Nonlinear nonresonant forces by radio-frequency waves in plasmas

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Nonresonant forces by applied rf waves in plasmas are analyzed. Along the background dc magnetic field, the force arises from the gradient of the ponderomotive potential. Only when the dc magnetic field is straight, however, is this parallel force completely consistent with that from the single particle picture, where the ponderomotive force depends on the gradients of rf fields only. Across the dc magnetic field, besides the ponderomotive force from the particle picture, additional Reynolds stress and polarization stress contribute to the total force. For waves with frequency much lower than the cyclotron frequency, the perpendicular forces from the particle and fluid pictures can have opposite signs. In plasmas with a symmetry angle (e.g., toroidal systems), nonresonant forces cannot drive net flow or current in the flux surface, but the radial force may influence macroscopic behavior of plasma. Moreover, nonresonant forces may drive flow or current in linear plasmas or in a localized region of toroidal plasmas. © *2007 American Institute of Physics*.
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The use of radio frequency (rf) power to drive plasma current and/or flow has received considerable discussion.[1–8] Although current drive and flow drive usually concerns different directions of the driving force on different species, the physics of the nonlinear force[9] by applied rf waves is fundamentally unified. Recently, Gao, Fisch, and Qin[10] (GFQ) discussed the rf force by low frequency waves in the picture of a single particle, a fluid element, and a kinetic plasma, respectively. The GFQ paper especially focused on the parallel force and found that the collisionless nonresonant force offered by the quasilinear electromagnetic force is completely cancelled by the nonlinear stress force. Therefore, in collisionless plasmas, none of the nonresonant forces by low frequency waves can drive parallel current and it negates the scheme of steady current driven by helicity injection of applied low frequency waves. A similar work was performed by Myra, Berry, D'Ippolito, and Jaeger,[11] (MBDJ) where both the nonlinear stress and Lorentz force are also included in a kinetic moment approach that was developed to treat rf-driven flows. While certain flow-drive effects could be treated generally, the nonlinear kinetic stress terms in the MBDJ paper were only given for the case of perpendicular forces and rf field gradients in a constant dc magnetic field. It was found that the forces driving flux-surface-averaged flows consisted of energy absorbing terms and dissipative stresses only.

These two works are complementary in the geometry and components of force considered. Also, the absence of the nonresonant force in the two papers is due to different mechanisms. The GFQ paper assumes that the gradient of the rf field is perpendicular to the dc magnetic field, so the nonresonant force is in the perpendicular direction and then the parallel force is totally due to resonant (i.e., dissipative) physics. In the MBDJ work, the nonresonant force disappears after projecting the local force onto the parallel and toroidal direction and averaging over a flux-surface. This operation is valid in toroidal plasmas since averaged forces and flows in the flux-surface, namely poloidal and toroidal, are usually what is important. However, the radial force, although driving no flow, affects the force-balance in magnetohydrodynamics (MHD) equilibrium, the radial electric field and accordingly the stability of the plasma. On the other hand, the flux-surface average is not appropriate in some cases, for example in linear devices or in a localized region of toroidal systems, where the local force may drive flow or current. Therefore, in some sense, an analysis of the local force is more general. The purpose of the present Brief Communication is to extend the local analysis of parallel forces in Ref. 10 to all directions in a general inhomogeneous background magnetic field.

The rf force includes nonresonant and resonant components. The latter arises from the resonant mechanism between waves and thermal particles, such as Landau damping or cyclotron damping. We can use a nonlinear kinetic approach to derive a detailed expression for the resonant force.[5,10,11] However, it is found that, from cold plasmas to thermal plasmas, except for the details, the nonresonant force is not changed in nature.[10,11] Therefore, the cold-fluid model is useful and convenient for analyzing nonresonant forces. In cold plasmas, the fluid velocity can be considered the same as the velocity of a single particle; on the other hand, when the kinetic theory is applied, the velocity given by the single particle picture can still be considered as the perturbation of the microcosmic canonical velocity variable by the rf field and the physics is not lost. Nonresonant forces by rf waves act on the bulk plasma rather than on selected particles, therefore they are usually referred to as ponderomotive forces. The "ponderomotive force" is a nonlinear force that a charged particle experiences in a rapidly oscillating, inhomogeneous electromagnetic field. Its mechanism can be understood by considering the difference between the restoring force at one turning point and that at the other in one cycle of oscillation due to the inhomogeneity in the field. The ponderomotive force does not rely on the wave-particle resonance; therefore its concept can be easily extended to the fluid picture. In the rest of this Brief Communication, we will discuss the local nonresonant force in the cold fluid model, as well as in the single particle picture. It is noted that, although the ponderomotive forces in both particle and fluid pictures have been derived before,[9] a further analysis will give us a more comprehensive understanding of nonlinear ponderomotive forces by rf waves.

