Study of Pentaquark and $\Lambda(1405)$

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Abstract. We perform a five-body calculation for pentaquark $(q^4\bar{q})$ state of $\Lambda(1405)$ as well as the three-body calculations for the ground state baryons and the $\Lambda(1405)$, and two-body calculations for mesons. The hamiltonian, which reproduces reasonably well the energies of ground state baryons (N,Λ,Σ,Ξ and Δ) and mesons (π,K,ρ,K^*), includes kinetic energy of semi-relativistic form, linear confinement potential, and the simplest form of color-magnetic interaction with Gaussian form factor. Flavor symmetry breaking ($m_s > m_{u,d}$) is taken into account. The energy calculated for $(q^4\bar{q})$ state of $\Lambda(1405)$ is lower than the energy for the (q^3) state. The present result suggests that the $\Lambda(1405)$ is a pentaquark-dominated state if the color-magnetic potential plays a leading role of the quark-quark and quark-antiquark interactions.

Keywords: Pentaquark, Quark model, Few-body systems
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INTRODUCTION

The $\Lambda(1405)$, $J^\pi = \frac{1}{2}^-$, has the smallest mass in the negative parity states of the baryon spectrum. Several constituent quark-model studies have been mentioning that the contribution of $(q^4\bar{q})$ configurations could be large in the $\Lambda(1405)$ [1, 2]. If this is the case, all of the five quarks may occupy their lowest $(0s)^5$ orbits, since an antiquark has the intrinsic negative parity. If we assume that the flavor $SU(3)$ is an exact symmetry, one can find that two kinds of the $(q^4\bar{q})$ states for $\Lambda(1405)$,

$$|\Lambda(1405)\rangle = [5]_R[222]_C[222]_F[32]_S, \tag{1}$$

give strongly attractive color-magnetic interactions: (The subscripts (RCF and S) stand for the position coordinates, color, flavor and spin space, respectively.)

$$\left\langle \sum_{i<j}(\lambda_i^C\cdot\lambda_j^C)(\sigma_i\cdot\sigma_j)\right\rangle = \begin{cases} 16 & (\text{ for } [22]_S \text{ of } (q^4)), \\ 80/3 & (\text{ for } [31]_S \text{ of } (q^4)), \end{cases} \tag{2}$$

where λ^C stands for the color $SU(3)$ generator, and σ is Pauli matrices for the spin. These attractive forces make lower the mass of the $(q^4\bar{q})$ state than that of the (q^3). However, the flavor symmetry is manifestly broken. A precise five-body calculation as well as the three-body calculation should be performed in order to clarify whether the mass of $(q^4\bar{q})$ state for the $\Lambda(1405)$ is still smaller than that of (q^3) state, even if the flavor symmetry breaking is taken into account. Therefore, the purpose of this study is to describe a five-body calculation of pentaquark $(q^4\bar{q})$ state for the $\Lambda(1405)$.

TABLE 1. Parameters of the present model.

$m_{u,d}$	m_s	V_0	C	α	Λ
0.34 GeV	0.5508 GeV	0.4534 GeV	0.08265 (GeV)^2	1.08	0.204 fm

INTERACTIONS AND METHOD

The hamiltonian is given by

$$H = \sum_{i=1}^{A} \sqrt{m_i^2 + \mathbf{p}_i^2} + \sum_{i<j} V_{ij}^{(\mathrm{conf})} + \sum_{i<j} V_{ij}^{(\mathrm{CM})}, \qquad \left(\text{with} \sum_{i=1}^{A} \mathbf{p}_i = 0\right), \tag{3}$$

where m_i and $\mathbf{p}_i$ are the mass and the momentum operator of the i-th quark (or antiquark). The two-body (qq or $q\bar{q}$) interaction consists of a confinement potential and a color-magnetic potential:

$$V_{ij}^{(\mathrm{conf})} = \left(-\frac{3}{8}\right)\left(\lambda_i^C \cdot \lambda_j^C\right)\left(-V_0 + C r_{ij}\right), \quad \text{and} \tag{4}$$

$$V_{ij}^{(\mathrm{CM})} = -\alpha \frac{2\pi}{3 m_i m_j}\left(\frac{\lambda_i^C}{2} \cdot \frac{\lambda_j^C}{2}\right)\left(\sigma_i \cdot \sigma_j\right) \times \frac{1}{\left(2\sqrt{\pi}\Lambda\right)^3} \exp\left\{-\frac{r_{ij}^2}{4\Lambda^2}\right\}, \tag{5}$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the interparticle coordinate. The delta-function in the color-magnetic potential is replaced by a Gaussian form factor with the size parameter Λ. All of the parameters of the present hamiltonian are given in Table 1.

