Semileptonic Decays of D, B mesons and CKM determinations from Unquenched Lattice QCD

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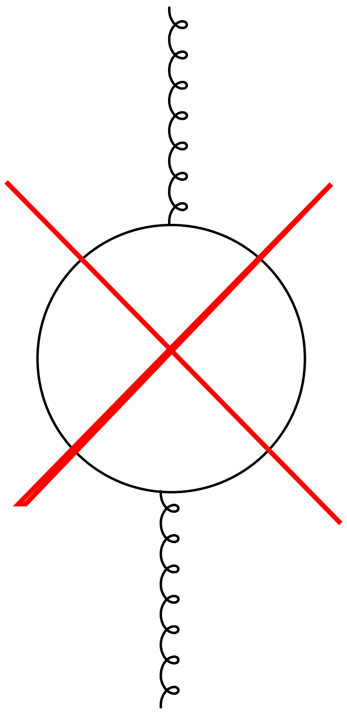
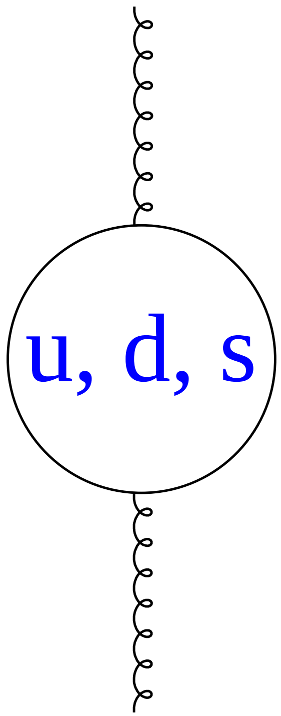
for Fermilab+MILC collaboration

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \end{pmatrix}$$

Status of Lattice QCD

Past $\Longrightarrow$ Now

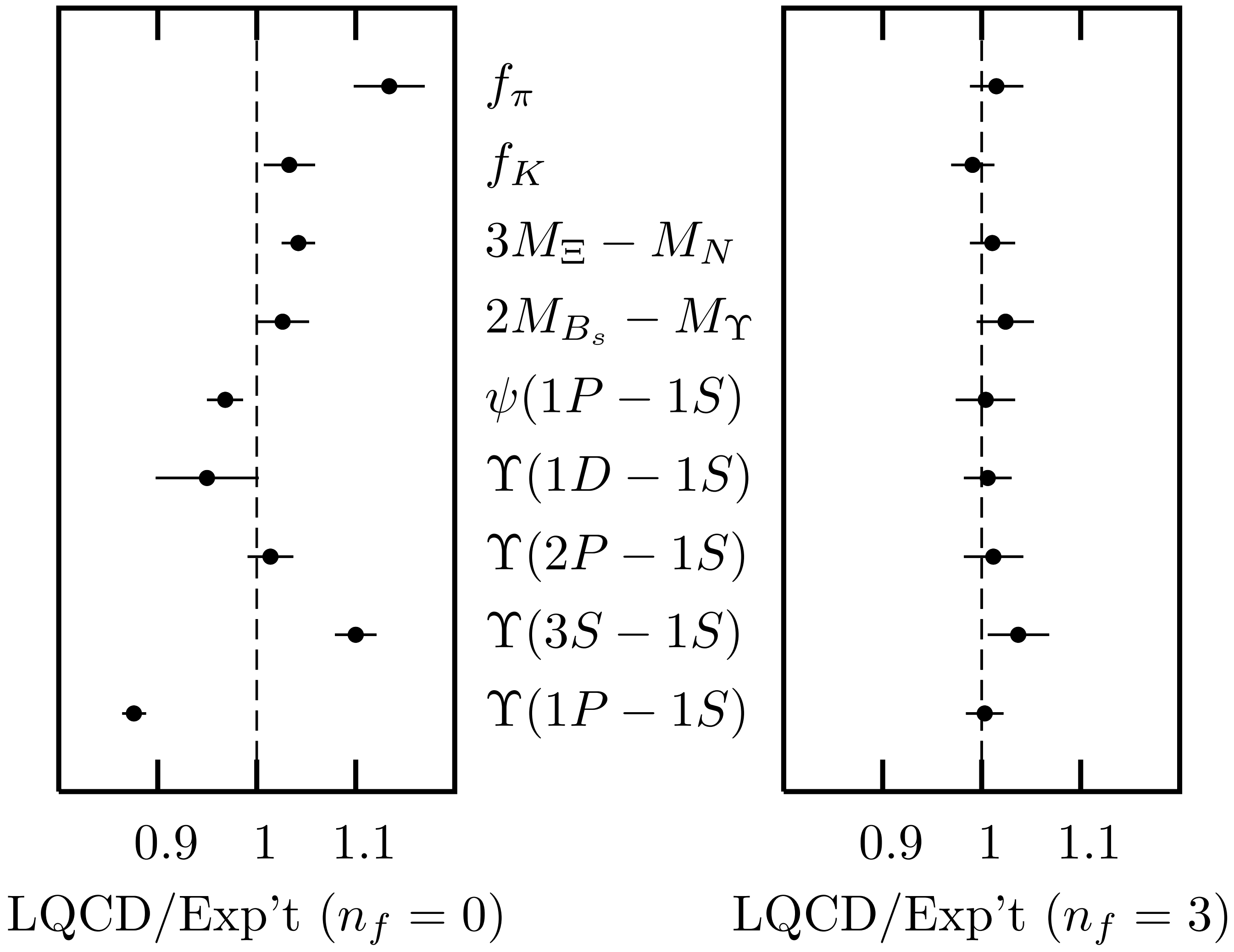




$n_f = 0$, "Quenched"

$n_f = 3$, "Unquenched"

(HPQCD+UKQCD+MILC+Fermilab Collab., PRL92,2004)



$n_f = 0$: $\approx$ 10% deviations from Exp't

$\Longrightarrow$ $n_f = 3$: agree to Exp't within 2-3%.

