Chapter 11, the first on cooling, of lectures to be given at Acc School in Vanderbilt in Jan 99, by R.B.Palmer.

11 Collider

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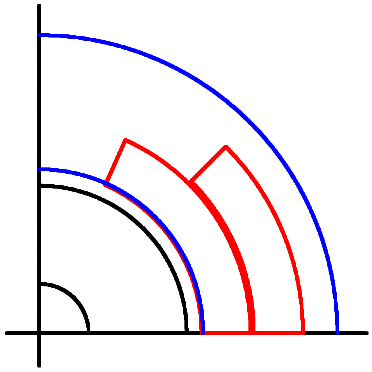
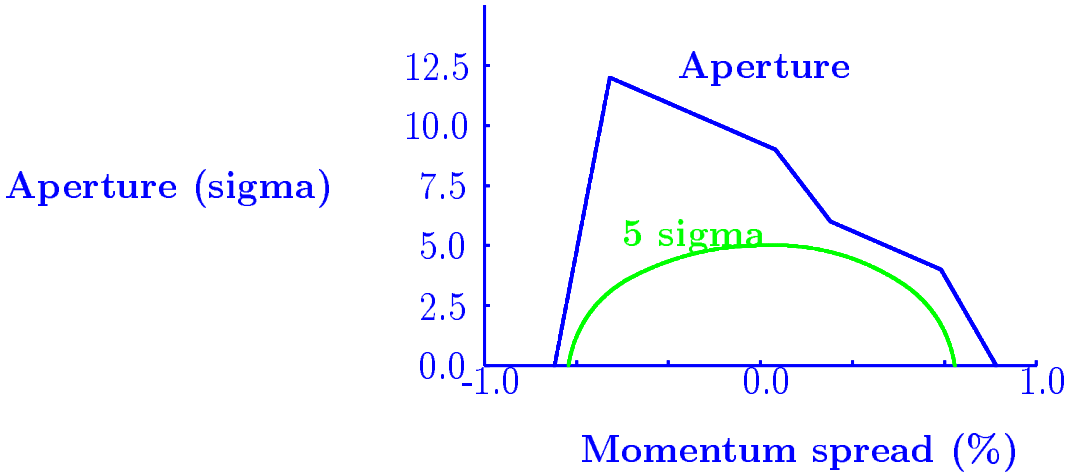
11.1 Introduction



- Low beta
  Local chromatic corr.
- Short Bunches
  Isochronous ring
  Flexible Momentum Compaction
- Tungsten Beam Shield
  6 cm at 3 TeV,
  3 cm at 100 GeV
- Status
  - feasibility demonstrated at 4 TeV
  - 100 GeV Lattices: Good Aperture



11.2 Luminosity & Tune Shift

11.2.1 Luminosity

$$\mathcal{L}_{\rm intantaneous} \;=\; \frac{\rm N_t^2\ f_{circ}}{4\pi\ \sigma^2} \;=\; \frac{\gamma\ \rm N^2\ f_{circ}}{4\pi\ \beta^*\ \epsilon_n}\ \rm H(A,D)$$

where $f_{inst} \;=\; \beta_v\ c/S$; S being the ring circumference.

The average luminosity is then

$$\mathcal{L} \;=\; \frac{\gamma\ \rm N_o^2\ f_{rep}\ n_{eff}}{4\pi\ \beta^*\ \epsilon_n}\ \rm H(A,D) \qquad (1)$$

where

$$n_{eff} \;=\; \frac{\beta_v\ c}{S}\int\left(\frac{N(t)}{N_o}\right)^2\ dt$$

$$=\; \frac{\beta_v\ c}{S}\ \int_o^\infty e^{\frac{2\ t}{\gamma\tau_\mu}}\ dt$$

$$n_{eff} \;=\; \frac{\gamma\ \tau\ \beta_v\ c}{2\ S} \;=\; \frac{\tau\ c^2}{4\ \pi\ (mc^2/e)} \;<B>\qquad (2)$$

and is independent of energy for a fixed average bending field $< B >$.

