Physics of the Δ(1232) resonance in chiral EFT vs dynamical models

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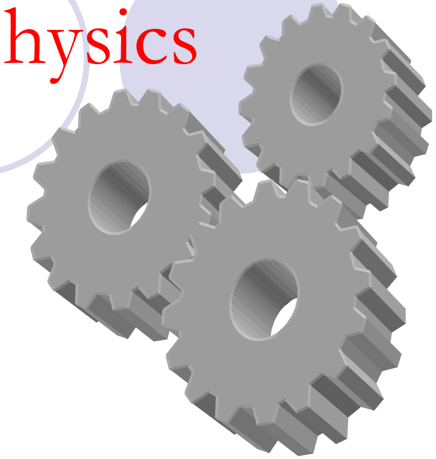
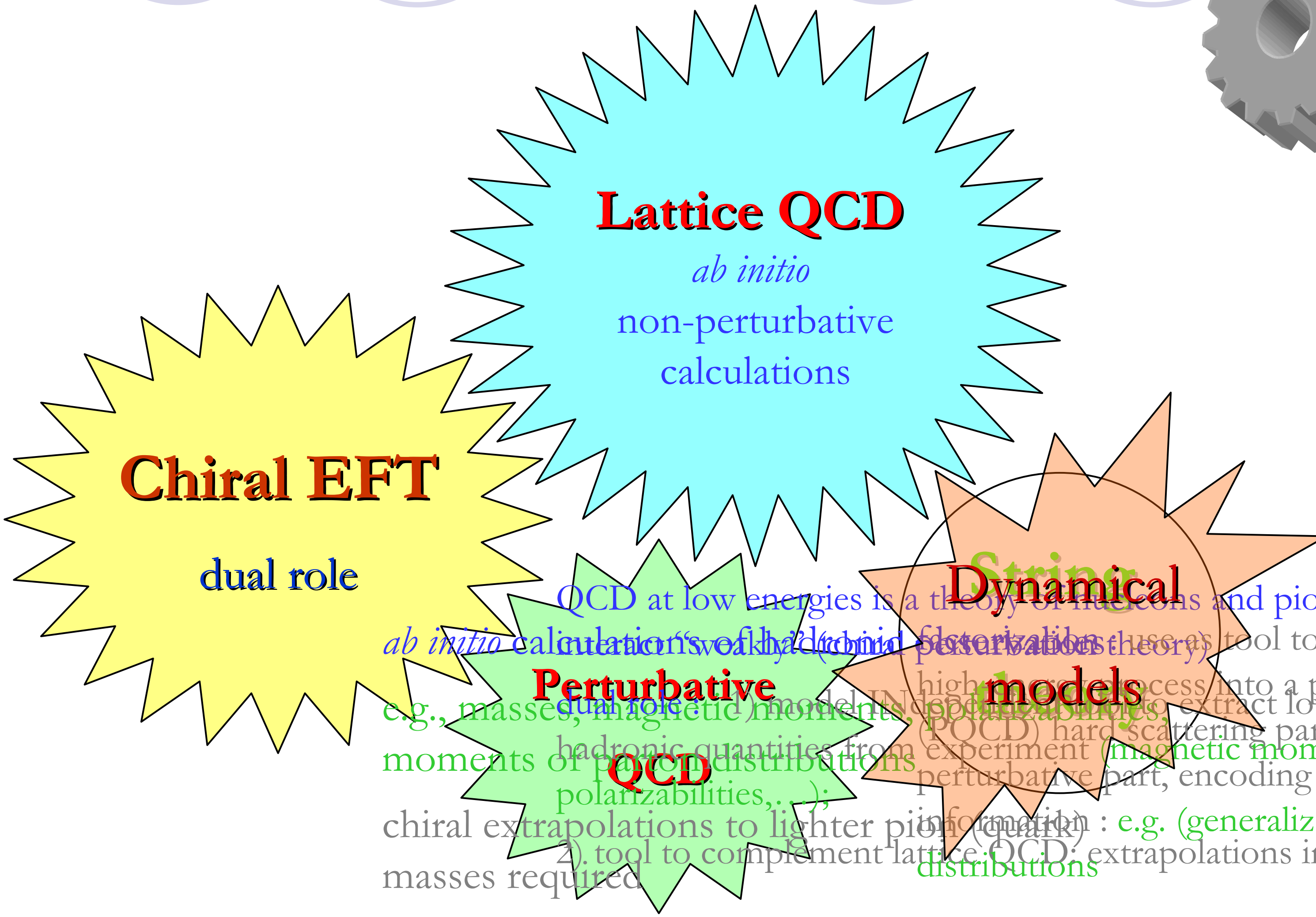
* *Supported by the U.S. Department of Energy*

@ "New Theoretical Tools for Resonance Analysis", Argonne, Aug 29 - Sep 2, 2005

Major theoretical tools in hadronic physics





Topics

❖ Relativistic spin-3/2 field:

[VP, PRD 58 (1998); VP & **Timmermans**, PRC 70 (1999); **Deser**, VP & **Waldron**, PRD 60 (2000), …]

❖ Power counting for the Δ resonance in chiral EFT (ChEFT):

[VP & **Phillips**, PRC 67 (2003); ibid. 68 (2003)].

