Fermilab Academic Lecture Series Chris Quigg

Solution to Exercise 1
March 17, 1995

1. (a) Use the connection

$$M_Z^2 = \frac{\pi\alpha}{G_F\sqrt{2}x_W(1-x_W)}$$

[*Gauge Theories* (6.3.58)] to determine the weak mixing parameter x_W from the mass of the Z^0. With the standard values $\alpha = 1/137.0359895$ and $G_F = 1.16639 \times 10^{-5}$ GeV^{-2}, I compute

$$x_W = 0.2122 \quad .$$

(b) The mass of the W-boson is now

$$M_W = (1-x_W)^{1/2}M_Z = 80.9374 \text{ GeV}/c^2$$

[*Gauge Theories* (6.3.57)] .

(c) The unit of decay rate for the Z^0-boson is

$$\Gamma(Z^0 \to \nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}} = 165.88 \text{ MeV}$$

[*Gauge Theories* (6.3.63)] . The number of light neutrino species is therefore

$$N_\nu = \Gamma_{\text{invisible}}/\Gamma(Z^0 \to \nu\bar{\nu}) = 3.013 \pm 0.021 \quad .$$

(d) To compute the remaining quantities, we require

$$\Gamma(Z \to \ell^+\ell^-) = \Gamma(Z^0 \to \nu\bar{\nu})\left((2x_W-1)^2 + (2x_W)^2\right) = 84.836 \text{ MeV}$$

[*Gauge Theories* (6.3.64)] ,

$$\begin{aligned}
\Gamma(Z \to u\bar{u}) &= 3\Gamma(Z^0 \to \nu\bar{\nu})\left(1 - \frac{8x_W}{3} + \frac{32x_W^2}{9}\right) = 295.715 \text{ MeV} \\
\Gamma(Z \to d\bar{d}) &= 3\Gamma(Z^0 \to \nu\bar{\nu})\left(1 - \frac{4x_W}{3} + \frac{8x_W^2}{9}\right) = 565.399 \text{ MeV},
\end{aligned}$$

[*Gauge Theories* (7.1.45)] and

$$\begin{aligned}
a_e &= -\frac{1}{2} \\
v_e &= \frac{4x_W - 1}{2} = -0.0756
\end{aligned}$$

[*Gauge Theories* (7.2.6)] . We calculate the hadronic decay width as

$$\Gamma(Z \to \text{hadrons}) = 2\Gamma(Z \to u\bar{u}) + 3\Gamma(Z \to d\bar{d}) = 1721.709 \text{ MeV},$$

including the decays into $u\bar{u}$, $c\bar{c}$, $d\bar{d}$, $s\bar{s}$, and $b\bar{b}$. The total width is

$$\Gamma_Z = \Gamma(Z \to \text{hadrons}) + 3\Gamma(Z^0 \to \nu\bar{\nu}) + 3\Gamma(Z \to \ell^+\ell^-) = 2.47386 \text{ GeV} \quad ,$$

where we are neglecting all fermion masses except m_t. In the same way, we can compute the width of the W-boson as

$$\Gamma_W = 9\Gamma(W \to \ell\nu) = 9\frac{G_F M_W^3}{6\pi\sqrt{2}} = 2.0878 \text{ GeV} \quad ,$$

to be compared with the current world average (PDG94), 2.08 ± 0.07 GeV.

Finally, the peak cross section

$$\sigma_{\text{had}}(\text{pole}) = \sigma_{\text{peak}}(e^+e^- \to \text{hadrons}) = \frac{12\pi\Gamma(Z \to e^+e^-)\Gamma(Z \to \text{hadrons})}{M_Z^2\Gamma_Z^2} = 42.125 \text{ nb}$$

[*Gauge Theories* (7.2.5)] .

The dominant component of the electroweak radiative corrections to these observables arises from the running of the fine structure constant. If we use $1/\alpha(M_Z^2) = 129.08 \pm 0.10$ [Morris L. Swartz, "Reevaluation of the Hadronic Contributions to $\alpha(M_Z^2)$," SLAC–PUB–6710] to determine $x_W = 0.2307$ from M_Z, we find $M_W = 79.9795$ GeV/c^2 and $\Gamma_W = 2.0145$ GeV. The modified Z^0 observables are compared with LEP measurements and the lowest-order theory in the following Table:

Observable	Measured	Leading-order	Running α
M_Z	91.1888 ± 0.0044 GeV/c^2	input	input
Γ_Z	2.4974 ± 0.0038 GeV	2.4739 GeV	2.4235 GeV
$\Gamma(Z \to \text{hadrons})$	1745.9 ± 4.0 MeV	1721.7 MeV	1675.6 MeV
$\sigma_{\text{had}}(\text{pole})$	41.49 ± 0.12 nb	42.13 nb	42.01 nb
$\Gamma(Z \to e^+e^-)$	83.85 ± 0.21 MeV	84.84 MeV	83.43 MeV
$\Gamma(Z \to \mu^+\mu^-)$	83.95 ± 0.30 MeV	84.84 MeV	83.43 MeV
$\Gamma(Z \to \tau^+\tau^-)$	84.26 ± 0.34 MeV	84.84 MeV	83.43 MeV
$\Gamma_{\text{leptonic}} = \Gamma(Z \to \ell^+\ell^-)$	83.96 ± 0.18 MeV	84.84 MeV	83.43 MeV
$\Gamma_{\text{invisible}}$	499.8 ± 3.5 MeV	497.6 MeV	497.6 MeV
a_e	-0.501 ± 0.001	-0.500	-0.500
v_e	-0.038 ± 0.002	-0.076	-0.039

References

Dorothée Schaile, CERN-PPE-94-162;
C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions.*

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The exercises to accompany these lectures will be available on World Wide Web. Point your web client to `http://chrisq.fnal.gov/Memos/Exercises.html`.