15

B-$\overline{B}$ Mixing and CP Violation

Observation of Mixing and the CP Program

Just as for K^0 and $\overline{K}^0$, there can be mixing between the B^0 and $\overline{B}^0$. In fact, this is possible for two distinct systems, the non-strange $B_d^0 = \overline{b}d$ and the strange $B_s^0 = \overline{b}s$. The mixing does not require CP violation, but depends only on the existence of common states to which both B^0 and $\overline{B}^0$ can couple. The B^0 favors decays to states like $\overline{D}\pi$, while the $\overline{B}^0$ prefers $D\pi$. Both however, can decay to $D\overline{D}$, or to any state composed of $c\ \overline{d}\ \overline{c}\ d$, albeit as a CKM-suppressed decay. Similarly, they both can decay virtually to $t\overline{t}$. Mixing arises both from real and virtual transitions.

Mixing depends on both the quark masses and the Kobayashi-Maskawa matrix. If the d, s, and b quarks were degenerate in mass, we could redefine them so that the u quark decayed entirely to d, c entirely to s, and t entirely to b. Then the Kobayashi-Maskawa matrix would be the unit matrix and there would be no intermediate quark states possible in Figure 15.1. The effect, then, depends critically on quark mass differences, emphasizing the importance of the heavy quarks.

The measured values of the CKM matrix show that jumping from the first generation to the second is suppressed in amplitude by roughly $\lambda = 0.22$. A second-to-third generation amplitude is reduced by $\lambda^2 \approx 0.05$, while a first-to-third transition is suppressed by roughly $\lambda^3 \approx 0.01$. As a result, it is much easier for B_s to reach the $c\overline{c}$ and $t\overline{t}$ intermediate states, which allow for mixing, than it is for a B_d. The virtual transitions dominate the mass splitting, Δm, which are proportional to $|V_{ts}|^2$ and $|V_{td}|^2$ for the B_s and B_d, respectively. The real transitions to $c\overline{c}$ states contribute to the lifetime difference. The lifetime difference between the two mass eigenstates of the $B_d - \overline{B}_d$ system, which is proportional to $|V_{cd}|^2$ is expected to be very small, but that for B_s, which is proportional to $|V_{cs}|^2$ should be about 10 - 20 % (??) of the B_s lifetime itself and directly observable.

For B_d we can ignore the lifetime difference. The mixing parameter, r, introduced in Chapter 7 as the ratio of wrong-sign semileptonic decays to right-sign

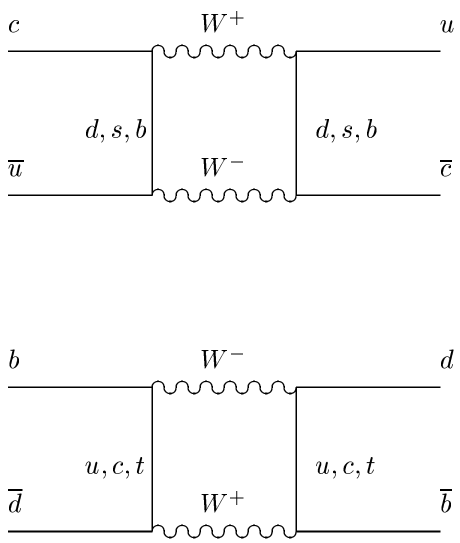

Figure 15.1: The diagrams contributing to mixing of D^0 with $\overline{D}^0$ and B^0 with $\overline{B}^0$. The relative strength of different contributions depends on the Kobayashi-Maskawa matrix and the quark masses. For the D^0, the s quark intermediate is most important and the matrix element is roughly proportional to $m_s^2(V_{cs}^* V_{us})^2$. For $B^0-\overline{B}^0$ mixing the t quark intermediate state dominates and the matrix element is proportional to $m_t^2(V_{tb}V_{td}^*)^2$, giving a result much greater than the matrix element for $D^0-\overline{D}^0$ mixing.

decays thus becomes

$$r = \frac{x^2}{2+x^2}$$

where $x = \Delta M/\Gamma$. An analogous equation holds for $D^0 - \overline{D}^0$ mixing.

