PLANETARY-INDUCED NUTATION OF THE EARTH - DIRECT TERMS

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Submitted to Astronomical Journal March 1995

2 Numbered Tables + 1 supplementary table (for CD ROM?)

No Figures

ABSTRACT

Small torques by the planets can alter the direction of the Earth's rotation axis in space. These direct planetary-induced nutations have been computed using a numerical technique. 55 nutation terms with periods between 0.2 and 243 yr are presented. The maximum nutation is 0.5 milliarcseconds. Also given are planetary-induced rates and accelerations of precession and obliquity.

1. INTRODUCTION

The moon and Sun dominate the gravitational torques on the oblate Earth. The planetary torques are much smaller. At Venus' closest approach to the Earth, its torque is four orders of magnitude smaller than the dominant bodies. Torques cause changes in the Earth's orientation in space, periodic nutation and secular precession of the equator and the rotation axis normal to it. Given the accuracy of modern observations, such small planetary effects are significant. The subject of this paper is the direct planetary-induced nutations which arise from torques on the Earth. The indirect nutations, which arise through orbit perturbations, are not considered here.

There is a history of computation of planetary nutation terms. Woolard (1953) and Kinoshita (1977) considered several indirect planetary terms. Vondrak (1982, 1983a,b) computed the first extensive series of direct and indirect terms rounded to 0.01 milliarcsecond (mas). Kinoshita and Souchay (1990) tabulated direct and indirect terms truncated at 0.005 mas. Hartmann and Soffel (1994) compared the analytically computed direct terms of Kinoshita and Souchay with nutations based on a Fourier analysis of tides computed using a numerically integrated ephemeris. Discrepancies up to 0.012 mas were noted in both nutation components for periods less than 20 yr and a longer period term was suspected. The present paper recomputes the direct terms using a different technique from either paper, resolves the discrepancies, and adds several terms including one 243 yr term of size 0.02 mas.

No planetary terms are present in the 1980 IAU nutation theory (Seidelmann 1982) and no planetary-induced rates are in the IAU precession theory (Lieske et al. 1977). For the most precise applications, the planetary influences are a necessary improvement.

2. BASIC EQUATIONS

The nutation of the equator plane is expressed as two angles: the change in the angle between the equator and ecliptic planes, the nutation in obliquity $\Delta\varepsilon$, and the change in the intersection (equinox) of the two planes measured along the ecliptic plane, the nutation in longitude $\Delta\psi$. For a rigid Earth the nutations can be given for three axes: the figure axis, the instantaneous spin axis, and the angular momentum axis. The first is most useful, but the last is the easiest to compute since the derivative of the angular momentum vector is equal to the torque. All three axes are close to one another and for nutations of small amplitude, like those of this paper, it is most convenient to compute the nutation of the angular momentum axis.

The expressions for the torque on the Earth were briefly derived in Williams (1994). The coordinates in those expressions are modified here to explicitly include the

Earth's precession. The torque depends on the attracting planet's geocentric coordinates. Here coordinates $\mathbf{R}_m = (X_m, Y_m, Z_m)$ have the X_m and Y_m axes in the ecliptic plane with the X_m axis pointing toward the intersection of the ecliptic plane with the moving equator at the equinox.

$$\sin\varepsilon \, d\Delta\psi/dt = K \left[(1/2)(Y_m^2 - Z_m^2)\sin 2\varepsilon + Y_m Z_m \cos 2\varepsilon \right]$$
$$d\Delta\varepsilon/dt = K \left[-X_m Z_m \cos\varepsilon - X_m Y_m \sin\varepsilon \right] \qquad (1)$$

The factor K depends on the product of the gravitational constant G and the planet's mass m, the Earth's moments of inertia C and A (with A<C), the spin rate ω_z about the symmetry axis , and the planet's geocentric distance R

$$K = 3 G m (C-A) / C \omega_z R^5 \qquad (2)$$

The geocentric coordinates depend on the difference between the heliocentric coordinates of the planet and the Earth $\mathbf{R} = \mathbf{R}_p - \mathbf{R}_e$. As the orbital elements of the planets are expressed in a fixed reference frame, it is necessary to transform $\mathbf{R}$ into the moving frame to give $\mathbf{R}_m$. The most important part of this transformation is the precession of the equinox. Apart from using the accumulated general precession p rather than the luni-solar precession, the time-varying rotation of the plane of the ecliptic is not explicit in the following rotation.

$$\begin{pmatrix} X_m \\ Y_m \\ Z_m \end{pmatrix} = \begin{pmatrix} X \cos p - Y \sin p \\ Y \cos p + X \sin p \\ Z \end{pmatrix} \qquad (3)$$

The retrograde 50.288 "/yr rate of p is small compared to the plane.t:try mean motions.

Solving the differential equations (1) by integration of the right-hand sides is sufficient to give a good first-order solution for the direct planetary-induced nutation and precession. The computation of the right-hand sides is the subject of the next section.

3. COMPUTATIONAL TECHNIQUE

The right-hand sides of the differential equations (1) is proportional to the torques. If each right-hand side is developed as a trigonometric series with each term of the form s sin v + c cos v, with v being a linear function of time, then the integration is easy. Classically the coefficients (s, c) would be developed using analytical expansions with power series in the planetary eccentricities and inclinations multiplying functions of the semimajor axes. Here the coefficients are determined by a technique which is more numerical than analytical.

Harmonic analysis is a Fourier analysis technique (Brouwer & Clemence 1961), but it is not Fourier analysis of a time series. For the. Earth and the torquing planet, the orbital semimajor axes, eccentricities, inclinations, nodes, and perihelion directions are considered fixed so the time variation of the torque T depends on the mean longitude of the planet L, the mean longitude of the Earth E, and p the general precession. in the final trigonometric series the arguments of the sines and cosines are linear combinations of the three angles

$$v = j E + j' L + j'' p \qquad (4)$$

where j, j', and j" are integers. Fourier analysis of a time series uses time as the independent variable. The torque T(t) is treated as a one-dimensional function. The rate of the linear combinations (4) gives the frequencies at which amplitudes are desired. in harmonic analysis the torque is treated as a three dimensional function T(E, L, p). Using the conventional expressions for elliptical orbits, the torque expressions are evaluated at even intervals of the three. angles L, E, and p with each angle being sampled over a full cycle. This even sampling permits the torques to be Fourier analyzed in terms of the linear combinations of the three angles in Eq. (4). The trigonometric terms are orthogonal to one another. The rates of the three angles do not enter the procedure until the trigonometric terms are integrated to solve the differential equations. The rates are treated as constant. Slight nonlinearities in L, E, and p are ignorable. Unlike Fourier analysis of a time series, there is no concern over separating terms with similar periods and there is no problem with angles with different periods (p has a 2.5,772 yr period while the planetary periods range from 0.24 to 165 yr). Individual and complete sampling of the three angles guarantees separation. In both types of Fourier analysis, the integration can raise a small, long-period term in the torque to importance.

