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Title: Structural Damage Detection Using Modal Test Data

Authors: Costas Papadimitriou

Marie Levine-West

Mark Milman

ABSTRACT

This study presents a methodology for updating the finite element model of a structure for damage detection purposes using an incomplete set of experimentally obtained modal frequencies and modeshapes. The proposed damage detection methodology involves a least squares minimization of the modal dynamic force balance residuals subject to quadratic inequality constrains introduced to properly account for the expected measurement and modeling errors. A three-step iterative procedure is proposed to iteratively predict the probable locations and size of significant, damage by updating the properties of the finite element model of the structure at the element level. Simulated modal data obtained on a three-dimensional truss structure are used to assess the strengths, limitations, and overall performance of the proposed damage detection methodology in relation to the number of measured modes, number and location of sensors, as well as location and magnitude of damage.

INTRODUCTION

In past years, several studies have been devoted in reconciling finite element models with measured modal data. The need for model updating arises because there are always errors associated with the process of constructing a theoretical model of a structure. This leads to uncertain accuracy in predicting the response, Another important application of model updating is in the prediction

Costas Papadimitriou, Instructor, Division of Engineering & Applied Science 104-44, California Institute of Technology, Pasadena, CA 91125

Marie Levine-West, Technical Group Leader, Science and Technology Development section, 4800 Oak Grove Dr., Jet Propulsion Laboratory, Pasadena, CA 91109-8099

Mark Milman, Member, Technical Staff, Automation and Control Section, MS 138-308, 4800 Oak Grove Dr., Jet Propulsion Laboratory, Pasadena, CA 91109-8099

of structural damage (see, for example, Natke and Yao 1988, Stubbs *et al* 1990, Yao and Natke 1994). The location and size of damage can be inferred by monitoring the reduction in stiffness and mass properties of the elements comprising the finite element model of the structure.

The general problem of model updating involves the selection of the best model from a parameterized class of models that best fits the modal data as judged by an appropriately selected measure of fit. The following are the difficulties associated with this inverse problem: 1) the modal data are contaminated by measurement error; 2) the chosen class of parametric finite element models is not representative of the actual structural behavior for all possible values of the model parameters; 3) the modal data are incomplete relative to the model complexity needed to produce physically meaningful models. For example, the set of observed DOF is usually a small subset, of the set of model DOF due to the limited number of sensors used or due to limited accessibility within a structure. Also, the number of identifiable modes of vibration is much less than the number of model DOF due to large measurement noise for higher modes, limited bandwidth in the response and hardware limitations. As a results of the aforementioned difficulties, the inverse problem leads to non-unique solutions and ill-conditioning (Berman 1984 and 1989, Beck 1989, Mottershead and Friswell 1993, Beck and Katafygiotis 1997).

A literature review of existing finite element model updating and damage detection methods can be found in the survey by Mottershead and Friswell (1993). Each method has its own advantages and shortcomings and there is no acceptable methodology for successfully treating the model updating and damage detection problem. Most methods address the problem by choosing some mathematical criteria which creates a unique optimal model while neglecting other models that can give an equally good fit to the measured data. Recently, new methods (Beck 1989, Beck and Katafygiotis 1997) based on Bayesian statistical inference have been developed for properly addressing the non-uniqueness by computing all (finite or infinite) models that can give acceptable fit to the data (Katafygiotis and Beck 1997, Katafygiotis and Beck 1991 and 1997, Beck *et al* 1994, Vanik 1997). The latter methods are powerful and have shown great promise in properly incorporating modeling and measurement errors, as well as properly addressing many of the difficulties encountered in the model updating problem, especially those associated with model and response prediction accuracy,

The problem of damage detection involves as a first step the identification of the location (or locations) of damage. Recently, various methods have been proposed for identifying probable damage locations in a structure using modal test data (Fahrat and Hemez 1993, Levine *et al* 1996). These are based on modeshape expansion techniques and when combined with appropriately-defined localized error measures, they have shown promise in predicting the locations in the structure that are more likely to be damaged. In particular, the least-squares error measure subject to inequality constrains proposed by Levine *et al* (1996 and 1997) properly accounts for the presence of measurement and modeling errors. The robustness and reliability of the resulting modeshape expansion technique for predicting the modeshape components at unmeasured points have been successfully evaluated in a previous study using actual experimental data obtained on the Jet Propulsion Laboratory micro-precision interferometer truss (Levine et al 1997). Compared with other modeshape expansion techniques, the least squares minimization technique with quadratic inequality constrains was found to provide the most reliable mode shape estimates and predictions of damage locations, even in adverse situations of significant measurement and model error.