If a $B^0\overline{B}^0$ pair is created and both mesons decay semileptonically, the B would be expected to give a positive lepton ($\overline{b} \to \overline{c}l^+\nu$) and the $\overline{B}$ a negative lepton. If there is $B^0-\overline{B}^0$ mixing, it is possible that both leptons will have the same sign. An unfortunate background arises from the chain $b \to c \to Xl\nu$ since the semileptonic decay of the c would give a lepton of the sign opposite that expected from a b decay. While some evidence for $B-\overline{B}$ mixing was found by UA-1 at the Sp$\overline{\text{p}}$S in the same-sign dilepton signal, clear convincing evidence was first obtained in an e^+e^- experiment.

The ARGUS Collaboration working at the Υ'''(**Ref. 15.1**) found one example of $\Upsilon''' \to B_d^0 B_d^0$, as demonstrated by specific semileptonic decays, each with a positive muon. Additional evidence for mixing was obtained by measuring the inclusive like-sign dilepton signal. A third independent measurement came from identifying complete B^0 decays and observing semileptonic decays of the accompanying meson. Finding a positive lepton opposite an identified B^0 is evidence for mixing. Combining the results of these measurements gave $r_d = 0.21 \pm 0.08$. The ARGUS

Collaboration revisited this measurement in 1992 with a data sample more than twice the size of the original one. Using much the same techniques, they confirmed the result with a refined determination: $r_d = 0.206 \pm 0.070$ or $x = 0.72 \pm 0.15$.

The mixing of B^0 and $\overline{B}^0$ is quite analogous to the mixing of K^0 and $\overline{K}^0$ and the mass eigenstates can be found by diagonalizing a matrix just like that considered in Chapter 7 using the analogous convention, $CP|B^0\rangle = |\overline{B}^0\rangle$:

$$\begin{pmatrix} M - i\dfrac{\Gamma}{2} & M_{12} - i\dfrac{\Gamma_{12}}{2} \\ \\ M_{12}^* - i\dfrac{\Gamma_{12}^*}{2} & M - i\dfrac{\Gamma}{2} \end{pmatrix}$$

The lifetime difference and thus Γ_{12} can be neglected for B_d but not for B_s. For B_d mixing then we find that the heavy and light eigenmasses are

$$\mu_H = M + i\frac{\Gamma}{2} + |M_{12}|$$
$$\mu_L = M + i\frac{\Gamma}{2} - |M_{12}|$$

so that the mass splitting is $\Delta m = 2|M_{12}|$. The eigenstates are

$$\begin{aligned} |B_H\rangle &= \frac{1}{\sqrt{2}}\left(|B^0\rangle + \frac{|M_{12}|}{M_{12}}|\overline{B}^0\rangle\right) \\ |B_L\rangle &= \frac{1}{\sqrt{2}}\left(|B^0\rangle - \frac{|M_{12}|}{M_{12}}|\overline{B}^0\rangle\right) \end{aligned}$$

and they evolve simply

$$\begin{aligned} |B_H\rangle &= e^{-i(M+\frac{\Delta m}{2}-i\frac{\Gamma}{2})}|B_H\rangle \\ |B_L\rangle &= e^{-i(M-\frac{\Delta m}{2}-i\frac{\Gamma}{2})}|B_L\rangle \end{aligned}$$

These states are analogous to K_L and K_S, except that the lifetime difference is ignored. If M_{12} were real, these states would be CP eigenstates; when M_{12} is complex CP is broken. A state that at $t = 0$ is purely B^0, will oscillate into $\overline{B}^0$:

$$|B^0_{phys}(t)\rangle = e^{-i(M-i\Gamma/2)t}\left[\cos\frac{\Delta m}{2}t\;|B^0\rangle - i\frac{|M_{12}|}{M_{12}}\sin\frac{\Delta m}{2}t\;|\overline{B}^0\rangle\right]$$

while its counterpart behaves as

$$|\overline{B}^0_{phys}(t)\rangle = e^{-i(M-i\Gamma/2)t}\left[\cos\frac{\Delta m}{2}t\;|\overline{B}^0\rangle - i\frac{M_{12}}{|M_{12}|}\sin\frac{\Delta m}{2}t\;|B^0\rangle\right]$$

A state that begins as a B^0 will produce semi-leptonic decays exponentially damped by $e^{-\Gamma t}$, with the "right" sign modulated by $\cos^2 \frac{1}{2}\Delta m t$ and with the "wrong" sign modulated by $\sin^2 \frac{1}{2}\Delta m t$.

