Details on Z-parameter determination and the study of $Z \to b\overline{b}, c\overline{c}$ at LEP are given in this note.

The standard 'lineshape' parameters of the Z are determined with increasing precision from an analysis of the production cross sections of these final states in e^+e^- collisions at LEP. The $Z \to \nu\overline{\nu}(\gamma)$ state is identified directly by detecting single photon production and indirectly by subtracting the visible partial widths from the total width. Inclusion in this analysis of the forward-backward asymmetry of charged leptons, $A_{FB}^{(0,\ell)}$, of the τ polarization, $P(\tau)$, and its forward-backward asymmetry, $P(\tau)^{fb}$, enables the separate determination of the effective vector ($\overline{g}_V$) and axial vector ($\overline{g}_A$) couplings of the Z to these leptons and the ratio ($\overline{g}_V/\overline{g}_A$) which is related to the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ (see the Standard Model review).

Determination of the b- and c-quark-related partial widths and charge asymmetries involves tagging the b and c quarks. Traditionally this was done by requiring the presence of a prompt lepton in the event with high momentum and high transverse momentum (with respect to the accompanying jet). Precision vertex measurement with silicon detectors enables one to do impact parameter and lifetime tagging. Lately, sophisticated neural-network techniques have been used to classify events as b or non-b on a statistical basis using event-shape variables.

Z-parameter determination at LEP

LEP is run at a few energy points on and around the Z mass constituting an energy 'scan.' The shape of the cross-section variation around the Z peak can be described by a Breit-Wigner *ansatz* with an energy-dependent total width [1]. The **three** main properties of this distribution, viz., the **position** of the peak, the **width** of the distribution, and the **height** of the peak, determine respectively the values of M_Z, Γ_Z, and $\Gamma(e^+e^-) \times \Gamma(f\overline{f})$, where $\Gamma(e^+e^-)$ and $\Gamma(f\overline{f})$ are the electron and fermion partial widths of the Z. The quantitative determination of these parameters is done by writing analytic expressions for these cross sections in terms of the parameters and fitting the calculated cross sections to the measured ones by varying these parameters, taking properly into account all the errors. Single-photon exchange and γ-Z interference are included, and the large ($\sim$30 %) initial-state radiation (ISR) effects are taken into account by convoluting the analytic expressions over a 'Radiator Function [1,2]' $H(s,s')$. Thus for the process $e^+e^- \to f\overline{f}$:

$$\sigma_f(s) = \int H(s,s')\ \sigma_f^0(s')\ ds' \tag{1}$$

$$\sigma_f^0(s) = \sigma_Z^0 + \sigma_\gamma^0 + \sigma_{\gamma Z}^0 \tag{2}$$

$$\sigma_Z^0 = \frac{12\pi}{M_Z^2}\ \frac{\Gamma(e^+e^-)\Gamma(f\overline{f})}{\Gamma_Z^2}\ \frac{s\ \Gamma_Z^2}{(s-M_Z^2)^2\ +\ s^2\Gamma_Z^2/M_Z^2} \tag{3}$$

$$\sigma_\gamma^0 = \frac{4\pi\alpha^2(s)}{3s}\ Q_f^2 N_c^f \tag{4}$$

$$\sigma_{\gamma Z}^0 = -\frac{2\sqrt{2}\alpha(s)}{3}\ (Q_f G_F N_c^f g_{Ve} g_{Vf}) \times \frac{(s-M_Z^2)M_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \tag{5}$$

where Q_f is the charge of the fermion, $N_c^f = 3(1)$ for quark (lepton) and g_{Vf} is the neutral vector coupling of the Z to the fermion-antifermion pair $f\overline{f}$.

Since $\sigma_{\gamma Z}^0$ is expected to be much less than σ_Z^0, the LEP collaborations have generally calculated the interference term in the framework of the Standard Model using the best known values of g_V. This fixing of $\sigma_{\gamma Z}^0$ leads to a tighter constraint on M_Z and consequently a smaller error on its fitted value.

Defining

$$A_f = 2\frac{g_{Vf}\cdot g_{Af}}{(g_{Vf}^2 + g_{Af}^2)} \tag{6}$$

where g_{Af} is the neutral axial-vector coupling of the Z to $f\overline{f}$, the lowest-order expressions for the various lepton-related asymmetries on the Z pole are [3] $A_{FB}^{(0,\ell)} = (3/4)A_e A_f$, $P(\tau) = -A_\tau$, $P(\tau)^{fb} = -(3/4)A_e$, $A_{LR} = A_e$. The full analysis takes into account the energy dependence of the asymmetries. Experimentally A_{LR} is defined as $(\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$ where $\sigma_{L(R)}$ are the $e^+e^- \to Z$ production cross sections with left- (right)-handed electrons.

In terms of g_A and g_V, the partial decay width of the Z to $f\overline{f}$ can be written as

$$\Gamma(f\overline{f}) = \frac{G_F M_Z^3}{6\sqrt{2}\pi}\ (g_{Vf}^2 + g_{Af}^2)\ N_c^f(1+\delta_{\rm QED})(1+\delta_{\rm QCD}) \tag{7}$$

where $\delta_{\rm QED} = 3\alpha Q_f^2/4\pi$ accounts for final-state photonic corrections and $\delta_{\rm QCD} = 0$ for leptons and $\delta_{\rm QCD} = (\alpha_s/\pi) + 1.409(\alpha_s/\pi)^2 - 12.805(\alpha_s/\pi)^3$ for quarks, α_s being the strong coupling constant at $\mu = M_Z$.

In the above framework, the QED radiative corrections have been explicitly taken into account by convoluting over the ISR and allowing the electromagnetic coupling constant to run [4]: $\alpha(s) = \alpha/(1-\Delta\alpha)$. On the other hand, weak radiative corrections that depend upon the assumptions of the electroweak theory and on the values of the unknown $M_{\rm top}$ and $M_{\rm Higgs}$ are accounted for by **absorbing them into the couplings**, which are then called the *effective* couplings $\overline{g}_V$ and $\overline{g}_A$ (or alternatively the effective parameters of the $\star$ scheme of Kennedy and Lynn [5]).

S-matrix approach to the Z

While practically all experimental analyses of LEP/SLC data have followed the 'Breit-Wigner' approach described above, an alternative S-matrix-based analysis is also possible. The Z, like all unstable particles, is associated with a complex pole in the S matrix. The pole position is process independent and gauge invariant. The mass, $\overline{M}_Z$, and width, $\overline{\Gamma}_Z$, can be defined in terms of the pole in the energy plane via [6]

$$\overline{s} = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z \tag{8}$$