Once the damage has been located, the magnitude of damage is predicted by updating the finite element model of the structure. The preferable methods of updating are usually the ones which preserve structural connectivity. The work by Farhat and Hemez (1993) combines modeshape expansion techniques with updating capabilities for predicting both the location and size of damage in a structure. It has been shown to work reasonably well for the cases which has been applied (Hemez and Farhat 1995). In a recent work, Alvin (1997) has pointed out potential problems of this method and suggested several modifications which are found to provide a more robust technique for locating and sizing errors in the finite element model of a structure. In this work, we formulate a model updating and damage detection methodology which is based on the mode-shape expansion method presented by Levine *et al* (1996). The measure of fit used in the proposed methodology for predicting the probable damage locations and size of damage accounts for the expected measurement error in both modal frequencies and mode shape components, as well as the expected modeling errors.

STRUCTURAL MODELS

The structure is modeled by the following general class of classically-damped linear models:

$$M(\theta)\,\ddot{x} + C(\theta)\,\dot{x} + K(\theta)x = f(t) \tag{1}$$

where the global mass and stiffness matrices $M(\theta)$ and $K(\theta)$ are respectively assembled, using a finite element analysis, from the element (or substructure) mass and stiffness matrices $M^e(\theta)$ and $K^e(\theta)$ as follows

$$M(\theta) = \sum^{e} M^e(\theta) \tag{2}$$

$$K(\theta) = \sum^{e} K^e(\theta) \tag{3}$$

The class of models has been parameterized using the parameter set θ which may represent mass and stiffness properties at the element or substructure level. The parameterization is chosen such that the undamaged finite element model of the structure corresponds to $\theta = 1$. Examples of finite element properties that can be included in the parameter set θ are: modulus of elasticity, cross-sectional area, thickness, moment of inertia and mass density of the finite elements comprising the model, as well as spring (translational or rotational) stiffnesses used to model fixity conditions at joints or boundaries.

In general, the system matrices $M(\theta)$ and $K(\theta)$ are nonlinear functions of θ. However, a parameterization which often arises in practical applications is the case for which both $M(\theta)$ and $K(\theta)$ are linear functions of θ, that is,

$$K(\theta) = K_0 + \sum_{i=1}^{p} K_i\theta_i \tag{4}$$

$$M(\theta) = M_0 + \sum_{i=1}^{p} M_i\theta_i \tag{5}$$

where K_0, K_i, M_0 and M_i are constant matrices independent of θ. Without loss of generality, the linear parameterization will be employed to demonstrate the methodology. However, the incorporation of a general nonlinear parameterization is straightforward and will not be discussed in detail.

DAMAGE DETECTION FORMULATION

The objective in a modal-based finite element model updating methodology is to find the values of the parameter set θ so that the modal data generated by the linear class of models best matches, in some sense, the experimentally obtained modal data. Thus, a model updating methodology involves minimizing a measure of fit between the model-based and the experimentally obtained modal data. A measure of fit that is explored herein is directly related to the modal dynamic force balance residuals $r(\omega, \phi, \theta)$, defined by

$$r(\omega, \phi, \theta) = [K(\theta) - \omega^2 M(\theta)]\phi \tag{6}$$

Note that the modal dynamic force balance residuals satisfy the equations

$$r(\omega_i(\theta),\ \varphi_i(\theta),\ \theta) = 0, \quad i = 1, \cdots, m \tag{7}$$

where $\omega_i(\theta)$ and $\varphi_i(\theta)$, i = 1, $\cdots$, m are respectively the modal frequencies and mass-normalized mode shapes of the first m modes of the structure generated by the model (1).

