Limits on X-Ray Imager Resolution Due to Photon Counting Statistics

T. J. Murphy
Plasma Physics Group, P-24
University of California, Los Alamos National Laboratory
Los Alamos, NM 87544

Abstract

Images obtained using backlighters and pinhole framing cameras on Nova and Omega often appear to have lower resolution than that calculated for the camera. The images also appear grainy. A simple calculational “experiment” demonstrates that these results are consistent with photon statistics, and that backlighters currently in use are several orders of magnitude too dim for use with pinhole cameras at current desired resolution, and that operational restrictions on NIF will prevent it from being any better than Nova. New x-ray imaging techniques need to be developed for use on Omega and NIF to allow increased photon collection if image resolution is not to be limited by photon statistics.

I. Introduction

The use of x rays to image targets, either in self-emission or from backlighters, is an important diagnostic technique in experiments on high-powered lasers. The ability to obtain high-resolution, high-quality images is necessary for the proper interpretation of the data, and for the verification of simulation codes used to model the experiments.

The resolution of the gated x-ray pinhole cameras used in our research has typically been expressed as that of the pinhole imaging element convolved with the inherent resolution of the microchannel plate, phosphor, and film. For very bright images, such as the core of imploded ICF capsules, photon counting statistics do not effect the quality of the image. However, recently the Laser Experiments program has begun efforts at imaging larger targets using backlighters. Recent examples are the experimental series recently begun by Cobble [1] and the Planar Defect Experiments performed by Goldman *et al.* [2]

The images obtained in these experiments are of insufficient quality for accurate comparisons to simulations. They appear grainy and do not appear to reach the predicted resolution capability of the cameras by a large margin.

In this report, I present a model and simple calculations for determining resolution limits set by photon statistics. I then verify this model with a computational experiment. Following this, I estimate the contribution of photon statistics to the Cobble experiments and show that the photon statistics are so poor as to limit the resolution to perhaps an order of magnitude worse resolution than that inherent to the instrument.

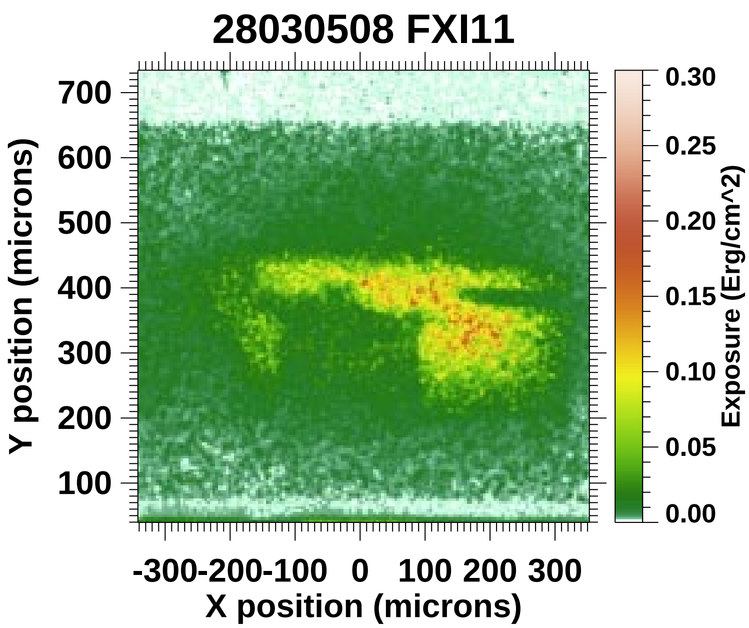

Fig. 1: Image from Cobble [1] showing the limited resolution and graininess of the images obtained.

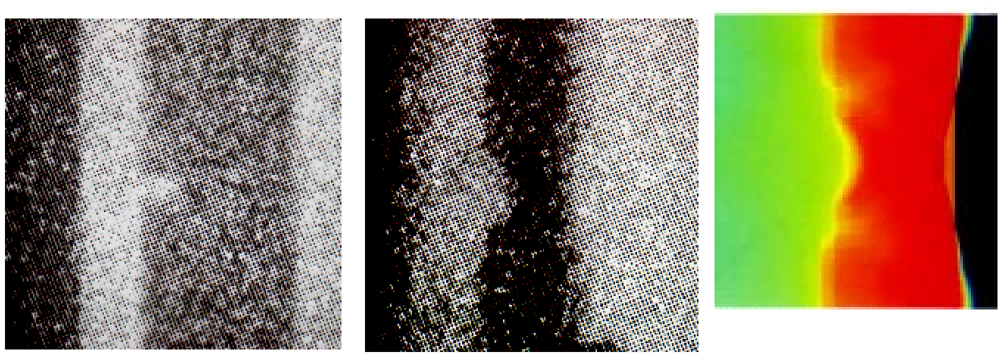

Fig. 2: Images of the initial target (left), the target (center) and Rage simulation (right) at shock breakout time of a gap target [2].