Next step :

Determination/Check of $\mathbf{V}_{\mathrm{CKM}}$
with $n_f = 3$ Lattice QCD

$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{pmatrix}
$$

Most $\mathbf{V}_{\mathrm{CKM}}$ can be determined from Lattice QCD + Experiment.

Next step :

Determination/Check of $\mathbf{V}_{\mathrm{CKM}}$
with $n_f = 3$ Lattice QCD

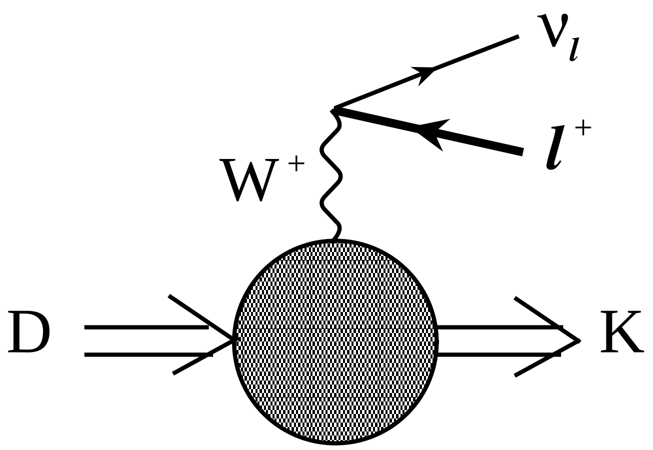
$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle &
\end{pmatrix}
$$

5 $\mathbf{V}_{\mathrm{CKM}}$ elements can be determined from semileptonic decays.

One example :

$\mathbf{V_{cs}}$ from semileptonic decay $D \to Kl\nu$

Experiment



$$\equiv \ \Gamma(D \to Kl\nu) \ \propto \ \int dq^2 \, |f_+(q^2)|^2 |\mathbf{V_{cs}}|^2$$

$$(q = p_D - p_\pi)$$

Lattice

$$\langle \pi(p_\pi) | V^\mu | D(p_D) \rangle \ = \ f_+(q^2) \left[p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2} \, q \right]^\mu + f_0(q^2) \, \frac{m_D^2 - m_\pi^2}{q^2} \, q^\mu$$

Error for $\mathbf{V}_{\rm CKM}$ now (PDG2002 etc.)

from B-physics

$$\delta\mathbf{V}_{\mathbf{ub}} \sim 20\% \ , \ \delta\mathbf{V}_{\mathbf{td}} > 10\% \ , \ \delta\mathbf{V}_{\mathbf{ts}} \sim \ ?\ \%$$

from D-physics

$$\delta\mathbf{V}_{\mathbf{cd}} \sim \ 7\% \ , \ \delta\mathbf{V}_{\mathbf{cs}} \sim \ 1\% \ , \ \delta\mathbf{V}_{\mathbf{cb}} \sim \ 5\%$$

$\mathbf{V}_{\rm CKM}$ from D's are more accurate than B's

CLEO-c exp't for D's (leptonic/semileptonic rates to 2-3%)

$$\Downarrow$$

D-physics is a good test ground for LQCD.

Our plans

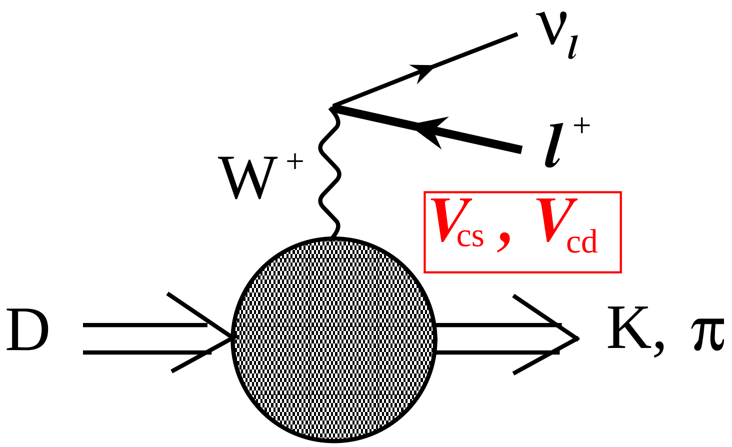
(1) **test** $n_f = 3$ **LQCD for** D**-physics**

(2) **predict** B**-physics with** $n_f = 3$ **LQCD**

This talk

First $n_f = 3$ **LQCD calculations for semileptonic decays of** D **mesons**

$\underline{D \to K l \nu \;\; , \;\; D \to \pi l \nu}$



and similarly for B mesons

$\underline{B \to \pi l \nu \;\; , \;\; B \to D l \nu}$ (PRELIMINARY)

$\Longrightarrow$ $|\mathbf{V_{cs}}|, |\mathbf{V_{cd}}|, |\mathbf{V_{ub}}|, |\mathbf{V_{cb}}|$ (**4** CKM from $n_f = 3$ LQCD!)

"Our" CKM matrix with $n_f = 3$ LQCD
(from semileptonic decays)

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \text{N/A} & \text{N/A} & 3.6(5)(4)(3)\times 10^{-3} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ 0.24(1)(2)(2) & 0.97(4)(8)(2) & 3.8(1)(1)(6)\times 10^{-2} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \text{N/A} & \text{N/A} & \text{N/A} \end{pmatrix}$$

value(stat)(syst)(expt)

4/**9** being determined with $n_f = 3$ LQCD.

