

# The Impact of Noisy Catch Data on Estimates of Efficient Output Derived From DEA and Stochastic Frontier Models: A Monte Carlo Comparison

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**Abstract** There is currently much national and international interest in measuring commercial fishing capacity. Two quantitative methods that will likely be used for this purpose are data envelopment analysis (DEA) and stochastic frontier (SF) production functions. Although both methods can be used to estimate a production frontier, their underlying assumptions and method of solving for the frontier are quite different. One substantial difference is how each model handles noisy data. An understanding of the implications of this difference is important because random variation is likely to exist in commercial fishery catch data. This research uses Monte Carlo simulations to investigate possible finite sample biases attributable to this type of noise when estimating fishing capacity. The results suggest that the mean bias associated with noisy data is often substantially larger for DEA than SF. However, the frequency distributions of the biases from each method show a wide variation in some cases.

**Keywords** Capacity, technical efficiency, fisheries, data envelopment analysis, stochastic production frontier, Monte Carlo analysis

## 1. INTRODUCTION

Problems associated with excess fishing capacity, including poor economic performance, biological overfishing, and the depletion or collapse of fisheries, have motivated recent policy initiatives focused on managing capacity. The FAO's International Plan of Action for the Management of Fishing Capacity (FAO 1999) urges countries to develop national fishery capacity management plans by 2002. The United States is currently developing a national fishery capacity management plan and the National Oceanic and Atmospheric Administration (NOAA) has adopted a formal planning objective of reducing the number of overcapitalized fisheries by fifteen percent by the year 2004 (NOAA 1999). A National Marine Fisheries Service (NMFS) taskforce is expected to recommend that estimates of fishing capacity be developed for each federally managed fishery (NMFS 1999).

Because the data necessary to estimate economic measures of capacity are generally not available for U.S. fisheries, most studies of fishing capacity will likely be based on a technical definition. Technical capacity is a measure of maximal output based on the full utilization of all variable inputs (Johansen, 1968), while economic capacity is usually defined in terms of cost functions (Klein 1960, Morrison 1985) or profit functions (Squires 1987). Two methods that have been proposed for estimating technical capacity are data envelopment analysis (DEA) (Färe, Grosskopf and Kokkelenberg

1989) and stochastic frontier (SF) production function models (Kalirajan and Salim 1997).<sup>1</sup> Each method has particular advantages and disadvantages. These are largely documented in: Charnes et al. (1994), and Seiford and Thrall (1990) for DEA; Bauer (1990) and Green (1993) for SF; and, both methods are discussed and contrasted in Coelli, Rao and Battese (1998) and Lovell and Schmidt (1988).

Both DEA and SF attempt to identify a production frontier for a group of production units (e.g., the vessels in a fishing fleet). DEA is a non-parametric method that uses linear programming to construct a piecewise surface (frontier) over the observed data. SF is a parametric method that estimates the parameters of a frontier production function. SF is designed to accommodate deviations from the production frontier due to both random variation in production rates that may neutrally move output levels either up or down, and technical inefficiency that lowers output relative to the production frontier. Therefore, the estimated SF production function may not envelop all output points. In contrast, DEA treats all deviations from the frontier as inefficiency. Consequently, the DEA frontier may be pushed outward because high output rates due to random variation are assumed to be achievable through efficient production.

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<sup>1</sup> DEA and SF can also be used when economic data are available (Coelli, Rao and Battese, 1998).

The impacts of noisy data are of particular interest for frontier applications to fisheries since there is often a great deal of random variation in catch rates due to such factors as unanticipated weather and resource conditions (Kirkley, Squires and Strand, 1998), and because the data collected is often imprecise. Therefore, an important policy question is how different levels of noise (random variation in output) affect the accuracy of the estimates from both methods. Although this issue has been raised in the literature, it has yet to be fully explored in a quantitative fashion.