The energies of various systems are calculated by the stochastic variational method (SVM)[3]. The trial function is given by a combination of basis functions:

$$\Psi = \sum_{k=1}^{K} c_k \varphi_k, \qquad \text{with} \qquad \varphi_k = \mathscr{A}\left\{G(\mathbf{x}; A_k)\, v_k^{L_k} \left[Y_{L_k}(\hat{\mathbf{v}}_k) \times \chi_{S_k}\right]_{JM} \eta_{kIM_I} \xi_{k(00)}\right\}. \tag{6}$$

Here $\mathscr{A}$ is an antisymmetrizer acting on the identical particles. For the spin (χ_k), the isospin (η_k), and the color (ξ_k) functions, all possible configurations are taken into account. The abbreviation $\mathbf{x} = (\mathbf{x}_1, \cdots, \mathbf{x}_{A-1})$ is a set of relative coordinates. For the spatial part, the basis function is constructed by the correlated Gaussian (CG), $G(\mathbf{x}; A_k)$, multiplied by the orbital angular momentum part. The CG is given by

$$G(\mathbf{x}; A_k) = \exp\left\{-\frac{1}{2}\sum_{i<j}^{A} \alpha_{kij}(\mathbf{r}_i - \mathbf{r}_j)^2\right\} = \exp\left\{-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{kij}\mathbf{x}_i \cdot \mathbf{x}_j\right\}. \tag{7}$$

The orbital angular momentum part, which is needed to the (q^3) model of $\Lambda(1405)$, is expressed by the global vector representation. The global vector, $\mathbf{v}_k$, is given by a linear combination of the relative coordinates:

$$\mathbf{v}_k = \sum_{i=1}^{A-1} (u_k)_i \mathbf{x}_i. \tag{8}$$

TABLE 2. Energies of baryons and mesons, given in units of MeV. The energies of the three-body model and the five-body model for $\Lambda(1405)$ are also given.

	Baryons					Mesons				$\Lambda(1405)$	
	N	Δ	Λ	Σ	Ξ	π	K	ρ	K^*	(q^3)	$(q^4\bar{q})$
Calc.	949	1266	1116	1208	1336	141	543	771	907	1405	1292
Expt.	939	1232	1116	1193	1318	137	496	776	892	1406 ± 4	

The A_k and u_k are sets of nonlinear parameters which characterize the spatial part of the basis function. The variational parameters are optimized by a stochastic procedure. The SVM with the above CG basis produces accurate solutions. The reader is referred to Refs.[4, 5] for details and recent applications.

RESULTS AND DISCUSSION

Table 2 lists the energies of three-body calculations (ground state baryons) and of two-body calculations (mesons). All of the calculated energies reasonably well reproduce the experimental values. The table also shows the energies of the three-body model and of the five-body model for the $\Lambda(1405)$. The present (q^3) model for the $\Lambda(1405)$ happens to reproduce the correct mass. However, this is not the point in the present study.

The remarkable result is seen in the five-body model of the $\Lambda(1405)$. The energy calculated for the $(q^4\bar{q})$ state is lower than that for the (q^3) state. Therefore, the present result suggests that the $\Lambda(1405)$ is a pentaquark-dominated state if the color-magnetic potential plays a leading role of the $q-q$ and $q-\bar{q}$ interactions. In the present model, the $(q^4\bar{q})$ state is a bound state since the energy obtained for the $(q^4\bar{q})$ state is lower than both the $\pi+\Sigma$ and the $\bar{K}+N$ thresholds, which are calculated to be 1348 MeV and 1492 MeV, respectively. More realistic model, e.g., taking account of effective meson-exchange force or coupling potential between (q^3) and $(q^4\bar{q})$, will be described in future publication. An attempt along this line has been made in Ref. [6].

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