# Effects of Topology on the Light Front

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We discuss how to construct theta vacua in the light-front field theories using the 1+1 dimensional Abelian Higgs model as an example. Unlike the non-gauged scalar field, zero modes of the Higgs field are in general dynamical as well as the gauge-field zero mode. While symmetry breaking is discussed in semi-classical treatment of the zero modes, the theta vacua are introduced in the quantum level by use of the large gauge symmetry.

## 1. "Vacuum physics" vs "light-front quantization"

Physics of longitudinal zero modes (ZMs) in the Discretized Light-Cone Quantization (DLCQ) approach [1] has been extensively studied with the aim of solving a problem how to realize the "vacuum physics" even with (almost) a trivial light-front vacuum. In DLCQ, we set the longitudinal direction $x^-$ finite with periodic boundary condition so that the ZMs can be safely treated. Then, for example, the ZMs of (non-gauged) scalar fields are found to be constrained and subject to relations called the *Zero-Mode Constraints* (ZMCs). It is now widely believed that nonperturbative treatment of ZMCs is crucial for describing spontaneous symmetry breaking (SSB) on the LF [2].

On the other hand, our understanding on the dynamics of the gauge-field ZM is too poor. Relevance of the gauge-field ZM in constructing the theta vacua is demonstrated only in very simple models such as the bosonized Schwinger model [3]. However, little is known about real effects of topology, which will involve quantum tunneling of the dynamical ZM. To observe such effects in (or give some basic framework for) more nontrivial models is our primary motivation. For that purpose, it is natural to consider the 1+1 dimensional Abelian Higgs model: This is familiar to us as an example whose instanton solutions drastically change a naive picture of the Higgs phase [4]. Before investigating the topological effects, we have to understand the Higgs mechanism on the light front. To our surprise, however, this subject has never been studied in DLCQ. It is not well-known that we cannot follow the same method as in SSB: solving the ZMCs.* To clarify this point is our secondary motivation. In the following, we first investigate canonical structure of the model and determine physical degrees of freedom in the vacuum sector [6]. With these ZMs, we then construct theta vacua and discuss quantum tunneling.

*If we ignore gauge-field ZM, we can discuss the Higgs mechanism by solving ZMC [5]. But this treatment is not suitable for our case because we want to see the dynamics of gauge-field ZM.

## 2. Abelian Higgs model on the light front

### 2.1. Canonical structure

In order to determine the true vacuum, it is very important to clarify which zero modes are dynamical. In simple scalar models, ZMs are not dynamical and subject to ZMCs. In the Abelian Higgs model, however, the situation is not so simple. To see this, let us consider the Euler-Lagrange equation for the Higgs field:

$$2D_-D_+\phi - ie\phi\ \Pi^- + \frac{\partial V}{\partial \phi^*} = 0, \tag{1}$$

where $D_\mu = \partial_\mu - ieA_\mu$, $\Pi^- = F_{+-} = \partial_+ A_- - \partial_- A_+$ and we take the potential so that the symmetry breaking occurs in the tree level: $V(\phi) = \lambda/4\,(\phi^*\phi - v^2)^2\,,\ v^2 > 0$. The covariant derivative $D_-$ can be replaced by a normal derivative $\partial_-$ if you introduce the (LF) spatial Wilson line

$$W(x^+, x^-) \equiv \exp\left\{ie\int_{-L}^{x^-} dy^- A_-(x^+, y^-)\right\}. \tag{2}$$

Then the Euler-Lagrange equation is equivalently rewritten as

$$\partial_-[2W^{-1}D_+\phi] = W^{-1}\left[ie\phi\ \Pi^- - \frac{\partial V}{\partial \phi^*}\right]. \tag{3}$$

Unlike the simple scalar case, integration of the left-hand-side does not necessarily vanish because the Wilson line is not periodic in general. This means that *only if the Wilson line is periodic* $W(-L) = W(L)$, the space integration of (3) generates a constraint analogous to the usual ZMC. Indeed we can confirm ourselves by carefully following the standard Dirac's procedure for constrained system. If we take the LC axial gauge, the only unfixed degrees of freedom is the ZM of $A_-$. The Wilson line is periodic when the ZM takes discrete values $\overset{\circ}{A}_- = \pi N/eL,\ N \in \mathbf{Z}$. Therefore the canonical structure of the model is summarized as follows:

- **Periodic Wilson line**: $W(-L) = W(L) \iff \overset{\circ}{A}_- = \pi N/eL$
  There is an extra constraint $\int_{-L}^{L} dx^-$ eq.(3) $= 0$ and one mode in the scalar field is a constrained variable.
- **Non-periodic Wilson line**: $W(-L) \neq W(L) \iff \overset{\circ}{A}_- \neq \pi N/eL$
  There is no extra constraint and all the modes of the scalar field are dynamical.