For convenience, let the subsets a and o be the sets of measured and unmeasured model degrees of freedom, respectively. The set $[a, o]$ contains the total number of degrees of freedom of the structural model. Each mode shape vector ϕ_i can be partitioned in the form $\phi_i^T = [\phi_{ai}, \phi_{oi}]$, where ϕ_{ai} and ϕ_{oi} are the components of the mode shape ϕ_i at the measured and unmeasured degrees of freedom, respectively.

Let now $\tilde{\omega}_i$ and $\tilde{\phi}_{ai}$ be the experimentally obtained frequencies and modeshapes of the structure at the measured degrees of freedom. The proposed method for model updating searches for the optimal model parameters θ which minimize an appropriately selected norm of the modal dynamic force balance residuals $r(\omega_i, \phi_i, \theta)$ subject to conditions that reflect the fact that both the modal frequencies ω_i and modeshapes ϕ_i are sufficiently close, depending on the experimental error expected, to the measured modal frequencies and modeshape components. Mathematically, the model updating problem is stated as

$$\min_{\theta,\phi_i} \sum_{i=1}^{m} \|r(\tilde{\omega}_i, \phi_i, \theta)\|_{R_i} = \min_{\theta,\phi_i} \sum_{i=1}^{m} \left\|(K(\theta) - \tilde{\omega}_i^2 M(\theta))\phi_i\right\|_{R_i} \tag{8}$$

subject to

$$\|P\phi_i - \tilde{\phi}_{ai}\|^2 \leq \alpha_i \|\tilde{\phi}_{ai}\|^2, \quad i = 1, \cdots, m \tag{9}$$

where $\|x\|^2 = x^T x$ is the usual Eucledian norm, $\|x\|_R = x^T Rx$, R_i is an appropriately selected weighting matrix which scales the contribution of each mode in the nwi.sure of fit (8), and P is a constant matrix of zeroes and ones satisfying $\phi_{ia} = P\phi_i$, i.e. it maps a mode-shape vector ϕ_i to a vector ϕ_{ia} that it icludes only the components Of ϕ_i at the measured degrees Of freedom .

The unknown quantities involved in the proposed error measure (8) include the model properties θ, as well as the components of the vector ϕ_i of the contributing modes at both measured and unmeasured model degrees of freedom. The optimal vector ϕ_i resulting from the minimization can be viewed as the expanded modeshape consistent with the measured modal data.

Next, the measure of fit in (8) is further analyzed in order to determine a reasonable choice for the weights R_i. The analysis will also provide insight into the relationship between the measure in (8) and other more direct measures of fit involving the differences between the model and measured modal frequencies as well as measures involving the mode-shape orthogonality conditions. For this, consider first the contribution $J_i(\phi_i, \theta) = \|r(\tilde{\omega}_i, \phi_i, \theta)\|_{R_i}$ from the i-th modal term in the overall measure of fit (8). Using the model modeshapes φ_j, $j = 1, \cdots, N$ and expanding the vectors ϕ_i in the form $\phi_i = \sum_{j=1}^{M} a_{ij}\varphi_j = \sum_{j=1}^{M} (\varphi_j^T M(\theta)\phi_i)\varphi_j$, the i-th modal residuals can be written in the form

$$J_i(\phi_i, \theta) = \sum_{j=1}^{N}\sum_{k=1}^{N} a_{ij}a_{ik}[\omega_j^2\omega_k^2 - \tilde{\omega}_i^2(\omega_k^2 + \omega_j^2) + \tilde{\omega}_i^4]\varphi_j^T M(\theta)R_i M(\theta)\varphi_k \tag{10}$$

This expression can be simplified considerably by choosing R_i such that

$$\varphi_j^T M(\theta)R_i M(\theta)\varphi_k = \gamma_i^4 \delta_{jk} \tag{11}$$

where δ_{jk} is the Kronecker delta, and γ_{ij} depends on the choice of R_i. Substitution of (11) into (1 O) gives

$$J_i(\phi_i, \theta) = \sum_{j=1}^{N} (\varphi_j^T M(\theta)\phi_i)^2 \frac{(\omega_j^2 - \tilde{\omega}_i^2)^2}{\tilde{\omega}_i^4}(\gamma_i\tilde{\omega}_i)^4 \tag{12}$$

The choice (11) was preferred because it is the only one that results in positive terms in the modal measure of fit (1 O).