Rest of this talk

- Lattice fermions
- How to compute $\langle\pi|V^\mu|D\rangle$ on the lattice
- Simulation parameters
- Extrapolation to u, d quark masses ($m_l \to m_{ud}$)
- $D \to \pi$ and $D \to K$ results
- Systematic errors
- ($B \to \pi$ and $B \to D$ results)

Lattice fermions

(1) "Naive" fermion

Simplest discretization of fermion:

$$\mathcal{L}_{\text{Naive}} = \bar{\psi}(x)(\gamma \cdot \Delta + m)\psi(x)$$

$$\Delta_\mu \psi(x) = \frac{1}{2a}\left[\psi(x + a_\mu) - \psi(x - a_\mu)\right]$$

Momentum space propagator:

$$aG^{-1}(p) = \gamma \cdot \sin(ap) + am$$

$2^4 = 16$ poles ("doubling"), rather than 1 pole, because

$$\sin^2(ap + \pi_\mu) = \sin^2(ap) \qquad (\mu = 0, 1, 2, 3)$$

(2) Wilson fermion

Remove unwanted 15$(= 16 - 1)$ poles by adding a 2nd derivative:

$$\mathcal{L}_{\rm Wilson} = \mathcal{L}_{\rm Naive} \; + \; \frac{1}{2}\, \bar{\psi}(x)\Delta^2\psi(x)$$

Momentum space propagator:

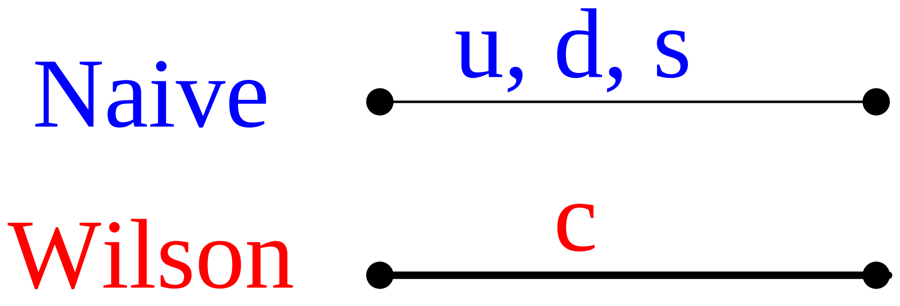
$$aG^{-1}(p) = \gamma \cdot \sin(ap) + am + 2\sin^2\left(\frac{ap}{2}\right)$$

$$\sin^2\left(\frac{ap + \pi_\mu}{2}\right) \neq \sin^2\left(\frac{ap}{2}\right)$$

$\Longrightarrow$ 15 poles get higher mass: $m + \frac{2}{a}\, n_\pi$

$\Longrightarrow$ only 1 pole has lowest energy.

In simulation, we use:



Wilson c quark removes unwanted 15 poles in heavy-light meson ($D = \bar{u}c$).

$$M_D \approx m_u + m_c, \;\; m_u + m_c + \frac{2}{a}, \;\; m_u + m_c + \frac{4}{a}, \cdots$$

But, why Naive light quark ? $\Longrightarrow$ CPU time.

CPU time $\propto 1/m_q$.   (very slow for u, d quarks)

(3) "1-spin" Naive fermion

Reduce CPU time by "spin diagonalization"

$$\psi(x) = \Omega(x)\chi(x) \ , \ \ \bar{\psi}(x) = \bar{\chi}(x)\Omega^{\dagger}(x) \tag{1}$$

$$\Omega(x) = \gamma_0^{x_0}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}$$

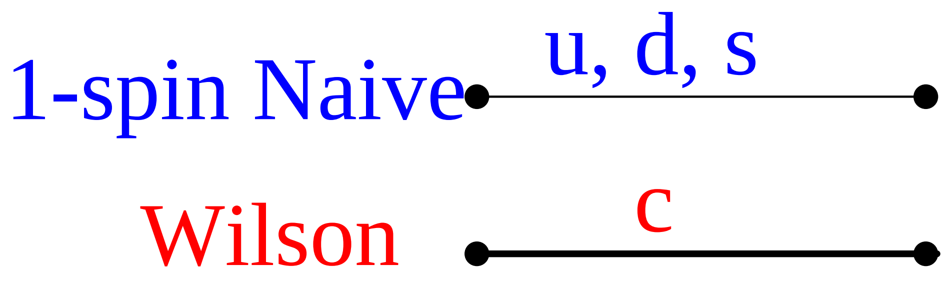
$$\bar{\psi}(x)(\gamma \cdot \Delta + m)\psi(x) = \bar{\chi}_\alpha(x)(\eta(x) \cdot \Delta + m)\delta_{\alpha\beta}\chi_\beta(x)$$

Then, drop spin index

$$\chi_\alpha \ \ (\alpha = 1-4) \ \ \Longrightarrow \ \ \chi$$

1-spin χ is much faster than 4-spin ψ in simulation

In simulation, we *actually* use:

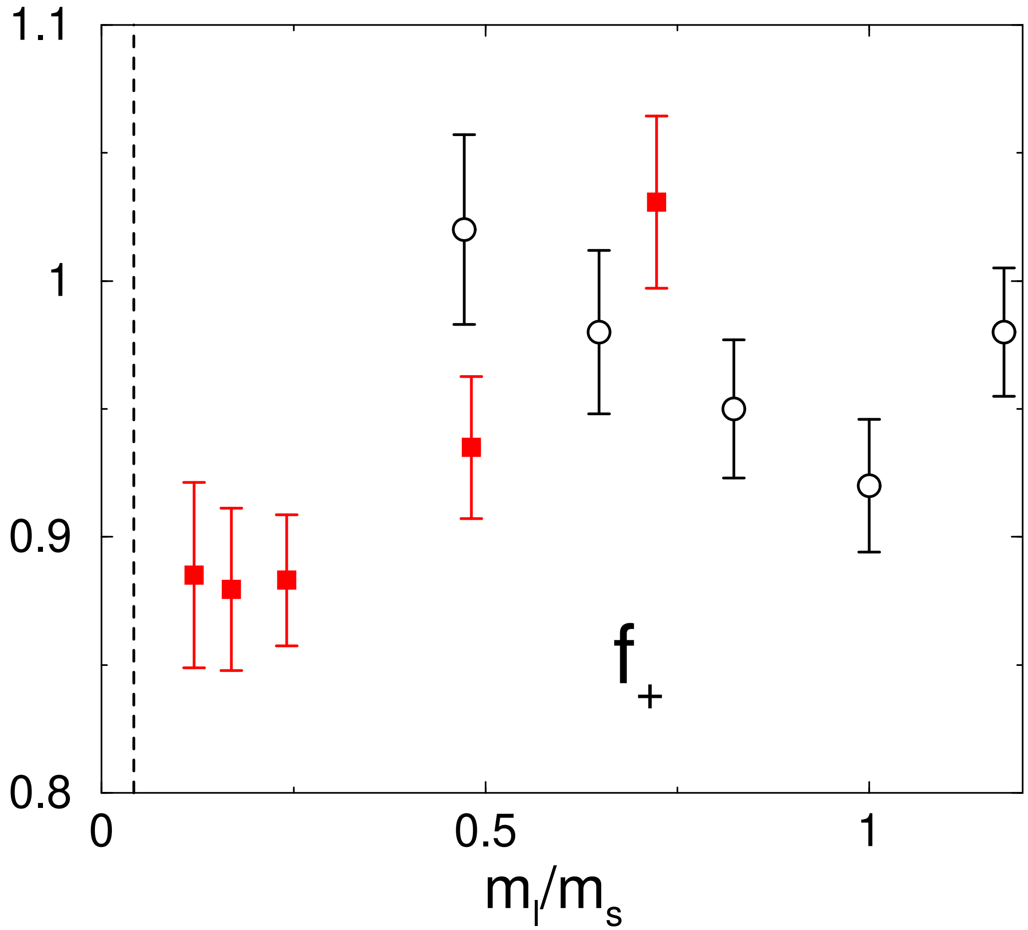


1-spin Naive u, d, s quarks are much faster than 4-spin Wilson u, d, s quarks.

$\Longrightarrow$ much lighter m_q $(< m_s/2)$ is accessible.

$\Longrightarrow$ more reliable extrapolation to $m_{u,d}$

How close can we reach to real m_{ud} quark mass ?



Wilson light [FNAL'01] $m_l/m_s \geq 1/2$ error $\approx$ 10%

Naive light [this work] $m_l/m_s < 1/2$ error $\approx$ 3%

How to compute $\langle \pi | V_\mu | D \rangle$

$D \to \pi$ 3-pt function :

$$C_3^{D\to\pi}(t_x, t_y) \quad \equiv \quad \langle \mathcal{O}_\pi(0) V_\mu(t_y) \mathcal{O}_D^\dagger(t_x) \rangle$$

$$\xrightarrow{t_x \gg t_y \gg 0} \quad \langle \mathcal{O}_\pi | \pi \rangle \frac{e^{-E_\pi t_y}}{2E_\pi} \langle \pi | V_\mu | D \rangle \frac{e^{-E_D(t_x - t_y)}}{2E_D} \langle D | \mathcal{O}_D \rangle$$

where

$$\mathcal{O}_D(t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \; \bar{c}(x)\gamma_5 u(x), \qquad V_\mu(t) = \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \; \bar{c}(y)\gamma_\mu d(y)$$

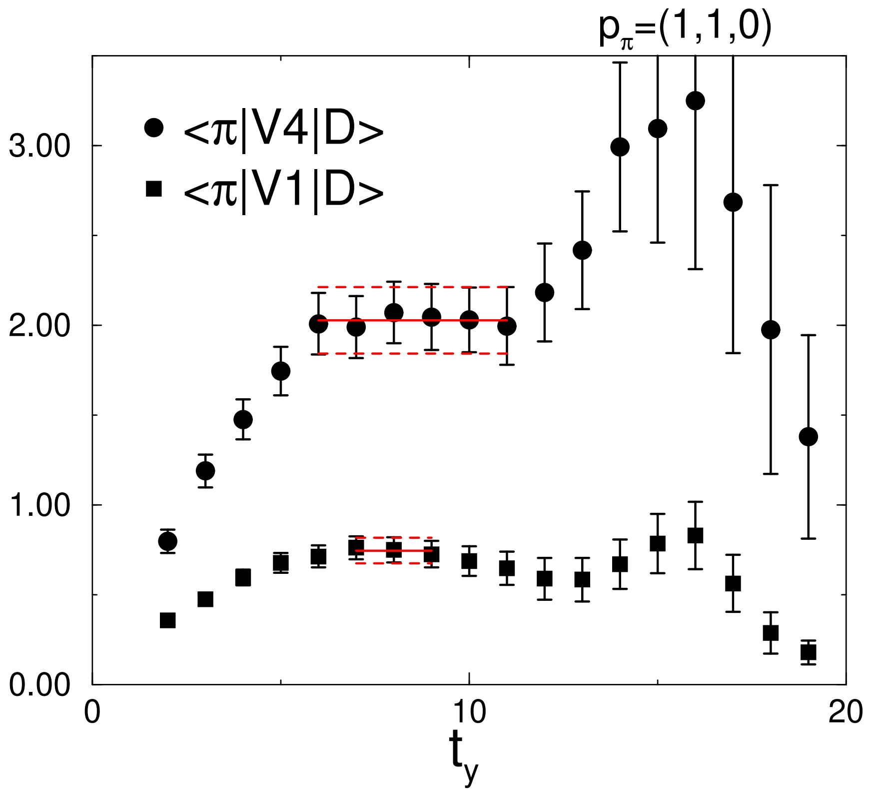
D, π 2-pt functions :

$$C_2^D(t) \;\equiv\; \langle \mathcal{O}_D(t) \mathcal{O}_D^\dagger(0) \rangle \quad \xrightarrow{t \gg 0} \quad \frac{e^{-E_D(t)}}{2E_D} |\langle \mathcal{O}_D | D \rangle|^2$$

$$\langle \pi | V_\mu | D \rangle \overset{t_x \gg t_y \gg 0}{\sim} \frac{C_3^{D\to\pi}(t_x, t_y)}{C_2^\pi(t_y)\ C_2^D(t_x - t_y)}$$

