File: HW95A.TEX, status July 27th 1995

The HW95 tidal potential catalogue

Computed by:

Torsten Hartmann
Theoretische Astrophysik
Universität Tübingen
Auf der Morgenstelle 10
D-72076 Tübingen
GERMANY
hartmann@tat.physik.uni-tuebingen.de

Hans-Georg Wenzel
Geodätisches Institut
Universität Karlsruhe
Englerstraße 7
D-76128 Karlsruhe
GERMANY
wenzel@gik.bau-verm.uni-karlsruhe.de

Abstract

This paper gives a short description of the HW95 tidal potential catalogue (Hartmann and Wenzel 1995) with 12935 tidal waves containing 19300 adjusted coefficients. The HW95 tidal potential catalogue has been computed using the DE200 numerical ephemeris of Jet Propulsion Laboratories, Pasadena, of the solar system bodies between 1850 and 2150. The computation has been carried out by an iterative procedure combining spectral analysis and least squares adjustment. The HW95 tidal potential catalogue has been validated using different gravity tide benchmark series. The error of gravity tides computed at mid latitude stations from HW95 is 0.0015 nm/s^2 = 0.15 ngal rms and 0.010 nm/s^2 = 1.0 ngal at maximum between years 1850 and 2150.

The HW95 tidal potential catalogue is distributed together with the necessary programs and benchmark series to public domain by INTERNET:

adress:	gik.bau-verm.uni-karlsruhe.de
	129.13.100.222
login:	ftp
password:	ftp
directory:	cd pub, cd hw95
description file:	hw95.txt

1. Introduction

The tidal potential $V_{(t)}$ due to a specific body at the surface of the Earth is given by (the second line accounts for the Earth flattening effect)

$$\begin{aligned} V_{(t)} &= GM_{\mathrm{pl}} \sum_{\ell=2}^{\ell=\ell_{\max}} \sum_{m=0}^{m=\ell} \frac{r^\ell}{r_{\mathrm{pl}}^{\ell+1}(t)} \frac{1}{2\ell+1} \overline{P}_{\ell m}(\cos\theta) \overline{P}_{\ell m}(\cos\theta_{\mathrm{pl}}(t)) \cos[m(\lambda - \Lambda_{\mathrm{pl}}(t))] \\ &+ GM_{\oplus} \frac{r_{\oplus}^2 r}{r_{\mathrm{pl}}^4(t)} J_{2\oplus} \left[\sqrt{\frac{3}{7}} \overline{P}_{10}(\cos\theta) \overline{P}_{30}(\cos\theta_{\mathrm{pl}}(t)) + \sqrt{\frac{2}{7}} \overline{P}_{11}(\cos\theta) \overline{P}_{31}(\cos\theta_{\mathrm{pl}}(t)) \cos(\lambda - \Lambda_{\mathrm{pl}}(t)) \right] \end{aligned}$$

Here, r, θ, λ denote the geocentric spherical coordinates of an arbitrary point on (or near) the Earth and $r_{\mathrm{pl}}, \theta_{\mathrm{pl}}, \Lambda_{\mathrm{pl}}$ are those of the center of mass of the specific planet, whose mass is denoted by M_{pl}. G is the gravitational constant. The fully normalised Legendre functions $\overline{P}_{\ell m}$ of degree ℓ and order m used are defined by (see section 6 for explicit formulas)

$$\overline{P}_{\ell m}(x) = (1-x^2)^{m/2}\frac{d^m}{dx^m}\left[\frac{1}{2^\ell \ell!}\frac{d^\ell}{dx^\ell}(x^2-1)^\ell\right]\sqrt{(2\ell+1)\frac{(\ell-m)!}{(\ell+m)!}(2-\delta_{m,0})}$$