Note that for $\phi_i = \varphi_i$, $i = 1, \cdots, m$, i.e. the case of perfectly correlated expanded and model mode-shapes, all but one term corresponding to $j = i$ in the modal measure $J_i(\phi_i, \theta)$ disappear. The modal measure $J_i(\phi_i, \theta)$ becomes proportional to the fractional difference between the squares of the model and n measured modal frequencies for mode i, weighted by $(\gamma_i\tilde{\omega}_i)^4$. This equivalence between the measure $J_i(\phi_i, 0)$ and the more direct measure involving tile difference between the squares of the model and measured modal frequencies was first reported in a recent study (Vanik 1997). In the general case for which $\phi_i \neq \varphi_i$, all terms in the modal error measure (12) are present. In particular, the terms in the modal error measure (12) corresponding to $j \neq i$ involve the mass orthogonality condition between the measured and model modeshapes, weighted by the factors $(\gamma_i\tilde{\omega}_i)^4(\omega_j^2 - \tilde{\omega}_i^2)^2/\tilde{\omega}_i^4$. Note that for a model which is well-corrclatfwl with the measured data, the factors $(\varphi_i^T M(\theta)\phi_i)^2 \approx 1$ and $(\varphi_j^T M(\theta)\phi_i)^2 \approx 0$ for $j \neq i$. Therefore, in the process of selecting tile optimal model, the mass orthogonality conditions are also enforced through the terms in (12) corresponding to j # i.

The term in (12) corresponding to $j = i$ provides insight, into the problem of choosing the weights R_i. From the aforementioned analysis, it becomes apparent that weighting the modal error measures $J_i(\phi_i, \theta)$ is equivalent to weighting the errors between the experimental and model medal frequencies. Thus, R_i can be selected such that errors between the experimental and model modal frequencies are weighted for each mode according to weights β_i. This is accomplished by choosing γ_i so that the factor $(\gamma_i\tilde{\omega}_i)^4$ is proportional to non-dimensional weights β_i. From a Bayesian statistical point of view, the weights β_i reflect the magnitude of the measurement errors expected between the experimental and model frequencies for each mode (Beck 1989, Vanik 1997). The size of these errors can be obtained from measurement data taken from repeated modal test analyses.

Several choices for the weights R_i can be made to satisfy condition (11) and, at the same time, guarantee that the factor $(\gamma_i\tilde{\omega}_i)^4$ is proportional to the non-dimensional quantity β_i. Attention is only given to the following two choices:

$$R_i = \beta_i M^{-1}(\theta)/\tilde{\omega}_i^4 \quad \Longrightarrow \quad (\gamma_i\tilde{\omega}_i)^4 = \beta_i \tag{13}$$

$$R_i = \beta_i K^{-1}(\theta) M(\theta) K^{-1}(\theta) \quad \Longrightarrow \quad (\gamma_i\tilde{\omega}_i)^4 = \beta_i\tilde{\omega}_i^4/\omega_j^4 \tag{14}$$

For the first choice it is assumed that $M^{-1}(\theta)$ is non-singular. Thus, it is not applicable for structures with zero mass degrees of freedom. However, this problem can easily be eliminated by applying Guyan model reduction to eliminate the degrees of freedom corresponding to zero mass. For the second choice it is assumed that the matrix $K(\theta)$ is non-singular and so it is not directly applicable for structures that are not supported or they are partially supported such as those employed in space or tested in the lab by suspending them by very soft springs. An advantage of the first choice is that it simplifies considerably the computation involved in updating θ in the case for which both $K(\theta)$ and $M(\theta)$ satisfy (4) and (5). This study explores the use of the second weight $R_i = \beta_i K^{-1}(\theta) M(\theta) K^{-1}(\theta)$ in the identification of the location and size of damage.