The enhancement factor $H(A, D)$, which is significant only in linear colliders, is

$$H(A,D) \approx 1+D^{1/4}\left[\frac{D^3}{1+D^3}\right]\left\{\ln\left(\sqrt{D}+1\right)+2\ln\left(\frac{0.8}{A}\right)\right\}, \qquad (3)$$

where $A = \sigma_z/\beta^*$ and the Disruption parameter D:

$$D = \frac{\sigma_z n_\mu}{\gamma\sigma_t^2} r_e\left(\frac{m_e}{m_\mu}\right) \qquad (4)$$

In the cases we are considering: A = 1, D $\approx$ .5 and H(A,D) $\approx$ 1[**?**].

11.2.2 Transverse Beam-Beam Tune Shift

From the section 3 discussion of tune shift in rings, we had, for test charge at radius r, with a cylindrically symmetric charge per unit length of $Q'(r)$ inside the radius r:

from the radial electric field

$$\mathcal{E}_{\rm r} = \frac{\rm Q'(r)}{2\pi\epsilon_{\rm o} \rm r}$$

but the magnetic field has the opposite sign, because of the opposite direction of the current:

$$B_\phi = +\frac{1}{\epsilon_o c^2}\, Q'(r)\, \frac{\beta_v c}{2\pi r}$$

now giving

$$F_\perp = \frac{Q'(r)}{\epsilon_o}\, \frac{e}{2\pi r}\, (1\ +\ \beta_v^2) \tag{5}$$

As before, the charge density inside r for $r << \sigma_r$

$$Q'(r)\ \approx\ \frac{r^2}{2\sigma^2}\, Q'$$

Thus

$$F_\perp\ =\ r\, \frac{e\ Q'}{4\pi\epsilon_o\ \sigma^2}\, (1\ +\ \beta^2)$$

This force is defocussing (proportional to r and outward) and gives an additional focussing strength per unit length dk' in the x direction of:

$$dk'\ =\ \frac{dk}{d\ell}\ =\ -\frac{F_\perp}{\gamma mp\ \beta_v^2\ c^2\ x}$$

and the same for y.

Noting that the clasical radius r_o is

$$r_o = \frac{e^2}{4\pi\epsilon_o mc^2}\ , \qquad Q'/e \ = \ dN/dz \quad \text{and} \quad \sigma_r^2 \ = \frac{\epsilon_n\ \beta}{\beta_v \gamma}$$

we get:

$$\frac{dk}{d\ell} \ = \ \frac{dN}{dz}\ \frac{r_o}{\epsilon_n\ \beta}\ \left(\frac{1\ +\ \beta_v^2}{\beta_v}\right) \tag{6}$$

Again, from Section 3,eq. **??**, we had that the change in tune of a ring is:

$$\Delta\nu \ = \ \frac{1}{4\pi}\int \beta\ dk'\ dz \ = \ \int \frac{\beta}{4\pi}\ \frac{dk}{d\ell}\ d\ell$$

$$\Delta\nu \ = \ \int \frac{\beta}{4\pi}\ \frac{dN}{dz}\ \frac{r_o}{\epsilon_n\ \beta}\ \left(\frac{1\ +\ \beta_v^2}{\beta_v}\right)\ d\ell$$

The β's cancel, and since the emittance ϵ_n is conserved, the integral is over the the particle density N':

$$\Delta\nu \ = \ \frac{r_o}{4\pi\ \epsilon_n}\ \left(\frac{1\ +\ \beta_v^2}{\beta_v}\right)\ \int\ \frac{dN}{dz}\ d\ell$$

Now, because the bunches are going in opposite directions, the length they pass through $d\ell$ is half the individual bunch lengths dz:

$$\int\ \frac{dN}{dz}\ d\ell \ = \ \frac{N}{2}$$

and

$$\Delta\nu \ = \ \frac{N\ r_o}{4\pi\epsilon_n}\ \left(\frac{1\ +\ \beta_v^2}{2\beta_v}\right) \tag{7}$$

or for $\beta_v \approx 1$

$$\Delta\nu \ = \ \frac{N\ r_o}{4\pi\epsilon_n} \tag{8}$$

Example:

Tevatron: $N = 6\ 10^{10}$, $\epsilon_N\ =\ 20\ \pi\ 10^{-6}\ m\ \ (95\%)\ =$ $3.3\ 10^{-6}\ m$ (rms), $\ r_o\ =\ 1.53\ 10^{-18}$ m:

$$\Delta\nu\ =\ 0.0022$$

In electron machines, in which there is synchrotron damping, $\Delta\nu\ \approx\ .05$ is common.

