CCM key comparison in the pressure range 0.05 MPa to 1 MPa (gas medium, gauge mode)

Phase A1: Dimensional measurements and calculation of effective area

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Abstract. The results obtained by five laboratories in the determination from dimensional measurements of the effective areas of two gas-operated 10 cm^2 piston-cylinder assemblies are presented. These measurements were carried out as phase A1 of a key comparison in the pressure range 0.05 MPa to 1 MPa under the auspices of the Consultative Committee for Mass and Related Quantities (CCM) of the Comité International des Poids et Mesures. The participants performed diameter, straightness and roundness measurements on each piston and cylinder bore and calculated the effective area of each piston-cylinder assembly using their own methods. The differences between diameters determined by the institutes are systematic and often greater than the uncertainties claimed by the participants. Nevertheless, all calculated effective areas agree with the reference values determined within the expanded uncertainties with a coverage factor 2, most of them even within their standard uncertainties. The choice of calculation method seems to be less important than the dimensional data themselves. The effective areas determined from the dimensional measurements are compared with those obtained in cross-float experiments with national pressure standards, in a comparison referred to as phase A2 and reported in a separate paper.

1. Introduction

The effective area, A_0, of piston-cylinder assemblies, used as primary pressure standards in pressure balances for the pressure range from about 0.05 MPa to 1 MPa, is most often calculated by national metrology institutes from the dimensional characteristics of the piston and cylinder. The effective areas determined in this way usually provide a basis for entire national pressure scales, because the pressure standards operating at lower or higher pressures are traceable to the primary pressure standards by stepping-down or stepping-up procedures. Therefore, the discrepancies observed for national pressure scales in previous international comparisons [1] might have had their origin in differences of the dimensional measurement techniques and/or the methods applied at the national metrology institutes for calculating effective areas.

To ensure the equivalence of the dimensional measurements and calculation methods of the institutes, the High Pressure Working Group of the CCM has organized a key comparison in the pressure range from 0.05 MPa to 1 MPa. One part of this comparison, referred to as phase A1, relates to dimensional measurements and calculation methods using two piston-cylinder units as transfer standards. This phase was accompanied by phase A2, in which the effective areas of the same piston-cylinder units had to be determined in cross-float experiments with national pressure standards. The results of pressure measurements obtained in phase A2 are reported in [2].

The CNR-IMGC (Italy), the BNM-LNE (France), the NIST (USA), the NPL (UK) and the PTB

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(Germany) participated in phase A1. With the exception of the NPL, the Dimensional and Pressure Metrology Sections of the individual institutes were involved. The participating laboratories had first to perform dimensional measurements and then to calculate effective areas. The NPL took part only in the dimensional measurements.

2. Transfer standards

Two gas-operated piston-cylinder assemblies with nominal effective areas of 9.8 cm^2, one manufactured by Desgranges et Huot (DH, France), and the other by DH Instruments (DHI, USA), serial nos. DH 6594 and DHI 107, were used as transfer standards. The main difference between them was the piston material: tungsten carbide for the DH, ceramic for the DHI units. The cylinders of both assemblies were made from tungsten carbide. Further details of the assemblies are given in [2].

The comparison started in summer 1995 and ended in autumn 1998. Each assembly was investigated at the beginning and end of the comparison at the same laboratory to check its stability: the DH at the PTB and the DHI at the BNM-LNE.

3. Dimensional measurements and procedures for effective area calculation

Dimensional measurement procedures were prepared by the PTB in its capacity as the pilot laboratory for phase A1 and were accepted by the participating laboratories. As obligatory measurements, determinations were defined for (i) four diameters in two azimuthal directions $\varphi = 0°$, 90°, and two horizontal planes with $z = -L/5$, $L/5$; (ii) roundness profiles in five equidistant horizontal sections with $z = -2L/5$, $-L/5$, 0, $L/5$, $2L/5$; and (iii) eight generatrices separated by 45° for each piston and cylinder; where L is the length of the cylinder and $z = 0$ is the middle of the cylinder length. Each roundness trace had to contain 360 points, and each generatrix line had to be recorded at steps of 0.5 mm. Each participating laboratory was allowed to perform additional dimensional measurements and to choose a method for the calculation of the effective area.