Example

$t_x = 20$ fixed. t_y varied $\Longrightarrow$ constant fit in t_y



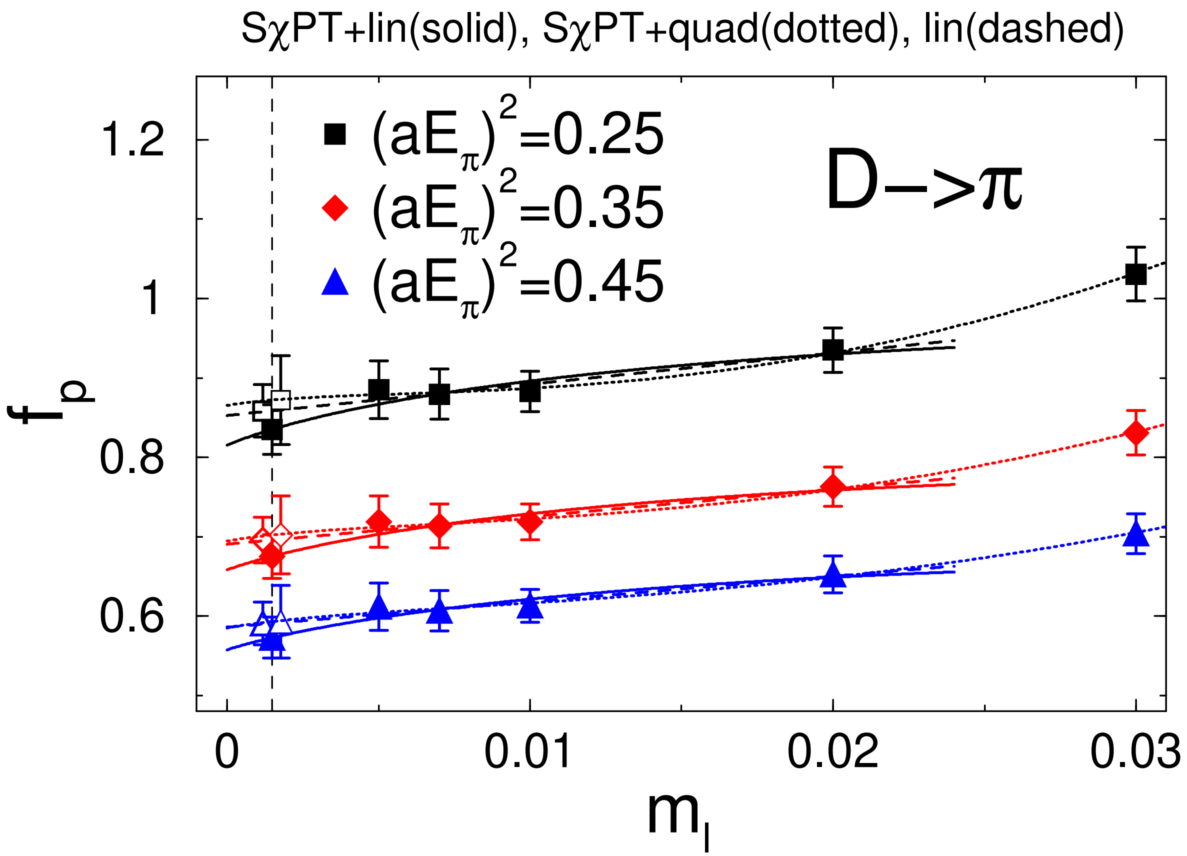
Simulation parameters

$n_f = 3$ unquenched lattice by MILC collab.

- Lattice spacing: $a \approx 1/8$ fm
- Lattice size: $L \times T = 20^3 \times 64$ ($La \approx 2.5$ fm)

- m_s, m_c fixed from K, D_s masses.
- m_u, m_d ($\approx \frac{1}{24} m_s$) extrapolated from $m_q \leq m_s/2$.
(*c.f.* previous studies: $m_q \geq m_s/2$)