❖ Chiral extrapolations in relativistic baryon ChPT

[VP, **Holstein** & **Vanderhaeghen**, PLB (2004); PRD (2005)]

❖ Pion electroproduction and γ N Δ form factors in ChEFT:

[VP & **Vanderhaeghen**, hep-ph/0508060].

❖ Radiative pion photoproduction and Δ's magnetic moment in ChEFT:

[VP & **Vanderhaeghen**, PRL 96 (2005)].

❖ OU dynamical model for pion photo- and electroproduction on the nucleon:

[VP & **Tjon**, PRC 70 (2004); **Caia**, VP, **Tjon** & **Wright**, PRC 70 (2004), …].

❖ 2-photon effects in γ N -> Δ transition: [**Carlson**, VP & **Vanderhaeghen**, in preparation].

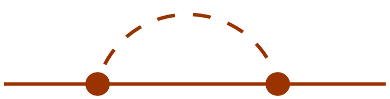
Chiral Effective Field Theory

Interaction of Goldstone bosons (of *spontaneous chiral symmetry breaking* in QCD), e.g. pions, is proportional to their 4-momentum, therefore is weak for small (relative to $\Lambda_{\chi SB}$) momenta: V » $(g_A/f_\pi)\, p$

One can set up a systematic expansion in powers of p/Λ – chiral PT.

Renormalization is done in the EFT sense [Weinberg, Physica A (1979)] – most general Lagrangian consistent with all the symmetries, etc.

Example: $$\mathcal{L}^{(1)}_{\pi N} = \frac{g_A}{2f_\pi}\bar{N}\,\gamma^\mu\gamma^5\,\tau^a\,N\,(\partial_\mu\pi^a)$$



$= O(p^3)$

$$M_N = M_{N0} + c_2 m_\pi^2 + \left(\frac{g_A}{4\pi f_\pi}\right)^2 [M_N m_\pi^2 + \chi m_\pi^3 + \ldots]$$

renormalization

after (on-mass-shell !) renormalization [Gegelia *et al.* (1999)],

i.e., *not* MS-bar as in [Gasser *et al.* (1989)].

Power counting for the Δ(1232) (δ-expansion)

- ✓ The excitation energy of the Δ resonance, $\Delta = M_\Delta - M_N$¼290 MeV resonance can also be treated as small: $\delta = \Delta/\Lambda_\chi$¿ 1.
- ✓ Feature of the "δ-expansion": scale hierarchy m_π¿ Δ ¿ Λ_χ. => δ=Δ/Λ , $m_\pi/\Lambda = \delta^2$
.
- ✓ This distinguishes the *low-energy* (p » m_π) and the *resonance* (p » Δ) regions.
- ✓ Crucial for *correct* counting of the One-Δ-reducible (OΔR) graphs:

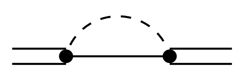
OΔR propagator

$$\frac{1}{p-\Delta}$$

»

p » m_π, $1/\Delta = O(1/\delta)$ [c.f., S_N» $1/p = O(1/\delta^2)$]

p » Δ , $1/(p\text{-}\Delta\text{-}\Sigma) = O(1/\delta^3)$

$\Sigma =$  + … = $O(p^3) = O(\delta^3)$

$$\mathcal{L} = \mathcal{L}_{\chi\mathrm{PT}} + \mathcal{L}_\Delta$$

$$\mathcal{L}_\Delta^{(1)} = \overline{\psi}_\mu \left(i\gamma^{\mu\nu\alpha} D_\alpha - M_\Delta \gamma^{\mu\nu} \right) \psi_\nu + \frac{ih_A}{2f_\pi M_\Delta} \left\{ \overline{N}\, T_a\, \gamma^{\mu\nu\lambda} (\partial_\mu \psi_\nu) D_\lambda \pi^a + \mathrm{H.c.} \right\}$$

$$\mathcal{L}_\Delta^{(2)} = \frac{e\mu_\Delta^{(0)}}{2M_\Delta} \overline{\psi}_\mu \psi_\nu F^{\mu\nu} + \frac{3\,i\,e\,g_M}{2M(M+M_\Delta)} \left\{ \overline{N}\, T_3\, \partial_\mu \psi_\nu \tilde{F}^{\mu\nu} + \mathrm{H.c.} \right\} + \frac{eh_A}{2f_\pi M_\Delta} \left\{ \overline{N}\, T_a^\dagger \gamma^{\mu\nu\lambda} A_\mu \psi_\nu\, \partial_\lambda \pi^a + \mathrm{H.c.} \right\}$$

