dynamical chiral symmetry breaking, Λ_χ, is around 1 GeV. It is conventional to call quarks heavy if $m > \Lambda_\chi$, so that explicit chiral symmetry breaking dominates, and light if $m < \Lambda_\chi$, so that spontaneous chiral symmetry breaking dominates. The c, b, and t quarks are heavy, and the u, d and s quarks are light. The computations for light quarks involve an expansion in m_q/Λ_χ about the limit $m_q = 0$, whereas for heavy quarks, they involve an expansion in Λ_χ/m_q about $m_q = \infty$. The corrections are largest for the s and c quarks, which are the heaviest light quark and the lightest heavy quark, respectively.

At high energies or short distances, nonperturbative effects such as chiral symmetry breaking are unimportant, and one can in principle analyze mass-dependent effects using QCD perturbation theory to extract the quark mass values. The QCD computations are conventionally performed using the $\overline{\rm MS}$ scheme at a scale $\mu \gg \Lambda_\chi$, and give the $\overline{\rm MS}$ "running" mass $\overline{m}(\mu)$. The μ dependence of $\overline{m}(\mu)$ at short distances can be calculated using the renormalization group equations.

For heavy quarks, one can obtain useful information on the quark masses by studying the spectrum and decays of hadrons containing heavy quarks. One method of calculation uses the heavy quark effective theory (HQET), which defines a HQET quark mass m_Q. Other commonly used definitions of heavy quark masses such as the pole mass are discussed in Sec. C. QCD perturbation theory at the heavy quark scale $\mu = m_Q$ can be used to relate the various heavy quark masses to the $\overline{\rm MS}$ mass $\overline{m}(\mu)$, and to each other.

For light quarks, one can obtain useful information on the quark mass ratios by studying the properties of the light pseudoscalar mesons using chiral perturbation theory, which utilizes the symmetries of the QCD Lagrangian Eq. (1). The quark mass ratios determined using chiral perturbation theory are those in a subtraction scheme that is independent of the quark masses themselves, such as the $\overline{\rm MS}$ scheme.

A more detailed discussion of the masses for heavy and light quarks is given in the next two sections. The $\overline{\rm MS}$ scheme applies to both heavy and light quarks. It is also commonly used for predictions of quark masses in unified theories, and for computing radiative corrections in the Standard Model. For this reason, we use the $\overline{\rm MS}$ scheme as the standard scheme in reporting quark masses. One can easily convert the $\overline{\rm MS}$ masses into other schemes using the formulæ given in this review.

C. Heavy quarks

The commonly used definitions of the quark mass for heavy quarks are the pole mass, the $\overline{\rm MS}$ mass, the Georgi-Politzer mass, the potential model mass used in ψ and Υ spectroscopy, and the HQET mass.

The strong interaction coupling constant at the heavy quark scale is small, and one can compute the heavy quark propagator using QCD perturbation theory. For an observable particle such as the electron, the position of the pole in the propagator is the definition of the particle mass. In QCD this definition of the quark mass is known as the pole mass m_P, and is independent of the renormalization scheme used. It is known that the on-shell quark propagator has no infrared divergences up to (at least) two-loop order [1], so this provides a definition of the quark mass up to order $\alpha_s(m_P)^2$. The pole mass cannot be used to arbitrarily high accuracy because of nonperturbative infrared effects in QCD. The full quark propagator has no pole because the quarks are confined, so that the pole mass cannot be defined outside of perturbation theory.

The $\overline{\rm MS}$ running mass $\overline{m}(\mu)$ is defined by regulating the QCD theory using dimensional regularization, and subtracting the divergences using the modified minimal subtraction scheme. The $\overline{\rm MS}$ scheme is particularly convenient for Feynman diagram computations, and is the most commonly used subtraction scheme.

The Georgi-Politzer mass $\widehat{m}(\xi)$ is defined using the momentum space subtraction scheme at the spacelike point $-p^2 = M^2$ [2], with $M^2 = (\xi + 1)m_P^2$. It is often used in computations involving QCD sum rules [3], which can be used to extract heavy quark masses. QCD sum rules are discussed in more detail in the next section on light quark masses.

Lattice gauge theory calculations can be used to obtain heavy quark masses from ψ and Υ spectroscopy. The quark masses are obtained by comparing a nonperturbative computation of the meson spectrum with the experimental data. The lattice quark mass values can then be converted into quark mass values in the continuum QCD Lagrangian Eq. (1) using lattice perturbation theory at a scale given by the inverse lattice spacing. A recent computation gives a b-quark pole mass of 4.94 ± 0.15 GeV [4].

Potential model calculations of the hadron spectrum also involve the heavy quark mass. There is no way to relate the quark mass as defined in a potential model to the quark mass parameter of the QCD Lagrangian, or to the pole mass. Even in the heavy quark limit, the two masses can differ by nonperturbative effects of order $\Lambda_{\rm QCD}$. There is also no reason why the potential model quark mass should be independent of the particular form of the potential used.

Recent work on the heavy quark effective theory [5–9] has provided a definition of the quark mass for a heavy quark that is valid when one includes nonperturbative effects and will be called the HQET mass m_Q. The HQET mass is particularly useful in the analysis of the $1/m_Q$ corrections in HQET. The HQET mass is not the same as the pole mass, a distinction that is often overlooked in the literature. Physical quantities such as hadron masses can in principle be computed in the heavy quark effective theory in terms of the HQET mass m_Q. The computations cannot be done analytically in practice because of nonperturbative effects in QCD, which also prevent a direct extraction of the quark masses from the original QCD Lagrangian, Eq. (1). Nevertheless, for heavy quarks, it is possible to parametrize the nonperturbative effects to a given order in the $1/m_Q$ expansion in terms of a few unknown constants that can be obtained from experiment. For example,