DEA and SF methods were originally developed to calculate the relative efficiency of different firms in an industry.<sup>2</sup> However various versions of the methods may be used to measure some type of frontier output. Since capacity is an output measure, we use an output measure that is easily derived from each model, technically efficient output. This facilitates comparison of the methods in terms of how they handle noisy data. We define technically efficient output as the maximum level of output that individual vessels, or the fleet as a whole, could produce with technically efficient use of its current input levels. The efficient usage of all inputs would, by definition, place a vessel on the production frontier. This should provide an upper bound on the level of catch a given fleet could take given observed levels of inputs. In the case where both fixed and variable inputs are constrained by regulations this would also provide a measure of maximum potential output. This level of output can be thought of as a measure of technical capacity somewhere between Johansen's (1968) technical capacity measure and current production. Our results using technically efficient output as the measure by which DEA and SF are compared apply in a qualitative fashion to other measures of frontier output, including models that more closely approximate Johansen's capacity output.<sup>3</sup>

Many authors have compared DEA and SF in an empirical setting. The comparisons are often based on the

<sup>2</sup> A small number of such applications of SF to fisheries appear in the literature. Kirkley, Squires and Strand (1995) use SF to estimate the technical efficiency in the Mid-Atlantic sea scallop fishery. Sharma and Leung (1998) and Kirkley, Squires and Strand (1998) use SF to analyze how vessel and managerial attributes affect technical efficiency. To the authors' knowledge, no applications of DEA to commercial fisheries have yet been published.

<sup>3</sup> For SF, one may solve for the full input utilization point on the estimated production function. Since these approaches are quite different, we have chosen a more similar comparison, technically efficient output. However, we also examine the impact of noise on the DEA model proposed by, Färe, Grosskopf and Kokkelenberg (1989) for estimating technical capacity.

ability of the models to correctly identify relative efficiency or returns to scale rather than technically efficient output. The results are inconclusive since there is substantial variation in the types of models and data used, and the results across studies are sometimes contradictory. Several empirical comparisons of DEA and SF found differences between their estimates (Cummins and Zi, 1998 and Kalaitzandonakes and Dunn, 1995) while other studies found the estimates to be somewhat close (Sharma, Leung and Zaleski, 1997 and Bjurek, Hjalmarsson and Forsund, 1990). Still other studies found mixed results (Hjalmarsson, Kumbhakar and Heshmati, 1996, and Ferrier and Lovell, 1990). These empirical findings amplify the need for Monte Carlo simulation analyses.

While a number of Monte Carlo studies have documented some of the small sample properties of SF and DEA when output data are noisy, none of these studies provides a complete and reliable picture of the impact of random noise on their estimates. Gstach (1998) found average efficiency estimates from standard DEA models in the presence of noisy data had an upward bias of 80% to 200% relative to the true levels, but he did not identify how the bias varied with different levels of noise. Coelli (1995) examined how different levels of noise and sample size impact SF estimates of efficiency. However, the study only looked at average bias of a parameter of the model. Our research presents the entire frequency distribution of the bias of estimates of technically efficient output in order to examine issues such as asymmetric or bimodal distributions. Gong and Sickles (1992) compare SF and DEA estimates of average efficiency as random noise varies relative to average efficiency, however they use panel data which tends to smooth out the bias of estimates, only the correlations of true and estimated efficiency are compared, and only 50 replications of each scenario are conducted. Banker et. al (1993) present a Monte Carlo experiment that addresses the issue of random noise in DEA and SF models; however, they perform only five replications of each scenario and use corrected ordinary least squares rather than the now popular maximum likelihood method to estimate the SF.

The remainder of the paper proceeds as follows. The DEA and SF models used in this study are presented followed by a description of the experimental design of our Monte Carlo simulation analysis. The results of the experiment, including an extension using a DEA model specifically designed to measure capacity, are presented next followed by a discussion of the implications of the results.

## 2. DEA AND SF MODELS

### 2.1 DEA Model

An output-oriented, constant-returns-to-scale DEA model is used in our analysis. The constant-returns-to-scale formulation conforms with the true technology used in the data generating process. This model, hereafter referred to as the CCR model, is an output-oriented version of the model proposed by Charnes, Cooper and Rhodes (1978). The model is solved independently for each of the  $j$  firms. The single output the model can be written as:

$$\text{Max } \theta_j \quad (1)$$

$$z_j$$

s.t.