How can we know which neutral B meson is present at some time? If a $B^+ = \overline{b}u$ is observed as well as a neutral B, it is clear that the latter began as $\overline{B}^0 = b\overline{d}$. If instead two neutral B's are produced there is ambiguity because of oscillations. When a B^0 decays semileptonically, it gives a positive lepton. However, because of oscillations, if we start with a B^0 and measure all its semileptonic decays, a fraction $(1+x^2/2)/(1+x^2) \approx 5/6$ will be to positive leptons and $(x^2/2)/(1+x^2) \approx 1/6$ will be to negative leptons. Thus if the complete decay of one neutral B is observed, and in the same event a decay of another neutral B giving a positive lepton is seen, the odds are about 5-to-1 that particle whose complete decay was observed began as a $\overline{B}^0$. This assumes the two neutral B's are uncorrelated, a good approximation if the B's are produced in a high energy collision.

Both LEP and Fermilab Tevatron Collider have studied oscillations with high energy B mesons. These mesons travel distances of the order of a millimeter before decaying so their time evolution can be measured. In this way Δm and not just $x = \Delta m/\tau$ can be determined.

The mixing of B and $\overline{B}$ provides an opportunity to explore CP violation just as the analogous mixing in the K system does. While it is also possible to measure CP violation by showing an inequality between the rate for B^+ decays to a state and B^- decays to the CP conjugate, decays of neutral B's can be analyzed more incisively.

The presence of phases in the CKM matrix is the source of CP violation in the Standard Model. These phases enter into decay matrix elements and into the mixing described by M_{12}. In the Wolfenstein parameterization, phases occur only in transitions between quarks of the first and third generation. One way to represent this is with the "unitarity triangle," shown in Figure xxx. The three angles of this triangle at the vertices 0, 1, and $\rho + i\eta$ are traditionally called γ, β, and α. From this we see that a transition $b \to u$ picks up the phase of $V_{ub} \propto \rho - i\eta$, which is $-\gamma$, while a transition $d \to t$ picks up the phase of $V_{td} \propto 1 - \rho - i\eta$, which is $-\beta$.

If a state that is initially B^0 decays at a later time into a final state f, there will be interference between the decay of the piece that has remained B^0 and the piece that has become $\overline{B}^0$. The phase between the two interfering amplitudes will depend on the relative phases of $\langle f|B^0\rangle$ and $\langle f|\overline{B}^0\rangle$ and on the phase of M_{12}.

Oscillations in the decay of a B to a CP eigenstate are especially interesting because $\langle f|B^0\rangle$ is then related to $\langle f|\overline{B}^0\rangle$ in a simple way. The weak interaction Hamiltonian is made up of many pieces $\mathcal{H}_j$: strangeness increasing, strangeness decreasing, charm increasing, charm decreasing, etc. Altogether the Hamiltonian must be hermitian so that the theory will be unitary (conserving probability). If CP is conserved, the Hamiltonian takes the form

$$\mathcal{H} = \sum_j \mathcal{H}_j + \sum_j \mathcal{H}_j^\dagger \tag{15.1}$$

where $CP\mathcal{H}_iCP = \mathcal{H}_i^\dagger$. On the otherhand, if CP is violated, the CKM matrix introduces phases into the currents that make up the weak interaction. The current that raises one quantum number has a phase opposite that of the current that lowers that quantum number. The Hamiltonian then takes the form

$$\mathcal{H} = \sum_j e^{i\phi_j}\mathcal{H}_j + \sum_j e^{-i\phi_j}\mathcal{H}_j^\dagger \tag{15.2}$$

where $CP\mathcal{H}_jCP = \mathcal{H}_j^\dagger$, but $CP\mathcal{H}CP \neq \mathcal{H}$.