Finally, the inequality constrains (9) are introduced to account for the expected measurement error in the mode-shape components, with α_i controlling the expected magnitude of these errors. The value of α_i can be computed from a statistical analysis of measurement data taken from repeated modal test analyses. It is worth pointing out that the methodologies presented by Farhat and

Hemez (1993) and Alvin (1997) are special cases of the measure (8) and condiion (9). In particular, both methods correspond to values $\alpha_i = 0$, which fail to directly incorporate tile expected measurement error. In contrast, the proposed inequality condition provides more flexibility in improving considerably the fit between model and measured modal data over the space of the parameter set θ.

The optimization can be performed using available inequality constaint optimization techniques. However, this is a complex and time-consuming nonlinear optimization problem. A more convenient iterative procedure is proposed next which avoids some of the computational difficult ies arising in the minimization of (8). In addition, the iterative procedure provides guidance in identifying the locations of damage and limiting the number of the parameters to be updating to only a few, thus reducing the problem of ill-conditioning and non-uniqueness expected when a large number of parameters is updated. Specifically, the parameters and the modeshapes are obtained using a three-step procedure. In the first step, a set of complete modeshapes is obtained by a model expansion method. In the second step, damage is localized in the structure using an appropriately defined clement strain energy error measurement to identify faulty elements. In the third step, the size of probable damage is predicted by updating the properties of the identified faulty elements using the set of expanded modeshapes obtained from the first stage. These three steps arc described in detailed as follows.

Step 1: Modeshape Expansion

Given the current model of the structure at the k-th iteration step, corresponding to the value of the parameter set θ, designated by $\hat{\theta}^{(k)}$, an expanded modeshapes is computed by solving the constrained minimization problem:

$$\min_{\phi_i} \sum_{i=1}^{m} \left\| (K(\hat{\theta}^{(k)}) - \tilde{\omega}_i^2 M(\hat{\theta}^{(k)}))\phi_i \right\|_{R_i} \tag{15}$$

subject to

$$P\phi_i - \tilde{\phi}_{ai} \;^2 \leq \alpha_i \;\; \tilde{\phi}_{ai} \;^2, \quad i = 1, \cdots, m \tag{16}$$

The minimization is performed with respect to the modeshape components at both measured and unmeasured degrees of freedom while holding the values of the model parameters θ fixed at their current values $\hat{\theta}^{(k)}$. Both the objective function and the inequality constrains are quadratic in the set of unknown parameters. It can be shown that a unique optimum exists (Levine *et al* 1997), denoted herein by $\hat{\phi}_i^{(k+1)}, i = 1, \cdots$, m. The algorithm for obtaining the unique solution is described in the work by Levine *et al* (1996, 1997).

Step 2: Location of Damage

The expanded modeshapes predicted in the first step are used to identify possible locations of damage by examining, for each finite element (or substructure), the difference in element strain energy between the expanded mode-shapes and the model mode-shapes. The modal element (or substructure) strain energy for a finite element (or substructure) designated by < (> is defined as

$$S^e(\phi_i) = \frac{1}{2}\phi_i^{eT} K^e \phi_i^e \tag{17}$$

where K^e is the element or substructure stiffness, and ϕ_i^e is the components Of the modeshape ϕ_i corresponding to the degrees of freedom of the element or the substructure. The following measure of modal strain energy error is used

$$\Delta S^e = \frac{S^e(\hat{\phi}_i^{e(k+1)}) - S^e(\varphi_i^{e(k)})}{S^e(\hat{\phi}_i^{e(k+1)})} \tag{18}$$

where $\varphi_i^{(k)} = \varphi_i(\theta^{(k)})$ is the modeshape computed form the current, structural model. It is expected that sufficiently large ΔS^e will be due to modeling errors in the particular element (or substructure) and will be indicative of probable damage in the element (or substructure). The properties of these elements (or substructures) are chosen to be updated if $|\Delta S^e| > tol_1$, where tol_1 is a user-specified threshold. These properties form an active subset of the parameter set θ, designated by $\theta_{act}^{(k+1)}$. The rest of the parameters in θ that are not included in the active set form the inactive set, designated by $\theta_{in}^{(k)}$. The properties of the finite element model included in the active set $\theta_{act}^{(k+1)}$ may differ from those in the set $\theta_{act}^{(k)}$ obtained form the previous iteration.