Our catalogue HW95 of the Earth tide generating potential has been computed by using an expansion of the form

$$V_{(t)} = \sum_{\ell=1}^{\ell=\ell_{\max}}\sum_{m=0}^{m=\ell}\left(\frac{r}{a}\right)^\ell \overline{P}_{\ell m}(\cos\theta)\sum_i\left[C_i^{\ell m}(t)\cos(\arg_i(t)) + S_i^{\ell m}(t)\sin(\arg_i(t))\right]$$

where a is the major semi axis of the Earth, t is the time from J2000 in Julian centuries, the time dependent tidal amplitudes (units $\mathrm{m}^2/\mathrm{s}^2$ and $\mathrm{m}^2/\mathrm{s}^2/\mathrm{J.cy}$) are given by

$$\begin{aligned} C_i^{\ell m}(t) &= \mathrm{C0}_i^{\ell m} + t\cdot \mathrm{C1}_i^{\ell m} \\ S_i^{\ell m}(t) &= \mathrm{S0}_i^{\ell m} + t\cdot \mathrm{S1}_i^{\ell m} \end{aligned}$$

and the arguments $\arg_i$ are computed from

$$\arg_i(t) = m\cdot\lambda + \sum_{j=1}^{j=11} k_{ij}\sum_{k=0}^{k=4}\frac{\partial^k\lambda_j(t)}{\partial t^k}\cdot t^k$$

The integer coefficients k_{ij} are given in our catalogue, while the expressions for the longitudes $\lambda_j = \tau, s, h, p, N'$, $p_s, L_{\mathrm{Mer}}, L_{\mathrm{Ven}}, L_{\mathrm{Mar}}, L_{\mathrm{Jup}}, L_{\mathrm{Sat}}$ are listed in Table 2.
The numerical standard ephemeris DE200 from JPL are used to compute the celestial geocentric coordinates for the time interval 1850 to 2150. The conversion to terrestial geocentric coordinates uses the well-known relation

$$\vec{r}_{\mathrm{terr.}} = \mathcal{P}\cdot\mathcal{N}\cdot\mathcal{S}\cdot\vec{r}_{\mathrm{cel.}}$$

The precession formulas of Simon et al. (1994) have been used (including corrections for modern masses, units are arcseconds)

$$\begin{aligned} \zeta(t) &= 2\,306.090\,99 + 0.302\,228\,t + 0.018\,018\,3\,t^2 - 5.830\cdot 10^{-6}\,t^3 - 2.85\cdot 10^{-7}\,t^4 - 2\cdot 10^{-10}\,t^5 \\ z(t) &= 2\,306.090\,99 + 1.095\,280\,t + 0.018\,266\,7\,t^2 - 2.821\cdot 10^{-5}\,t^3 - 3.01\cdot 10^{-7}\,t^4 - 1\cdot 10^{-10}\,t^5 \\ \theta(t) &= 2\,004.201\,98 - 0.426\,568\,t - 0.041\,823\,8\,t^2 - 7.310\cdot 10^{-6}\,t^3 - 1.27\cdot 10^{-7}\,t^4 + 4\cdot 10^{-10}\,t^5 \end{aligned}$$

The mean obliquity has been also taken from that paper

$$\varepsilon = 84\,381.412 - 46.809\,56\,t - 1.52\cdot 10^{-4}\,t^2 + 1.9989\cdot 10^{-3}\,t^3 - 5.1\cdot 10^{-7}\,t^4 - 2.5\cdot 10^{-8}\,t^5$$

The IAU 1980 Nutation theory was used with the Delaunay arguments replaced by those of Simon et al. (1994), see Table 2 for their numerical values.
The Earth rotation uses the theory of Aoki et al. (1982), eqn.(14), for Greenwich Mean Sidereal Time

$$\mathrm{GMST} = 67\,310.54841 + (3.155\,760\,000 + 8\,640\,184.812\,866)\,t + 0.093\,104\,t^2 - 6.2\cdot 10^{-6}\,t^3$$

and adds the equation of equinoxes to obtain the Greenwich Apparent Siderial Time

$$\mathrm{GAST} = \mathrm{GMST} + \Delta\Psi \cdot \cos\varepsilon$$

No correction for the polar motion has been applied. The time base for all parts is TDB.