11.2.3 Tune Shift Luminosity Limit

The average luminosity (from eq.1 is

$$\mathcal{L} \approx \frac{\gamma \, N_o^2 \, f_{rep} \, n_{eff}}{4\pi \, \beta^* \, \epsilon_n}$$

Substituting $\Delta\nu$:

$$\mathcal{L} = \frac{\Delta\nu}{r_o} \frac{\gamma \, N \, n_{eff} \, f}{\beta^*} = \frac{\Delta\nu \, n_{eff}}{r_o} \frac{P_{beam}}{(m_\mu \, c^2/e)} \qquad (9)$$

11.3 Final Focus

11.3.1 Final Triplet Design

From Section 4, equation **??**, we had the relation between maximum angle and maximum amplitude for a Li lens:

$$\theta_{max} = \sqrt{\frac{c\ B_{surface}\ a_{max}}{(pc/e)}}$$

or in general:

$$\theta_{max} = K_\theta \sqrt{\frac{c\ B_{surface}\ a_{max}}{(pc/e)}} \tag{10}$$

where K_theta is a constant depending on the focus system. For the monopole, $K_a = 1$. This scaling law is true for any linear magnetic focus system with fixed geometry. It is, for instance, true for a quadrupole triplet.

Alternatively, we can write:

$$a_{max} = \frac{pc/e)}{B\ c}\ \theta^2_{max}$$

And in general we can write

$$a_{max} = K_a\ \frac{pc/e)}{B\ c}\ \theta^2_{max} \tag{11}$$

Again K_a is a constant depending on the focus system, where, for the monopole, $K_a = 1$.

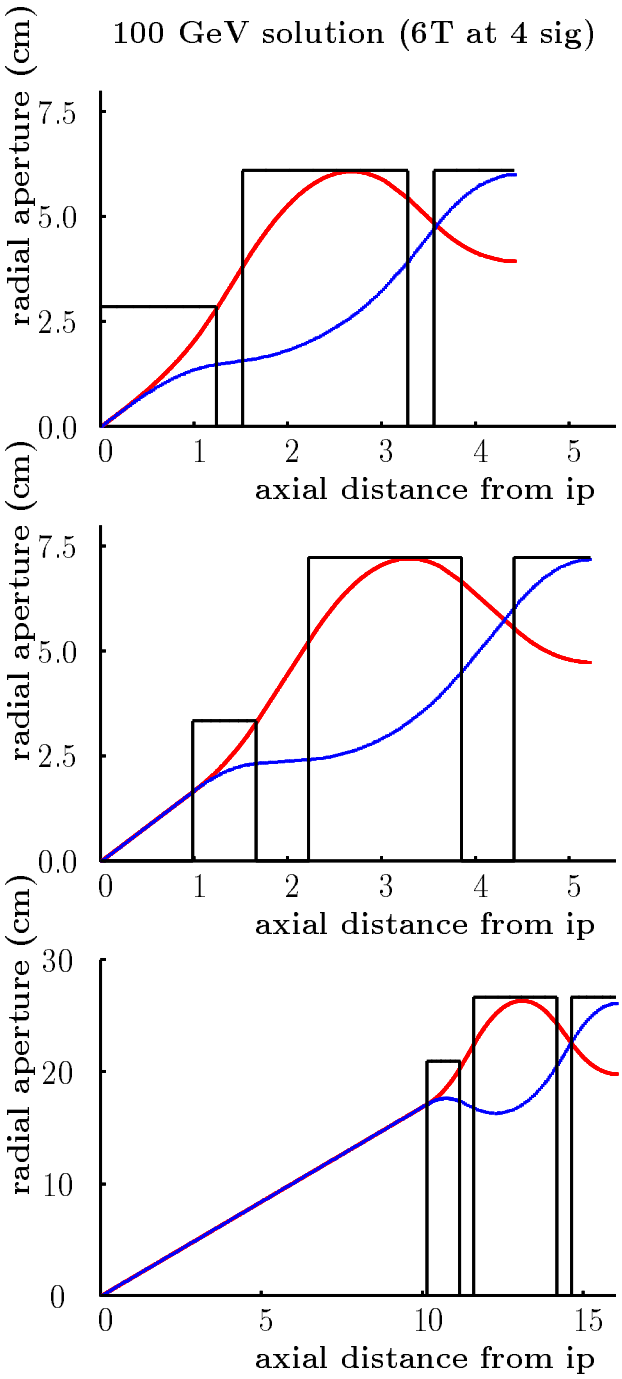
The smallest K_a for a given triplet is obtained:

a) if the first magnet extends down to the focus,

b) there are no spaces between the magnets, and

b) if the maximum transverse dispalcements in x and y are equal.