First, we consider a single particle, with mass m and charge q, under a background dc magnetic field $\mathbf{B}$ and a rf wave with electric and magnetic fields, $\mathbf{E}_1$, $\mathbf{B}_1 \propto \exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$. If the spatial perturbed displacement and the average drift displacement are small compared to the scale of the field, the ponderomotive force on a single particle can be represented as

$$\begin{aligned}\mathbf{f}_{\rm sp} &= \tfrac{1}{2}\,\mathrm{Re}[(\mathbf{p}_1\cdot\nabla)\mathbf{E}_1^* + \mathbf{p}_1\times(\nabla\times\mathbf{E}_1^*)] \\ &= \tfrac{1}{2}\,\mathrm{Re}[(\nabla\mathbf{E}_1^*)\cdot\mathbf{p}_1], \end{aligned}\tag{1}$$

where $\mathbf{p}_1 \equiv q\boldsymbol{\delta}_1 = \boldsymbol{\chi}\cdot\mathbf{E}_1$ is the dipole moment with a perturbed displacement $\boldsymbol{\delta}_1$. When the polarizability $\boldsymbol{\chi}$ is Hermitian, the force can be rewritten as

$$\mathbf{f}_{\rm sp} = \tfrac{1}{4}[\nabla(\mathbf{E}_1^*\cdot\mathbf{p}_1) - \nabla_\chi(\mathbf{E}_1^*\cdot\mathbf{p}_1)]. \tag{2}$$

Here the operator ∇_χ denotes a spatial derivative holding $\mathbf{E}_1$ fixed (both the explicit $\mathbf{E}_1$ and the one inside $\mathbf{p}_1$). Therefore, the ponderomotive force on a single particle is only due to the inhomogeneity of rf fields, which, in fact, can be seen from Eq. (1) directly. Using the single particle motion equation,

$$-m\omega^2\boldsymbol{\delta}_1 = q\mathbf{E}_1 - i\omega q\boldsymbol{\delta}_1\times\mathbf{B}, \tag{3}$$

we can rewrite the last term of Eq. (2) and the single particle ponderomotive force becomes

$$\mathbf{f}_{\rm sp} = \frac{1}{4}\left[\nabla(\mathbf{E}_1^*\cdot\mathbf{p}_1) + \frac{\omega}{q}(\nabla\mathbf{B})\cdot(i\mathbf{p}_1^*\times\mathbf{p}_1)\right]. \tag{4}$$

When the inhomogeneity of the equilibrium magnetic field is much smaller than that of the applied rf field, the force reduces to the well-known form—the gradient of the ponderomotive potential.

For a fluid element, the electric field force due to the displacement of a single particle is replaced by a force due to the charge accumulation by the divergence of the flow, and the Reynolds stress should be included as well. The Reynolds stress describes the net transfer of momentum across a surface in a turbulent fluid because of fluctuations in fluid velocity, therefore its divergence gives a volume force. The total force on a fluid element (strictly speaking the force density) is

$$\mathbf{F}_{cf} = \tfrac{1}{2}\,\mathrm{Re}[\rho_1\mathbf{E}_1^* + nq\mathbf{v}_1\times\mathbf{B}_1^* - \nabla\cdot(nm\mathbf{v}_1\mathbf{v}_1^*)], \tag{5}$$

where $\rho_1 = -\nabla\cdot n\mathbf{p}_1$ is the charge density, $\mathbf{v}_1 = -i\omega\boldsymbol{\delta}_1$ is the perturbed velocity and $\mathbf{B}_1 = (1/i\omega)\nabla\times\mathbf{E}_1$ is the rf magnetic field. This force is connected with the single particle ponderomotive force by

$$\mathbf{F}_{cf} = n\mathbf{f}_{\rm sp} - \tfrac{1}{2}\,\mathrm{Re}[\nabla\cdot(n\mathbf{p}_1\mathbf{E}_1^* + nm\mathbf{v}_1\mathbf{v}_1^*)]. \tag{6}$$