2. Numerical constants

Table 1: IERS 1992 numerical constants used for the HW95 catalogue

Item	Numerical value	Comment
f	1 / 298.256 421 984 6	flattening of the Earth's ellipsoid
a	6 378 136.3 m	equatorial radius of the Earth
$J_{2\oplus}$	$1.082\,636\,2 \cdot 10^{-3}$	2nd degree zonal harmonic coeff. of the Earth's gravity field
GM	$3.986\,004\,418 \cdot 10^{14}$ $\mathrm{m^3/s^2}$	geocentric gravitational constant
M/Ea	0.012 300 034	ratio of mass of Moon to that of the Earth
S/Me	6 023 600	ratio of mass of Sun to that of the Mercury
S/Ve	408 523.71	ratio of mass of Sun to that of the Venus
S/Ea	332 946.045	ratio of mass of Sun to that of the Earth
S/Ma	3 098 708	ratio of mass of Sun to that of the Mars
S/Ju	1 047.348 6	ratio of mass of Sun to that of the Jupiter
S/Sa	3 497.90	ratio of mass of Sun to that of the Saturn

Table 2: Fundamental arguments after Simon et al. (1994), units are: ° and °/1000 yrs

j	1	t	t^2	t^3	t^4
1.	242.149 804 529 99	127 037 328.885 530 56	0.176 961 11	−0.001 831 40	0.000 088 24
2.	218.316 645 629 99	4 812 678.811 957 50	−0.146 638 89	0.001 851 40	−0.000 153 55
3.	280.466 450 160 02	360 007.697 488 06	0.030 322 22	0.000 020 00	−0.000 065 32
4.	83.353 243 119 98	40 690.136 352 50	−1.032 172 22	−0.012 491 68	0.000 526 55
5.	234.955 444 990 00	19 341.362 619 72	−0.207 561 11	−0.002 139 42	0.000 165 01
6.	282.937 340 980 01	17.194 576 67	0.045 688 89	−0.000 017 76	−0.000 033 23
7.	252.250 905 519 99	1 494 740.721 722 33	0.030 349 84	0.000 018 11	−0.000 065 32
8.	181.979 800 850 00	585 192.129 533 30	0.031 013 95	0.000 014 90	−0.000 065 32
9.	355.432 999 580 02	191 416.963 702 97	0.031 051 87	0.000 015 64	−0.000 065 32
10.	34.351 518 740 03	30 363.027 748 48	0.022 329 72	0.000 037 01	−0.000 052 14
11.	50.077 444 300 00	12 235.110 686 22	0.051 907 83	−0.000 029 85	−0.000 097 40

Table 3: Fundamental frequencies after Simon et al. (1994) in °/hour

1.	14.492 052 120 18
2.	0.549 016 519 73
3.	0.041 068 639 91
4.	0.004 641 813 41
5.	0.002 206 406 87
6.	0.000 001 961 51
7.	0.170 515 710 90
8.	0.066 757 030 52
9.	0.021 836 295 20
10.	0.003 463 726 64
11.	0.001 395 746 14

3. Formulas for the tidal acceleration

The tidal acceleration $\vec{f_S}$ due to part V_{lm} of the tidal potential in a spherical coordinate system is given by

$$
\begin{aligned}
f_{S.x} &= -\frac{1}{r}\frac{\partial}{\partial\theta}V_{lm} && = -\frac{1}{r}\frac{1}{\overline{P}_{lm}}\left(\frac{\partial}{\partial\theta}\overline{P}_{lm}\right)V_{lm} \\
f_{S.y} &= \frac{1}{r\cos\theta}\frac{\partial}{\partial\lambda}V_{lm} && \\
f_{S.z} &= \frac{\partial}{\partial r}V_{lm} && = \frac{l}{r}V_{lm}
\end{aligned}
$$

while its components $\vec{f_E}$ in an ellipsoidal coordinate system are given by

$$
\begin{aligned}
f_{E.x} &= \cos\alpha\, f_{S.x} - \sin\alpha\, f_{S.z} \\
f_{E.y} &= f_{S.y} \\
f_{E.z} &= \sin\alpha\, f_{S.x} + \cos\alpha\, f_{S.z}
\end{aligned}
$$