The following figures show three such a scalable optimal designs. The first with no initial gap, and the following ones with longer gaps. The values of K_a's for the three are: 7.84, 8.21, and 13.9, respectively; all far worse than the value of 1.0 for a Li lens. The values of K_θ for the same triplets are: 0.36, 0.35, and 0.27.



1) zero initial gap

2) small initial gap

3) long initial gap

11.3.2 Chromaticity

Chromaticity is defined as the change in tune $d\nu$ for a given hange in $\delta = dp/p$

$$\xi = \frac{\Delta\nu}{\Delta p/p} \tag{12}$$

From eq. **??**

$$\Delta\nu = \oint \frac{\beta}{4\pi}\, dk'\, dz$$

Now

$$k' \propto \frac{1}{p}$$

so

$$\frac{dk'}{k'} = -\frac{dp}{p}$$

and

$$dk' = -k'\,\delta$$

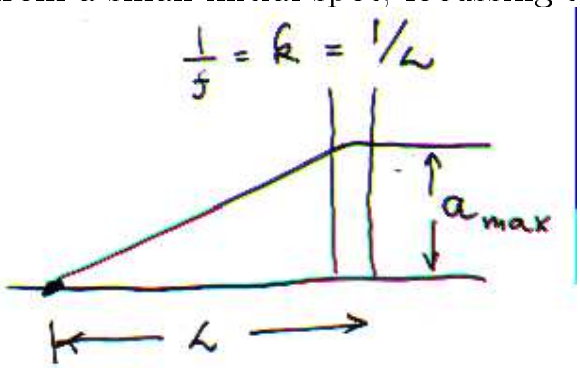
So

$$\Delta\nu = \oint -\frac{\beta}{4\pi}\, k'\,\delta\, dz$$

and

$$\xi = \oint -\frac{\beta}{4\pi}\, k'\, dz \tag{13}$$

Consider the chromaticity of a thin lens at a distance L from a small initial spot, focussing to a parallel beam.



To focus:

$$\int k' dz \;=\; k \;=\; \frac{1}{L}$$

$$\xi \;=\; \frac{\hat{\beta}\, k}{4\pi} \;=\; \frac{1}{4\pi}\,\frac{\hat{\beta}}{L}$$

So if

$$\xi \;=\; K_\xi\,\frac{\hat{\beta}}{L} \tag{14}$$

then for a thin lens, $K_\xi \;=\; 1/(4\pi)$, and it is similar to this in cases such as the triplets shown above. ***

$$\beta \;=\; \beta^*\left(1 \;+\; \left(\frac{L}{\beta^*}\right)^2\right) \;\approx\; \frac{L^2}{\beta^*}$$

so

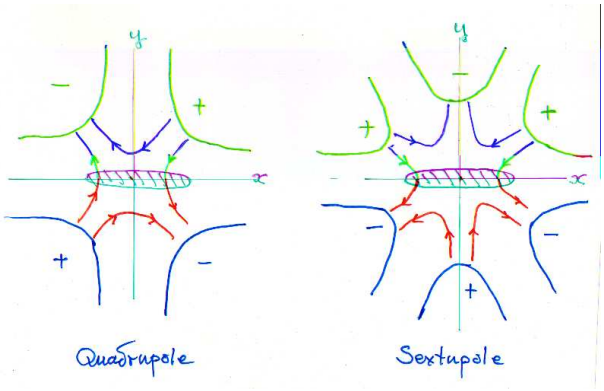
$$\frac{\hat{\beta}}{L} \;=\; \sqrt{\frac{\hat{beta}}{\beta^*}} \;=\; \frac{\hat{\sigma}}{\sigma^*}$$

so, for a thin lens:

$$\xi \;=\; \frac{1}{4\pi}\,\frac{\hat{\sigma}}{\sigma^*} \tag{15}$$

and, this, too is approximately true for the triplets.