If one single part $\mathcal{H}_j$ of the weak Hamiltonian is responsible for the decay $B^0 \to f$ then

$$\begin{aligned}
\langle f|\mathcal{H}|B^0\rangle &= \langle f|e^{i\phi_j}\mathcal{H}_j|B^0\rangle \\
&= \langle f|e^{i\phi_j}CP\mathcal{H}_j^\dagger CP|B^0\rangle \\
&= \eta_f e^{2i\phi_j}\langle f|e^{-i\phi_j}\mathcal{H}_j^\dagger|\overline{B}^0\rangle \\
&= \eta_f e^{2i\phi_j}\langle f|\mathcal{H}|\overline{B}^0\rangle
\end{aligned}$$

where η_f is the value of CP for the state f.

Interference in the decay of a neutral B depends on the weak phases ϕ_j, which come from the CKM matrix, and on the phase introduced by M_{12}. Mixing results from the processes shown in Fig. 15.1. For M_{12} itself, the dominate diagram has t quark intermediates and $M_{12} \propto (V_{tb}V_{td}^*)^2$ with a negative coefficient of proportionality with the convention $CP|^0\rangle = |\overline{B}^0\rangle$. It follows that $|M_{12}|/M_{12} = e^{-2i\beta}$. Combining all these results we find

$$\begin{aligned}
\langle f|B^0_{phys}(t)\rangle &\propto e^{-\Gamma t/2}A\left[\cos\frac{\Delta m}{2}t + i\lambda\sin\frac{\Delta m}{2}t\right] \\
\langle f|\overline{B}^0_{phys}(t)\rangle &\propto e^{-\Gamma t/2}\overline{A}\left[\cos\frac{\Delta m}{2}t + i\frac{1}{\lambda}\sin\frac{\Delta m}{2}t\right]
\end{aligned}$$

where

$$A = \langle f|\mathcal{H}|B^0\rangle; \qquad \overline{A} = \langle f|\mathcal{H}|\overline{B}^0\rangle$$

and where

$$\begin{aligned}
\lambda &= -\frac{|M_{12}|}{M_{12}}\frac{\overline{A}}{A} \\
&= \eta_f e^{-2i\beta}e^{-2i\phi_{wk}}
\end{aligned}$$

where ϕ_{wk} is the single weak phase in the amplitude for $B^0 \to f$. We see that $|\lambda| = 1$, a consequence of our assumption that only one mechanism contributes to the decay. The decay rate is then governed by

$$|\langle f|B^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma t}\,[1 + \eta_f \sin 2(\beta+\phi)\sin \Delta m t]$$
$$|\langle f|\overline{B}^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma t}\,[1 - \eta_f \sin 2(\beta+\phi)\sin \Delta m t]$$

What is remarkable here is that there are no unknown matrix elements involving hadrons. When just a single weak phase occurs, the hadronic uncertainty disappears.

A particularly important example is the decay $B \to J/\psi K_S$. Since the J/ψ with $CP = +1$ and the K_S with $CP = +1$ must be combined in a p-wave ($CP = -1$), we have $\eta_f = -1$. Here the underlying transition is $\overline{b} \to \overline{c}c\overline{s}$. Because this involves only second and third generation quarks, no weak phase is introduced. Thus the weak phase ϕ is zero. This process, then, measures the phase of δm, which is predicted by the Standard Model to be 2β.

Unfortunately, this simplicity is not general. Consider, for example, the decay $B \to \pi\pi$, for which $n_f = +1$. This decay will result from the rather suppressed process $\overline{b} \to \overline{u}u\overline{d}$. This introduces the CKM matrix element V^*_{ub} and thus the phase γ. However, there is another way to reach the same final state, through a penguin process analogous to $b \to s\gamma$ discussed in Chapter 11.