II. Model of photon-counting limited imaging

In order to assess the effects of photon statistics on image resolution, I assume a simple model (Fig. 3) in which the image is made up of pixels of area *A* with photons distributed in the pixels to form an image. I assume that each photon contributes equally to the image, and that the photons follow Poisson statistics.

In the model, I assume that we wish to resolve two contiguous pixels. The first has a photon density of *n* photons per unit area; the second has $n(1-\delta)$ photons per unit area. I refer to δ as the contrast between the pixels.

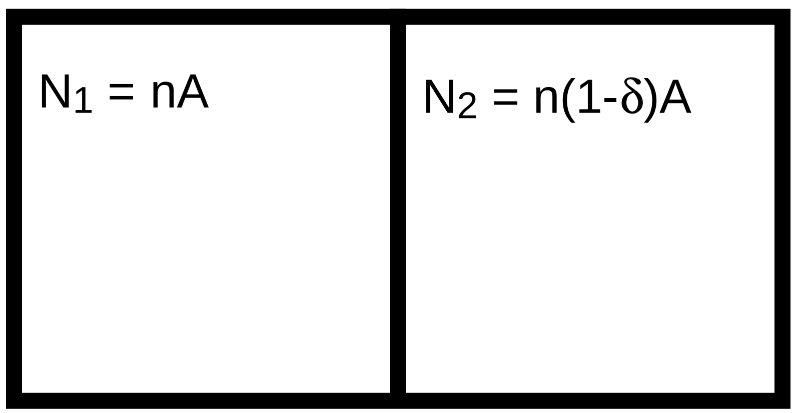

Fig. 3: Contiguous pixels which must be resolved. The photon density in the left pixel is n; that in the right pixel is $(1-\delta)n$. The pixels have the same area A.

In order to resolve the pixels, the difference in the number of photons in the two pixels must be greater than the statistical uncertainty in the difference. The difference in the number of photons in each pixel is given by

$$\begin{aligned} \Delta N \equiv N_1 - N_2 &= nA - nA(1-\delta) \\ &= nA\delta \end{aligned} \tag{1}$$

The statistical uncertainty in that difference, can be approximated by

$$\begin{aligned} \sigma_{\Delta N} &= \sqrt{\sigma_N^2 + \sigma_N^2} \\ &= \sqrt{nA + nA(1-\delta)} \\ &= \sqrt{nA(2-\delta)} \end{aligned} \tag{2}$$

Requiring the difference to be greater than the statistical uncertainty in the difference gives a requirement on the number of photons per resolution element

$$\begin{aligned} \Delta N &> \sigma_{\Delta N} \\ nA\delta &> \sqrt{nA(2-\delta)} \\ nA &> \frac{2-\delta}{\delta^2} \end{aligned} \tag{3}$$

This establishes a lower limit on the photon flux needed to resolve the difference between adjacent pixels of different mean exposure.

From the above equation, we can see that the minimum area of a resolution element can be determined by the average photon flux in the image. That is,

$$A > \frac{2-\delta}{n\delta^2}. \tag{4}$$

If the photon flux is sufficiently low, A may be larger than the size of a resolution element inherent in the imaging instrument. Also, the lower the contrast that needs to be resolved, the greater the number of photons needed in a resolution element. Therefore, at a given photon flux, the ability to measure small values of contrast requires larger pixel areas, and therefore reduced resolution.

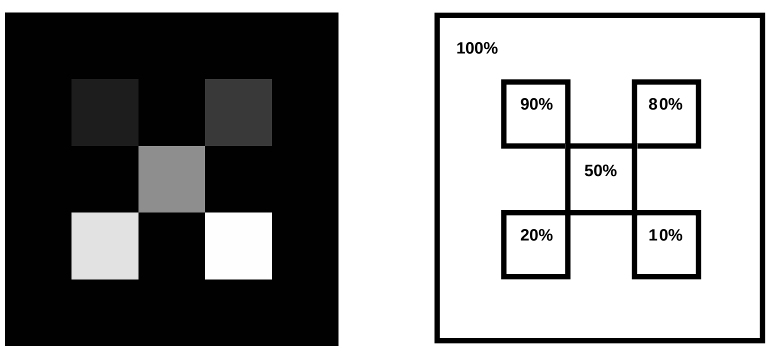

Fig. 4: Test pattern used to investigate the effect of photon statistics on image resolution.