11.3.3 Chromatic Correction



Consider a sextupole at a location with $\beta_x \gg \beta_y$ so that $\sigma_x \gg \sigma_y$. The deflections in y will be negligible, but:

$$\Delta x' = k_s\, x^2$$

and if at that location, there is dipersion D_x in x, so that

$$x(p) = x(0) + \frac{\Delta p}{p} D_x = x(0) + \delta\, D_x$$

then

$$\Delta x'(p) = k_s\, (x(0) + \delta\, D_x)^2$$

Now if at π phase advance from this location we ahve another similar sextupole with the same dispersion and same β's, then since the signs of x' and x are reversed:

$$\Delta x'(p) = -k_s\, (x(0) + \delta\, D_x)^2 + k_s\, (-x(0) + \delta\, D_x)^2$$

$$\Delta x'(p) = k_s\, 4\, x(0)\, \delta\, D_x$$

which may be copared with the deflection for a quadrupole of strength k_q:

$$\Delta x' = k_q\, x(0)$$

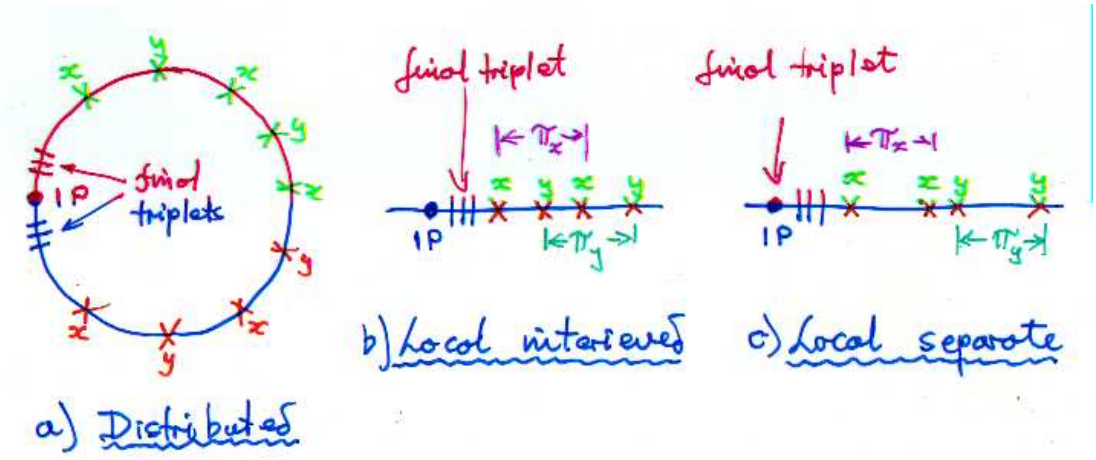
so the effect of the two sextupoles is of a quadrupole with strength:

$$k_q = 4\, \delta\, D_x\, k_s \qquad (16)$$

i.e a focus that is momentum dependent, in which the dependence can be chosen to correct the natural chromaticity of other elements.

Clearly one needs another such pair of sextupoles to correct the y chromaticity. The sextupoles may be

- may be more in number, and distributed round the ring (as in conventional collider rings);
- interterlieved (as in the current μ collider designs), to minimize length; or
- placed one after the other (as in NLC final focus designs).



The above ideal situation corrects chromaticity with no higher order terms, but aberations arise from:

- finite length of sextupoles
- chromatiity in the π phase advance between the sextupoles
- aberatins between the sextupoles, such as a member of the other pair , if they are interlieved.
- end effects
- aberations beween the correction and elements that are to be corrected

11.3.4 Triplet examples

Parameters of examples of final triplets suitble for a) a 4 TeV collider, and b) a 100 GeV Higgs Factory. examples.

