Cosmology Course

Classical Cosmology:

The Big Bang Nucleosynthesis

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Bibliography

- Cosmological Physics, John Peacock (1999)
- The Early Universe, Edward Kolb and Michael Turner (1990)
- Gravitation and Cosmology, Steven Weinberg (1972)
- Review of Big Bang Nucleosynthesis and Primordial Abundances, David Tytler, John M. O'Meara, Nao Suzuki & Dan Lubin (astro-ph/0001318)

Nuclear Statistical Equilibrium (NSE)

In *kinetic* equilibrium, the number density of a very nonrelativistic nuclear species $A(Z)$ is given by

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_A - m_A}{T}\right)$$

In *chemical* equilibrium,

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

The binding energy of the nuclear species $A(Z)$ is

$$B_A \equiv Z m_p + (A - Z) m_n$$

The mass fraction contributed by nuclear species $A(Z)$

$$X_A \equiv \frac{A n_A}{n_N}$$
$$\sum_i X_i = 1$$

The present baryon-to-photon ratio is given by

$$\eta \equiv \frac{n_N}{n_\gamma} = 2.68 \times 10^{-8} \left(\Omega_B\, h^2\right)$$

$$Q \equiv m_n - m_p = 1.293\,\mathrm{MeV}$$

In NSE the mass fraction of species $A(Z)$ is given by

$$X_A = g_A \left[\zeta(3)^{A-1}\pi^{(1-A)/2}2^{(3A-5)/2}\right] A^{5/2} \times \left(\frac{T}{m_N}\right)^{3(A-1)/2} \zeta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$$

^AZ	B_A	g_A
^{2}H	2.22 MeV	3
^{3}H	6.92 MeV	2
^{3}He	7.72 MeV	2
^{4}He	28.3 MeV	1
^{12}C	92.2 MeV	1

A rough estimate of when a nuclear species $A(Z)$ becomes thermodynamically favoured is given by

$$T_{NUC} \simeq \frac{B_A}{(A-1)\left[1.5\ln\left(\frac{m_N}{T}\right) - \ln(\eta)\right]}$$

where $X_A \simeq X_n \simeq X_p \simeq 1$ has been assumed.

Initial Conditions

$(T \gg 1\,\mathbf{MeV}, t \ll 1\,\mathbf{sec})$

The balance between neutrons and protons is kept by the weak interactions:

$$\begin{array}{ccccccccc} & & n & \longleftrightarrow & p & + & e^- & + & \bar{\nu} \\ \nu & + & n & \longleftrightarrow & p & + & e^- & & \\ e^+ & + & n & \longleftrightarrow & p & + & \bar{\nu} & & \end{array}$$

When $\Gamma_{\text{weak}} \gtrsim H$, *chemical* equilibrium $\Longrightarrow$

$$\frac{n}{p} \equiv \frac{n_n}{n_p} = \frac{X_n}{X_p} = \exp\left[-\frac{Q}{T} - \left(\frac{\mu_e - \mu_\nu}{T}\right)\right] \approx \exp\left(-\frac{Q}{T}\right)$$

In the low-temperature and high-temperature limits

$$\Gamma_{pe\to\nu n} \to \begin{cases} 1.636\frac{G_F^2}{2\pi^3}(1+3g_A^2)m_e & T \ll Q, m_e \\ \frac{7}{60}\pi(1-3g_A^2)G_F^2T^5 \simeq G_F^2T^5 & T \gg Q, m_e \end{cases}$$

By comparing Γ_{weak} with $H(\simeq 1.66 g_*^{1/2}T^2/m_{Pl})$ we find

$$\frac{\Gamma_{\text{weak}}}{H} \approx \left(\frac{T}{0.8\,\text{MeV}}\right)$$

$$
\begin{aligned}
X_n/X_p &= \exp(-Q/T) \\
X_2 &= 16.3(T/m_N)^{3/2}\eta\exp(B_2/T)X_nX_p \\
X_3 &= 57.4(T/m_N)^3\eta^2\exp(B_3/T)X_nX_p^2 \\
X_4 &= 113(T/m_N)^{9/2}\eta^3\exp(B_4/T)X_n^2X_p^2 \\
X_{12} &= 3.22\times10^5(T/m_N)^{32/2}\eta^{11}\exp(B_{12}/T)X_n^6X_p^6 \\
1 &= X_1+X_2+X_3+X_4+X_{12}
\end{aligned}
$$

^AZ	T_{NSE} (MeV)
^{2}H	0.07
^{3}He	0.11
^{4}He	0.28
^{12}C	0.25

@ $T \approx 0.1$ MeV the low abundances of D and ^{3}He delay nucleosynthesis briefly!