Note that harmonic analysis gives zero frequency terms which are secular rates, analogous to Gauss' method for orbits. The rates and very long period terms (j=j'=0) are discussed in section 7.

4. REQUIREMENTS, CONSIDERATIONS> AND ACCURACIES

What are the practical requirements for harmonic analysis? Choices must be made for the range of integers (j, j', j") in the Fourier analysis and the number of samples per angle. a) The maximum absolute values of j, j', and j" cannot exceed the integers for the Nyquist frequency for N samples per angle. That is, $N > 2\,\mathrm{Max}(|j|, |j'|, |j''|)$, To give an example, detecting the nearly commensurate term between Venus and Earth which involves the argument 8V–13E, where V is the mean longitude of Venus, requires that N be at least 27 when sampling the Earth's longitude. b) N should also be large enough to sample (j, j', j") of any term of significant size. If N is too small, significant terms outside of the sampled range of integers will alias. Aliased terms change markedly when N is changed and as a practical matter when N is large enough there is no problem.

Planetary-induced nutations were computed for the seven attracting planets Mercury, Venus, Mars-Neptune. The two planets adjacent to the Earth require the most care. For Mars, integers (j, j') up to 15 (since 8E–15M has a 40 yr period) and samples of 31 were used. For Venus, integers (j, j') up to 16 (a near commensurability 8V-13E has a 239 yr period) and N=33 were used. The other planets used integers up to 6, and 13 samples per angle. When substituted into Eq. (1), the trigonometric functions of p in Eq. (3) are raised to the powers 0, 1, and 2, so one expects $-2 \leq j'' \leq 2$ and this range was used. Of the three integers (j, j', j"), two should sample positive and negative values, but the negative values for the third can be excluded. That is, terms with negative and positive frequencies are not independent. A check case reproducing solar nutations was run by placing the sun in a planetary orbit with zero radius.

The planetary orbital elements are known with great accuracy and are input to the harmonic analysis scheme. Orbit elements have been summarized in Simon et al. (1994) based on analytical planetary theories.(Bretagnon 1982; Simon 1983; Simon and Bretagnon 1984; Bretagnon and Francou 1988) adjusted to the DE200 numerical fit and integration of the planetary data (Standish 1982, 1990). For evaluating the final series, the Simon et al. (1994) set of polynomial expressions for planetary mean longitudes with respect to the fixed J 2000 equinox are recommended. The general precession p is accumulated from J2000 and its rate is 50.288 "/yr.

What accuracy is needed? For the coefficients there is computational noise, round off, and truncation limits. Existing observational and data analysis accuracies provide a guide.

It has been a common practice to treat the accuracy of the $\Delta\psi$ and $\Delta\varepsilon$ series alike. But the observable parameters are the displacements of the pole of rotation, $\Delta\varepsilon$ and $\sin\varepsilon\,\Delta\psi$. It is argued here that the desired uncertainty of the $\Delta\varepsilon$ series should be a factor of $\sin\varepsilon = 0.40$ smaller than the $\Delta\psi$ series. When there are non-zero $\Delta\varepsilon$ coefficients, their magnitude is larger than $0.4\,\Delta\psi$ [see functions of ε in Eq. (1)]. Here, for any individual nutation argument, the coefficients of both series are retained if the $\Delta\psi$ amplitude is above the truncation limit. When fitting data with the IAU nutations, the equal truncation limit for the two series should lead to more noise (in a χ^2 sense) for $\Delta\varepsilon$.

A rigid-body nutation series is the goal of this paper, but the transformation to a non-rigid-body theory predicts a resonance which can increase the size of some terms (Wahr 1981; Dehant 1990; Mathews et al. 1991). To reach a particular goal for the non-rigid theory requires knowing smaller rigid-body terms near the resonance. The resonance is estimated to be near 430 d retrograde (Herring et al. 1991).

Analyses of different VLBI data sets give formal uncertainties of 0.01 to 0.03 mas in $\Delta\varepsilon$ and 0.03 to 0.07 mas in $\Delta\psi$ when solving for individual (luni-solar) nutation series components (Charlot et al. 1995; Souchay et al. 1995). This is equivalent to the noise expected from a spectral analysis of the data. Spectral accuracy should improve with increasing time span of observations. Kinoshita and Souchay's (1990) and Vondrak's (1982) truncation limit for individual coefficients was 0.005 mas. Hartmann and Soffel (1994) used 0.0025 mas. A truncation limit of 0.0025 mas is adopted here for $\Delta\psi$.

The harmonic analysis technique is very accurate and the computations of this paper should have noise on individual coefficients well under 1 microarcsecond. This is more than adequate for the direct terms, but it is well to consider the full nutation series. The errors in each coefficient of a large number of trigonometric terms will add together in some manner when the series is evaluated. The simplest approximation is that they add randomly so that n terms with accuracy σ for each coefficient give. a final rms (root mean square) uncertainty of $\sigma\sqrt{n/2}$. Errors may not add randomly. For example, the evaluation of the planetary terms peaks up at certain times and their errors also peak up (see section 6). Kinoshita and Souchay have a total of nearly 400 luni-solar and planetary terms. Thus, the final evaluation of the nutation series should have at least 14 times the noise of individual coefficients.

5. NUTATION RESULTS

The direct planetary-induced nutations of the Earth were computed for attracting planets other than Pluto. All four coefficients resulting from the. harmonic analysis were recorded to the nearest 0.1 μas (microarcsecond) if the amplitude (root sum square of sine and cosine coefficients) in $\Delta\psi$ was ≥ 0.5 μas (158 terms). A summary tabulation of the 55 largest terms is given in Table 1, where the coefficients are given in microarcseconds and are rounded to the nearest microarcsecond. The tabulation lower limit was set at 2.5 μas in amplitude for $\Delta\psi$. Table 1 gives 55 terms: 41 from Venus, 4 from Mars, 8 from

Jupiter, and 2 from Saturn I'or the argument, the first letter of each planet is used to indicate the mean longitude. As a refinement, an (lppolzcr-like transformation was applied to correct the nutation coefficients from the angular momentum axis to the figure axis, but this is a very small effect (no more than 0.1 pas) which will, at most, cause the last digit to round differently. The nutation coefficients should be accurate to all digits given in '1'able 1.