Step 3: Size of Damage

The stiffness, mass and geometrical properties at the identified probable locations of significant errors in the properties of the finite element model of the structure are updated using the latest estimates $\hat{\phi}_i^{(k+1)}$, $i = 1, \cdots, m$ of the complete modeshapes. This step involves the updating the values of the active model parameters θ_{act} identified in step 2. The values of the inactive set are kept constant and equal to those in the set $\theta^{(k)}$ determined in the previous iteration. The reduction in the values of θ_{act} of the parameter set is indicative of possible damage. The following unconstrained minimization problem:

$$\min_{\theta_{act}^{(k+1)}} J(\theta^{(k+1)}) = \min_{\theta_{act}^{(k+1)}} r(\tilde{\omega}_i, \hat{\phi}_i^{(k+1)}, \theta^{(k+1)}) \tag{19}$$

$$= \min_{\theta_{act}^{(k+1)}} \sum_{i=1}^{m} \|(K(\theta^{(k+1)}) - \tilde{\omega}_i^2 M(\theta^{(k+1)}))\hat{\phi}_i^{(k+1)}\|^2_{R_i} \tag{20}$$

gives the optimal values $\hat{\theta}_{act}^{(k+1)}$. This is a nonlinear optimization problem which can be solved using available iterative schemes such as modified Newton method. For the case for which $K(\theta)$ and $M(\theta)$ are linear function of θ and R_i is independent of θ, the objective function $J(\theta)$ is quadratic in θ and the unique solution $\hat{\theta}_{act}^{(k+1)}$ can be obtained without iterations by solving a linear algebraic system in θ. It can be readily shown that the system for the optimal $\hat{\theta}_{act}^{(k+1)}$ has the form:

$$H(\hat{\Phi}^{(k+1)})\, \hat{\theta}_{act}^{(k+1)} = h(\hat{\Phi}^{(k+1)}) \tag{21}$$

where the (j,k) component of H and the j-th component of h are given by

$$H_{jk}(\Phi) = \sum_{i=1}^{m}(L_{ik}\phi_i)^T R_i(L_{ij}\phi_i) \tag{22}$$

and

$$h_j(\Phi) = \sum_{i=1}^{m}(L_{i0}\phi_i)^T R_i(L_{ij}\phi_i) \tag{23}$$

respectively, in which $L_{ij} = K_j - \hat{\omega}_i^2 M_j$, $j = 0, 1, \cdots, n$.

Since tile updated finite element model contains inaccuracies clue to the fact that the expanded modeshapes are based on a current inaccurate model, the three step procedure is repeated using the new updated finite element model until convergence is reached. Specifically, the iterative process is terminated when

$$\left\| \frac{\hat{\theta}^{(k+1)} - \hat{\theta}^{(k)}}{\hat{\theta}^{(k+1)}} \right\| < tol_2$$

where tol_2 is a user-specified threshold level.

APPLICATIONS

The model updating technique has been implemented in matlab to enhance the capabilities of the existing integrated Modeling of Optical Systems (IMOS) software package developed at Jet Propulsion Laboratory. Simulated modal data generated from a ~llulti-degree-c)f-frccc]o~~l truss structures are used to assess the performance of the proposed model updating methodology in relation to the number of measured modes, number and location of sensors, as well as magnitude and location of errors in the properties of the initial finite element model. The role of measurement and modeling errors on the resolution of the location and size of errors in the properties of the finite element model is also examined.

The model of the undamaged structure is a three-dimensional truss shown in Figure 1. It consists of 135 axial rod elements (1 per strut) with total of 120 degrees of freedom (3 per node). The structure is supported at the right end by restraining all degrees of freedom of nodes 1 to 4. The modulus of elasticity, cross-sectional area and mass density for all members are chosen to be the same and with values such that the first eight modal frequencies of the model range from 10 Hz to 200 Hz. The element 63, 72 and 108, located at different places on the structure as shown in Figure 1 are damaged by reducing the cross-sectional area of these clement by 50%. Simulated modal data are obtained for the damaged structure and are contaminated by 1 % and 5% noise level in the values of the modal frequencies and mode-shapes respectively. The first eight modes of the damaged structure are taken as the measured modes.

