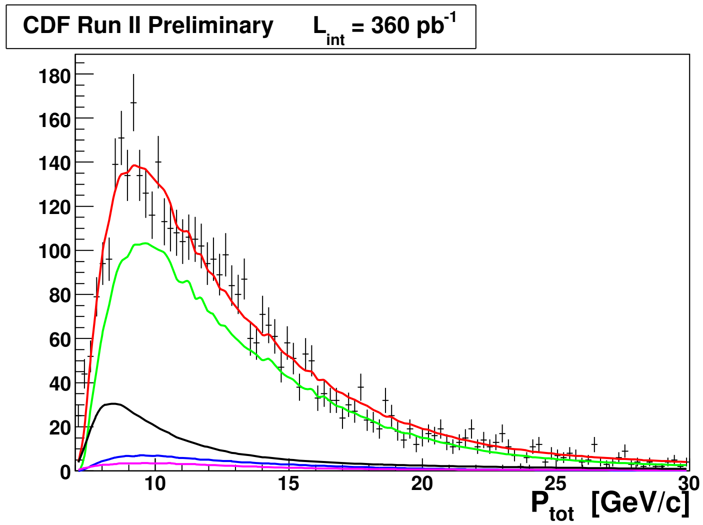


FIG. 8: p_{tot} Projection

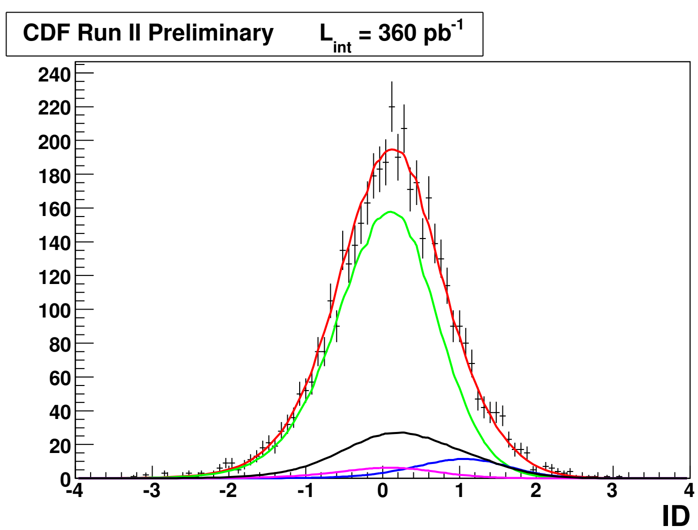


FIG. 9: ID Projection

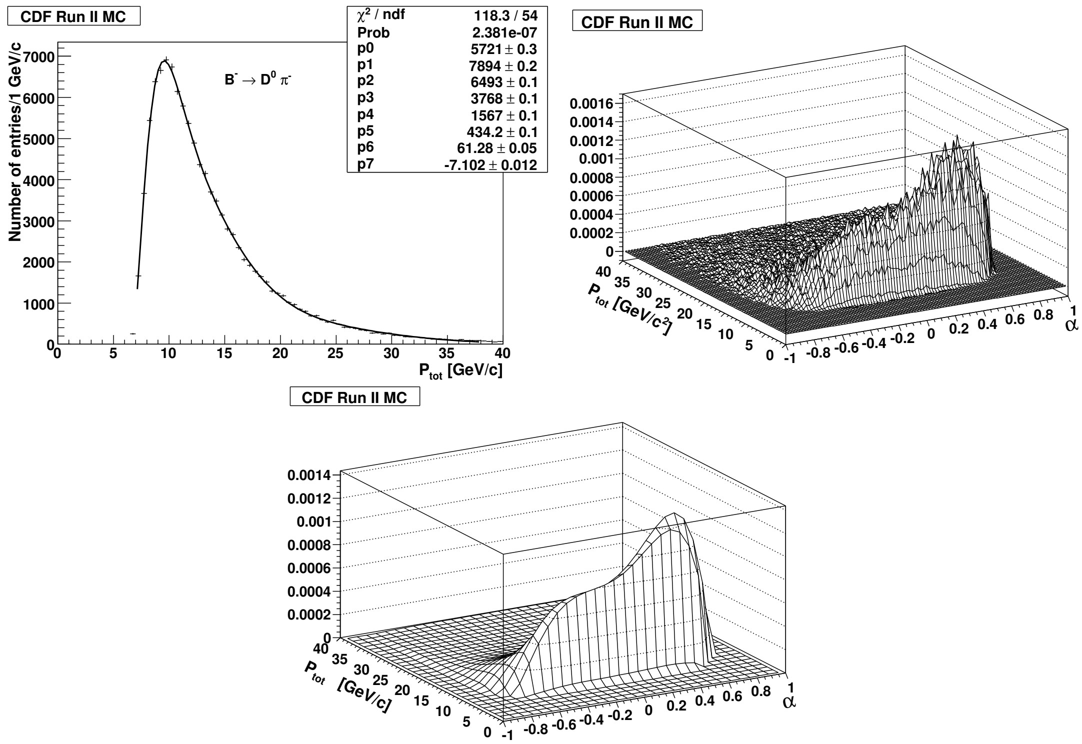


FIG. 10: Determination of the joint (α, p_{tot}) density for the $B^- \to D^0\pi^-$. Top left: Fit of p_{tot} distribution. Top right: distribution of $\alpha - p_{tot}$ in simulated events. Bottom: Two dimensional fit result.

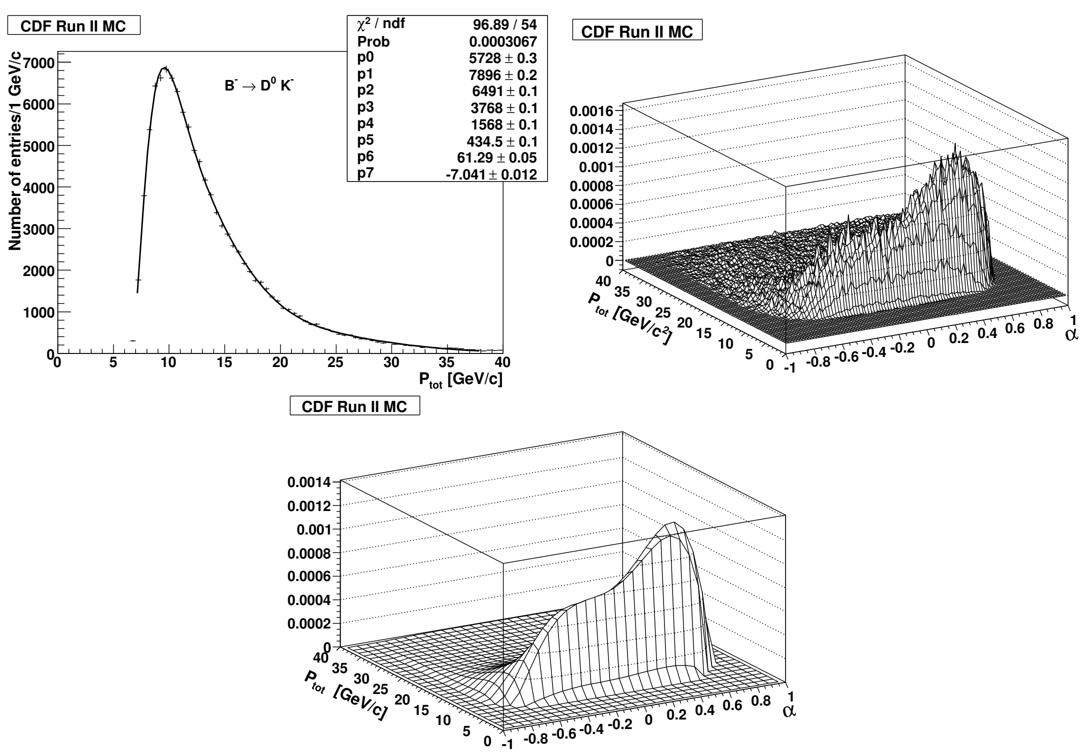


FIG. 11: Determination of the joint (α, p_{tot}) density for the $B^- \to D^0K^-$. Top left: Fit of p_{tot} distribution. Top right: distribution of $\alpha - p_{tot}$ in simulated events. Bottom: Two dimensional fit result.