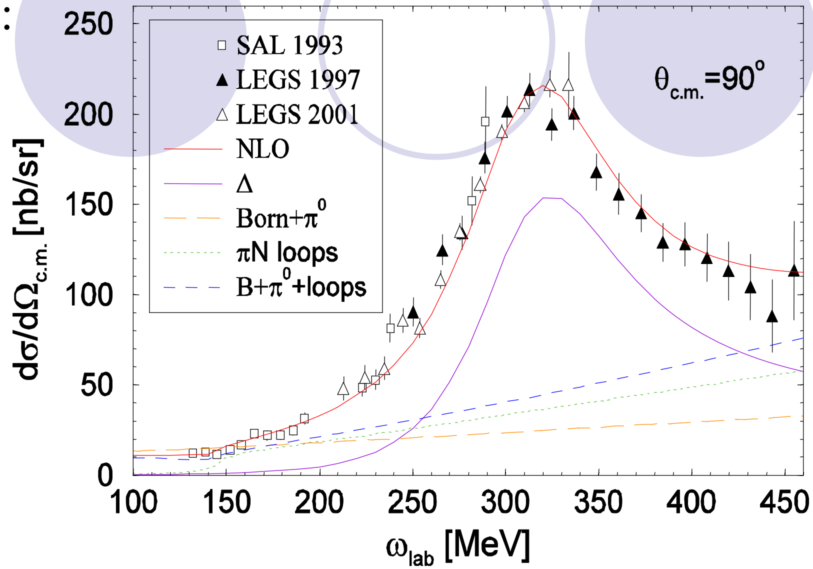
$$\mathcal{L}_\Delta^{(3)} = \frac{-3\,e\,g_E}{2M(M+M_\Delta)} \left\{ \overline{N}\, T_3\, \gamma_5\, \partial_\mu \psi_\nu F^{\mu\nu} + \mathrm{H.c.} \right\},$$

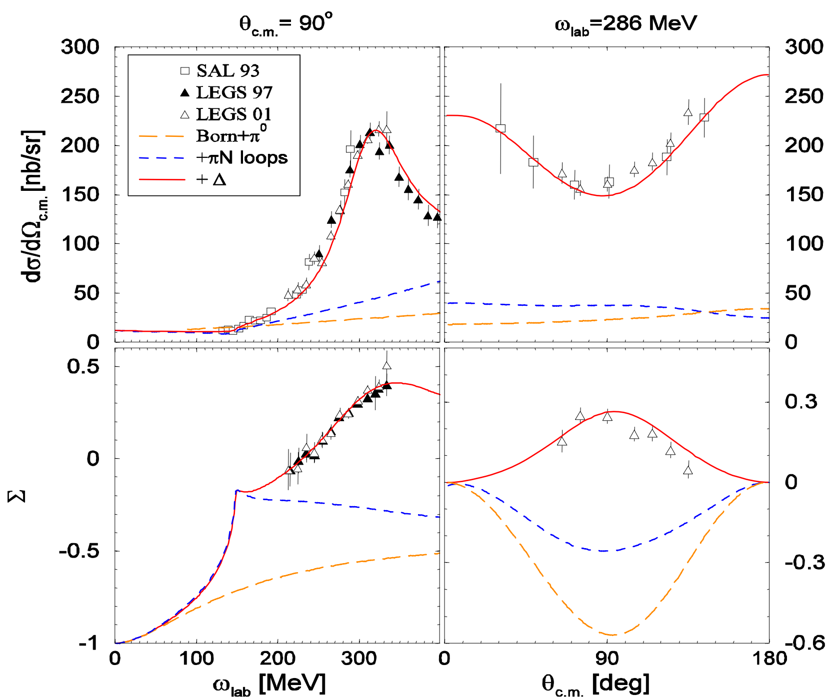
where $D_\mu = \partial_\mu + ieA_\mu$,
$\gamma^{\mu\nu\alpha} = \frac{1}{2}(\gamma^\mu\gamma^\nu\gamma^\alpha - \gamma^\alpha\gamma^\nu\gamma^\mu)$, $\gamma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$.

$$\psi_\mu(x) \rightarrow \psi_\mu(x) + \partial_\mu \varepsilon(x)$$

Ensures the decoupling of the spin-1/2 components of the Rarita-Schwinger field -- correct spin d.o.f. content of the theory.