$$\sum_{j=1}^J z_j y_j \geq \theta_j y_j \quad (2)$$

$$\sum_{j=1}^J z_j x_{jn} \leq x_{jn} \quad (3)$$

$$z_j \geq 0, j = 1, 2, \dots, J \quad (4)$$

where:

$y_j$  is the output level for firm  $j$   
 $x_{jn}$  are the levels of inputs indexed by  $n$  for the input and  $j$  for the firms  
 $z_j$  are weights, solved for by the model, for each of the  $j$  firms  
 $\theta_j$  is a measure of technical inefficiency solved for by the model that indicates the amount by which firm  $j$  could increase its output if it operated on the frontier. It takes a minimum value of 1 when the firm is operating on the frontier and the maximum value is unbounded.

This linear programming problem can be solved with widely available software.

Technically efficient output for each firm  $j$  is defined by  $y_j \theta_j$ . This gives the amount each firm could produce if it operated on the frontier given its current levels of all inputs  $n$ . The efficient output for the fleet is therefore given by  $\sum_j y_j \theta_j$ .

### 2.2 SF Model

The stochastic frontier model used follows those first proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). In general terms this model can be expressed as

$$y_j = f(x_j, \beta) + \varepsilon_j, \quad (5)$$

where  $y_j$  is output,  $x_j$  is a vector of inputs and  $\beta$  is a vector of parameters to be estimated. Actual output deviates from potential output due to the effect of an individual specific error term,  $\varepsilon_j$ . The error is assumed to be induced by, possibly, two sources. One source is due to inefficient input use. This error is denoted by  $u_j$ , and is drawn from an independent distribution with a weakly positive range. The other source of error is due to neutral random, positive or negative, deviations from the production function. This may be due to purely random variations in output unrelated to efficiency (e.g., luck, weather, etc.), measurement error in output, or data coding errors and is directly analogous to the error term in a standard regression model. This error is denoted by  $v_j$ , and is drawn from a mean zero, independent, unbounded distribution. Therefore,  $\varepsilon_j$  can be written as

$$\varepsilon_j = v_j - u_j. \quad (6)$$

It is usual to specify  $v_j$  as coming from an iid normal distribution and  $u_j$  as coming from a half normal or truncated normal distribution. For other distributional assumptions see Green (1990) and Stevenson (1980). Most current applications of SF models use maximum likelihood to estimate the parameters, and that is the method employed in this study. The stochastic frontier models are estimated in logged form with a Cobb-Douglas functional form. We do not impose constant returns to scale in the estimation.

Technically efficient output for each firm  $j$  is defined by  $f(x_j, \hat{\beta})$  where  $\hat{\beta}$  is the estimated parameter vector. Given the Cobb-Douglas form and log transformation used in our estimations,  $f(x_j, \hat{\beta}) = \exp(\hat{\beta} x_j)$ . This gives the amount each firm could produce if it operated on the frontier given its current levels of all inputs. The efficient output for the fleet is therefore given by  $\sum_j f(x_j, \hat{\beta})$ .

## 3. EXPERIMENTAL DESIGN

The DEA and SF models are estimated under several data generating processes to investigate their ability to correctly determine technically efficient output. The study focuses on three issues that are quite fundamental to the estimation of technically efficient or capacity output, and may highlight some differences between the models' capabilities: 1) the effect of different levels of random variation in output; 2) the effect of the number of vessels in the fleet; and 3) the effect of a having different proportions of the fleet operating on the production frontier. This third issue also relates to the robustness of

SF estimates when deviations from the assumed error structure are present.

A single output,  $Y$ , constant-returns-to-scale Cobb-Douglas production function is used to generate the data. Production is dependent on two inputs: capital ( $K$ ) and labor ( $L$ ). For each observation,  $j$ , the maximum production level during a period of time is given by the following equation.

$$Y_j = \alpha_0 K_j^{\alpha_k} L_j^{\alpha_l} \quad (7)$$

For purposes of the simulation,  $K$  and  $L$  are drawn from independent uniform distributions with a range [50, 100], and  $\{\alpha_0, \alpha_k, \alpha_l\} = \{1, .5, .5\}$ .