III. Testing the model

In order to test this model, a test image was simulated (Fig. 4) consisting of a square divided into a 25 by 25 array of pixels. Within this image are five square regions made of 5 by 5 arrays of pixels of varying exposure relative to the background, and ranging from 90% to 10% of the background exposure. From this test image, nine images were created with differing mean photon density in the unattenuated region (Fig. 5). The mean exposure was varied from 1 photon / pixel to 10,000 photons / pixel by factors of the square root of 10. Poisson statistical noise was added to each pixel.

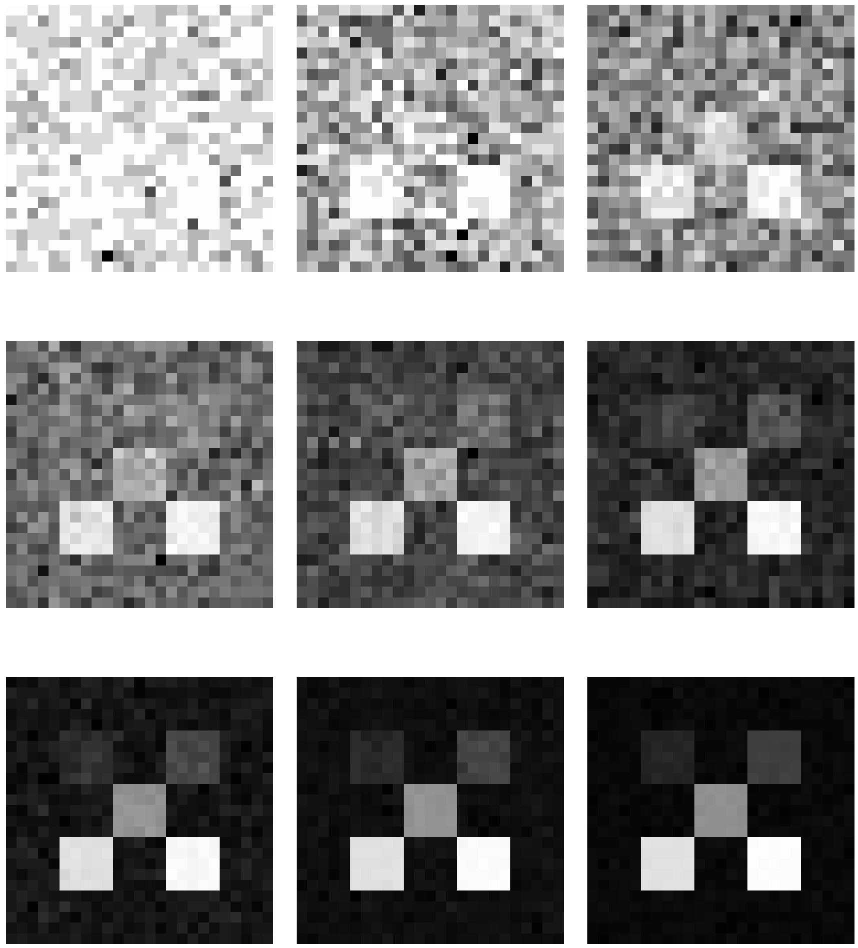

Fig. 5: The test image from Fig. 4 with Poisson statistics applied to the images. From left to right and then top to bottom, the exposure varies from 1 photon/pixel to 10,000 photons/pixel by factors of the square root of 10.

The images in Fig. 5 demonstrate that photon statistics can play a major role in determining whether or not an image can be resolved. The different contrast regions become apparent at different photon densities.

If we compare the photon numbers at which the features become discernible (Fig. 6), we obtain good agreement with the model above.