Because of the large variation in the distance of Venus from the Earth, the nutation signatures arc complex. in Table 1 there arc sequences with arguments j(V-E) and jV-(j+2)E-2p which arc encoding these signatures. The former sequence gives a 1.6 yr (synodic period between Venus and Earth) signature in $\Delta\psi$ resembling a rounded sawtooth. If the planetary-inctuccd precession (section 7) is also included, then the signature resembles rounded steps with rapid change at closest approach. The second sequence includes the dominant 3V–5E–2p term at 8.1 yr. The other terms in the second sequence give an 8.1 yr modulation of a synodic-period signature. Venus makes slightly more than 13 revolutions to the Earth's 8 and the Earth-Venus geometry nearly repeats after 8 yr. The 3V-5E term gives this fundamental period since, using rates, 3(13/8)-5 = – 1 /8 cycles/yr. Five synodic periods is also 8 yr. Thus, the Venus-induced nutations will show considerable repetition every 8 yr. The second largest nutation term is the 5.9 yr Jupiter term, and it has a 22 yr beat period with the 8.1 yr Venus term. These features may be seen in the plots of direct terms in Souchay and Kinoshita (1991) and 1 Iartmann and Soffel (1994).

The planetary torques can give rise to terms with the same arguments as the solar nutations. Venus causes the only significant contribution and gives a ha]f-year term with argument 2E+2p in Table 1 which has the same argument (2E+2p=2L'=2F+2Ω–2D) as the largest solar nutation. '1'here is also a Venus-induced term in $\Delta\psi$ of 3 pas sin ℓ', where ℓ' is the solar/Earth mean anomaly, corresponding to the second largest solar term.

The extended file of terms with amplitudes ≥0.5 μas has 158 entries: 1 from Mercury, 103 from Venus, 26 from Mars, 22 from Jupiter, 5 from Saturn, and 1 from Uranus. For the three planets not represented in Table. 1, the largest amplitudes arc 1.1 pas for Mercury (2403 d period), 0.7 pas for Uranus (15294 d), and 0.4 μas (29900 d) for Neptune. Venus terms with periods as short as 53 d (11V–11E) and 54 d (14V–16E–2p) arc present (both 0.6 μas). Computation of selected Venus terms with different sampling indicates that the noise for some coefficients in the extended file may reach O. 1μas. The extended file is available on request.

The series for the two nutation components were evaluate.d for the time interval 1950 to 2050. For $\Delta\psi$ and $\Delta\varepsilon$ the extreme deviations from zero were 0.46 mas and 0.15 mas, respectively, and the rms values were 0.18 mas and 0.08 mas, respectively.

6. COMPARISONS AND DISCUSSION

The results in '1'able 1 have been compared with Tables 14 and 15 of Kinoshita and Souchay (1990). Most coefficients compare within 2 pas and arc considered to agree. Noteworthy discrepancies arc marked in the last column (KS). Three terms not present in Kinoshita and Souchay which arc above their 5 pas truncation limit arc indicated with note 1. '1'here is no note for the many terms smaller than 5 μas for $\Delta\varepsilon$ or $\Delta\psi$. The missing 243 yr Venus term has a 23 μas amplitude. The term results from a small torque amplified by the long-period near commensurability. The missing 243, 8.1, and 7.8 yr terms share a common characteristic. In an analytical expansion, they would arise from powers of the eccentricities and inclinations greater than one. The discrepancies of notes 2, 3, and 5 arc misprints in the published paper (J. Souchay, private communication 1 994). The two terms of note 2 were incorrectly listed as V–E- 2p and V–2E–2p, though the slow rate of p makes this a minor problem for numerical evaluation. The Venus term of note 3 has an extraneous 12 μas out-of-phase (sine) coefficient in $\Delta\varepsilon$. This largest term also has differences of 3 pas for the in-phase $\Delta\varepsilon$ and 5 μas for in-phase $\Delta\psi$. The Mars term of note 4 is in error by 11 μas (amplitude). Note 5 results from a Jupiter term listed as E-J rather than E- 3J–2p. Kinoshita and Souchay also listed the 29.5 yr Saturn term of note 6, below their nominal cutoff limit, which has the sine and cosine coefficients interchanged.

Several of the foregoing corrections, including the larger ones, arc also evident in the comparisons done by Hartmann and Soffel (1994). They were tentative about the 29.5 yr Saturn term and the 243 yr Venus term because of their long periods. They had a good estimate for the former, but their latter term, though about the right magnitude, was phase shifted (the sine coefficients for both nutation components have reversed signs). To compare with Hartmann and Soffel it is necessary to combine terms which have the same planetary arguments, but different precession (p) factors. When this combination is done with the computations of this paper (extended file), the differences arc no larger than 1 pas for all terms except the 243 yr one.

Vondrak's (1982) series, truncated at 5 pas and rounded to the nearest 10 pas, results from an analytical expansion through the first power of the planetary eccentricities (c) and inclinations (i). It shows reasonable agreement, but there arc some differences. The missing terms 3V–5E, 5V–8E–p, and 8V-13E–2p arc expected since they arise from powers of c and i greater than onc in the analytical expansions. These three terms arc also missing in Kinoshita and Souchay. Two terms (3V–4E and 6V–9E–2p) near Vondrak's truncation limit would be below it according to the computations of this paper, but such effects can be explained by noise. More notable, the terms V-t E+2p, E+J+2p, and V–E arc too large.

The amplitudes below the 2.5 pas cutoff of Table 1 have a root-sum-square (rss) value of 13 pas for $\Delta\psi$ and 5 pas for $\Delta\varepsilon$. Since p is near zero, it is appropriate to compute the error by combining terms which differ only in p factors. The rss values of the truncated amplitudes arc then 10 pas and 4 pas, respectively. If randomly phased these truncated terms would contribute rms errors of 7 pas anti 3 μas, respectively, to $\Delta\psi$ and $\Delta\varepsilon$, but Venus dominates so, given the sequences of related terms, larger peak errors arc certain and different rms errors arc possible. The series of truncated terms was evaluated 20 times pcr year for the time interval 1950 to 2050. To $\Delta\psi$ and $\Delta\varepsilon$ they contributed rms errors of 8 pas and 4 pas, respectively, but the peak errors (absolute values) were 29 pas and 17 pas, respectively. The peak error is an order of magnitude larger than the truncation limit! Examining the time-varying nutations, the largest errors tend to be associated with “corners” adjacent to “steps” of rapid change.