α is the angular difference between the ellipsoidal latitude and the geocentric latitude. $-f_{E.z}$ is the gravity tide, while $f_{E.x}$ and $f_{E.y}$ are the tilt tide in North-direction and East-direction, respectively.

4. The new HW-Format for tidal catalogues

For a better representation the following "HW-Format" was introduced. Most other catalogues (Doodson, CTED-73, Büllesfeld, Tamura, Xi-89) have been transformed into that format. Each data line consists of

```
FORTRAN-77:
  I6,1X,A2,I2,11A3,F12.8,2F11.0,2F9.0

ANSI-C:
  "%6i %2s%2i%3i%3i%3i%3i%3i%3i%3i%3i%3i%3i%3i%12.8f%#12.0f%#12.0f%#10.0f%#10.0f\n"
```

where the entries denote

```
Col.  1...  6: sequence number (1...12935; 999999 = end of file)
Col.        7: blank
Col.  8...  9: body generating the potential. MO = Moon, SU = Sun,
               ME = Mercury, VE= Venus, MA = Mars, JU= Jupiter,
               SA = Saturn, FM = quadrupole moment of the Earth on Moon,
               FS = quadrupole moment of the Earth on Sun.
Col. 10... 11: l = degree of the spherical harmonic development.
Col. 12... 14: m = order  of the spherical harmonic development,
               equal
               k1 = integer argument number for the mean local Moontime
Col. 15... 17: k2 = integer argument number for the mean longitude of the Moon
Col. 18... 20: k3 = integer argument number for the mean longitude of the Sun
Col. 21... 23: k4 = integer argument number for the mean longitude of the lunar perigee
Col. 24... 26: k5 = integer argument number for the negative mean longitude of the lunar ascending node
Col. 27... 29: k6 = integer argument number for the mean longitude of the solar perigee
Col. 30... 32: k7 = integer argument number for the mean longitude of Mercury
Col. 33... 35: k8 = integer argument number for the mean longitude of Venus
Col. 36... 38: k9 = integer argument number for the mean longitude of Mars
Col. 39... 41: k10= integer argument number for the mean longitude of Jupiter
Col. 42... 44: k11= integer argument number for the mean longitude of Saturn
Col. 45... 56: fr=  frequency of the tidal wave at J2000 in degree per hour.
Col. 57... 68: C0 = COS-coefficient of the tidal potential in
                    10**-10 m**2/s**2. The C0 coefficient has to be multiplied
                    with the COS of the argument.
Col. 69... 80: S0 = SIN-coefficient of the tidal potential in
                    10**-10 m**2/s**2. The S0 coefficient has to be multiplied
                    with the SIN of the argument.
Col. 81... 90: C1 = t*COS-coefficent of the tidal potential in 10**-10
                    m**2/s**2 per Julian century. The C1 coefficient has to
                    be multiplied with the time difference between the epoch
                    and J2000 (in Julian centuries) and with the COS of
                    the argument.
Col. 91...100: S1 = t*SIN-coefficent of the tidal potential in 10**-10
                    m**2/s**2 per Julian century. The S1 coefficient has to
                    be multiplied with the time difference between the epoch
                    and J2000 (in Julian centuries) and with the SIN of
                    the argument.
Col.      101: blank
Col.102...105: nam= Darwin name of the tidal wave (for very few main tidal
                    waves available only).
```