In all laboratories, the diameter measurements were carried out mechanically, by contacting the cylinder or piston surfaces at diametrically opposite points by ball- or plane-ended styluses, where the distance travelled was determined by laser interferometer systems. Apparatuses equipped with ball-shaped styluses were calibrated with reference standard gauge blocks or fused-silica box standards. Contact forces were smaller than 0.06 N and 2.5 N for ball-shaped and plane-ended styluses, respectively. The temperature was kept within (20 ± 0.3) °C. Measurement results and their uncertainties refer to zero elastic distortion and temperature of 20 °C.

All the approaches applied by the participants to determine the effective area can be divided into three groups.

(a) The effective area was calculated as a mean of piston-cylinder cross-sectional areas found from directly measured diameters (method 1).

(b) The participants linked the roundness and straightness deviations to diameters to obtain absolute diameter values along generatrix and circle traces. Again, the effective area was calculated by averaging all piston and cylinder cross-sectional areas (method 2).

(c) The effective area was calculated from the data describing the shapes of piston and cylinder bore using the theory of Dadson et al. [3, equations 22, 32] (method 3).

Although all the methods used by the participants were basically the same, there were many differences as regards the number of directly measured diameters and roundness and straightness data, methods of linking the shape deviations to the diameters, data-processing procedures, and uncertainty estimation of the effective area, among others. To correctly treat the results reported and to distinguish the effects by differences in the dimensional data and in the calculation methods for the effective area applied by the participants, it was decided that the PTB should calculate the effective area using the same method on the basis of dimensional data sets supplied by the participating laboratories.

Supplementary dimensional measurements and special features of the calculation methods of the participants are given below.

3.1 Istituto di Metrologia "G. Colonnetti"

The CNR-IMGC, in addition to the obligatory dimensional measurements specified above, determined diameters at $z = -2L/5$, 0, $2L/5$, each for $\varphi = 0°$, 90°, and measured straightness with a height step of 0.25 mm. When method 2 was applied, the effective area was calculated twice: once using a restricted number of generatrix line radii and once using all radial values available for the generatrix and circle traces. For the calculation of the effective area by method 3, two models were used. In the first, a linear pressure distribution in the clearance between piston and cylinder was assumed. In the second, it was assumed that the applied gauge pressure tends to zero or that the pressure-transmitting medium is incompressible [3, equation 35].

3.2 Laboratoire National d'Essais

The BNM-LNE considered the opposite generatrix traces to be parallel and alternatively linked the shape deviations to one of four diameters measured at

$z = -L/5$, $L/5$ and $\varphi = 0°$, $90°$. With methods 2 and 3, the effective area was calculated from each of the four data sets obtained and the mean and its standard deviation were determined. With method 3, the effective area was calculated for four pressure ratios $p(-L/2)/p(L/2) = 1$, 2, 5 and 10.

3.3 National Physical Laboratory

The NPL only determined diameters at $z = -L/5$, $L/5$, each for $\varphi = 0°$, $90°$, and maximal roundness deviations at heights $-2L/5$, $-L/5$, 0, $L/5$ and $2L/5$ for the piston and cylinder of the two transfer standards.

3.4 National Institute of Standards and Technology

The NIST, in addition to the obligatory dimensional measurements, determined diameters at $z = 0$. When method 3 was applied, the dimensional data were fitted by cylindrical harmonics to obtain an analytical function for the piston and cylinder radii. The effect on the effective area of a possible change of the viscous flow regime to a molecular one in the piston-cylinder gap was analysed.

3.5 Physikalisch-Technische Bundesanstalt

The PTB performed additional measurements yielding nine circle traces for the cylinder and eleven for the piston separated by 4 mm, and generatrices with a height step of 0.1 mm. Two approaches to linking the dimensional data and to describing the shape of piston and cylinder bore were studied: (i) in terms of diameters connecting opposite points of generatrix or circle traces; and (ii) in terms of radii corresponding to the shape of each particular generatrix or circle.