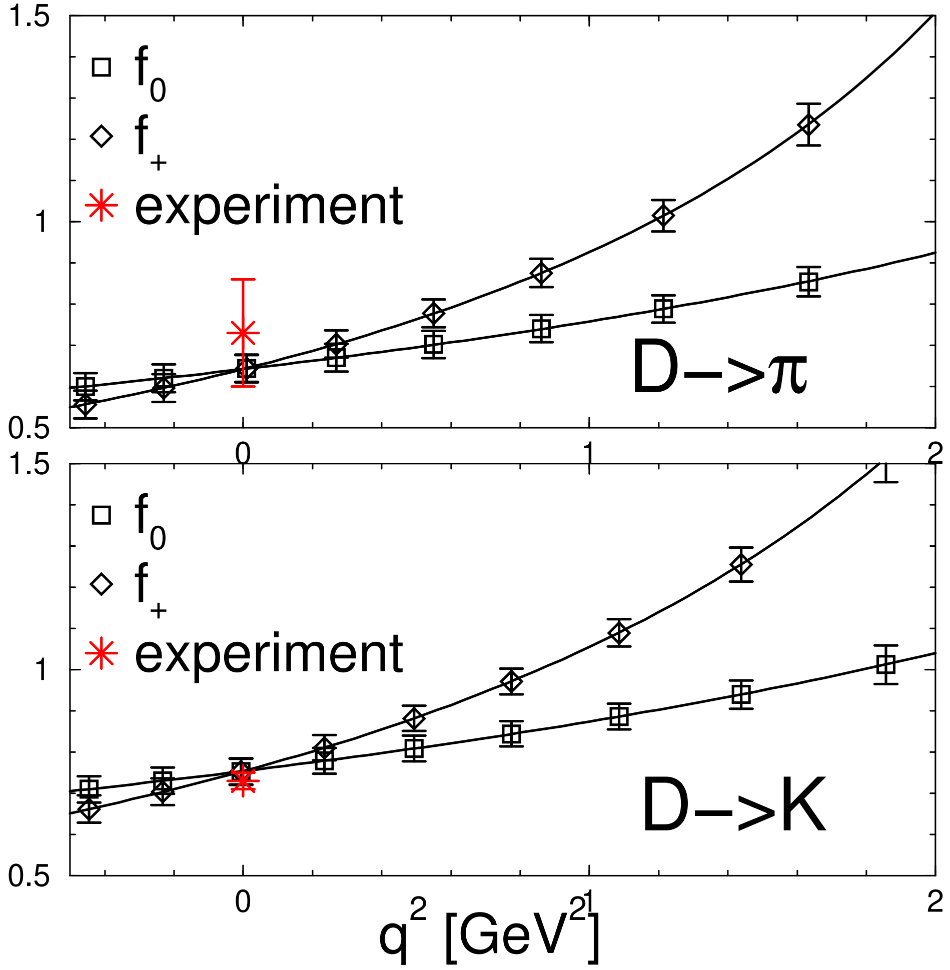
Exptrapolation of $m_l \to m_{ud}$ ("chiral fit")



χPT+lin fit (solid)	$f = A(1 + \delta f^{\chi PT}) + Bm_l$	central value
χPT+quad fit (dotted)	$f = A(1 + \delta f^{\chi PT}) + Bm_l + Cm_l^2$	+(0-3)%
linear fit (dashed)	$f = A + Bm_l$	+3%

Main results

$$\langle\pi|V^\mu|D\rangle \quad = \quad f_+(q^2)\left[p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2}\, q\right]^\mu \;+\; f_0(q^2)\,\frac{m_D^2 - m_\pi^2}{q^2}\, q^\mu$$



$$f_+^{D\to\pi}(0) = 0.64(3)() \qquad , \qquad f_+^{D\to K}(0) = 0.73(3)()$$

Systematic errors

- Lattice actions $(\alpha_s = 0.3,\ \Lambda = 0.4\ GeV)$

  Naive u, d, s quarks $\mathcal{O}(\alpha_s a^2 \Lambda^2) \approx 2\%$

  Wilson c quark $\Longrightarrow\ \mathcal{O}(\alpha_s a \Lambda, a^2 \Lambda^2) \approx 7\% \approx \mathcal{O}(\alpha_s \frac{\Lambda}{m_c}, \frac{\Lambda^2}{m_c^2})$

- χfits : 3 fits [SχPT fits, linear] differ by 3%.

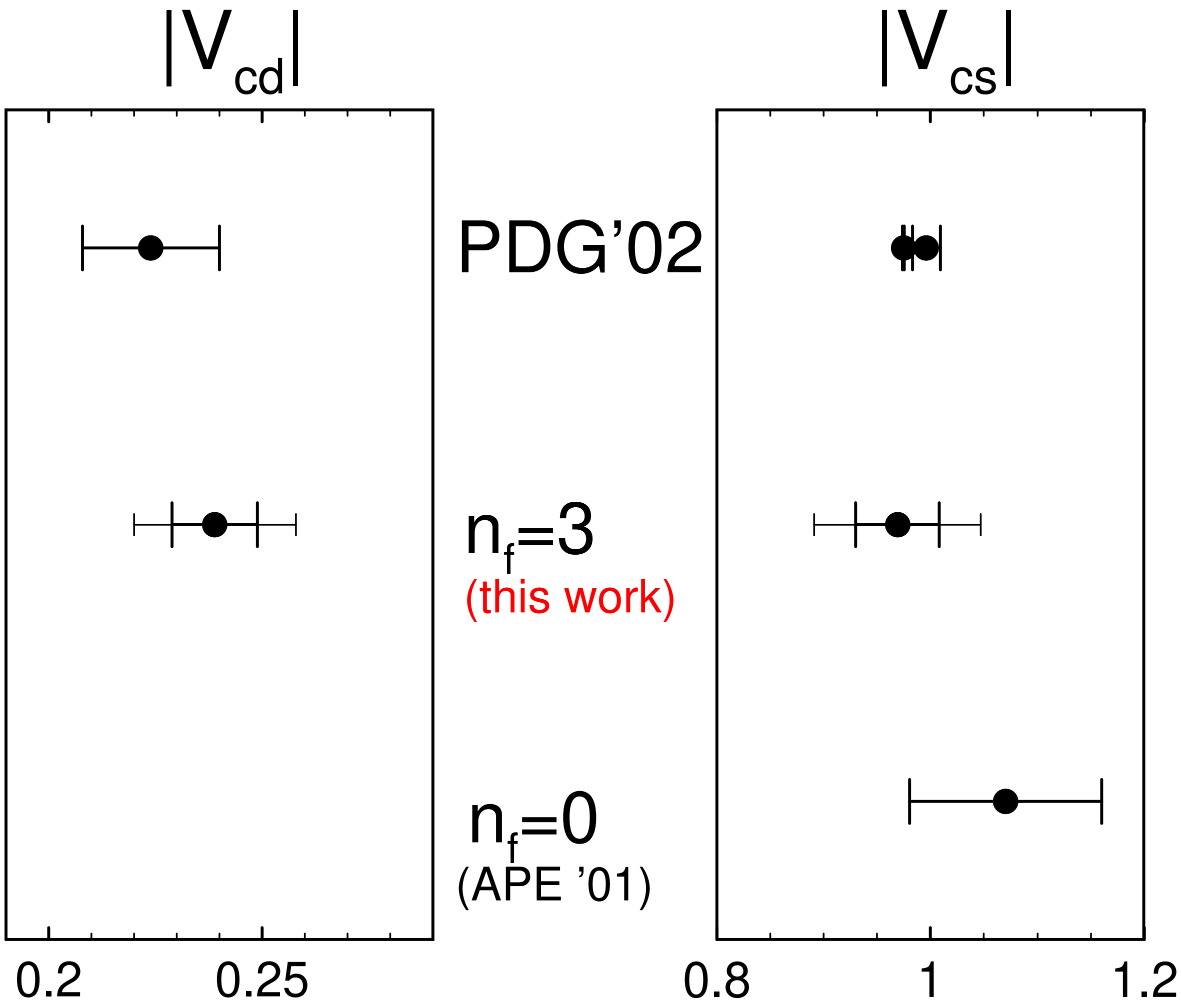
- other errors : 3-pt fitting (3%), a^{-1}(2%)