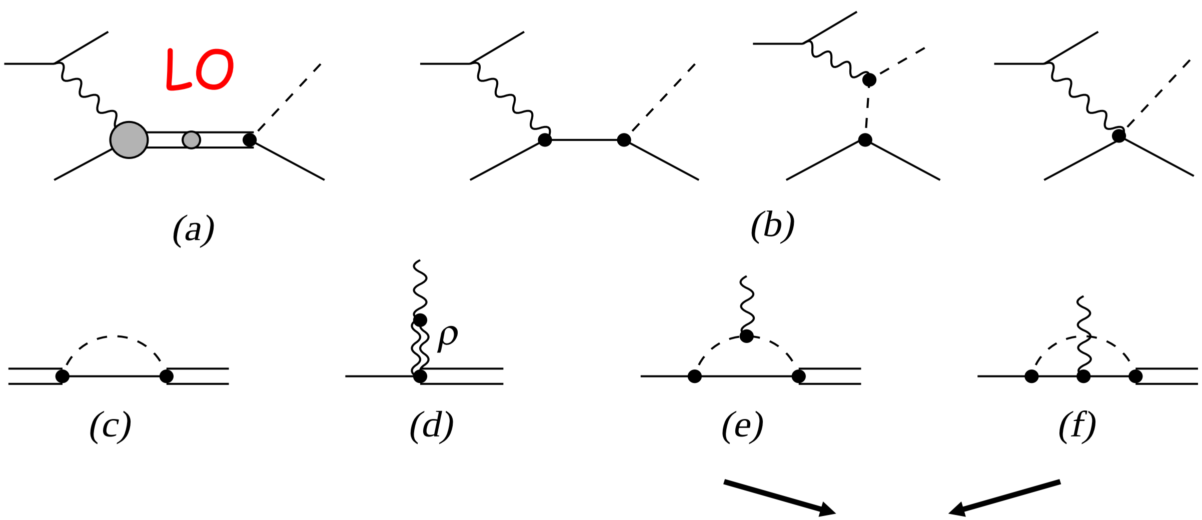
Compton scattering observables to NLO in δ-expansion:





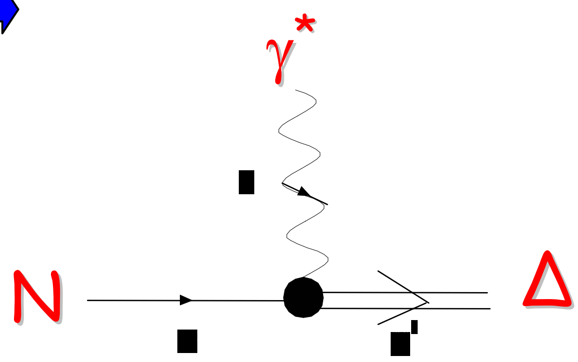
Chiral effective field theory calculation of the $e\ p \to e\ p\ \pi^0$ process in $\Delta(1232)$ region

➡ calculation to NLO in δ expansion (powers of δ) [VP & Vdh, hep-ph/0508060]



vertex corrections : unitarity & gauge invariance exact to NLO

$\gamma^* N \Delta$ vertex



3 electromagnetic transitions :