Sample:

```
number    l  m  k  k  k  k  k  k  k  k  k  k  frequency          C0          S0        C1        S1 nam
                2  3  4  5  6  7  8  9 10 11  (deg/hour)
C*****************************************************************************************************
     1 MO 2  0  0  0  0  0  0  0  0  0  0  0  0.00000000-5944286666.          0. -1935360.        0. MO
     2 MO 4  0  0  0  0  0  0  0  0  0  0  0  0.00000000     375384.          0.      706.        0.
     3 SU 2  0  0  0  0  0  0  0  0  0  0  0  0.00000000-2751200925.          0.  -890704.        0. SO
     4 ME 2  0  0  0  0  0  0  0  0  0  0  0  0.00000000       -686.          0.        0.        0.
     5 VE 2  0  0  0  0  0  0  0  0  0  0  0  0.00000000     -33359.          0.        0.        0.
     .
     .
     .
999999
```

5. Explicit formulas for the Legendre functions and their derivatives

ℓ	m	$\overline{P}_{\ell m}(\cos\theta)$	$\partial/\partial\theta\,\overline{P}_{\ell m}(\cos\theta)$
1	0	$\sqrt{3}\,\cos\theta$	$-\sqrt{3}\,\sin\theta$
1	1	$\sqrt{3}\,\sin\theta$	$\sqrt{3}\,\cos\theta$
2	0	$\sqrt{\frac{5}{4}}\,(3\cos^2\theta-1)$	$-\sqrt{45}\,\sin\theta\cos\theta$
2	1	$\sqrt{15}\,\sin\theta\cos\theta$	$\sqrt{15}\,(1-2\sin^2\theta)$
2	2	$\sqrt{\frac{15}{4}}\,\sin^2\theta$	$\sqrt{15}\,\sin\theta\cos\theta$
3	0	$\sqrt{\frac{7}{4}}\,\cos\theta\,(5\cos^2\theta-3)$	$\sqrt{\frac{63}{4}}\,\sin\theta\,(1-5\cos^2\theta)$
3	1	$\sqrt{\frac{21}{8}}\,\sin\theta\,(5\cos^2\theta-1)$	$\sqrt{\frac{21}{8}}\,\cos\theta\,(4-15\sin^2\theta)$
3	2	$\sqrt{\frac{105}{4}}\,\sin^2\theta\cos\theta$	$\sqrt{\frac{105}{4}}\,\sin\theta\,(3\cos^2\theta-1)$
3	3	$\sqrt{\frac{35}{8}}\,\sin^3\theta$	$\sqrt{\frac{315}{8}}\,\sin^2\theta\cos\theta$
4	0	$\sqrt{\frac{9}{64}}\,(3-30\cos^2\theta+35\cos^4\theta)$	$\sqrt{\frac{225}{4}}\,(3-7\cos^2\theta)\sin\theta\cos\theta$
4	1	$\sqrt{\frac{45}{8}}\,\sin\theta\cos\theta\,(7\cos^2\theta-3)$	$\sqrt{\frac{45}{8}}\,(3-27\cos^2\theta+28\cos^4\theta)$
4	2	$\sqrt{\frac{45}{16}}\,\sin^2\theta\,(7\cos^2\theta-1)$	$\sqrt{45}\,(7\cos^2\theta-4)\sin\theta\cos\theta$
4	3	$\sqrt{\frac{315}{8}}\,\sin^3\theta\cos\theta$	$\sqrt{\frac{315}{8}}\,\sin^2\theta\,(4\cos^2\theta-1)$
4	4	$\sqrt{\frac{315}{64}}\,\sin^4\theta$	$\sqrt{\frac{315}{4}}\,\sin^3\theta\cos\theta$
5	0	$\sqrt{\frac{11}{64}}\,\cos\theta\,(15-70\cos^2\theta+63\cos^4\theta)$	$\sqrt{\frac{2475}{64}}\,\sin\theta\,(-1+14\cos^2\theta-21\cos^4\theta)$