The physics is clear. Besides the single particle ponderomotive force, a fluid element encounters two surface forces, one coming from the Reynolds stress and the other from the polarization stress. Using the cold fluid momentum equation, i.e., the single particle motion equation, Eq. (3), we have

$$\mathbf{F}_{cf} = n\mathbf{f}_{\rm sp} + \frac{1}{2}\,\mathrm{Re}\left[\nabla\cdot\left(in\frac{\omega}{q}\mathbf{p}_1\mathbf{p}_1^*\times\mathbf{B}\right)\right]. \tag{7}$$

Since $\nabla\cdot\mathbf{B}=0$, we have $(\nabla\mathbf{B})\cdot(\mathbf{p}_1^*\times\mathbf{p}_1) = \mathbf{p}_1\times(\mathbf{p}_1^*\cdot\nabla)\mathbf{B} - \mathbf{p}_1^*\times(\mathbf{p}_1\cdot\nabla)\mathbf{B}$. Then, the $\nabla\mathbf{B}$ term in $\mathbf{f}_{\rm sp}$ is cancelled, and Eq. (7) is rewritten as

$$\mathbf{F}_{cf} = \frac{n}{4}\nabla(\mathbf{E}_1^*\cdot\mathbf{p}_1) + \frac{1}{2}\,\mathrm{Re}\left\{\frac{\omega}{q}[\nabla\cdot(in\mathbf{p}_1\mathbf{p}_1^*)]\times\mathbf{B}\right\}. \tag{8}$$

Using the identity $\nabla\times(\boldsymbol{\alpha}\times\boldsymbol{\beta}) = \nabla\cdot(\boldsymbol{\beta}\boldsymbol{\alpha}-\boldsymbol{\alpha}\boldsymbol{\beta})$ valid for arbitrary vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, we can recast Eq. (8) into the form

$$\mathbf{F}_{cf} = \frac{n}{4}\nabla(\mathbf{E}_1^*\cdot\mathbf{p}_1) - \frac{\omega}{4q}\mathbf{B}\times\nabla\times(in\mathbf{p}_1^*\times\mathbf{p}_1). \tag{9}$$

For a system with dc magnetic field symmetry in the direction of a periodic variable ζ (such as the toroidal direction in a tokamak or the azimuthal direction in a linear device), the torque is

$$\begin{aligned} R\mathbf{e}_\zeta\cdot\mathbf{F}_{cf} = {}& \frac{n}{4}R\mathbf{e}_\zeta\cdot\nabla(\mathbf{E}_1^*\cdot\mathbf{p}_1) + \frac{\omega}{4q}\nabla\cdot[(R\mathbf{e}_\zeta\times\mathbf{B}) \\ & \times(in\mathbf{p}_1^*\times\mathbf{p}_1)]. \end{aligned}\tag{10}$$

Here we have used $\nabla\times(R\mathbf{e}_\zeta\times\mathbf{B}) = -\nabla\times\nabla\psi = 0$ where ψ is a magnetic flux function and R is the (major) radius. The flux surface average of the first term in Eq. (10) vanishes because of periodicity in ζ, and the flux surface average of the second term vanishes because $R\mathbf{e}_\zeta\times\mathbf{B}$ is normal to the flux surface. The flux surface average of $\mathbf{B}\cdot\mathbf{F}_{cf}$ also vanishes since the flux-surface average will annihilate any result of the operator $\mathbf{B}\cdot\nabla$ on any function. Then, we conclude that flux surface averages of all nonresonant force components in the surface vanish, in agreement with MBDJ.

Now, we return to the analysis of the local force. From Eq. (8) or Eq. (9) we can see that, along the dc magnetic field, the nonlinear force on a fluid element is strictly the gradient of the ponderomotive potential, multiplied by the density n. Note, in this case, that the inhomogeneity of the background magnetic field also contributes to the force through the polarization $\mathbf{p}_1 = \boldsymbol{\chi}\cdot\mathbf{E}_1$. From Eq. (7), we see that the parallel force exactly equals the single particle force only when the background magnetic field is straight. In this case the parallel force depends on the parallel gradient of the rf field.

In the GFQ paper, the rf field is uniform along the dc magnetic field, so the parallel nonresonant force is zero. However, if an asymmetry such as a gradient in the potential in the parallel direction is artificially constructed, the current drive then arises from rearranging particle phase space through Hamiltonian forces under energy absorption.[8] We have shown that the parallel force will disappear when averaged over the flux surface in toroidal systems. Therefore, parallel current/flow drive by nonresonant force is effective only in linear devices or in a localized region in toroidal devices.