Two cases are used to assess the performance of the method in relation to the number and location of sensors. In the first case, designated by Case A, a large number of 99 sensors are used. These sensors are places at nodes 5 through 37 and provide measurements for all three degrees of freedom for each *node.* For the second case, designated by Case B, only 15 sensors are used. A set of three sensors is placed at each of the nodes 5, 13, 21, 29 and 37 to provide measurements for all three degrees of freedom per node. The properties in the parameter set θ to be updated are the cross- sectional area of each member. The methodology was slightly modified to consider as acceptable changes in the values of the parameter set θ only those which correspond to reduction in the closs-sectional area of the members. The predicted location and size of damage is shown in Figures 2 and 3 for the cases A and B, respectively. For each case, the mean and the standard deviation of the prediction is shown. Only five samples were used in the estimates provided in these figures.

For case A, the predicted mean reductions in the cross-sectional area of members 63, 72 and 108 are 45%, 28% and 52%, respectively. These members have the highest mean reduction in cross-sectional area. The relatively small values of the standard deviation of these estimates is indicative of the relatively high confidence that damage has occurred in these members. In contrast, the standard deviation estimates of the rest of the members with non-zero mean reduction of cross-sectional area are relatively large. This is due to the fact that only a small percentage of the samples have resulted in non-zero reduction in cross-sectional area of these members. Specifically, 3 to 4 out of the 5 samples predicted no reduction or almost insignificant reduction in the cross-sectional area for these members. The small size of samples used resulted in relatively high mean reduction values. It is expected that as the number of samples is increased, the mean values and the standard deviation for these members will decrease. For case B, the predicted mean reductions in the cross-sectional area of members 63, 72 and 108 are 58%, 32% and 33%, respectively. It is worth noting that the resolution of size of damage at element 108 is not as good as for the Case A since sensors are not directly placed in the vicinity of the member 108. However, the elements 63, 72 and 108 have been correctly identified as the damaged elements.

Numerical results have also shown that the accuracy of the prediction increases as the number of measured modes increases, or as the level of the measurement error decreases. Location of number of sensors play also a role in the resolution of location and size of damage.

CONCLUSIONS

The proposed model updating methodology is suitable for damage detection purposes. It is based on an iterative scheme which provides estimates of probable locations and size of damage by updating the properties of the finite element model at the element level. The identification of the probable locations of damage is based on element strain energy error ~lwa.sure between the expanded modeshapes predicted at a given iteration and model modeshapes predicted from the previous iteration. The size of the damage is then updated using the predicted expanded mode-shapes. These estimates are iteratively updated until convergence is reached. A study using simulated data demonstrated that the methodology is promising for reliably predicting both the location and the size of damage in a structure. Measurement error was incorporated in the data by adding noise in the simulated data. The noise levels considered are similar to those expected in practical applications. Although the method suggested herein works well with simulated modal data and simulated measurement error, the practical use of this method with real data requires further study.

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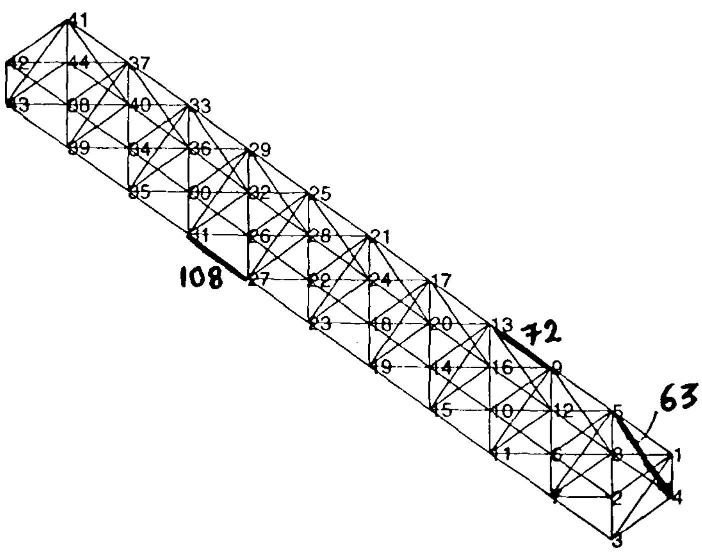