For the computation of the numerical coefficients of ‘1’able 1 harmonic analysis was used instead of analytic expansions. The analytic point of view is valuable for interpreting the results. The differential equations (1) depend on terms in the numerator (Y_m^2, $X_m Y_m$, etc.) and the R^5 in the denominator. For circular, coplanar orbits the distance depends on the semimajor axes of the Earth and the attracting planet and the difference in their mean longitudes $R^2 = a_e^2 + a_p^2 - 2\, a_e a_p \cos(E - L)$. Inverted and raised to a power, the resulting series contains only a constant and cosines of the multiples $j(E–L)$, where j is an integer. For circular, coplanar orbits the terms in the numerator contain a constant (for the $\Delta\psi$ differential equation, but not $\Delta\varepsilon$), and trigonometric terms involving 2E+2p, 2L+2p, and E+L+2p. The product of the constant terms in the numerator and denominator gives precession, the constant in the numerator times the periodic terms from the denominator gives nutation terms in $\Delta\psi$ with arguments of the form j(E–L), while mixing the periodic terms from the numerator with the expansion of the denominator gives nutation arguments jE–(j+2)L–2p and (j+2)E–jL+2p. If the longitudes had been defined as being measured from the moving equinox, the 2p would not bc necessary. For circular, coplanar orbits, the nutation arguments have integer multipliers of E and L which sum to -2, 0, or 2, and the periodic terms give onl y sines in $\Delta\psi$ and cosines in $\Delta\varepsilon$. The arguments of the largest terms in Table 1 fit these patterns and the corresponding coefficients arc dominated by the zero-eccentricity, zero-inclination part of our imaginary expansion.

In an expansion using the planetary eccentricities (c) and inclinations (i) it is useful to refer to the leading power of c and i as the degree. Terms dominated by the circular, coplanar effects of the previous paragraph would be of degree zero. in Eq. (1) the terms which are linear in Z_m give degree one terms in i with a single p or –p in the arguments. Many examples arc to bc found in Table 1. The largest is 3V–5E–p at 8.1 yr. in the torque, terms of increasing degree should bc smaller, but when integrated to give the nutations it is possible for higher-degree terms to achieve prominence if of long period. Terms of non-zero degree will normally have out-of-phase coefficients (cosines in $\Delta\psi$ and sines in $\Delta\varepsilon$) because the nodes and perihelion directions introduce a phase. Using the notation of Eq. (4) the degree of a term is |j+j'–j "|, except if this sum is zero and j"=±1 when the term is of degree two. The largest terms of degree onc in c arc 5V–8E–2p at 7.8 yr and J at 11.9 yr. The largest two terms of second degree arc 3V–5E at 8.1 yr and 5V–8E–p at 7.8 yr. The 243 yr term with argument 8V–13E–2p is of degree three and would bc made up of the combinations c^3 and ci^2, where the eccentricities of both Earth and Venus enter to give six combinations. The 241 yr term 8V–13E–p is of degree four. For analytical expansions see Vondrak (1982) and Kinoshita and Souchay (1 990).

Long-period terms get amplified during the integration. There is a fifth-degree, 40 yr Mars term with argument 8E–15M–2p and amplitude 1.5 pas. The sixth-degree, 26 yr term 9E–17M–2p was checked and has an amplitude of 0.4 pas. There arc linear combinations of angles giving long periods which arc not represented in the table and which were not searched for. It is argued that terms such as E–12J–2p or 17E–32M–2p arc small because they enter at high degree: degree 9 for the former and degree 13 for the latter.

Fixed planetary orbits were used for the computations of Table 1, but there arc perturbations on the orbits. Periodic perturbations of the four inner planet orbits arc small, but there arc sizable perturbations of the outer planet orbits. Jupiter's mean longitude is perturbed with an amplitude of 0.006 radians due to the nine century "great inequality" with argument 2J–5S. Mixed with the largest Jupiter-induced nutation term in the table, this perturbation causes largest terms (in pas)

$$
\begin{aligned}
\Delta\psi &= 0.6\ \sin(4J-5S+2p) + 0.1\ \cos(4J-5S+2p) \\
&\quad - 0.6\ \sin(5S+2p) + 0.1\ \cos(5S+2p)
\end{aligned}
$$

$$
\Delta\varepsilon = -0.3\ \cos(4J-5S+2p) + 0.3\ \cos(5S+2p) \qquad (5)
$$

This class of planetary-induced nutations is small enough that it dots not require further investigation here. The influence of secular orbital changes is considered in the next section.

7. RATES AND ACC1{I.IXA'1'1ONS

in addition to nutation, the planetary torques also give rise to rates and accelerations of precession and obliquity. Their values also result from the harmonic analysis procedure. These rates and accelerations permit refinements in precession theory (Williams 1994). in addition to an improved numerical representation of precession, refinements in the theory will give a more accurate value for the moment of inertia combination (C- A)/C.

When integrated, the zero frequency coefficients in the harmonic analysis give pure secular (t) terms in precession and obliquity. The harmonic analysis also gives very long period terms with arguments of p and 2p (periods of 25772 and 12886 yr, respectively, at .12000). These very long period terms arise from the finite planetary inclinations and eccentricities and the motion of the equinox with respect to the corresponding nodes and perihelion directions. The terms depending on planetary nodes in section 4 of Williams (1994) arc the first approximation of the p terms. The very long period terms arc presented here as rates and accelerations. l'able 2 gives the contribution of each planet to the precession and obliquity rates at .12000 from the combined pure secular and two very long period terms. Total rates arc 313.645 μas/yr for precession and --13.520 μas/yr for obliquity. Table 2 updates table 2 in Williams (1994) and is more accurate. The changes from the former table arc --4.6 μas/yr in precession and 0.7 μas/yr for obliquity rate. The earlier table did not include the influence of the planetary eccentricities and it used a first-degree computation for the inclinations. The direct planetary contribution to precession rate may also be compared with Vondrak (1982), Kinoshita and Souchay (1 990), and Hartmann and Soffel (1 994). The contributions to the precession rate from the pure secular, p, and 2p terms arc 320.905 μas/yr, -8.184 μas/yr, and 0.924 μas/yr, respectively, and the corresponding contributions to the obliquity rate arc O, – 13.356 μas/yr, and --0.165 μas/yr.