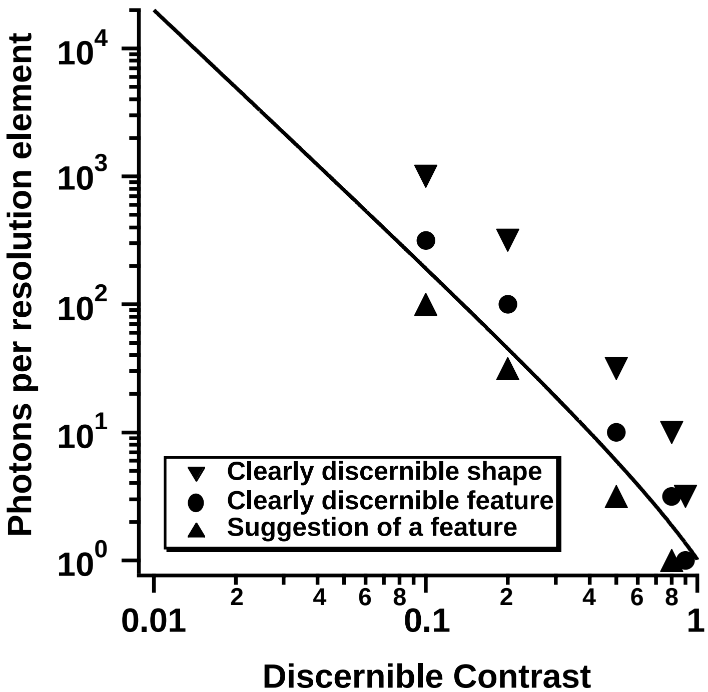

Fig. 6: Photon numbers at which pixels of a given contrast become resolvable according to the model presented in this report, along with data points from the above simulations for when the regions of different contrast just become discernible.

IV. Estimate of photon statistics for a typical experiment

We can estimate the photon statistics for the experiment in Fig. 1. The parameters for the experiment are given in Table 1.

Matthews *et al.* [3] have measured the efficiency of exciting the 4.7 keV Ti line at 1.1 x 10^{12} photons into a sphere per incident joule of laser radiation.[*] From this, we can calculate a backlighter brightness of 7.7 x 10^{25} photons/cm^2/sr/s. The pinholes and filters used limit the flux onto the detector to 3.4 x 10^{15} photons/cm^2/s, or, in units that are more appropriate to the situation, 3.4 x 10^{-5} photons/µm^2/ps. The quantum efficiency of gold photocathodes similar to those used in the imagers has been measured [5] at this wavelength to be about 6%.

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[*] Yaakobi *et al.* [4] measured a somewhat higher backlighter efficiency of about 8 x 10^{12} photons into a sphere per incident joule of laser radiation. Using this higher value, we obtain 4.3 photons per 5 µm resolution element, giving a 16 µm effective resolution at for a discernible contrast of 20%.

Using these values, we obtain a detected number of photons of 0.59 photons per resolution element, where the resolution element is defined as 5 µm at the target (see Appendix).

From the above equations, we see that to detect a 20% contrast, we require about 45 photons per resolution element. Thus, we need to increase the linear dimensions of our resolution elements by a factor of about 9. Thus, for the backlighter, pinholes, and filter used for this experiment, photon statistics limit the resolution at the target to about 45 µm for a discernible contrast of 20%.

Parameter	Nova	NIF
Backlighter energy, J	5,000	360,000
Pulse length, ns	2	3
Spot size, µm	600	1200
Backlighter material	Ti	Ti
Imaging pinhole diameter, µm	5	5
Pinhole-to-detector distance, cm	35	120
Magnification	12	12
Framing time, ps	80	80
Desired resolution element size		
At the target, µm	5	5
At the detector, µm	60	60
Filter thickness		
Beryllium, mil	31	31
Titanium, mil	0.5	0.5

Table 1: Parameters of the experiment shown in Fig. 1, along with NIF parameters for similar experiments.

An important point to make is that NIF will not be any better. If we assume, as on Nova, that we use 20% of NIF energy to drive a backlighter, and that the NIF backlighter spot size is about twice that of Nova (since the individual beamlets are f/16, rather than f/4, this is probably reasonable), and use the longer 3 ns pulse length, we get an intensity on the NIF backlighter that is 12 times that on the Nova backlighter. However, the standoff distance for diagnostics on NIF will be at least 10 cm. Therefore, the pinhole cannot be any closer than 10 cm from the target. For 12X magnification, the detector is then 120 cm from the pinholes, with a resulting decrease in image brightness of

11.8, nearly exactly compensating for the increased brightness of the backlighter.

V. Possible solutions

In order to obtain useful information on these experiments, some changes need to be made in how the x-ray imaging is done. The simplest solution is to use larger pinholes.

If one changes from 5 µm pinholes to 10 µm pinholes, the brightness of the image will increase by a factor of 4. Since the resolution of the imager is then degraded, the digitizing of the film can also be performed over a larger area, gaining another factor of 4. The length of the backlighter can be shorted to 1 ns, doubling the intensity. This increases the number of photons per 10-µm resolution element to about 18. This will allow detection of regions of 30% contrast with 10 µm resolution.

A better solution would be to replace the pinholes with x-ray optics with higher collection efficiency. P-24 has started a number of efforts to investigate imaging systems with better collection efficiency, including both 1- and 2-D KB microscopes [6-8], curved crystal transmission imagers [9], and Fresnel zone plate imagers [10,11]. Each of these would have the advantage of higher light collecting efficiency, and would also be able to focus the images, allowing improved resolution over pinhole imagers.