Productions of the Light Elements

$(T = 10\,\mathbf{MeV}, t = 10^{-2}\,\mathbf{sec})$

- The energy density is dominated by radiation, $e^{\pm}$ and 3 neutrino species $\Longrightarrow g_* = 10.75$
- $\Gamma/H \gg 1 \Longrightarrow (n/p) = (n/p)_{EQ}$
- $T_\nu = T$

For $\eta = 10^{-9}$

X_n, X_p	X_2	X_3	X_4	X_{12}
0.5	2×10^{-12}	2×10^{-23}	2×10^{-34}	2×10^{-126}

$(T = 10\,\mathbf{MeV}, t = 1\,\mathbf{sec})$

- The three neutrino species decouple from the plasma
- $e^{\pm}$ pairs annihilate and their entropy is transfered into the photons $\Longrightarrow T = (11/4)T_\nu$
- $\Gamma/H \leq 1 \Longrightarrow (n/p) = \exp(-Q/T_F) = 1/6$

X_n	X_p	X_2	X_3	X_4	X_{12}
$1/7$	$6/7$	10^{-12}	10^{-23}	10^{-28}	10^{-108}

$(T = 0.3\,\textbf{to}\,0.1\,\textbf{MeV}, t = 1\,\textbf{sec.}\;\;\textbf{to}\,3\,\textbf{min})$

- g_* decreases to its value today 3.36
- (n/p) decreases from $\sim 1/6$ to $\sim 1/7$
- The number densities at earlier times are too low to allow nuclei to be built up directly from many-body collisions like $2n + 2p \rightarrow {}^4\text{He}$

$$
\begin{array}{ccccccccccc}
p & + & n & \longleftrightarrow & \text{D} & + & \gamma & & & & \\
\text{D} & + & \text{D} & \longleftrightarrow & {}^3\text{He} & + & n & \longleftrightarrow & {}^4\text{He} & + & p \\
{}^3\text{He} & + & \text{D} & \longleftrightarrow & {}^4\text{He} & + & \text{n} & & & &
\end{array}
$$

- @ $T \simeq 0.5\,\text{Mev}$ the value of ^{4}He falls bellow its NSE value because the reactions are not fast enough (low number density + Coulomb barrier)
- While the abundances of D, ^{3}He and ^{4}He are $\gtrsim$ the NSE abundances, these are still very small: $X_i = 10^{-12}, 2 \times 10^{-19}, 5 \times 10^{-19}$

- Until $T \simeq 0.1\,\mathrm{Mev} \simeq T_{NUC}$ the reactions cannot produce enough ^{4}He to establish its NSE abundances
- The Coulomb-barrier + absence of tighly-bound isotopes with mass = 5 and 8 + low nucleon density for allowing the triple-alpha reaction
- ^{7}Li is synthesized, $^7\mathrm{Li/H} \approx 10^{-10}\,\mathrm{to}\,10^{-9}$
- Substantial amounts of D and ^{3}He are left unburnt (d, $^3\mathrm{He/H} \sim 10^{-5}\,\mathrm{to}\,10^{-4}$)

Observations

- $(\mathrm{D/H})_P = (3.4 \pm 0.3) \times 10^{-5}$
- $(^3\mathrm{He/H})_P = (0.3 \pm 1.0) \times 10^{-5}$
- $\mathrm{Y}_P = 0.244 \pm 0.002$
- $(^7\mathrm{Li/H})_P = (1.7 \pm 0.15) \times 10^{-10}$
- $\eta = (5.5 \pm 0.5) \times 10^{-10} \Longrightarrow$
  $(\Omega_B\, h^2)_{BBN} = (0.020 \pm 0.002)$

From Boomerang $(\Omega_B\, h^2)_{CMB} = (0.030 \pm 0.004)$