5	1	$\sqrt{\frac{165}{64}}\,\sin\theta\,(1-14\cos^2\theta+21\cos^4\theta)$	$\sqrt{\frac{165}{64}}\,\cos\theta\,(29-126\cos^2\theta+105\cos^4\theta)$
5	2	$\sqrt{\frac{1155}{16}}\,\sin^2\theta\cos\theta\,(3\cos^2\theta-1)$	$\sqrt{\frac{1155}{16}}\,\sin\theta\,(1-12\cos^2\theta+15\cos^4\theta)$
5	3	$\sqrt{\frac{385}{128}}\,\sin^3\theta\,(9\cos^2\theta-1)$	$\sqrt{\frac{3465}{128}}\,\sin^2\theta\cos\theta\,(15\cos^2\theta-7)$
5	4	$\sqrt{\frac{3465}{64}}\,\sin^4\theta\cos\theta$	$\sqrt{\frac{3465}{64}}\,\sin\theta\,(-1+6\cos^2\theta-5\cos^4\theta)$
5	5	$\sqrt{\frac{693}{128}}\,\sin^5\theta$	$\sqrt{\frac{17325}{128}}\,\cos\theta\sin^4\theta$
6	0	$\sqrt{\frac{13}{256}}\,(-5+105\cos^2\theta-315\cos^4\theta+231\cos^6\theta)$	$\sqrt{\frac{5733}{64}}\,\sin\theta\cos\theta\,(-5+30\cos^2\theta-33\cos^4\theta)$
6	1	$\sqrt{\frac{273}{64}}\,\sin\theta\cos\theta\,(5-30\cos^2\theta+33\cos^4\theta)$	$\sqrt{\frac{273}{64}}\,(-5+100\cos^2\theta-285\cos^4\theta+198\cos^6\theta)$
6	2	$\sqrt{\frac{1365}{512}}\,\sin^2\theta\,(1-18\cos^2\theta+33\cos^4\theta)$	$\sqrt{\frac{1365}{128}}\,\sin\theta\cos\theta\,(19-102\cos^2\theta+99\cos^4\theta)$
6	3	$\sqrt{\frac{1365}{128}}\,\sin^3\theta\cos\theta\,(-3+11\cos^2\theta)$	$\sqrt{\frac{12285}{128}}\,\sin^2\theta\,(1-15\cos^2\theta+22\cos^4\theta)$
6	4	$\sqrt{\frac{819}{256}}\,\sin^4\theta\,(11\cos^2\theta-1)$	$\sqrt{\frac{819}{64}}\,\cos\theta\sin\theta\,(-13+46\cos^2\theta-33\cos^4\theta)$
6	5	$\sqrt{\frac{9009}{128}}\,\sin^5\theta\cos\theta$	$\sqrt{\frac{9009}{128}}\,\sin^4\theta\,(6\cos^2\theta-1)$
6	6	$\sqrt{\frac{3003}{512}}\,\sin^6\theta$	$\sqrt{\frac{27027}{128}}\,\sin^5\theta\cos\theta$

References

- Hartmann, T. and Wenzel, H.-G. (1994): *"Catalogue of the Earth tide generating potential due to the planets"*, Bulletin d'Information Marees Terrestres, **118**, 8847–8880.
- Hartmann, T. and Wenzel, H.-G. (1995): *"The HW95 tidal potential catalogue"*, submitted to Geophysical Research Letters for publication.
- McCarthy, D. (1992): *"IERS Standards 1992"*, IERS Technical note no. 13, Central Bureau of International Earth Rotation Service, Observatoire de Paris.
- Simon, J.L., Bretagnon, P., Chapront, J., Chapront-Touze, M., Francou, G. and Laskar, J. (1994): *"Numerical expressions for precession formulae and mean elements for the Moon and the planets"*, Astron. Astrophys., **282**, 663–683.