Note that we can meet the same kind of 'strange' structure in any dimensions [6] and even in non-Abelian gauge theories [7]. Some people discuss when the gauge field takes the above discrete values, but it corresponds to a very special case and it is very likely that such restriction misses important physics. In the following, we discuss only the nonperiodic case because we are interested in the dynamics of the gauge-field ZM.

### 2.2. Symmetry breaking and quantization around classical vacua

Let us confine ourselves to the nonperiodic Wilson line case. At first one may wonder how to describe SSB without solving ZMC. However, our situation here is rather similar to the usual calculation. We just evaluate the vacuum energy and find the true vacuum.

Before developing a full-quantum theory, let us incorporate the effects of symmetry breaking in the tree (classical) level. It is easy to find classical configurations which minimize the LF energy. The LF energy becomes zero if and only if field configuration is given by $\phi = v\ \mathrm{e}^{i\pi N\frac{x^-}{L}},\ \overset{\circ}{A}_-= \pi N/eL \quad (N \in \mathbf{Z})$. This configuration gives nonzero vev to the scalar field. Then we can proceed to quantum theory analogously to the *background field method*. Let us introduce "fluctuations" by $A_\pm = A^{(N)}_{\pm\text{b.g.}} + V_\pm$ and $\phi_N = v + \varphi_N$ where $A^{(N)}_{+\text{b.g.}} = 0,\ A^{(N)}_{-\text{b.g.}} = \frac{\pi N}{eL}$. Introduction of the twisted scalar field $\phi_N = \mathrm{e}^{-i\pi N\frac{x^-}{L}}\phi$ makes the decomposition (into ZM and non-ZM) $V_\pm = \overset{\circ}{V}_\pm + \tilde{V}_\pm$ and $\varphi_N = \overset{\circ}{\varphi}_N + \tilde{\varphi}_N$ useful. The use of $\phi_N$ and $V_\pm$ corresponds to performing the *large gauge transformation*

$$\overset{\circ}{A}_- \longrightarrow \overset{\circ}{A}_- + \frac{\pi n}{eL}, \quad \phi \longrightarrow \mathrm{e}^{i\pi n\frac{x^-}{L}}\phi, \qquad n \in \mathbf{Z}. \tag{4}$$

Since the theory is symmetric under (4), there is no effect in the Lagrangian. To avoid the singularity $\overset{\circ}{V}_-= 0$, we consider only $0 < \overset{\circ}{V}_-< \pi/eL$. This restriction with a given $N$ means that we work in a fixed sector with respect to the large gauge transformation.

Following the Dirac quantization in the LC axial gauge, we obtain complicated results for commutators between physical variables: $\overset{\circ}{V}_-,\ \overset{\circ}{\Pi}^-,\ \varphi_N$, and $\varphi_N^\dagger$ [6]. However, such complication can be reduced if we introduce new variables $q = \frac{eL}{\pi}\ \overset{\circ}{V}_-,\ p = \frac{2\pi}{e}\ \overset{\circ}{\Pi}^-$, and $a_0 = \sqrt{4\pi q}\ (v+ \overset{\circ}{\varphi}_N)$ instead of $\overset{\circ}{V}_-,\ \overset{\circ}{\Pi}^-$, and $\overset{\circ}{\varphi}_N$. Then commutators among ZM variables are simply $[q,\ p] = i,\ [a_0, a_0^\dagger] = 1$ and the others are zero. Moreover, a clever mode expansion of the nonZM can simplify the algebra $[a_n,\ a_m^\dagger] = \delta_{n,m},\ [b_n,\ b_m^\dagger] = \delta_{n,m}$, where

$$\tilde{\varphi}_N(x) = \frac{1}{\sqrt{4\pi}}\sum_{n>0}\left[\frac{1}{\sqrt{n+q}}\ a_n\ \mathrm{e}^{-\frac{i\pi n}{L}x^-} + \frac{1}{\sqrt{n-q}}\ b_n^\dagger\ \mathrm{e}^{\frac{i\pi n}{L}x^-}\right]. \tag{5}$$

Note also that the ZM variables $(p,\ q,\ a_0)$ and nonZM variables $(a_n,\ b_n)$ commute.