Suppose the $\overline{b}$ turns into $\overline{t}W^+$ and then this virtual pair recombines to make $\overline{d}$. Before recombining, the $\overline{t}$ could emit a gluon, which itself could become $u\overline{u}$. The overall result would be $\overline{b} \to \overline{d}u\overline{u}$, the same final state as before. Here, however, the phase would come from V_{td}, i.e. $-\beta$. With two different weak phases present, the simple analysis above fails. If the latter contribution, the so-called "penguin" could be ignored, the decay's time dependence would be

$$\begin{aligned} |\langle \pi^+\pi^-|B^0_{phys}(t)\rangle|^2 &\propto e^{-\Gamma t}\,[1 + \sin 2(\beta+\gamma)\sin \Delta m t] \\ &\propto e^{-\Gamma t}\,[1 - \sin 2\alpha \sin \Delta m t] \end{aligned}$$

assuming from the unitarity triangle the relation $\alpha + \beta + \gamma = \pi$. Unfortunately, penguin contributions may be sizeable. To separate out the penguin effects requires measuring isospin-related processes like $B \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$, or $B^0 \to \rho^+\pi^-, \rho^-\pi^+, \rho^0\pi^0$.

At a hadron collider, it is possible to use time-integrated measurements. From

$$\begin{aligned} |\langle J/\psi K_S|B^0_{phys}(t)\rangle|^2 &\propto e^{-\Gamma t}\,[1 - \sin 2\beta \sin \Delta m t] \\ |\langle J/\psi K_S|\overline{B}^0_{phys}(t)\rangle|^2 &\propto e^{-\Gamma t}\,[1 + \sin 2\beta \sin \Delta m t] \end{aligned}$$

we find integrating from $t = 0$ to $t = \infty$.

$$\begin{aligned} N(B^0 \to J/\psi K_S) &\propto 1 - \tfrac{x}{1+x^2}\sin 2\beta \\ N(\overline{B}^0 \to J/\psi K_S) &\propto 1 + \tfrac{x}{1+x^2}\sin 2\beta \end{aligned}$$

where $x = \Delta m/\Gamma = 0.73 \pm 0.03$. The asymmetry is connected to $\sin 2\beta$ by

$$\frac{N(B^0 \to J/\psi K_S) - N(\overline{B}^0 \to J/\psi K_S)}{N(B^0 \to J/\psi K_S) + N(\overline{B}^0 \to J/\psi K_S)} = -\frac{x}{1+x^2}\sin 2\beta \qquad (15.3)$$

In Run I at the Tevatron Collider, which lasted from 1991 to 1996, CDF demonstrated that such measurements can be made in the intense environment of a hadron collider. Not only can B decays that include a J/ψ be reconstructed, it is possible to tag, with a few percent efficiency, either the decaying B itself or the other B to determine whether the decaying B began as a B^0 or as a $\overline{B}^0$. Because the b quark in a $\overline{B}^0$ decays through the sequence $b \to c \to s$, a K^- is the sign of a $\overline{B}^0$. The semileptonic decay $b \to c\ell^-\overline{\nu}$ makes a negative lepton a tag for a $\overline{B}^0$.

Whatever means is used to determine whether the B observed as $J/\psi K_S$ began as a B^0 or $\overline{B}^0$ will be imperfect. If it is wrong a fraction w of the time, a distribution that should be $1 - A\sin\Delta mt$ will instead appear as $(1-w)(1-A\sin\Delta mt)+w(1+A\sin\Delta mt) = 1 - DA\sin\Delta mt$, where the dilution D is just $1-2w$. A figure of merit for an experiment is $Q = \sum \epsilon_i D_i^2$, where the ith tagging category captures a fraction ϵ_i of the neutral B events and has a dilution D_i.

In Run I, CDF used three methods of tagging. One looked for a track whose momentum was nearly along the direction of the reconstructed neutral B. If this latter began with a b quark. Adding a $d\overline{d}$ pair would give B^0 plus an extra d, which could appear in a negative meson, but not a positive one. Thus an associated negative meson points to B^0 and not $\overline{B}^0$. CDF found this method to have a dilution of about 0.17, that is it gave the wrong answer about 41% of the time.

