leading to the relations

$$\overline{M}_Z = M_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \tag{9}$$

$$\approx M_Z \ - 34 \text{ MeV} \tag{10}$$

$$\overline{\Gamma}_Z = \Gamma_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \tag{11}$$

$$\approx \Gamma_Z \ - 0.9 \text{ MeV} \tag{12}$$

Some authors [7] choose to define the Z mass and width via

$$\overline{s} = (\overline{M}_Z - \frac{i}{2}\overline{\Gamma}_Z)^2 \tag{13}$$

which yields $\overline{M}_Z \approx M_Z - 26$ MeV, $\overline{\Gamma}_Z \approx \Gamma_Z - 1.2$ MeV.

The L3 collaboration at LEP (ADRIANI 93H, ADRIANI 93M) have analyzed their data using the S-matrix approach as defined in Eq. (8), in addition to the conventional one. As expected, they observe a downward shift of ≈ 34 MeV in the Z mass.

Handling the large-angle e^+e^- final state

Unlike other $f\overline{f}$ decay final states of the Z, the e^+e^- final state has a contribution not only from the s-channel but also from the t-channel and s-t interference. The full amplitude is not amenable to fast calculation, which is essential if one has to carry out minimization fits within reasonable computer time. The usual procedure is to calculate the non-s channel part of the cross section separately using the Standard Model program ALIBABA [8] using the 'most reasonable' values of M_{top}, and M_{Higgs} and add it to the s-channel cross section calculated as for other channels. This leads to two additional sources of error in the analysis: firstly, the theoretical calculation in ALIBABA itself is known to be accurate to ~ 0.5 %, and secondly, there is uncertainty due to unknown M_{top} and M_{Higgs}. These additional errors are propagated into the analysis by including them in the systematic error on the e^+e^- final state.

Errors due to uncertainty in LEP energy determination [9]

The systematic errors related to the LEP energy measurement can be classified as:

- The absolute energy scale error;
- Energy-point-to-energy-point errors due to the non-linear response of the magnets to the exciting currents;
- Energy-point-to-energy-point errors due to possible higher-order effects in the relationship between the dipole field and beam energy;
- Energy reproducibility errors due to various unknown uncertainties in temperatures, tidal effects, corrector settings, RF status *etc.* Since one groups together data taken at 'nominally same' energies in different fills, it can be assumed that these errors are uncorrelated and are reduced by $\sqrt{\overline{N}_{\text{fill}}}$ where $\overline{N}_{\text{fill}}$ is the (luminosity weighted) effective number of fills at a particular energy point.

At each energy point the last two errors can be summed into one point-to-point error.

Choice of fit parameters

The LEP collaborations have chosen the following primary set of parameters for fitting: M_Z, Γ_Z, σ^0_{hadron}, R(lepton), $A_{FB}^{(0,\ell)}$, where $R(\text{lepton}) = \Gamma(\text{hadrons})/\Gamma(\text{lepton})$, $\sigma^0_{\text{hadron}} = 12\pi\Gamma(e^+e^-)\Gamma(\text{hadrons})/M_Z^2\Gamma_Z^2$. With a knowledge of these fitted parameters and their covariance matrix, any other parameter can be derived. The main advantage of these parameters is that they form the **least correlated** set of parameters, so that it becomes easy to combine results from the different LEP experiments.

Thus, the most general fit carried out to cross section and asymmetry data determines the **nine parameters**: M_Z, Γ_Z, σ^0_{hadron}, $R(e)$, $R(\mu)$, $R(\tau)$, $A_{FB}^{(0,e)}$, $A_{FB}^{(0,\mu)}$, $A_{FB}^{(0,\tau)}$. Assumption of lepton universality leads to a **five-parameter fit** determining M_Z, Γ_Z, σ^0_{hadron}, R(lepton), $A_{FB}^{(0,\ell)}$. The use of **only** cross-section data leads to six- or four-parameter fits if lepton universality is or is not assumed, *i.e.*, $A_{FB}^{(0,\ell)}$ values are not determined.

In order to determine the best values of the effective vector and axial vector couplings of the charged leptons to the Z, the above mentioned nine- and five-parameter fits are carried out with added constraints from the measured values of A_τ and A_e obtained from τ polarization studies at LEP and the determination of A_{LR} at SLC.

Combining results from the four LEP experiments [10]

Each LEP experiment provides the values of the parameters mentioned above together with the full covariance matrix. The statistical and experimental systematic errors are assumed to be uncorrelated among the four experiments. The sources of **common** systematic errors are i) the LEP energy uncertainties, and ii) the effect of theoretical uncertainty in calculating the small-angle Bhabha cross section for luminosity determination and in estimating the non-s channel contribution to the large-angle Bhabha cross section. Using this information, a full covariance matrix, V, of all the input parameters is constructed and a combined parameter set is obtained by minimizing $\chi^2 = \Delta^T V^{-1}\Delta$, where Δ is the vector of residuals of the combined parameter set to the results of individual experiments.

Study of $Z \to b\overline{b}, c\overline{c}$ at LEP

These studies lead to the experimental determination of the ratios of the partial widths $\Gamma(b\overline{b})/\Gamma(\text{hadrons})$, $\Gamma(c\overline{c})/\Gamma(\text{hadrons})$, and the forward-backward (charge) asymmetries $A_{FB}^{(0,b)}$ and $A_{FB}^{(0,c)}$. Each LEP experiment has used more than one technique of b,c tagging, and then used certain assumptions, some model dependent, to extract the final numbers. This makes the task of **combining** these LEP results quite daunting. A first step in this direction was made recently [10] for a joint LEP presentation for the 1993 summer conferences. In principle, the task is the same as for combining results on the lineshape: identification of the **common systematic errors**, which in this case exist not only among different LEP experiments, but