Fig. 1. Three-dimensional truss structure

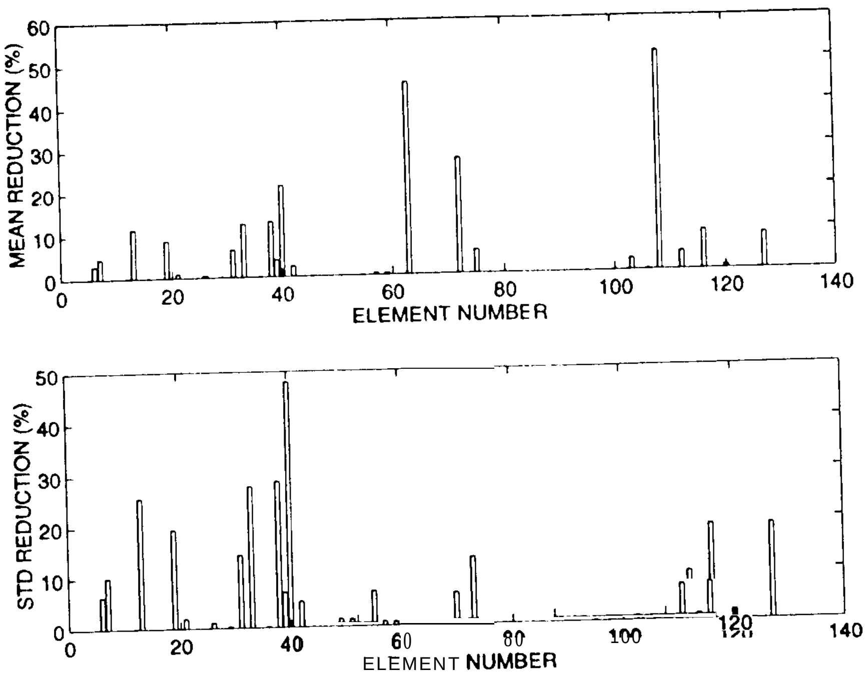

Fig. 2. Predicted location and magnitude of damage; Case A

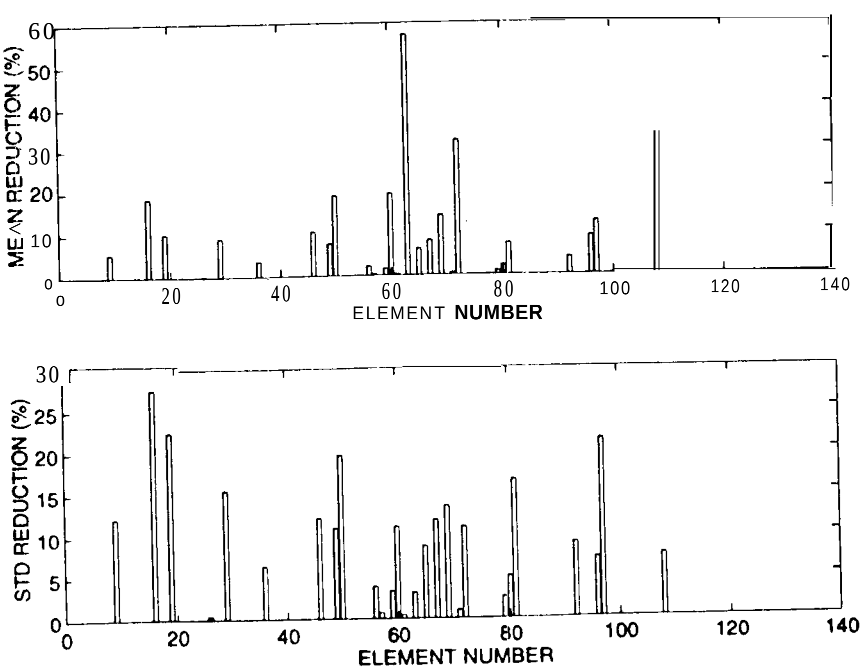

Fig. 3. Predicted location and magnitude of damage; Case B