Actual output deviates from potential output (1) due to the effect of an individual specific error term,  $\varepsilon_j$ , where  $\varepsilon_j = v_j - u_j$  as in equation (6) above. The one-sided error term,  $u_j$ , is drawn from an independent half-normal distribution with a zero mean and weakly positive range. The two-sided error term,  $v_j$ , is drawn from a mean zero, independent normal distribution.

The realized or actual output level,  $\tilde{Y}_j$ , is therefore given by

$$\tilde{Y}_j = \alpha_0 K_j^{\alpha_k} L_j^{\alpha_l} e^{\varepsilon_j} \quad (8)$$

where  $e$  is the exponential operator.

The effect of the different levels of random variation in output are investigated by fixing the standard deviation of  $u$  at 0.25 and using three different standard deviations for  $v$  (0.05, 0.20 and 0.35). This experimental design allows for the isolation of the effect of various levels of random noise while holding the distribution of relative fleet efficiency constant. These comprise the three base error structure scenarios and correspond to the data generating process of a standard type SF model with half-normal and normal errors.

The effect of deviations from this structure is also investigated by creating a separate set of cases with 30 percent of the vessels placed on the frontier. In this data generating process 30% of the  $u_j$  are set equal to zero. These scenarios characterize a fishery with a concentration of fully efficient vessels.<sup>4</sup> The effect of placing 30% of the vessels on the frontier is to change the shape of the distribution, sometimes just slightly, and shift it to the right.

<sup>4</sup> Banker, Gadh and Gorr (1993) also placed a similar percentage of the sample on the frontier in their Monte Carlo simulation.

For each of these data generating processes, data sets are generated with fleet sizes of 50, 100, 200 and 400. This design creates twenty-four combinations [(3 random error levels)  $\times$  (2 percentages of the fleets placed on the frontier)  $\times$  (4 fleet sizes)] of the three issues under investigation. Each scenario is simulated 1000 times. In order to reduce unnecessary variability across the four sample sizes, one draw of 400 observations is made for each replication. The other sample sizes for that replication are created using the first 50, 100 and 200 observations of the original 400 observations.

#### 4. RESULTS

We begin the analysis of the results by examining the mean bias of each method in estimating the technically efficient output of the fleet. The DEA and SF mean biases are presented in Tables 1 and 2 (next page), respectively.<sup>5</sup> The results of the DEA models indicate that random noise in output data can generate considerable upward bias in estimates of the fleet's technically efficient output. The level of bias in output estimates increases sharply with the standard deviation of the random noise  $v$ . The bias also increases with fleet size. This is because there is a greater probability of outliers pushing out the frontier. For example, the mean bias for a fleet size of 100 when the level of noise is "small" (std. dev.  $v = .05$ ) is 3.3%. When the level of the noise increases to the "middle" and "high" values (std. dev.  $v = 0.20$  and  $0.35$ ), the mean biases increase to 36.5% and 89.2%, respectively. Likewise, when the sample size increases from 50 to 400 vessels, the mean biases increase from 1.1% to 7.0% when the noise level is "small", and from 72.5% to 124.4% when the noise level is "high". As would be expected, the level of mean bias is higher when the data generating process places 30 percent of the fleet on the frontier, and the same pattern of bias associated with the level of noise and fleet size applies.

We also investigated whether a log-log specification would perform better than the DEA model described above. This specification has been suggested as more appropriate if the production function is believed to be Cobb-Douglas. The results from this specification were very similar to those described above. With noise in the data the average level of upward bias was in general slightly higher (0.5% to 1.5%) than it was with the CCR model and the differences between the models tended to decline with sample size.

<sup>5</sup> All DEA estimates were obtained using the GAMS software package using the MINOS5 primal simplex method on a Unix based computer. The SF estimates were obtained using Gauss for Windows and the CML (constrained maximum likelihood) application module with analytical derivatives supplied by the authors.

**Table 1.** Mean bias of technically efficient output estimates from the DEA model with varying levels of noise and fleet size.