TOTAL systematic error $\approx (7 + 3 + 3 + 2)\% \approx 8\%$

$$f_+^{D\to\pi}(0) = 0.64(3)(5)\ ,\quad f_+^{D\to K}(0) = 0.73(3)(6)$$

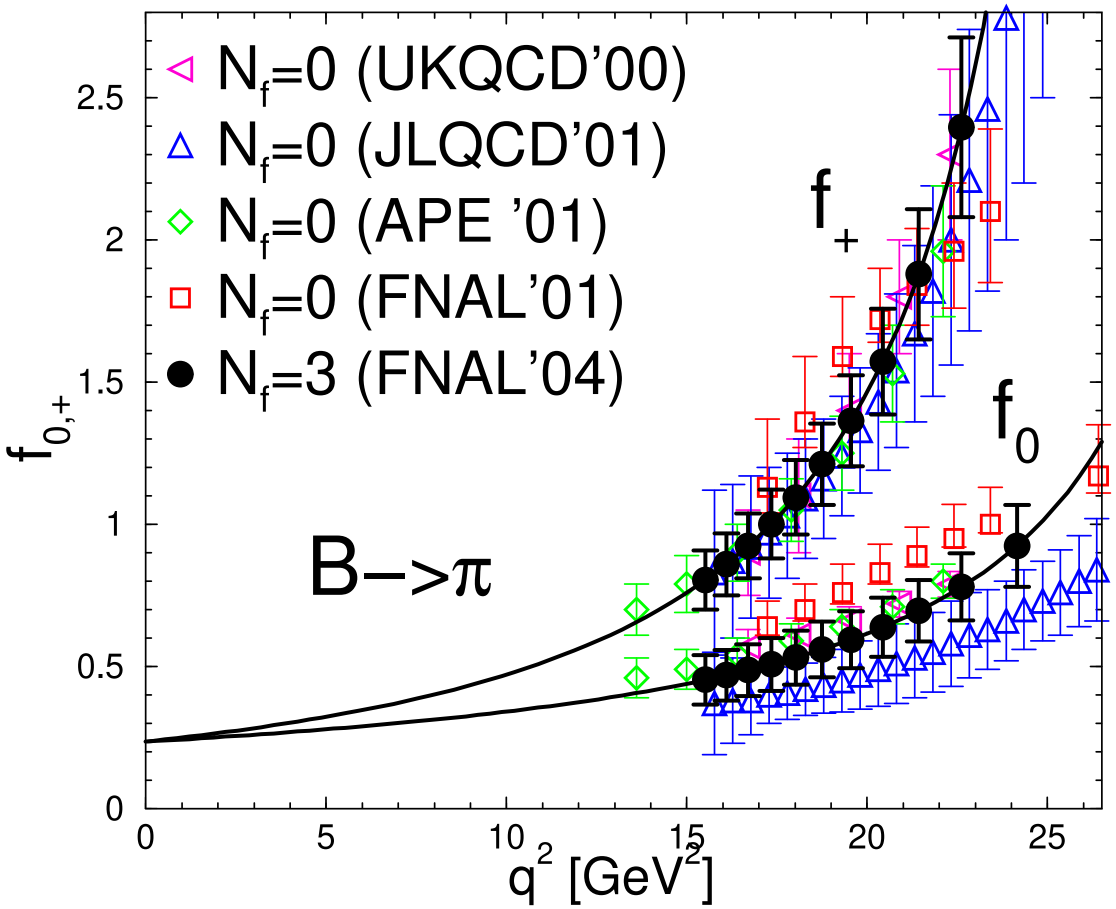
CKM matrix

$|V_{cq}|^2 = \frac{\Gamma_{\rm exp}}{(\Gamma/|V_{cq}|^2)_{\rm lat}}$, $(\Gamma/|V_{cq}|^2)_{\rm lat} \propto \int dq^2 |f_+(q^2)|^2.$



$$|V_{cd}| = 0.239(10)(19)(20) \quad , \quad |V_{cs}| = 0.969(39)(78)(24)$$

$B \to \pi$ results (preliminary)



Using CLEO's branching ratio

$$|V_{ub}| \times 10^3 = 2.88(36)(29)(59) \quad \text{with } \mathcal{B}(q^2 \geq 16\ \mathrm{GeV}^2)$$

$$|V_{ub}| \times 10^3 = 3.60(52)(36)(30) \quad \text{with } \mathcal{B}(\text{all } q^2)$$