References

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Appendix: Calculation of flux onto a detector from a backlighter foil

The flux onto a detector from a backlighter can be calculated easily.

A given spot on the detector can see only the part of the backlighter that is not obscured by the pinhole. We assume that the backlighter fills that view through the pinhole. The backlighter will appear to be just as bright regardless of its distance from the detector, so we can calculated the flux onto the detector using the backlighter brightness applied to the pinhole aperture. Then, the flux onto a detector element is given by:

$$F = B\frac{\pi\left(d_{ph}/2\right)^2}{D_d^2}\eta \tag{A1}$$

where B is the brightness of the backlighter in photons per unit area per steradian, and η is the filter transmission at the appropriate x-ray energy.

The calculation can also be done using the flux from the backlighter area that is visible to the detector. In this case, the diameter of the backlighter spot is given by

$$d_{bl} = d_{ph}\frac{D_{bl}+D_d}{D_d} \tag{A2}$$

and the distance to the backlighter is given by $D_d + D_{bl}$. Using this diameter and distance, one obtains an identical result to that in Eqn. 1.

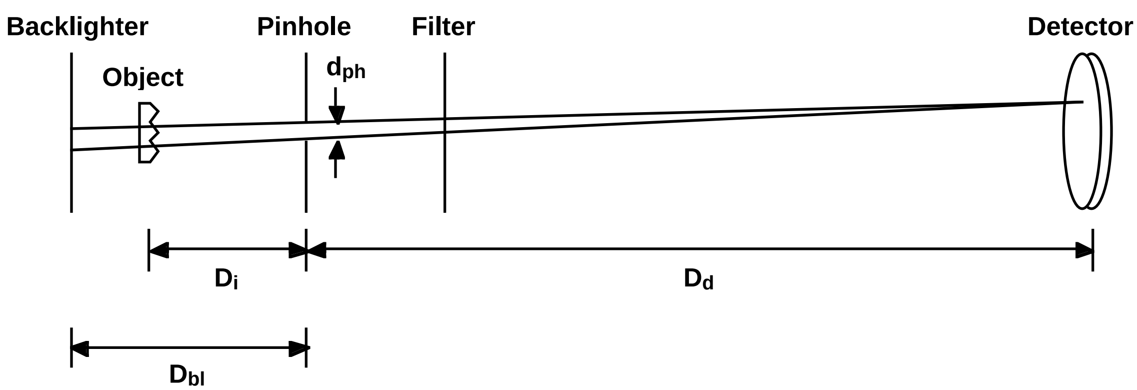

Fig. 7: Geometry used to calculate the flux from a backlighter onto a detector element.

The number of photons per detector element can then be expressed:

$$N_\gamma = F\tau\lambda_d^2\varepsilon. \tag{A3}$$

were τ is the integration time, λ_d is the resolution element in the detector plane, and ε is the photocathode efficiency for detecting photons.

The brightness of the backlighter for the parameters in Table 1 and the conversion efficiencies in Matthews *et al.* [3] of 1.1 x 10^{12} photons into a sphere per incident joule of laser radiation is

$$B = 1.1\times10^{12}\,\frac{\text{photons}}{\text{sphere J}}\times\frac{1\text{ sphere}}{4\pi\text{ sr}}\times\frac{5000\text{ J}}{2\text{ ns}}\times\frac{1}{\pi(600\ \mu\text{m}/2)^2}$$
$$= 7.7\times10^{25}\,\frac{\text{photons}}{\text{sr cm}^2\text{ s}} \tag{A4}$$

The flux onto a detector element, from Eqn. A1, is then

$$F = 7.7\times10^{25}\,\frac{\text{photons}}{\text{sr cm}^2\text{ s}}\times\frac{\pi\left(5\ \mu\text{m}/2\right)^2}{\left(35\text{ cm}\right)^2}\times27.3\%$$
$$= 3.4\times10^{15}\,\frac{\text{photons}}{\text{cm}^2\text{ s}} \tag{A5}$$

Therefore, the number of photons per detector element from Eqn. A3 is

$$N_\gamma = 3.4\times10^{15}\,\frac{\text{photons}}{\text{cm}^2\text{ s}}\times80\text{ ps}\times\left(\frac{60\ \mu\text{m}}{\text{resolution element}}\right)^2\times6\%$$
$$= 0.59\,\frac{\text{detected photons}}{\text{resolution element}} \tag{A6}$$

which is the result reported in Section IV.