4. Results and discussion

The participants' results for diameter measurements as well as for effective area calculations are presented and analysed in terms of their deviations from respective reference values. All the reference values were determined as non-weighted averages. Two results of the institutes which twice performed dimensional measurements on the same transfer standard in 1995 and in 1997, the BNM-LNE on the DHI and the PTB on the DH standards, were averaged before a reference value was calculated so that ultimately only one value from each participant was taken into account. From several values for the effective area calculated by each participant, only the one indicated by the laboratory as the most reliable was used in calculating the reference value. The uncertainties of the reference values were calculated from the uncertainties claimed by the participants, considering them to be uncorrelated.

With this approach, the relative standard uncertainties of the effective area reference values of the DH and DHI transfer standards were 2.5×10^{-6} and 2.7×10^{-6}, respectively.

Table 1 gives the diameters reported by the institutes for each piston and cylinder of the two transfer standards at heights $-L/5$ and $L/5$ in the directions 0° and 90°, in terms of deviations from the respective reference values. Twenty-six of the eighty deviations are greater than their expanded uncertainties with a coverage factor $k = 2$.

Table 2 summarizes all the laboratory results of the effective area calculations for the DH and DHI piston-cylinder units. This table is in two parts: (i) the results reported by the participating institutes; and (ii) the effective areas calculated at the PTB from the dimensional data supplied by the participants using the same calculation procedure.

Table 1. Deviations of participants' diameters from reference values for the DH and DHI piston and cylinder measured at heights $-L/5$ and $L/5$ and angle directions 0° and 90° (ΔD), standard uncertainties of these deviations, $s(\Delta D)$, and maximum differences of the diameters $(D_{\max} - D_{\min})$.

Institute	ΔD/nm				$s(\Delta D)$/nm	ΔD/nm				$s(\Delta D)$/nm
	$-L/5$, 0°	$-L/5$, 90°	$L/5$, 0°	$L/5$, 90°		$-L/5$, 0°	$-L/5$, 90°	$L/5$, 0°	$L/5$, 90°	
	DH unit, piston					*DHI unit, piston*				
CNR-IMGC	49	50	83	–5	47	50	120	42	126	154
BNM-LNE	–134	–140	–189	–148	47	–141	–143	–147	–121	52
NIST	132	116	177	145	47	156	64	182	135	52
NPL	–112	–79	–125	–48	65	–118	–38	–146	–182	60
PTB	65	52	55	55	34	72	16	89	59	42
$(D_{\max} - D_{\min})$/nm	266	256	366	293		297	263	329	317	
	DH unit, cylinder					*DHI unit, cylinder*				
CNR-IMGC	16	12	48	28	46	9	–46	–7	–38	103
BNM-LNE	–59	–98	–34	–50	46	–104	–143	–138	–62	48
NIST	2	26	–18	15	52	45	53	28	44	54
NPL	5	67	–16	2	55	–60	–40	–46	–27	56
PTB	38	–9	19	5	34	91	108	94	67	36
$(D_{\max} - D_{\min})$/nm	97	165	82	78		195	251	232	129	

Table 2. Relative deviations of particular effective areas of the DH and DHI transfer standards from the reference values and relative combined standard uncertainties of the effective areas reported. The values in bold are the final results of the participants. The results were obtained with the following models and/or data: (1) diameters measured directly; (2) selected diameters along generatrices; (3) diameters along generatrices and circles; (4) linear pressure distribution in p-c clearance; (5) zero gauge pressure; (6) diameters along generatrices; (7) diameters along circles; (8) radii along generatrices and circles.