Alternatively, CDF looked for signs of the decay of the other B meson, neutral or charged in the event. The clearest sign would come from a lepton, indicating a semileptonic B decay. This produced a reliable tag, with D about 0.6, but was only 6% efficient, because of the low semileptonic branching ratio.

Finally, CDF attempted to measure the sign of the unreconstructed B by forming a "jet charge", weighting each particle in a jet by its charge and by other factors designed to optimize the identification. This technique achieved a dilution of about 24%.

When an event was tagged by more than one method, the results were combined to form a single tag. In 1998, CDF reported (**15.2**) a value $\sin 2\beta = 1.8 \pm 1.1 \pm 0.3$, using only the same-side tagging method and taking only events in which both the J/ψ-decay muons were seen by the SVX.

In 2000, using all three tagging methods. The data sample included about 400 events in which the J/ψ was seen in its $\mu^+\mu^-$ decay mode and the K_S^0 was seen in $\pi^+\pi^-$. In about half of the events, the muons were measured by the silicon vertex detector (SVX) providing precise information on the distance traveled before the decay to $J/\psi K_S^0$. With this much larger dataset, an improved result (15.3), $\sin 2\beta = 0.79^{+0.41}_{-0.44}$, was reported.

The $\Upsilon(4S)$, which provided such an excellent source of Bs at CESR, can be used to study CP violation as well. However, in contrast to the production of $B\overline{B}$ pairs at a hadron collider, which can be regarded as incoherent, the production at the $\Upsilon(4S)$ is completely coherent. If at some instant, say $t = 0$, one B is known to be a B^0, then at the same time the other must be a $\overline{B}^0$. This follows from Bose statistics, which requires that the odd spatial wave function (for angular momentum one) must be balanced by a wave function odd under particle interchange. The result is that if the state f is observed at t (which can be either positive or negative), the time dependence is

$$|\langle f|B^0_{phys}(t)\rangle|^2 \propto e^{-\Gamma|t|}\left[1 - \sin 2(\beta + \phi)\sin \Delta m t\right]$$

Integrating over all t, positive and negative, cancels the asymmetry.

To measure the asymmetry, then, the actual time dependence must be seen. This is hardly possible in a collider like CESR. There the $\Upsilon(4S)$ is produced at rest and the Bs it yields go about 30 μm on average before decaying. Such decay lengths are too short to be measured with sufficient accuracy to see the oscillations.

To overcome this, asymmetric e^+e^-colliders have been built at SLAC and at the Japanese high energy physics facility, KEK. The general features of the accelerators and detectors at the two locations are quite similar. At SLAC the energy of the electron beam is about 9 GeV and that of the positron beam is near 3 GeV. This will produce an $\Upsilon(4S)$ resonance with a relativistic factor $\beta\gamma = 0.56$. At BELLE, $\beta\gamma = 0.42$. The typical B path length at SLAC is 250 μm. Such distances can be measured reliably with a silicon vertex detector. High precision tracking is not the only requirement for these detectors. The reconstruction of full decays, especially ones with small branching ratios demands excellent particle identification and good energy resolution for photons and electrons. Particle identification relies primarily on Cerenkov radiation, either as a threshold device or with imaging to reconstruct the angle of the Cerenkov cone. Crystals of CsI provide electromagnetic calorimetry with the requisite precision.

The new asymmetric colliders at KEK and SLAC reached luminosities of order $10^{33}\mathrm{cm}^{-2}\mathrm{s}{-1}$ remarkably quickly and by March 2001 both the Belle and BaBar Collaborations reported new values for $\sin 2\beta$ or $\sin 2\phi_1$ as it is called by Belle. The Belle result (**15.4**) was $0.58\,^{+0.32}_{-0.34}\,(\mathrm{stat})\,^{+0.09}_{-0.10}\,(\mathrm{sys})$ while that from BaBar (**15.5**) was $0.34 \pm 0.20 \pm 0.05$. Combining the CDF, Belle, and BaBar results gave 0.49 ± 0.16, strongly indicating a non-zero result, but still too limited by statistics to provide a sharp test of the Standard Model. A few months later, the BaBar Collaboration announced a result (**15.6**) $0.59 \pm 0.14 \pm 0.05$ that taken alone was enough to establish CP violation in the B system.