Precession and obliquity accelerations arise in several ways. '1'here arc long-time-scale variations in the planetary orbits which change the "pure secular" and vey long period (p and 2p) rates, and there arc the time. expansions (t for the torques or t^2 for the integrated quantites) of the very long period p and 2p terms. The consequence of orbital element rates was computed by changing the elements in the harmonic analysis program by onc century, computing precession and obliquity rates, and differencing from the J2000 values. The ecliptic motion is not explicit in Eq. (3), but was allowed for by differencing the motion of the orbit planes of the attracting planets and Earth. The computation gives accelerations in precession of –t).?, --22.7, and 0.9 μas/century^2 from the pure secular, p, and 2p contributions. For obliquity acceleration, the corresponding figures arc 0, 2.0, and --().'7 μas/century^2. From the expansion of the p and 2p terms the accelerations arc 60.9 and 1.8 μas/century^2 for precession and - 1().6 and 1.9 μas/century^2 for obliquity. Accelerations also arise from mixing the precession and obliquity rates with the. secular change in the obliquity (-46.83 "/century) through the trigonometric functions in Eq. (1). For precession the pure secular and 2p terms depend on cos ε and the p terms depend on $\cos 2\varepsilon / \sin \varepsilon$. For obliquity the 2p terms depend on sin ε and the p terms depend on cos ε. Those accelerations are --13.6, --().4, and 0 μas/century^2 for precession and O, -0.1, and O μas/century^2 for obliquity. The latter type of accelerations would automatically be a part of a solution of the differential equations for precession variables using functions of a time-varying obliquity (see section 8 of Williams 1 994). Here the rates and accelerations have been split into their three components so that they can be used in such a precession solution. The total direct planetary-induced precession acceleration is 25 pas/cent ury^2 and the obliquity acceleration is -8 μas/century^2.

8. SUMMARY

Torques from the planets contribute to nutations of the Earth. Though small, they should not be ignored for processing high accuracy observations. This paper concentrates on the direct planetary nutations which arise from gravitational torques on the Earth (indirect nutat ions from orbit perturbations are also important, but are not considered here). The differential equations for direct nutations are given in section 2.

The differential equations are solved using a numerical technique (section 3). The resulting nutation terms are described in section 5 and those with amplitudes larger than 2.5 pas are given in Table 1. Some corrections and additions to earlier work are noted in section 6, The largest addition is a 23 μas term with 243 yr period. The direct planetary torques also give rise to rates and accelerations of precession and obliquity (section 7). Table 2 gives the rates due to each planet. improvements in the nutation and precession theories should benefit the fitting and interpretation of high-accuracy data types such as interferometry and ranging.

ACKNOWLEDGMENTS

I thank J. Souchay and H. Kinoshita for discussions of their work. The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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Table 1. Direct planetary-induced nutations with amplitudes larger than 2.5 μas. Each argument consists of a linear combination of the mean longitudes of the five planets Venus-Saturn, and the general precession p accumulated from J2000. The column headed KS lists notes on the comparison with Kinoshita and Souchay (1990).

Argument						Δψ (pans)		Δε (μas)		Period		KS
V	E	M	J	S	p	sin	Cos	sin	cos	days	years	
1	-1	0	0	0	1	-2	-6	-3	1	583.89	1.599	2
1	-1	0	0	0	0	85	0	0	0	583.92	1.599	
1	-1	0	0	0	--1	-2	9	-5	1	583.96	1.599	
0	1	0	0	0	0	0	-3	0	0	365.26	1.000	
1	1	0	0	0	2	-3	0	0	2	139.11	0.381	
2	--2	0	0	0	1	--1.	-3	-1	0	291.95	0.799	
2	-2	0	0	0	0	35	0	0	0	291.96	0.799	
2	-2	0	0	0	--1	-1	5	-3	1	291.97	0.799	
1	-2	0	0	0	0	0	-9	0	0	975.38	-2.670	2
0	2	0	0	0	1	1	-3	-2	0	182.62	0.500	
0	2	0	0	0	2	-9	0	0	4	182.62	0.500	
3	-3	0	0	0	0	19	0	0	0	194.64	0.533	
3	-3	0	0	0	-1	-1	3	-2	0	194.64	0.533	
2	-3	0	0	0	0	0	14	0	0	1454.94	3.983	
1	-3	0	0	0	-1	-1	--5	2	1	-265.73	-0.728	
1	--3	0	0	0	-2	17	0	0	7	-265 ."/3	-0.728	
4	-4	0	0	0	0	11	0	0	0	145.98	0.400	
3	--4	0	0	0	0	0	4	0	0	416.69	1.1.41	
2	-4	0	0	0	-1	-2	-8	4	-1	-48"/ .66	-1..335	
2	-4	0	0	0	-2	35	0	0	15	-48"/ .64	-1.335	
5	-5	0	0	0	0	7	0	0	0	116. "/8	0.320	
3	-5	0	0	0	0	7	-3	0	0	-2959.21	--8.102	1
3	-5	0	0	0	-1	-1o	43	23	-6	--2958.28	-8.099	
3	-5	0	0	0	-2	215	0	0	93	-'295'1 .35	--8.097	3
6	-6	0	0	0	0	4	0	0	0	97.32	0.266	
4	-6	0	0	0	-)	2	9	--5	1	'/2"] .52	1.992	
4	-6	0	0	0	-2	-50	0	0	22	"/'2'/.58	1.992	
3	-6	0	0	0	-2	0	3	-1	0	-325.10	-0.890	
' /	- ' /	0	0	0	0	3	0	0	0	83.42	0.228	
5	-'/	o	0	0	-1	1	3	--2	0	323.93	0.887	
5	-'/	o	0	0	-2	-20	0	0	-9	323.94	0.887	
4	-7	0	0	o	-2	0	7	--3	0	'733.47	-2.008	
6	-8	0	0	0	-2	-1.2	0	0	-5	208.35	0.570	
5	-8	0	0	0	-1	-6	1	-1	-3	2863.02	7.839	1
5	-8	0	0	0	-2	1	27	12	0	2863.89	7.841	
7	-9	0	0	o	-2	-7	0	0	-3	153.56	0.4'20	
6	-9	0	0	0	-2	0	-4	2	0	485.03	1.328	
8	-10	0	0	0	-2	-5	0	0	-2	121.59	0.333	
9	-11	0	0	0	-2	-3	0	0	-1	100.63	0.276	
8	-13	0	0	0	-1	4	-2	1	2	88081.94	241.155	
8	-13	0	0	0	-'2	6	22	-9	3	88913.94	243.433	1
0	1	-2	0	0	0	-9	5	0	0	-5'/ 64.01	-15.781	4
0	1	-1	0	0	0	3	0	0	0	'/"/9.94	2.135	
0	2	-4	0	0	-2	5	0	0	2	-2880.24	--7.886	

0	3	-6	0	0	-2	2	-1	1	1	-1920.55	-5.258	
0	0	0	1	0	0	34	-5	0	0	4332.59	11.862	
0	0	0	1	0	2	4	2	1	-2	4328.60	11.851	
0	0	0	2	0	1	1	5	3	-1	2165.80	5.930	
0	0	0	2	0	2	-106	0	0	46	2165.30	5.928	
0	0	0	3	0	2	-12	2	1	5	1443."15	3.953	
0	1	0	-4	0	-2	-3	-1	0	-1	551.16	1.509	
0	1	0	-3	0	-2	-11	0	0	-5	488.96	3.339	5
0	1	0	-1	0	0	12	0	0	0	398.88	1.092	5
0	0	0	0	1	0	0	-4	0	0	10759.23	29.457	6
0	0	0	0	2	2	-12	0	0	5	5373.47	14.712	

Notes to column KS in table 1.
1) Not present in KS above 5 μas. 2) Different integer factor for p in $\Delta\psi$. 3) Different coefficient for $\Delta\varepsilon$. 4) Different coefficient for $\Delta\psi$. 5) Mistaken argument for $\Delta\varepsilon$. 6) Coefficients interchanged.