### 2.3. Large gauge symmetry and theta vacua

Now let us discuss the vacuum structure. First of all, the Fock vacuum with respect to the nonzero modes is defined by $a_n|\tilde{0}\rangle_N = b_n|\tilde{0}\rangle_N = 0,\ n > 0$, where we attached a subscript $N$ in order to remind that we are working in a fixed background. In general, the true vacuum in the "$N$" sector will be constructed only by the zero modes on the Fock vacuum: $|\text{vac}\rangle_N = f(\alpha^\dagger, a_0^\dagger)|0\rangle_0 \otimes |\tilde{0}\rangle_N$ where $\alpha^\dagger = (q - ip)/\sqrt{2}$ is the creation operator of the gauge-field ZM and $|0\rangle_0$ is the Fock vacuum of the vacuum sector $a_0|0\rangle_0 = \alpha|0\rangle_0 = 0$.

In general, when we perform regularization of composite operators, it is favorable to respect residual symmetries as much as possible. The residual symmetry here is the large gauge transformation (4). However, it is known that a naive point-splitting regularization causes undesirable anomaly [8]. In order to avoid such problem, we use only the background field $A^{(N)}_{-\text{b.g.}} = \pi N/eL$ in the string operator of the point splitting. This modified point splitting keeps the large gauge transformation because it is essentially a shift operation of $N$ of the background field into $N + n$. Using this, the global Gauss law operator $\overset{\circ}{G} = \left\langle ie(\phi_N^\dagger \Pi_{\phi_N^\dagger} - \Pi_{\phi_N}\phi)\right\rangle_0$ is regularized as

$$\overset{\circ}{G}_{\text{reg}} = \frac{e}{2L}\left[a_0^\dagger a_0 + \sum_{n>0}\left(a_n^\dagger a_n - b_n^\dagger b_n\right)\right] - \frac{e}{2L}\left[\frac{eL}{\pi}A^{(N)}_{-\text{b.g.}} - 1\right]. \tag{6}$$

If we were to apply the physical state condition $\overset{\circ}{G}_{\rm reg}\,|{\rm phys}\rangle = 0$ on the vacuum, a part of the structure of the true vacuum will be determined

$$|{\rm vac}\rangle_N = \psi(\alpha^\dagger)\frac{1}{\sqrt{(N-1)!}}\,(a_0^\dagger)^{N-1}|0\rangle_0 \otimes |\tilde{0}\rangle_N. \tag{7}$$

The wave function $\psi(\alpha^\dagger)$ should be determined from the dynamics.

If we further impose the large gauge symmetry on the true vacuum, then it is natural to consider the linear combination of $|{\rm vac}\rangle_N$:

$$|\theta\rangle = \sum_{N\in\mathbf{Z}} {\rm e}^{iN\theta}|{\rm vac}\rangle_N. \tag{8}$$

This is the theta vacua labeled by one parameter $\theta$.

## 3. Discussion

We have formally constructed the theta vacua by *imposing* the large gauge symmetry. Strictly speaking, however, it should *not* be imposed but should be checked after we examine the dynamics of the gauge-field ZM. If the potential for the gauge-field ZM is infinitely high, there would be no tunneling and we need not construct the theta vacua. For example, it is known that the large gauge symmetry of QED is spontaneously broken and the massless photon can be understood as a NG boson [9]. On the other hand, the Higgs phase can be interpreted as the "symmetric" phase of the large gauge symmetry and so the gauge field can acquire mass [10]. Therefore it is necessary to determine the shape of the potential or, equivalently, to calculate the vacuum equation $H_{\rm ZM}|\theta\rangle = E(\theta)|\theta\rangle$, $H_{\rm ZM} = {}_N\langle 0|H|0\rangle_N$. Nevertheless, such task is practically very hard because it suffers from complicated renormalization. So we are now trying to estimate the vacuum equation in some effective approximations with less degrees of freedom.

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