$B \to Dl\nu$ **decay**

$$\langle D|V^\mu|B\rangle = \sqrt{m_B m_D} \times [h_+(w)(v_B + v_D)^\mu + h_-(w)(v_B - v_D)^\mu],$$

where $w = v_B \cdot v_D$.

$$\begin{aligned} \frac{d\Gamma(B \to Dl\nu)}{dw} &\propto |\mathcal{F}_{B\to D}(w)|^2 |\mathbf{V_{cb}}|^2 \\ \mathcal{F}_{B\to D}(w) &= h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w). \end{aligned}$$

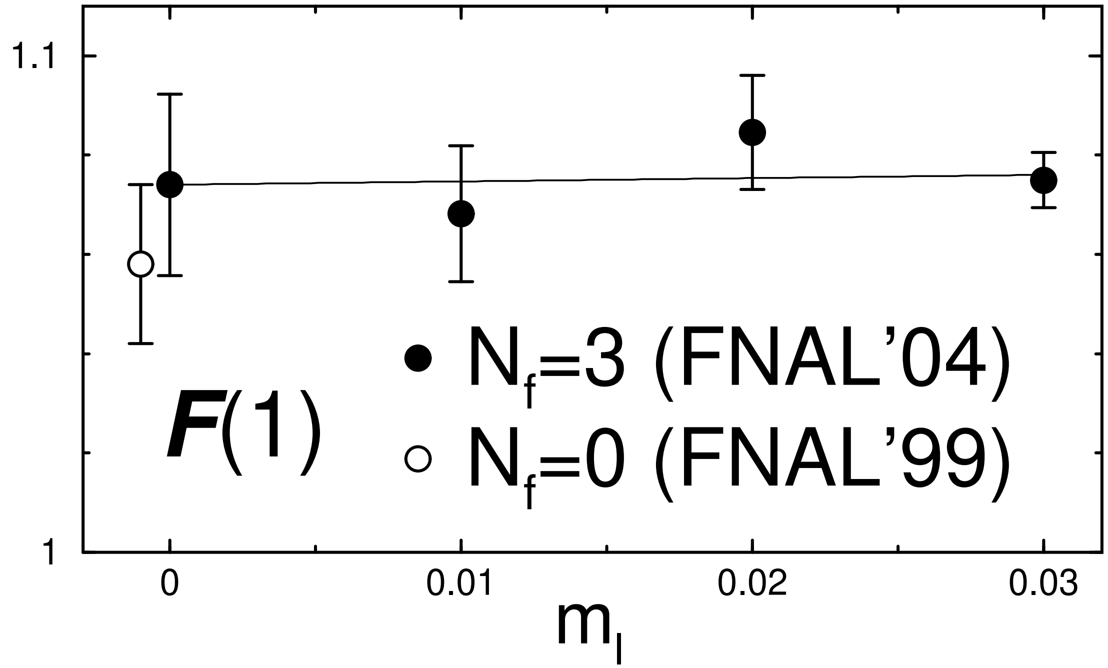
We focus on the zero recoil limit ($w = 1$).

Ratio method (S. Hashimoto *et.al.* '99)

$$\frac{C^{DV_0B}(t)C^{BV_0D}(t)}{C^{DV_0D}(t)C^{BV_0B}(t)} \to \frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle} = |h_+^{B\to D}(1)|^2$$

$\mathcal{F}(1) = 1$ in $B = D$ limit.

$B \to D$ results (preliminary)



$$\mathcal{F}^{n_f=3}_{B\to D}(1) = 1.074(18)(15)$$

Using Belle's branching ratio

$$|V_{cb}| \times 10^2 = 3.83(07)(06)(64)$$

Summary

First $n_f = 3$ LQCD results for D, B decays.

$\mathbf{D \to K l \nu}$, $\mathbf{D \to \pi l \nu}$

- $f_+^{D\to\pi}(0) = 0.64(3)(5) \quad (\Longrightarrow \mathbf{V_{cd}})$
- $f_+^{D\to K}(0) = 0.73(3)(6) \quad (\Longrightarrow \mathbf{V_{cs}})$

$\mathbf{B \to \pi l \nu}$, $\mathbf{B \to D l \nu}$ (preliminary)

- $f_+^{B\to\pi}(0) = 0.24(3)(2) \quad (\Longrightarrow \mathbf{V_{ub}})$
- $\mathcal{F}_{B\to D}(1) = 1.07(2)(2) \quad (\Longrightarrow \mathbf{V_{cb}})$

"Our" CKM matrix with $n_f = 3$ LQCD
(from semileptonic decays)

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathrm{N/A} & \mathrm{N/A} & 3.6(5)(4)(3)\times 10^{-3} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ 0.24(1)(2)(2) & 0.97(4)(8)(2) & 3.8(1)(1)(6)\times 10^{-2} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \mathrm{N/A} & \mathrm{N/A} & \mathrm{N/A} \end{pmatrix}$$

value(stat)(syst)(expt)

4/**9** being determined with $n_f = 3$ LQCD.

“Our” CKM matrix with $n_f = 3$ LQCD
(from semileptonic decays)

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathrm{N/A} & \mathrm{N/A} & 3.6(5)(4)(3)\times 10^{-3} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ 0.24(1)(2)(2) & 0.97(4)(8)(2) & 3.8(1)(1)(6)\times 10^{-2} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \mathrm{N/A} & \mathrm{N/A} & \mathrm{N/A} \end{pmatrix}$$

value(stat)(syst)(expt)

4/**9** being determined with $n_f = 3$ LQCD.

LQCD unitarity check!

$$(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(4)(8)(2)$$