TABLE 2. Precession and obliquity rates at J2000 from direct planetary torques on the Earth's bulge. Uses (C–A)/C = 0.0032737634.

Planet	ψ rate μas/yr	ε rate μas/yr
Mercury	3.697	-0.088
Venus	181.565	-16.813
Mars	5.998	0.356
Jupiter	117.068	2.804
Saturn	5.188	0.219
Uranus	0.300	0.001
Neptune	0.029	0.001
Total	313.645	-13.520

Extended File. Direct plancta~-induced nutations with amplitudes larger than 0.5 μas. Each argument consists of a linear combination of the mean longitudes of the seven planets Mercury-Uranus, and the genera] precession p accumulated from J2000. The degree is the implicit power of the planetary eccentricities and inclinations.

Argument								Δψ μas)		Δε (μas)		Period		Deg
Q	V	E	M	J	S	U	p	sin	cos	sin	cos	days	years	
1	o-4		0	0	0	0	-2	0.4	1.0	-0.4	0.2	2402.79	6.578	1
0	1	0	0	0	0	0	0	0.0	-1.2	0.0	0.0	224.70	0.615	1
0	2	0	0	0	0	0	1	0.4	-1.5	-0.8	-0.2	112.35	0.308	1
0	2	0	0	0	0	0	2	-1.6	0.0	0.0	0.7	112.35	0.308	0
0	3	- 1	o	o	0	0	2	-0.9	0.0	0.0	0.4	94.22	0.258	0
0	3	-1	0	0	0	0	1	0.'2	--1.0	-0.5	--0.1	94.22	0.258	1
0	2	-1	0	0	0	0	0	0.0	-0.6	0.0	0.0	162.26	0.444	1
0	1	-1	0	0	0	0	1	-1.5	-6.4	-3.4	0.8	583.89	1.599	1
0	1	-1	0	0	0	0	0	84.6	0.0	0.0	-0.1	583.92	1.599	0
0	1	-1	0	0	0	0	-1	-2.2	9.5	-5.1	-1.2	583.96	1.599	1
0	0	1	0	0	0	0	0	0.0	-2.7	0.0	0.0	365.26	1.000	1
0	1	3	0	0	0	0	1	0.5	-2.3	-1.2	--0.3	139.12	0.381	1
0	1	1	0	0	0	0	2	-3.3	0.0	0.0	1.5	139.11	0.381	0
0	4	- 2	0	0	0	0	1	0.2	-0.7	-0.4	-0.1	81.13	0.222	1
0	2	- 2	0	0	0	0	1	-0.6	-2.5	--1.3	0.3	291.95	0.799	1
0	2	-2	o	0	0	0	0	35.0	0.0	0.0	0.0	291.96	0.799	0
0	2	-2	0	0	0	0	- 1	-1.2	5.1	--2.7	-0.6	291.97	0.799	1
0	1-2		0	0	0	0	1	-0.9	0.2	0.1	0.5	--975.48	-2.671	2
0	1	-2	0	0	0	0	0	0.1	-8.7	0.0	0.0	--975.38	-2.670	1
0	1	-2	0	0	0	0	-1	0.7	0.2	-0.1	0.4	--9"? 5.28	-2.670	2
0	0	2	0	0	0	0	0	-0.5	-0.2	0.0	0.0	182.63	0.500	2
0	0	2	0	0	0	0	1	0.8	-3.2	-1 .7	-0.4	182.62	0.500	1
0	0	2	0	0	0	0	2	-8.7	0.0	0.0	3. 7	182.62	0.500	0
0	3-3		0	0	0	0	1	-0.3	-1.3	--0.7	0.2	194.64	0.533	1
0	3-3		0	0	0	0	0	18.7	0.0	0.0	0.0	194.64	0.533	0
0	3-3		0	0	0	0	-1	-0.8	3.4	--1.8	-0.4	194.64	0.533	1
0	2	-3	0	0	0	0	1	1.3	-0.3	--0.2	-().'/	1454.71	3.983	2
0	2-3		0	0	0	0	0	-0.2	13.7	0.0	0.0	1454.94	3.983	1
0	2	- 3	0	0	0	0	- - 1	-1.8	-0.3	0.2	-1.0	1455.16	3.984	2
0	2	- 3	0	0	0	0	--2	-0.5	-1.1	0.5	-0.2	1455.39	3.985	1
0	1	-3	0	0	0	0	0	0.8	-0.3	0.0	0.0	-265.74	-0.728	2
0	1	-3	0	0	0	0	--1	-1.1	-4.6	2.4	-0.6	-265.73	-0.728	1
0	1	- 3	0	0	0	0	--2	17.0	0.0	0.0	7.3	-265.73	-0.728	0
0	4-4		0	0	0	0	1	-0.2	-0.7	-0.4'	0.1	145.98	0.400	1
0	4-4		0	0	0	0	0	11.0	0.0	0.0	0.0	145.98	0.400	0
0	4-4		0	0	0	0	--1	-0.6	2.4	-1.3	-0.3	145.98	0.400	1
0	3-4		0	0	0	0	0	-o.)	3.9	0.0	0.0	416.69	1.141	1
0	3	- 4	0	0	0	0	--1	-0.7	-0.3	0.1	-0.4	416.71	1.141	2
0	3-4		0	0	0	0	--2	-0.1	-0.5	0.2	-0.1	416.73	1.141	1
0	2-4		0	0	0	0	0	1.3	-0.5	0.0	0.0	-487.69	-1.335	2
0	2	-4	0	0	0	0	--1	-1.9	-7.9	4.2	-1.0	-487.66	-1.335	1
0	2-4		0	0	0	0	--2	34.9	0.0	0.0	15.1	-487.64	--1.335	0
0	1	-4	0	0	0	0	--2	0.0	0.8	-0.3	0.0	-153.82	-0.421	1