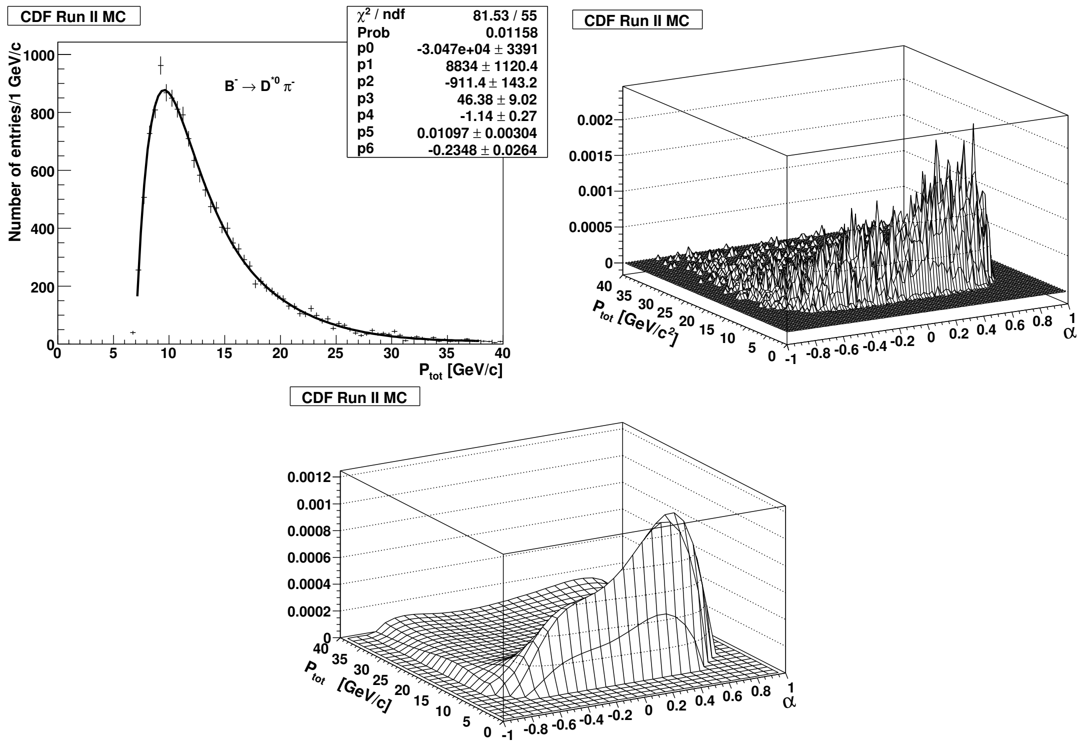


FIG. 12: Determination of the joint (α, p_{tot}) density for the $B^- \to D^{*0}\pi^-$. Top left: Fit of p_{tot} distribution. Top right: distribution of $\alpha - p_{tot}$ in simulated events. Bottom: Two dimensional fit result.

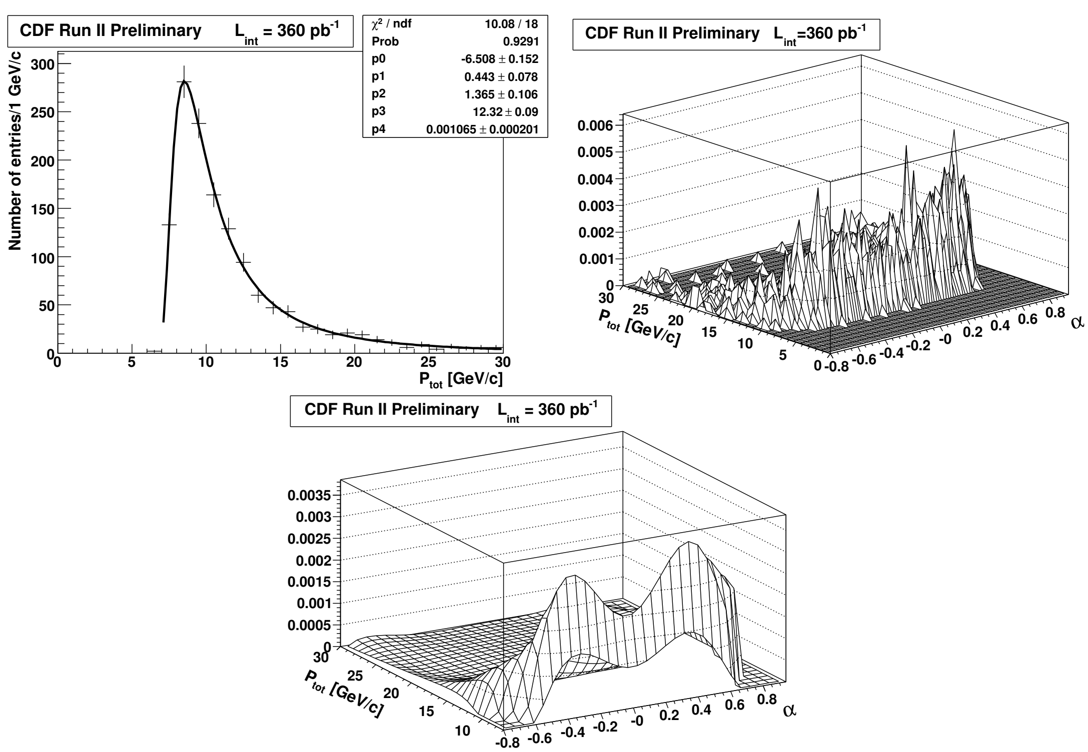


FIG. 13: Determination of the joint (α, p_{tot}) density for the combinatorial background. Top left: Fit of p_{tot} distribution. Top right: distribution of $\alpha - p_{tot}$ in simulated events. Bottom: Two dimensional fit result.