a) 4 TeV solution (8T at 4 sig)

p(GeV)	2000		
norm emit (10^{-6} m)	50.0		
beta = sigz (mm)	3.0		
rms openimg angles (mr)	0.9		
4 sigma angles (mr)	3.7		
gaps (m)	3.7417	1.8708	1.8708
mag lengths (m)	5.9118	11.6934	5.7078
apertures (cm)	4.7600	9.3800	9.3800
pole B Fields (T)	-8.0	8.0	-8.0
max size (cm) at 4 sigmas	4.6097	9.3146	9.3457
max betas (km)	50.59428	206.5735	207.9592

b) 100 GeV solution with lower angle and aperture (6T at 4 sig)

p(GeV)	50		
norm emit (10^{-6} m)	136.0		
beta = sigz (mm)	16.0		
openimg angles (mr)	4.2		
4 sig angles (mr)	16.9		
gaps (m)	1.9998	1.4083	0.4507
mag lengths (m)	0.4788	1.6891	1.0165
apertures (cm)	4.7600	10.7576	10.7576
pole B Fields (T)	-6.0	6.0	-6.0
max size (cm) at 4 sigmas	4.5075	10.3804	10.6883
max betas (km)	.4446256	2.358016	2.500016

11.4 Longitudinal Stability

11.4.1 Flexible Momentum Compaction

11.4.2 Wakes/Impedance

11.4.3 rf Correction

11.5 Transverse Stability

11.5.1 Wakes/Impedance

11.5.2 BNS Damping

11.6 Ring Magnets

11.6.1 Decay Heating

11.6.2 Cos(θ)

11.6.3 High Field

11.7 Appendix: Longitudinal tune shift

Consider the change of energy of a test charge in bunch 1 as it passes through bunch 2. We assume that the two bunches have the same parameters and that $\gamma >> 1$ so that there is negligible axial space charge. z is the axial position of the test chargerelative to the intersction point (ip), ℓ_1 is the axial positadial p sitionion of the test charge in bunch 1 relative to the center of that bunch, ℓ_2 will be the axial position bunch 2 relative to its center. If $s_1 = -s$ is the axial position of the center of bunch 1 relative to the ip and $s_2 = s$ is the opsition of the other bunch center, then:

$$z \;=\; \ell_1 - s \tag{17}$$

$$\ell_2 = \ell_1 \;- 2\; s \tag{18}$$

If $< \theta >$ is the average radial divergence angle of the test particle in bunch 1, i.e. $< \theta >=< dr_1/dz >$, and $\mathcal{E}_\perp$ is the radial space charge in bunch 2 at the r and z of the test charge, then the average change of energy of the test charge as it makes one pass through the opposing bunch is:

$$< dE(r_o, \ell_1) > \;=\; \int_{-\infty}^{+\infty} \mathcal{E}_\perp(\mathrm{r,z},\ell_1) \;< \theta > (\mathrm{r,z}.\ell_1)\; \mathrm{dz} \tag{19}$$

where r_o is the mean radius at test charge at the ip, and

$$\mathcal{E}_\perp \;=\; \mathrm{q'_r}\; \frac{1}{2\pi\epsilon_\mathrm{o}\; < \mathrm{r} >} \tag{20}$$

where q'_r is the linear charge density inside the radius $< r >$. If β_o is the Courant Schneider parameter at the ip then their average $< r >$ at other positions will be

$$< r > \;=\; r_o \sqrt{1 + \left(\frac{z}{\beta_o}\right)^2} \tag{21}$$

$$< \theta > \;=\; \frac{r_o}{\beta_o^2}\; \frac{-z}{\sqrt{1 + \left(\frac{z}{\beta_o}\right)^2}} \tag{22}$$

and

$$q'_r \;=\; q' \left(1 - e^{\left(\frac{-r_o^2}{2\sigma_r^2}\right)}\right) \tag{23}$$

where σ_r is the rms beam size at the ip; i.e. $\sigma_r = \sqrt{\frac{\epsilon_n \beta_o}{\beta_v \gamma}}$, where ϵ_n is the normalized emittance, β_v is the velocity/c, and γ is the relativistic factor. The total axial charge density, including its variation due to decay, is given approximately by:

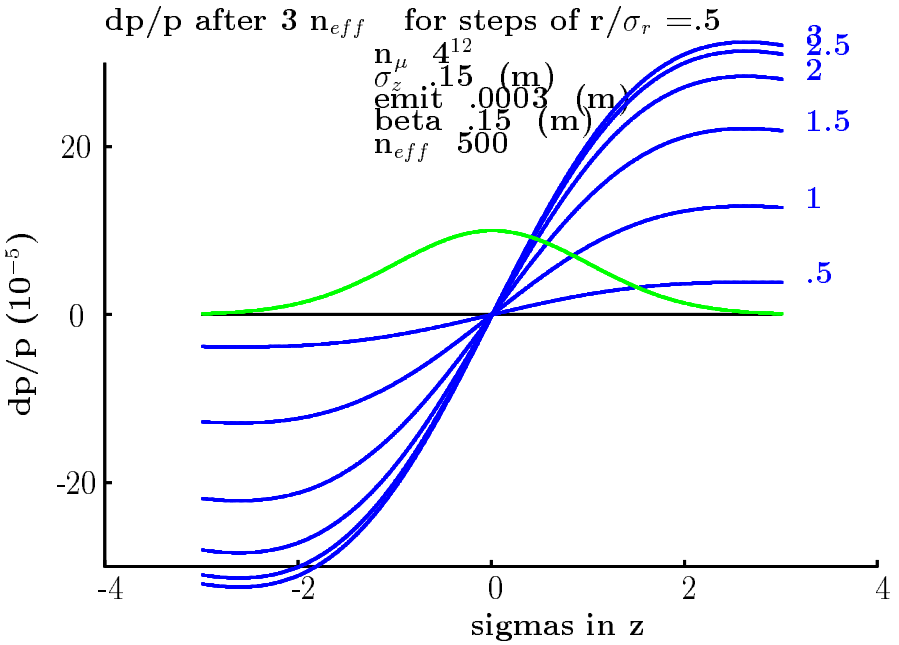
$$q' \;=\; Q_o\; e^{-\frac{n_{turns}}{n_{decay}}}\; \frac{1}{\sqrt{2\pi}\sigma_z}\; e^{\left(\frac{-z_{\ell_2}^2}{2\sigma_z^2}\right)} \tag{24}$$

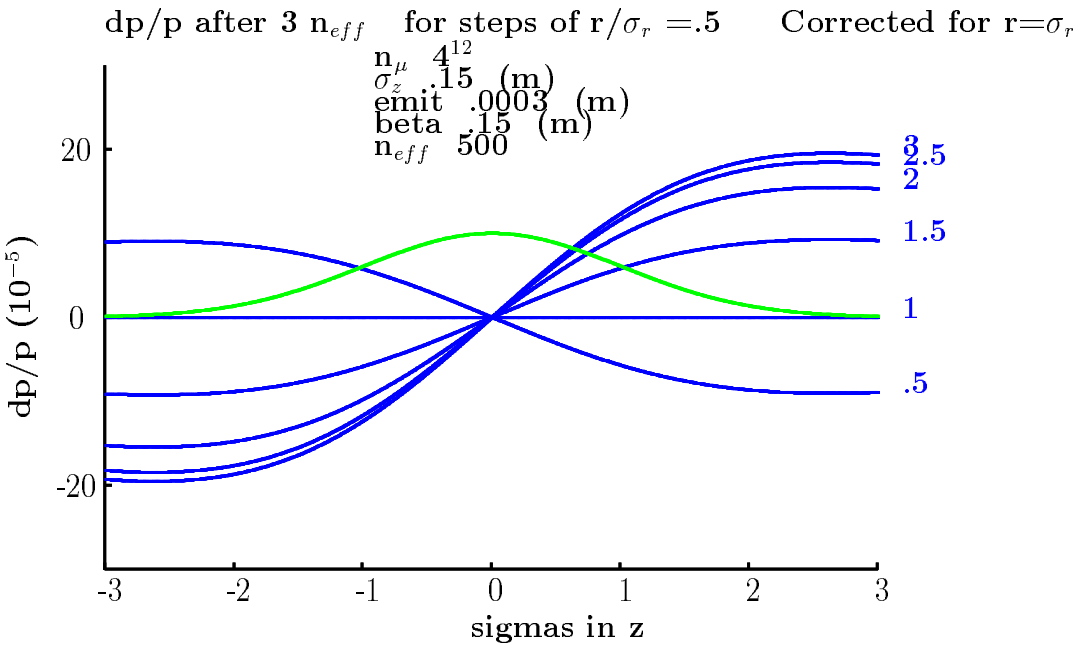
where Q_o is the total bunch charge, $n_{decay} = \tau_\mu \gamma / C$ where C is the ring circumference, and σ_z is the rms bunch length (assumed Gaussian). The total energy change after n_{turns} will be:

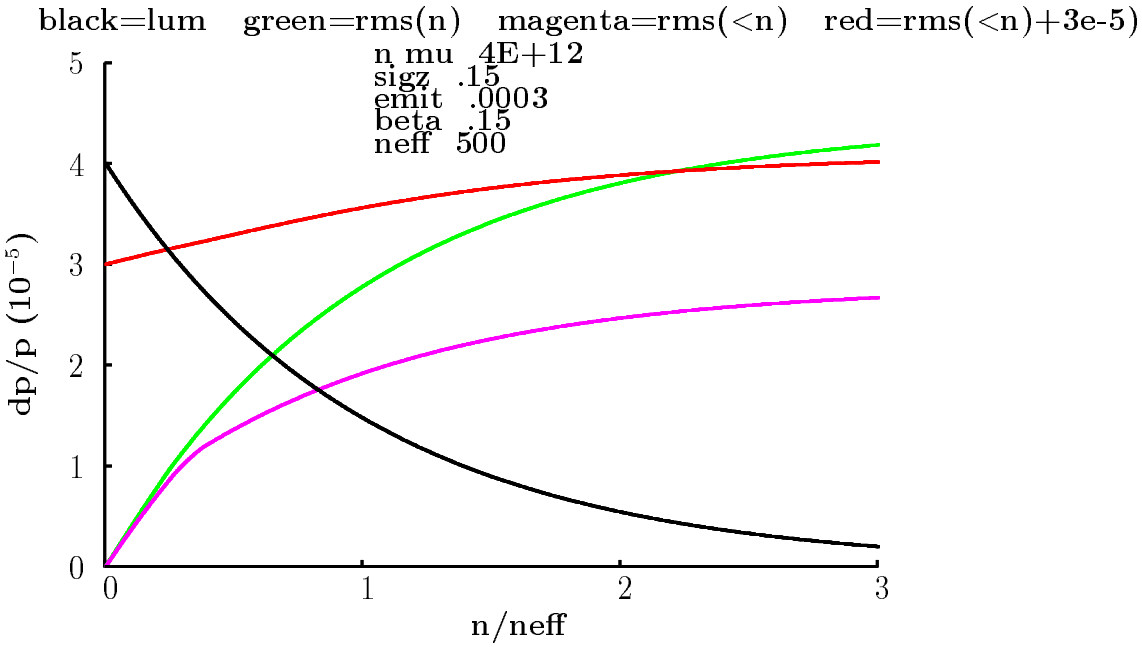
$$\Delta E(r_0, \ell_1, n_{turns}) \;=\; \sum_{1}^{n_{turns}} dE \tag{25}$$

These energy changes are monotonic in ℓ_1 (see figure 1) and larger at larger radii r_o, yielding an average monotonic energy shift vs. the position in the bunch. Since the energy vs. bunch length will have to be controlled for other reasons, it is reasonable to correct it to minimize the overal energy spread. To do this it is found best to subtract an energy equal to the shift for $r_o = \sigma_r$. Then

$$\Delta E_{corr}(r_0, \ell_1, n_{turns}) \;=\; \sum_{1}^{n_{turns}} dE \;-\; \Delta E(\sigma_r, \ell_1, n_{turns}) \tag{26}$$


dp/p after 3 n_{eff} for steps of r/σ_r =.5
n_μ 4^{12}
σ_z .15 (m)
emit .0003 (m)
beta .15 (m)
n_{eff} 500
3
2.5
2
1.5
1
.5
20
0
-20
dp/p (10^{-5})
-4
-2
0
2
4
sigmas in z


dp/p after 3 n_{eff} for steps of r/σ_r =.5 Corrected for r=σ_r
n_μ 4^{12}
σ_z .15 (m)
emit .0003 (m)
beta .15 (m)
n_{eff} 500
3
2.5
2
1.5
1
.5
20
0
-20
dp/p (10^{-5})
-3
-2
-1
0
1
2
3
sigmas in z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n mu 4E+12
sigz .15
emit .0003
beta .15
neff 500
dp/p (10^{-5})
n/neff
0
1
2
3
4
5