o	5 -5	0	0	0	0	0	6.8	0.0	0.0	0.0	1.16.78	0.320	0
0	5 -5	0	0	0	0	-1	-0.4	1.7	-0.9	-0.2	116.79	0.320	1
0	4 -5	0	0	0	0	0	0.0	2.1	0.0	0,0	243.16	0.666	1
0	3 -5	0	0	0	0	0	7.2	-2.7	0.0	0.0	2959.21	-8.102	2
0	3 - 5	0	0	0	0	-1	-10.3	-43.1	23.0	-5,5	2958.28	-8.099	1
0	3 --5	0	0	0	0	- 2	215.0	0.1	-0.1	93.2	2957.35	-8.097	0
0	2 -5	0	0	0	0	- 2	-0.1	1.4	-0.6	0.0	-208.83	-0.572	1
0	6 - 6	0	0	0	0	0	4.4	0.0	0.0	0.0	97.32	0.266	0
0	6 - 6	0	0	0	0	- 1	-0.3	1.3	-0.7	-0.2	97.32	0.266	1
0	5 -6	0	0	0	0	0	0.0	1.3	0.0	0.0	171.67	0.470	1
0	4 -6	0	0	0	0	0	-1.6	0.6	0.0	0.0	727.47	1.992	2
0	4 -6	0	0	0	0	- 1	2.2	9.3	-4.9	1.2	727.52	1.992	1
0	4 - 6	0	0	0	0	- 2	-50.3	0.0	0.0	-21.9	727.58	1.992	0
0	3 -6	0	0	0	0	-1	0.7	-0.1	0.1	0.4	-325.11	-0.890	2
0	3 - 6	0	0	0	0	-2	-0.1	2.6	-1.1	0.0	-325.10	-0.890	1
0	7 -7	0	0	0	0	0	2.8	0.0	0.0	0.0	83.42	0.228	0
0	7 -7	0	0	0	0	-1	-0.2	0.9	-0.5	-0.1	83.42	0.228	1
0	6 -“?	0	0	0	0	0	0.0	0.9	0.0	0.0	132.67	0.363	1
o	5 7	0	0	0	0	0	--0.6	0.2	0.0	0.0	323.92	0.887	2
o	5 -7	0	0	0	0	-1	0.9	3.5	-3.9	0.5	323.93	0.887	1
o	5 -7	0	0	0	0	-2	-20.4	0.0	0.0	-8.9	323.94	0.887	0
o	4 -7	0	0	0	0	-1	1.5	-0.3	0.2	0.8	--”/33.53	-2.008	2
o	4 -7	0	0	0	0	-2	-0.3	6.6	-2.9	-0.1	--”/33.47	-2.008	1
0	8 -8	0	0	0	0	0	1.9	0.0	0.0	0.0	72.99	0.200	0
0	8 --8	0	0	0	0	- - 1	-0.2	0.7	--0.4	-0.1	72.99	0.200	1
0	7 -8	0	0	0	0	0	0.0	0.6	0.0	0.0	108.11	0.296	1
0	6 - 8	0	0	0	0	-1	0.5	1.9	-1.0	0.2	208.35	0.570	1
0	6 -8	0	0	0	0	--2	--11.6	0.0	0.0	-5.1	208.35	0.570	0
0	5 -8	0	0	0	0	0	-0.3	-0.9	0.0	0.0	2862.15	7.836	3
o	5 -8	0	0	0	0	--1	-5.7	1.1	-0.6	-3.0	2863.02	7.839	2
o	5 -8	0	0	0	0	- - 2	1.1	--26.8	11.6	0.5	2863.89	7.841.	1
0	9 -9	0	0	0	0	0	1.2	0.0	0.0	0.0	64.88	0.178	0
0	9 -9	0	0	0	0	--1	--0.1	0.5	-0.3	-0.3	64.88	0.178	1
0	7 -9	0	0	0	0	--1	0.3	1.2	-0.6	0.2	153.56	0.420	1
0	7 -9	0	0	0	0	--2	--7.3	0.0	0.0	-3.2	153.56	0.420	0
0	6 -9	0	0	0	0	-1	-0.9	0.2	-0.1	-0.5	485.00	1.328	2
0	6 -9	0	0	0	0	-2	0.2	--4.5	2.0	0.1	485.03	1.328	1
0	5 - 9	0	0	0	0	-2	-0.7	0.2	-0.1	-0.3	-418.65	-1.146	2
0	10-10	0	0	0	0	0	0.8	0.0	0.0	0.()	58.39	0.160	0
0	8-10	0	0	0	0	-1	0.2	0.’/	-0.4	0.1	121.58	0.333	1
0	8-10	0	0	0	0	- 2	-5.0	0.0	0.0	-2.2	121.59	0.333	0
0	7-10	0	0	0	0	-2	0.1	--2.4	1.0	0.0	264.95	0.725	1
o	6-10	0	0	0	0	-1	0.2	0.5	-0.3	0.1	1479.37	-4.050	3
0	6-10	0	0	0	0	- 2	-2.4	0.5	-0..2	-1.1	1479.14	-4.050	2
0	11-11	0	0	0	0	0	0.6	0.0	0.0	0.0	53.08	0.145	0
0	9-11	0	0	0	0	-1	0.1	0.5	-0.3	0.s	100.63	0.276	1
o	9-11	0	0	0	0	-2	-3.4	0.0	0.0	-1.5	100.63	0.276	0
0	8-11	0	0	0	0	-2	0.1	-1.5	0.7	0.0	182.25	0.499	1
0	“/-13	0	0	0	0	-2	1.6	-0.3	0.1	0.’/	964.79	2.641	2
o	10-12	0	0	0	0	-2	-2.4	0.0	0.0	-1.0	85.84	0.235	0
0	9-12	0	0	0	0	-2	0.0	-1.0	0.5	0.0	138.90	0.380	1
0	8-12	0	0	0	0	-2	0.6	-0.1	0.0	0.2	363.76	0.996	2
0	11-13	0	0	0	0	-2	-1.7	0.0	0.0	-0.’/	74.84	0.205	0
0	10-13	0	0	0	0	-2	0.0	-0.8	0.3	0.0	112.21	0.307	1