Fleet Size	0% On Frontier			30% On Frontier		
	Std.Dev $v=0.05$	Std.Dev $v=0.20$	Std.Dev $v=0.35$	Std.Dev $v=0.05$	Std.Dev $v=0.20$	Std.Dev $v=0.35$
50	1.1%	27.8%	72.5%	5.6%	36.3%	86.2%
100	3.3%	36.5%	89.2%	7.6%	45.0%	104.3%
200	5.2%	44.1%	107.4%	9.4%	52.6%	121.1%
400	7.0%	52.1%	124.4%	11.0%	60.5%	139.7%

**Table 2.** Mean bias of technically efficient output estimates from the SF model with varying levels of noise and fleet size.

Fleet Size	0% On Frontier			30% On Frontier		
	Std.Dev $v=0.05$	Std.Dev $v=0.20$	Std.Dev $v=0.35$	Std.Dev $v=0.05$	Std.Dev $v=0.20$	Std.Dev $v=0.35$
50	-0.84%	-2.39%	0.59%	6.24%	5.75%	6.96%
100	-0.14%	-2.61%	-1.36%	6.61%	5.55%	5.64%
200	-0.06%	-2.02%	-2.20%	6.50%	6.54%	4.95%
400	-0.01%	-1.26%	-2.76%	6.40%	7.55%	4.76%

The mean biases from the SF models show that random noise in output data generates smaller biases than from DEA, especially at the “middle” and “high” levels of noise (Table 2). In fact, the largest (in absolute value) mean bias from the SF model when no vessels are placed on the frontier is -2.76%. This level of bias is smaller than the bias of DEA estimates in all but one scenario. In contrast to the DEA results, the SF results generally show less bias as the sample size increases, but the relationship varies and is highly nonlinear. The general pattern is similar to the results found by Coelli (1995) when he examined one parameter from the SF model. The bias is negative when the level of noise is small, then becomes positive as the level of noise increases. With 30 percent of the vessels on the frontier, a similar pattern of bias associated with the level of noise and fleet size applies, however, in this case the bias is shifted upward and is always positive.

Examining the frequency distribution of the biases across the 1000 simulations run for each model and scenario reveals a more complete and informative picture. Examining the DEA frequency distributions, it is apparent that there is considerable variation in the bias of efficient output estimates among the 1000 trials done for each data generating process. The 95 percent bounds, calculated as the upper limits of the 2.5 and 97.5 percentiles, provide an indication of the range of the possible bias that may be expected from each data generating process. An increase in the sample size leads to a large, positive shift in the 95 percent bounds, even in the case of the “small” level of noise. When the standard deviation of  $v$  is 0.05 and the sample size is 50, the lower 95 percent bound is -3.3% and the upper 95 percent bound is 5.9%. When the sample

size increases to 400, the lower bound increases to 4.0% and the upper bound increases to 10.6%. Perhaps surprisingly, an increase in the sample size holding the level of noise constant, has little effect on the difference between the upper and lower bound of bias. However, the spread of the bias increases substantially with the standard deviation of  $v$ . For example, even when the sample size is 50, an increase in the standard deviation of  $v$  from 0.05 to 0.35 increases the lower 95 percent bound from -3.3% to 33.9%, and the upper bounds from 5.9% to 140%.

The SF frequency distributions of bias reveal that an increase in the level of noise increases the dispersion of the bias estimates, although not by as much as the DEA estimates. Perhaps the most striking feature of the SF bias distributions is that they can be bimodal with “higher” levels of noise and a “smaller” sample size. However, even when the sample size is 400 in our simulations, the distributions show a bimodal characteristic when the noise is at the “middle” level, and this characteristic is quite pronounced when the level of the noise is “high.” Therefore, although the mean biases from the SF models are usually quite close to zero, there is likely to be a high occurrence of over or under estimates. Although this characteristic diminishes as the sample size increases, it may require a fairly large sample if the level of noise in the data is “high”. It may be difficult to establish when and under what conditions this is likely to be true using real world data sets.

## 5. RESULTS FOR A DEA CAPACITY MODEL

A modification of the DEA model was proposed by Färe, Grosskopf, and Kokkelenberg (FGK) (1989) as a method of estimating capacity and capacity utilization assuming unconstrained use of variable inputs. The only difference between the FGK model and the CCR model is that the constraints on the variable inputs are dropped. Consequently firms with the highest ratio of output to fixed inputs make up the frontier. This model may be a more appropriate model for measuring technical fishing capacity since it more closely fits standard definitions (*e.g.*, Johansen, 1968) which assumes full utilization of variable inputs. Our Monte Carlo analysis also includes the application of the FGK model to a subset of the data generating processes to determine how random noise impacts the capacity estimates of this model.