Next, we focus on the perpendicular component of the force. Besides the single particle ponderomotive force, the nonlinear stress contributes significantly to the perpendicular force. In Eq. (8) the force from the first gradient term is obviously along the direction of the inhomogeneity, while the second term provides the possibility of "off-diagonal" drive. An example of this "off-diagonal" drive is that the inhomogeneity of rf fields along the dc magnetic field can induce local forces in the perpendicular plane, which can be easily identified from Eq. (8). However, the perpendicular gradient of rf fields results in a local force only in the direction of the gradient, since the binormal component of the force vanishes as pure imaginary due to the structure of $\nabla\cdot(in\mathbf{p}_1\mathbf{p}_1^*)\times\mathbf{B}$. Specifically, we consider the case where the electric field is inhomogeneous across the dc magnetic field and this inhomogeneity dominates that of the dc magnetic field and other equilibrium profiles. Then, we can decompose the rf electric field as $\mathbf{E}_1=E_{1x}(x)\hat{\mathbf{x}}+E_{1y}(x)\hat{\mathbf{y}}+E_{1z}(x)\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}=\mathbf{b}$ is the direction of local dc magnetic field, $\hat{\mathbf{x}}$ is the direction of inhomogeneity and $\hat{\mathbf{y}}=\hat{\mathbf{z}}\times\hat{\mathbf{x}}$. According to Eqs. (2) and (8), the force in the single particle picture is

$$\langle n\mathbf{f}_{\mathrm{sp}}\rangle=-\frac{nq^2}{4m\omega}\left[\frac{\omega}{\omega^2-\Omega^2}\frac{\partial(|E_{x1}|^2+|E_{y1}|^2)}{\partial x}-\frac{2\Omega}{\omega^2-\Omega^2}\,\mathrm{Im}\,\frac{\partial(E_{x1}^*E_{y1})}{\partial x}+\frac{1}{\omega}\frac{\partial|E_{z1}|^2}{\partial x}\right]\hat{\mathbf{x}},\tag{11}$$

and the total fluid force is

$$\langle\mathbf{F}_{cf}\rangle=-\frac{nq^2}{4m\omega}\left[\frac{\omega(\omega^2+\Omega^2)}{(\omega^2-\Omega^2)^2}\frac{\partial(|E_{x1}|^2+|E_{y1}|^2)}{\partial x}-\frac{4\Omega\omega^2}{(\omega^2-\Omega^2)^2}\,\mathrm{Im}\,\frac{\partial(E_{x1}^*E_{y1})}{\partial x}+\frac{1}{\omega}\frac{\partial|E_{z1}|^2}{\partial x}\right]\hat{\mathbf{x}}.\tag{12}$$

For high frequency waves $\omega\gg\Omega$, the forces in particle and fluid pictures are almost the same. However, for low frequency waves $\omega\ll\Omega$ and taking $E_{x1}=E_{z1}=0$ or $E_{y1}=E_{z1}=0$, these two forces have opposite signs!

All these nonresonant forces, in principle, may drive flow or current locally. As seen previously, in toroidal systems, the toroidal and poloidal force, namely the force in the flux surface, will disappear after the flux surface average and only the radial force survives. The radial force does not drive flow since its flux-volume integral is zero, but it will influence the force balance in MHD equilibrium and instability as well. For example, when a laser is injected into plasmas, the self-focusing phenomena and the filament instability are due to the radial ponderomotive force.

In summary, the nonresonant force by applied rf waves is analyzed in cold plasmas. Along the dc magnetic field, the force is the product of the density and the gradient of the well-known ponderomotive potential, so the parallel force exists when the potential has an inhomogeneity in the parallel direction. The parallel force is exactly equal to the ponderomotive force on single particles, only when the dc magnetic field is straight, and in that case it depends on the inhomogeneity of rf fields only. Across the dc magnetic field, besides the single particle ponderomotive force, the nonlinear stress, including the Reynolds stress and the polarization stress contributes to the force. These additional forces will modify the conventional ponderomotive forces, and can even negate its sign. All these nonresonant forces, in principle, may drive flow or current locally. However, the nonresonant forces will be annihilated in the toroidal and poloidal direction after a flux-surface average in toroidal plasmas, but the radial force survives. Therefore, the nonresonant force may be more important in linear plasmas or in a localized region in toroidal confined plasmas. Here, we have not discussed the resonant force, which arises from the resonant mechanism between wave and thermal particles, such as Landau damping or cyclotron damping and survives after the flux-surface average. Detailed expressions of the resonant force can be obtained using the nonlinear kinetic approach.[5,10,11]

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