o	8	-13	0	0	0	0	0	0.4	0.5	0.0	0.0	8"/265 .36	238.920	5
0	8	-13	0	0	0	0	-1	4.1	-2.2	1.2	2.2	88081.94	241.155	4
0	8	-13	0	0	0	0	-2	6.1	21.7	--9.4	2.6	88913.94	243.433	3
0	12	-14	0	0	0	0	- 2	-1.2	0.()	0.0	-0.5	66.34	0.182	0
0	11	-14	0	0	0	0	-2	0.0	-0.6	0.2	0.0	94.12	0.258	1
0	13	-15	0	0	0	0	-2	--0.9	0.0	0.0	-0.4	59.57	0.163	0
0	14	-16	0	0	0	0	- 2	-0.6	0.0	0.0	-0.3	54.05	0.148	0
0	0	0	1	0	0	0	0	0.9	0.5	0.0	0.0	686.98	1.881	1
0	0	1	-3	0	0	0	-2	1.1	0.0	0.0	0.5	-613.74	-1.680	0
0	0	1	-2	0	0	0	- 1	-0.2	1.3	-0.7	-0.1	-5760.48	-15.771,	2
0	0	1	-2	0	0	0	0	-8.9	5.0	0.0	0.0	--5764.01	-15.781	1
0	0	1	-2	0	0	0	1	-0.8	-0.3	-0.2	0.4	-5767.54	-15.791	2
0	0	J	- l	o	0	0	0	3.2	0.0	0.0	0.0	779.94	2.135	0
0	0	2	- 5	0	0	0	-'2	0.6	--0.4	0.2	().3	-554.68	-1.519	1
0	0	2	-4	0	0	0	--2	5.2	0.3	0.0	2.2	-2880.24	-7.886	0
0	0	2	- 4	0	0	0	-1	0.6	0.9	-0.5	0.3	-2881.12	-7.888	1
0	0	2	- 4	0	0	0	0	-1.0	1.6	0.0	0.0	-2882.00	-7.890	2
0	0	2	-3	0	0	0	0	1.4	--0.8	0.0	0.0	901.99	2.470	1
0	0	2	-2	0	0	0	0	1.2	0.0	0.0	0.0	389.97	1.068	0
0	0	3	- 6	0	0	0	-2	2.4	-1.3	0.6	1.0	-3920.55	--5.258	1
0	0	3	-6	0	0	0	-1	0.5	0.2	--0.1	0.3	-1920.94	-5.259	2
0	0	3	- 5	0	0	0	-2	-1.7	0.0	0,0	-0.7	1069.56	2.928	0
0	0	3	-5	0	0	0	0	0.4	-0.6	0.0	0.0	3069.32	2.928	2
0	0	3	-4	0	0	0	0	0.6	-0.3	0.0	0.0	418.2"?	1.145	1
0	0	3	- 3	0	0	0	0	0.6	0.0	0.0	0.0	259.98	0.712	0
0	0	4	-8	0	0	0	-2	0.6	-1. .0	0.4	0.3	--1440.56	-3.944	2
0	0	4	- 7	0	0	0	--2	-1.6	0.9	--0.4	-0.7	1313.25	3.595	1
0	0	4	- 6	0	0	0	--2	-0.6	0.0	0.0	-0.2	451.04	1.235	0
0	0	5	- 9	0	0	0	-2	-0.8	1.2	--0.5	-0.3	1700.73	4.656	2
0	0	5	-8	0	0	0	--'2	-0.5	0.3	--0.1	-0.2	489.33	1.340	1
0	0	6	--11	0	0	0	- - 2	-0.1	1.1	--0.5	0.0	2412.59	6.605	3
0	0	7	--13	0	0	0	- - 2	0.4	0.8	-0.4	0.2	41.49.36	11.360	4
0	0	8	-15	0	0	0	--2	1.2	0.9	-0.4	0.5	14812.45	40.554	5
0	0	0	0	1	0	0	--1	-0.3	-0.8	0.4	-0.1	4334.58	11.867	2
0	0	0	0	1	0	0	0	33.9	-5.2	0.0	0.0	4332.59	11.862	1
0	0	0	0	1	0	0	1.	0.0	0.6	0.3	0.0	4330.60	11.857	2
0	0	0	0	)	0	0	2	4.4	1.7	0.7	-1.9	4328.60	11.851	1
0	0	0	0	2	0	0	0	1.2	-0.4	0.0	0.0	2166.29	5.931	2
0	0	0	0	2	0	0	1	0.9	4.9	2.6	-0.5	2165.80	5.930	1
0	0	0	0	2	0	0	2	-106.4	0.1	0.0	46.1	2165.30	5.928	0
0	0	0	0	3	0	0	1	0.2	0.5	0.3	-0.3	3443.97	3.953	2
0	0	0	0	3	0	0	2	--12.0	2.2	).0	5.2	1443,75	3.953	1
0	0	0	0	4	0	0	2	-1.0	0.4	0.2	0.4	1082.90	2.965	2
0	0	1	0	- 5	0	0	-2	-0.5	-0.2	0.1	-0.2	631.49	1.729	2
0	0	1	0	- 4	0	0	-2	-3.1	--0.6	0.2	-1.3	551.16	1.509	1
0	0	1	0	-3	0	0	- 2	--11.3	0.0	0.0	-4.9	488.96	1.339	0
0	0	1	0	- 3	0	0	-1	0.1	-0.5	0.3	0.1	488.93	1.339	1
0	0	1	0	-2	0	0	- 2	0.5	-0.1	0.1.	0.2	439.37	1.203	1
0	0	1	0	-2	0	0	0	1.9	0.3	0.0	0.0	439.33	1.203	1
0	0	1	0	-1	0	0	0	11.7	0.0	0.0	0.0	398.88	1,092	0
0	0	1	0	0	0	0	0	0.6	-0.1	0.0	0.0	365.26	1.000	1
0	0	1	0	1	0	0	2	-1.6	0.0	0.0	0.7	336.83	0.922	0
0	0	2	0	-5	0	0	-2	-0.6	-0.1	0.0	-0.2	231.41	0.634	1
0	0	2	0	- 4	0	0	-2	-1. .7	0.0	0.0	-0.7	219.68	0.601	0

0	0	2	0	-2	0	0	0	1.4	0.0	0.0	0.0	199.44	0.546	0
0	0	0	0	0	1	0	0	-0.2	-4.0	0.0	0.0	30759.23	29.457	1
0	0	0	0	0	1	0	2	0.0	0.7	0.3	0.0	10734.69	29.390	1
0	0	0	0	0	2	0	1	0.4	0.9	0.5	-0.2	5376.54	14.720	1
0	0	0	0	0	2	0	2	-12.3	0.0	0.0	5.3	5373.47	14.712	0
0	0	0	0	0	3	0	2	0.1	1.5	0.7	0.0	3583.68	9.812	1
0	0	0	0	0	0	2	2	-0.7	0.0	0.0	0.3	15294.38	41.874	0