The constant returns to scale specification is appropriate for estimating technically efficient output, but is arguably not appropriate for the FGK model. With only the capital input constrained, the model will pick as the frontier firms those with the highest output to capital ratio and will make capacity estimates for all firms based on this ratio. As a result, the model will tend to pick frontier firms with a low level of capital and a high level of labor. The capacity predictions implicitly assume that non-frontier firms can raise their actual labor input to the point where the labor to capital ratio would equal that of the frontier firms. This may well be a higher level of labor than was allowed in the data generating process which is described below. Therefore we present results both from a constant returns to scale and a variable returns to scale version of the FGK model. The variable returns to scale (VRS) models include the following additional constraint:

$$\sum_{j=1}^J z_j = 1$$

This constraint, proposed by Banker, Charnes and Cooper (1984), essentially ensures that firms are only benchmarked against other firms of similar size.

The FGK model was used to generate capacity estimates with a fleet size of 200 and four different level of two-sided random error. The DEA model is designed to calculate a measure of capacity, that reflects “full utilization” of the variable input. Theoretically there is no full utilization point with this Cobb-Douglas specification, however, the DEA model is not designed to find the point where marginal product of the variable input is zero, only the level of variable inputs used by firms with the highest ratio of outputs to fixed inputs. Since the level of labor that corresponds with full utilization is not defined, it is unclear what exactly the true capacity should be for computing a level of bias. Rather than calculating a level of bias of capacity estimates we simply present the estimates of total frontier output calculated by the CRS and VRS versions of the CCR and FGK models (Table 3).

The results in Table 3 demonstrate that the impacts of noise on capacity estimates derived by projecting firms onto the frontier mapped out by the FGK model are similar to those from the CCR model. Estimates of frontier output from both models are greatly increased as more noise is introduced into the data. As with the CCR model, capacity estimates range widely over the 1000 replications done for each data generating process. It is also apparent that assumptions on returns to scale can change capacity estimates considerably. Allowing for variable returns to scale results in significantly lower estimates of capacity

**Table 3.** Fleet output and capacity utilization estimates from constant (CRS) and variable (VRS) returns of scale versions of CCR and FGK models with varying levels of noise and a fleet size of 200.

Model	v=0.0	v=0.05	v=0.20	v=0.35
CRS-FGK	19,336	19,869	25,980	36,945
VRS-FGK	16,391	16,973	22,271	31,473
CRS-CCR	14,718	15,643	21,410	30,823
VRS-CCR	14,505	15,158	19,826	27,442

## 6. DISCUSSION

The results of the Monte Carlo analysis raise some serious concerns regarding the accuracy of DEA and SF in identifying overall levels of technically efficient output in commercial fisheries. DEA models may incorrectly interpret random noise (exogenous shocks beyond the control of the production unit) and measurement error, as inefficiency. DEA may consequently provide significant overestimates of efficient or capacity output at levels of noise that appear to be plausible in real world fisheries data.<sup>6</sup> Although the sensitivity of the DEA frontier to random noise has been acknowledged in the literature, many practitioners may not be aware of the extent to which estimates of efficient or capacity output can be biased. If random variation in catch rates are thought to be significant, it may be advisable to seek alternatives to the standard DEA model.

Parametric methods such as SF are one alternative, however there are variations of the DEA model that may prove useful. If panel data is available it may be possible to reduce the bias caused by random noise through use of catch data averaged or aggregated over time for a given individual vessel. However, if the level of random noise is large, capacity estimates may still exhibit significant upward bias. A variety of more sophisticated techniques have been developed to accommodate random noise in DEA models (e.g., Gstach 1998 and Land, Lovell and Thore 1993) though they are not available with standard commercial software. Further research and investigation of the accuracy of these and other techniques are clearly needed.<sup>7</sup>

The average biases of SF estimates of technically efficient output are small relative to DEA estimates when the data are noisy. However, as the bimodal frequency distribution of bias across replications demonstrates, the probability of deriving estimates of technically efficient output well above or below the true level may be quite

<sup>6</sup> In fact, estimated standard deviations of  $v$  (calculated by the present authors from reported point estimates of SF model parameters) in two recent SF applications to fisheries (Kirkley, Squires and Strand 1998 and Sharma and Leung 1998) are close to our “middle” value (0.20).