Because of their better particle identification, the e^+e^-colliders have a much better chance of measuring the process $B \to \pi\pi$ than the all-purpose detectors at hadron colliders. Detectors dedicated to this physics can be built at hadron colliders by adopting a geometry that allows space for particle identification through

Cerenkov counters. Particular care needs to be taken to assure excellent electromagnetic calorimetry if such detectors are to have the necessary neutral pion capability. The e^+e^-machines are not effective for studying the B_s since there is no resonance at which it is produced copiously. In the long term, $e^+e^-$and hadron colliders offer complementary capabilities for studying CP violation in B decays.

Oscillations of B_s^0 are similar in principle to those of the ordinary $B = B_d$. However, the replacement of the d quark by an s quark results in some dramatic changes. The decays $\overline{b}s \to \overline{c}c\overline{s}s$ and $b\overline{s} \to \overline{c}c\overline{s}s$ are CKM favored. Because B_s and $\overline{B}_s$ communicate significantly, just as K^0 and $\overline{K}^0$ do through $\pi\pi$, the off-diagonal term $\delta\gamma$ is not so small. This leads the expectation of a significant difference between the lifetimes of the mass eigenstates, which could be measurable.

The other off-diagonal piece of the $B_s - \overline{B}_s$ mass mixing matrix, δm, is also enhanced. This is so because the factor V_{td}^2 that occurs in δm for B_d becomes V_{ts}^2. In the Wolfenstein parameterization, V_{ts} is suppressed by one fewer power of $\lambda = 0.22$. From this alone, we would expect Δm_s to be perhaps 20 times larger than Δm_d. This will lead to very fast oscillations. Indeed, a limit has been set already by the LEP experiments ALEPH, DELPHI, and OPAL: $\Delta m_s > 10.6\ \mathrm{ps}^{-1}$, compared to the measured value $\Delta m_d = 0.472 \pm 0.017\ \mathrm{ps}^{-1}$. There are good prospects for measuring Δm_s in Run II at the Tevatron Collider.

The Standard Model provides a potential explanation for CP violation through the phases in the CKM matrix. This explanation will be tested in detail by measurements at both $e^+e^-$and hadron colliders over the coming years. The test of the Standard Model will include many measurements of decays that do not involve CP violation, but which determine the values of parameters that appear in CP violation predictions. CP violation will be examined not just in B mesons, but in the traditional venue of K mesons, especially in rare decays like $K \to \pi\ell^+\ell^-$ and $K \to \pi\nu\nu$.

The fascination with CP violation is not due just to the fundamental nature of this broken symmetry. As Andrei Sakharov first recognized in 1967, CP violation is required to explain the evident baryon - antibaryon asymmetry of the Universe if one supposes that this asymmetry was not present at the outset. The CP violation of the Standard Model seems not to be large enough to explain the measured ratio of photons to baryons, however. This suggests that there are additional sources of CP violation besides those provided through the CKM matrix. It remains to be seen whether such sources will show up in CP violation measured in B meson decays.

EXERCISES

15.1 Show that if a $B^0\overline{B}^0$ pair is produced in e^+e^- annihilation in association with other particles far above the $B\overline{B}$ threshold, if both Bs decay semileptonically, the like to unlike sign ratio is

$$\frac{N(l^+l^+) + N(l^-l^-)}{N(l^+l^-)} = \frac{2r}{1+r^2}$$

but if the pair is produced by the $\Upsilon'''(4^3S_1)$ the ratio is simply r.