Institute	$10^6 \times$ Participants' results				$10^6 \times$ PTB calculation from participants' data				$10^6 \times (A_{0\max} - A_{0\min})/A_0$	
	Average from diameters		Dadson's theory		Average from diameters		Dadson's theory			
	DH	DHI	DH	DHI	DH	DHI	DH	DHI	DH	DHI
CNR-IMGC	3.3 ± 6.2$^{(1)}$ 0.6 ± 10.7$^{(2)}$ 2.6 ± 10.8$^{(3)}$	3.1 ± 11.2$^{(1)}$ 0.1 ± 8.5$^{(2)}$ 1.1 ± 8.6$^{(3)}$	6.7 ± 10.5$^{(4),(8)}$ **4.5 ± 10.5**$^{(5),(8)}$	1.4 ± 11.1$^{(4),(8)}$ **0.6 ± 11.1**$^{(5),(8)}$	0.4 ± 4.1$^{(1)}$	–0.6 ± 7.7$^{(1)}$	2.8 ± 5.6$^{(3)}$ 3.0 ± 8.6$^{(8)}$	–0.5 ± 13.5$^{(3)}$ –1.8 ± 14.0$^{(8)}$	6.3	4.9
BNM-LNE	**–3.4 ± 2.4**$^{(6)}$ –3.9 ± 2.8$^{(7)}$	**–4.3 ± 2.3**$^{(6)}$ –5.7 ± 4.1$^{(7)}$	**–4.6 ± 2.2**$^{(3)}$	**–2.3 ± 2.2**$^{(3)}$	–7.6 ± 3.7$^{(1)}$	–8.7 ± 4.7$^{(1)}$	–5.5 ± 3.5$^{(3)}$	–6.0 ± 4.8$^{(3)}$	4.2	6.4
NIST	2.7 ± 2.8$^{(1)}$ 4.9$^{(3)}$	3.0 ± 3.0$^{(1)}$ 6.9$^{(3)}$	**1.1 ± 4.4**$^{(8)}$	**6.1 ± 4.5**$^{(8)}$	2.6 ± 3.6$^{(1)}$	2.6 ± 5.2$^{(1)}$		5.5 ± 4.4$^{(3)}$ 5.4 ± 4.6$^{(8)}$	3.8	4.3
NPL					–3.7 ± 3.4$^{(1)}$	–7.0 ± 4.6$^{(1)}$				
PTB	0.4 ± 3.7$^{(1)}$	1.9 ± 4.2$^{(1)}$	2.2 ± 3.9$^{(3)}$ **1.6 ± 3.7**$^{(8)}$	5.8 ± 4.4$^{(3)}$ **4.6 ± 4.1**$^{(8)}$	0.4 ± 3.7$^{(1)}$	1.9 ± 4.2$^{(1)}$	2.2 ± 3.9$^{(3)}$ 1.6 ± 3.7$^{(8)}$	5.8 ± 4.4$^{(3)}$ 4.6 ± 4.1$^{(8)}$	1.8	3.9
$10^6 \times (A_{0\max} - A_{0\min})/A_0$	8.8	12.6	11.3	8.4	10.2	11.3	8.5	11.8		

The effective areas of the DH and DHI assemblies calculated by the participants using different methods agree within 8.4 parts in 10^6 to 12.6 parts in 10^6. When the effective areas are calculated at the PTB from dimensional data of the participants applying the same evaluation procedure, the maximum relative differences between the results are very similar: 8.5 parts in 10^6 to 11.8 parts in 10^6. This means that the observed discrepancies are caused by differences in the dimensional measurement data and not in the calculation methods.

The last two columns of Table 2 contain maximum relative differences between the effective areas calculated by different methods from geometrical data of the same institute for the DH and DHI units. It can easily be shown that these relatively large differences, of up to 6.3 parts in 10^6, are not attributable to the methods (i.e. diameter averaging versus Dadson's theory), but rather to the very different amounts of input information used (for example, measurement of only four diameters or description of the whole assembly shape).