<sup>7</sup> It was suggested to us that the bias of DEA estimates caused by noise could be largely eliminated by searching for and removing outliers from the data. However, our concern is not with outliers in the typical sense which might be due to errors in the data or inclusion of observations from different production processes. We are assuming that there is inherent randomness in production. To remove the bias completely would require removing all observations with an error term sufficiently positive to distort the frontier, and this is clearly neither possible nor reasonable.

high if the sample is “small” or the level of noise is “high”. While random noise clearly results in an upward bias in DEA estimates, it may be difficult to determine the direction of bias in an empirical SF application. However, larger sample sizes are clearly better than smaller sample sizes.

Although these results are based on models that predict technically efficient output as opposed to maximum potential output for a given capital stock, they can be expected to carry over to estimates of technical capacity that assume full utilization of variable inputs. The DEA model proposed by Färe, Grosskopf and Kokkelenberg (1989) for measuring capacity, that simply drops the constraints on variable inputs from the model, is subject to the same qualitative effects from noise as the standard DEA model<sup>8</sup>. Regardless of whether the model includes variable inputs, efficient or capacity output will still be determined by the firms with the highest output to input ratio and this ratio may be biased upward by random noise. Predictions of capacity output from SF (i.e., the maximum of the SF production function for a given set of fixed inputs) can also be expected to show a bias that is similar to estimates of technically efficient output. The bias in SF estimation of efficient output is directly related to biased parameter estimates. Therefore, any results found here apply to any use of the model’s parameter estimates, including capacity output.

It would be incorrect to assume that empirical DEA estimates of technically efficient output will always be upwardly biased in the presence of random noise or that they will typically exceed SF estimates. This can depend on the extent to which data are available to fully map out the frontier and the specifications chosen. The DEA frontier may in fact be downwardly biased if a sufficient number of efficient firms with varying input levels are not included in the data. This is particularly likely if the number of inputs and outputs is large and may not be mitigated by large sample sizes<sup>9</sup> (Tauer and Hanchar 1995, Pedraja-Chapporo, Salinas-Jimenez and Smith 1999). DEA models that allow for variable returns to

<sup>8</sup> Färe, Grosskopf and Kokkelenberg (1989) propose as a measure of capacity utilization the ratio of frontier output from a standard output oriented DEA model to frontier output from their capacity output model. Since the impact of noisy data tends to cancel out in the ratio, this measure of capacity utilization may remain relatively free from noise induced bias although the estimate of capacity output may be highly biased.

<sup>9</sup> Banker et al. (1989) suggest a heuristic that sample size should exceed three times the sum of inputs and outputs, however Pedraja-Chapporo, Salinas-Jimenez and Smith (1999) show that sample size must increase exponentially with the number of inputs to preserve a given level of model performance.

scale (VRS) will generally provide lower estimates of technically efficient output than those that assume constant returns to scale (CRS). This is because, in determining the relevant frontier for a particular firm, the VRS version of DEA is restricted to considering firms of similar size and may eliminate comparisons to more efficient firms that are larger or smaller. Empirical applications (e.g., Sharma, Leung and Zaleski 1997, Hjalmarsson, Kumbhakar and Heshmati 1996, Bjurek, Hjalmarsson and Forsund 1990) have found CRS and VRS estimates of efficiency from DEA models that straddle estimates from SF. SF estimates of technically efficient output will also vary with the specifications chosen. They will tend to be lower when more flexible functional forms are used. Both DEA and SF estimates of technically efficient output may be downwardly biased if catch is under-reported.

The results of this study should not be interpreted as an indication of the general superiority of either DEA or SF. While DEA may confound noise with inefficiency, SF may confound inefficiency with specification error. The accuracy of estimates of efficiency, or technically efficient or capacity output depends on many factors. The choice of the appropriate method should depend on a careful weighing of these factors as well as the likely presence of noise in the data.

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