15.2 Compare the results from the ARGUS Collaboration for r_d, viewed as a measurement of $x_d = \Delta m_d/\Gamma_d$ with those obtained from a direct measurement of Δm_d using Bs produced at higher energies, combined with measurements of the B^0 lifetime. Consult the *Review of Particle Propoerties*, either in its published form or on-line.

15.3 Determine the eigienstates $|B_H\rangle$ and $|B_L\rangle$ including the first order corrections in $\delta\gamma/\delta m$. Use this result to show that

$$\frac{N(BB) - N(\overline{BB})}{N(BB) - N(\overline{BB})} = -\frac{|\frac{q}{p}|^4 - 1}{|\frac{q}{p}|^4 + 1} \approx \Im \frac{\Gamma_{12}}{M_{12}}.$$

15.4 The transition $\overline{B}^0 \to B^0$ occurs, at the quark level, through box diagrams where the intermediate states are $t\bar{t}, t\bar{c}, t\bar{u}, c\bar{c}\ldots$ etc. The sum of all the diagrams would vanish if the quark masses were zero (or just all identical). The result then is dominated by the t quark contribution and is given by

$$M_{12}^{SM} = -\frac{G_F^2}{12\pi^2}(B_B f_B^2) m_B m_t^2 \eta (V_{tb}V_{td}^*)^2 f(x_t) \tag{15.4}$$

where

- $G_F = 1.166 \times 10^{-5}\mathrm{GeV}^{-2}$
- $B_B \approx 1.3$ is the bag parameter, relating the matrix element of a quark operator between physical states to the value obtained naively. This is obtained in lattice calculations.
- $f_B \approx 175 \pm 25$ MeV is the decay constant for the B meson. This is also obtained from lattice calculations.
- $\eta = 0.55$ is a QCD correction.
- $$f(x_t) = \frac{4 - 11x_t + x_t^2}{4(1-x_t)^2} - \frac{3x_t^2 \ln x_t}{2(1-x_t)^3}$$

  is a kinematical factor with $x_t = m_t^2/m_W^2$. With $m_t = 175$ GeV, we find $f(x_t) = 0.54$.
- $V_{tb} \approx 1$,

and where the phase convention $CP|B^0\rangle = |\overline{B}^0\rangle$ is used. Show that the value $\Delta m = 0.472\ \mathrm{ps}^{-1} = 3.1 \times 10^{-13}$ GeV gives the estimate $|V_{td}| = 0.009$.

15.5 Using uncritically the value obtained in Problem [15.4], $|V_{td}| = 0.009$, and the values of the other CKM matrix elements given above, predict the angle β.

15.6 In fitting a distribution $f(t; A)$ normalized so $\int dt f(t; A) = 1$, the expected uncertainty in A with N data points is given by

$$\sigma_A^{-2} = \int dt \frac{1}{f}\left(\frac{\partial f}{\partial A}\right)^2$$

If there are several distributions f_i into which the data fall, the result is similarly

$$\sigma_A^{-2} = \sum_i \int dt \frac{1}{f_i}\left(\frac{\partial f_i}{\partial A}\right)^2$$

Apply this to the determination of the asymmetry in $B \to J/\psi K_S^0$. Show that with perfect tagging

$$\sigma_A^{-2} = N \int_0^\infty du e^{-u} \frac{\sin^2 xu}{1 - A^2 \sin^2 xu} \approx N \frac{2x^2}{1 + 4x^2}$$

where the approximation applies for small A^2 and where $x = \Delta m/\Gamma$. How does the result change if there is a dilution $D \neq 1$?

Use this result to estimate the uncertainty you would expect for the BaBar data set and compare to the reported statistical uncertainty. Use only the "golden events" $J/\psi K_S^0$ to avoid consideration of the large background in $J/\psi K_L^0$.

Suppose instead of measuring the time distribution, one just counts events, comparing the number of $B^0 \to J/\psi K_S^0$ to $\overline{B}^0 \to J/\psi K_S^0$ in an experiment like CDF. Find the analogous general expression for the error in a counting experiment and apply again to this process. Show that for a fixed number of events, the uncertainty using just counting is larger, on average, than using the actual time distribution.

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