In Figure 1, relative deviations of the calculated effective areas from the reference values are shown for the DH and DHI transfer standards, with vertical bars indicating the standard uncertainties of these deviations. Although a few participants' diameters disagree (Table 1) and the diameter uncertainties are dominant in the A_0 uncertainty budget, most of the A_0 results deviate from the reference value by no more than the standard uncertainties of the deviations; all of them would agree with the reference value within the expanded uncertainties with a coverage factor $k = 2$. This is because the disagreement in some diameters is compensated by others being in agreement and also because the A_0 uncertainty includes additional contributions, mainly arising from uncertain straightness and roundness measurements, and the dependence of A_0 on pressure and the angular position of the piston in the cylinder.

The results obtained by each laboratory with the two transfer standards are reproducible within about 5 parts in 10^6. The two dashed lines in Figure 1 indicate the positions of reference values for the effective areas determined from the cross-float experiments with national pressure standards in phase A2 [2]. The reference values in phase A2 are greater than those in phase A1 by 3.1 parts in 10^6 and 4.1 parts in 10^6 for the DH and DHI transfer standards, respectively. This is to some extent because the NPL took part in phase A1 but not in phase A2. If the reference values of phase A1 had been calculated without the NPL results, the relative differences between $A_{0\text{ref}}$ (phase A2) and $A_{0\text{ref}}$ (phase A1) would have been 2.2 parts in 10^6 and 2.4 parts in 10^6 for the DH and DHI transfer standards, respectively. These differences are smaller than their standard uncertainties.

Table 3 presents the relative differences between the effective areas of each pair of laboratories and relative standard uncertainties of the differences. For the DH piston-cylinder assembly, all the results with the exception of those from the BNM-LNE and the PTB agree with one another within their standard uncertainties. All results agree within the expanded uncertainties. For the DHI piston-cylinder assembly, results in the pairs BNM-LNE/NIST and NPL/NIST

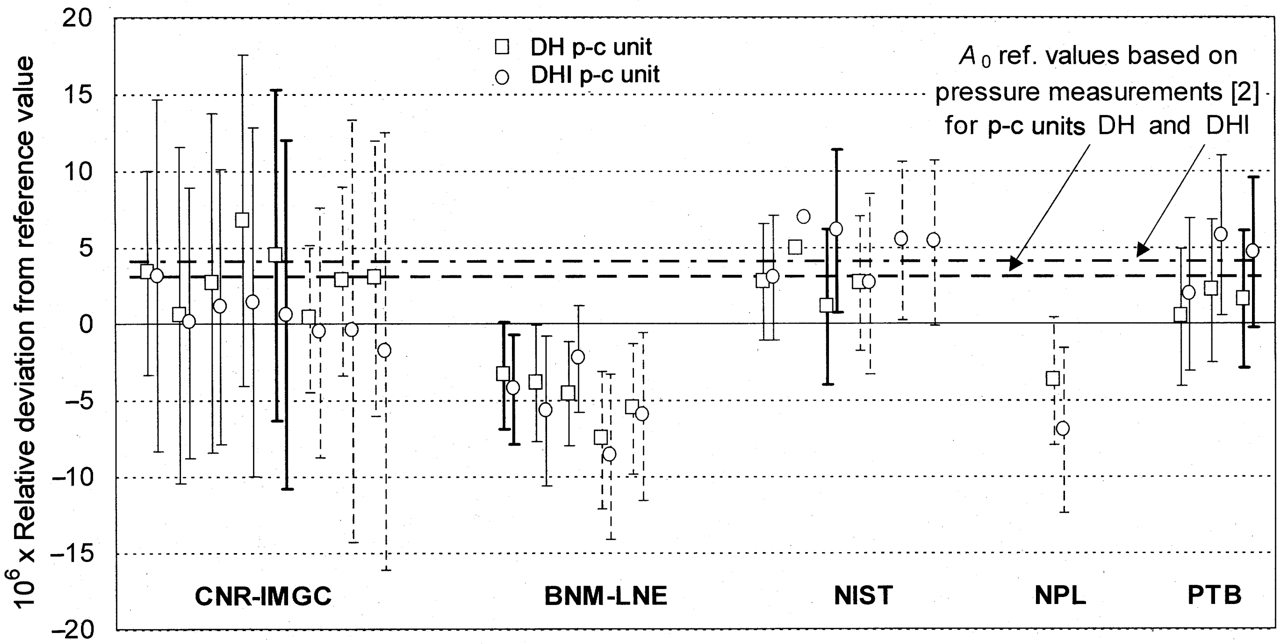

Figure 1. Relative deviations of effective areas from reference values with standard combined uncertainties of these deviations (vertical bars). Particular points to note are: thin bars – values calculated by the participants using different methods; thick bars – participants' final results; dashed bars – calculated by the PTB from dimensional data supplied by the participants.

differ by more than the expanded uncertainties. In addition, results in the pairs BNM-LNE/PTB and NPL/PTB differ by more than the standard uncertainties.

Table 3. Relative differences of the effective areas of the DH and DHI transfer standards determined by two laboratories (upper values) and their relative standard uncertainties (lower values). Values referred to as NPL are calculated from the NPL diameters.

10^6 × Rel. diff. in A_0		DHI				
10^6 × Rel. std uncertainty		CNR-IMGC	BNM-LNE	NIST	NPL	PTB
	CNR-IMGC		4.8 11.3	–5.5 12.0	7.5 12.0	–4.1 11.9
	BNM-LNE	–7.9 10.8		–10.4 5.1	2.7 5.1	–8.9 4.8
DH	NIST	–3.4 11.4	4.5 5.0		13.1 6.4	1.4 6.2
	NPL	–8.2 11.2	–0.3 4.5	–4.8 5.8		–11.6 6.2
	PTB	–2.9 11.1	4.9 4.4	0.5 5.7	5.3 5.3	

5. Stability of transfer standards

For the DHI transfer standard, the maximum changes in diameter of piston and cylinder bore measured by the BNM-LNE at the beginning and end of the comparison are 56 nm and 111 nm, respectively. These are smaller than the expanded uncertainty $(k = 2)$ of their determination, 114 nm. The relative change in effective area calculated by the BNM-LNE, -2.2×10^{-6}, is even smaller than the standard uncertainty of its determination, 3.2×10^{-6}, so this transfer standard may be considered to be stable.

For the DH transfer standard studied by the PTB at the beginning and end of the comparison, the cylinder was found to be stable: the maximum diameter change of 31 nm is smaller than its standard uncertainty, 35 nm. However, the dimensional properties of the piston changed during the comparison. The roundness measurement results of the PTB and the other participants showed that the maximum deviations from roundness in horizontal sections close to the piston cap continuously increased during the comparison. For the section $z = 2L/5$, the maximum deviation from roundness increased from 36 nm in 1995 to 179 nm in 1997. Nevertheless, as the changes in diameter in vertical sections $\varphi = 0°$ and 90° had almost the same magnitude but different signs, $\Delta D(-L/5) = 41$ nm and –41 nm and $\Delta D(L/5) = 84$ nm and –98 nm at 0° and 90°, respectively, the effective area changed by only 0.3 parts in 10^6. This change is substantially smaller than its standard uncertainty, 5 parts in 10^6, and thus the participants' results for effective area may be considered comparable.

6. Conclusions

- The effective areas determined by the participants from dimensional data agree within two standard uncertainties with the reference values determined for the DH and DHI transfer standards. However, if the participants' results are compared with one another, there are two cases where they differ by more than their expanded uncertainties with a coverage factor $k = 2$.

- The reference effective areas determined from the dimensional measurements are smaller than those found in cross-float experiments with national pressure standards in phase A2 of the comparison [2], by 3.1 parts in 10^6 and 4.1 parts in 10^6 for the DH and DHI transfer standards, respectively.
- Twenty-six of the eighty diameters reported differ from their respective reference values by more than two standard uncertainties, clearly showing the need for harmonization of diametric measurements between national metrological laboratories.
- The deviations of participants' effective areas from the reference values obtained for the two transfer standards are reproducible within 5 parts in 10^6.
- For the effective area calculated, dimensional data, especially diameters, are dominant. The choice of calculation method does not seem to be very important.

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