M1 -> G_M^*

E2 -> G_E^*

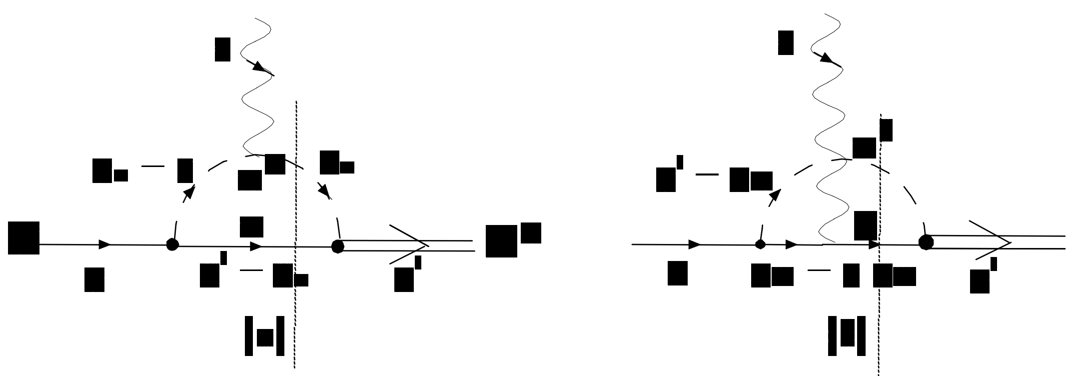
C2 -> G_C^*

$$R_{EM} \equiv -\frac{G_E^*}{G_M^*}$$

$$R_{SM} \equiv -\frac{|\vec{q}|}{2\,M_\Delta}\,\frac{G_C^*}{G_M^*}$$

q : photon momentum in Δ rest frame

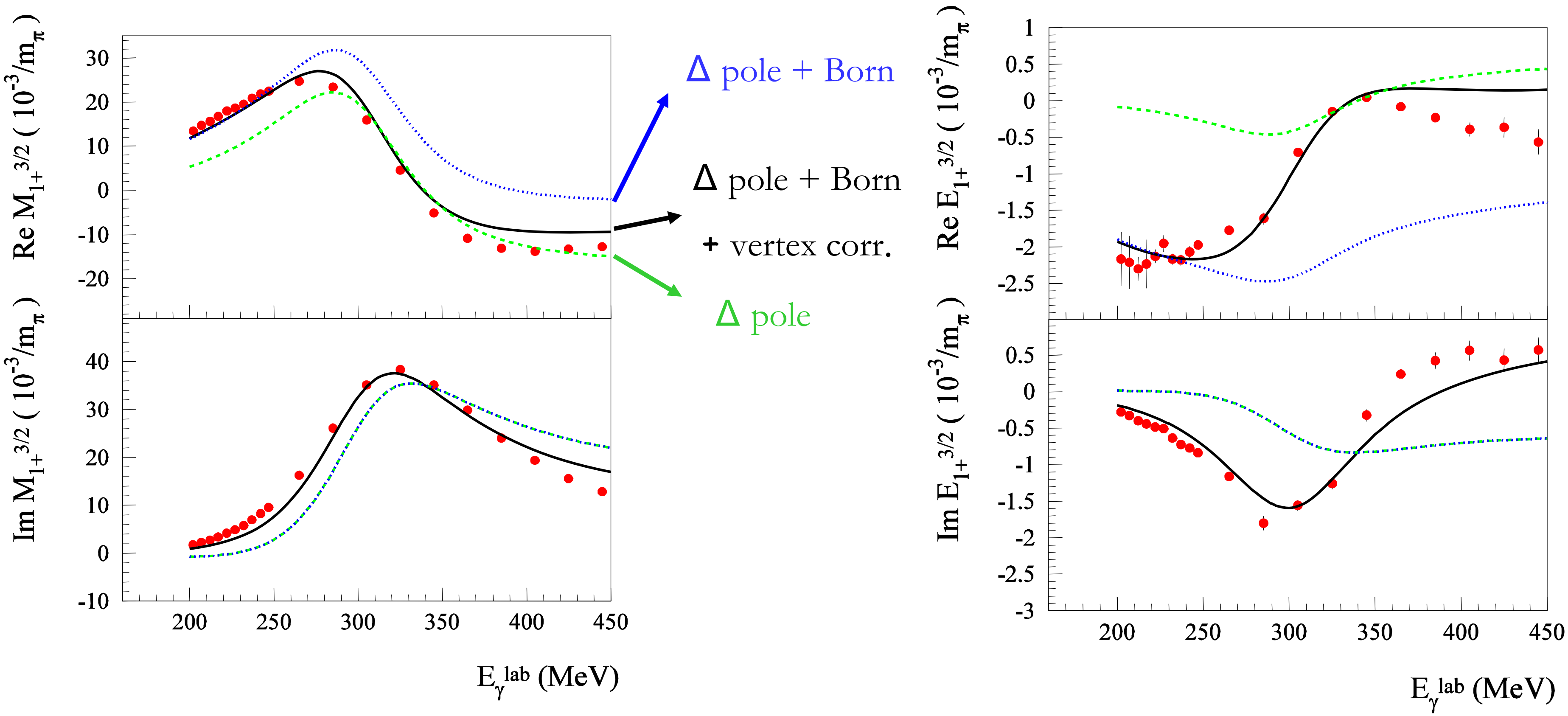
vertex corrections : fully relativistic loop calculation : equivalent to a sideways dispersion relation



imaginary part is model independent prediction in chiral EFT framework

Photoproduction: magnetic (M1) & electric (E2) N – Δ transition (resonant multipoles)

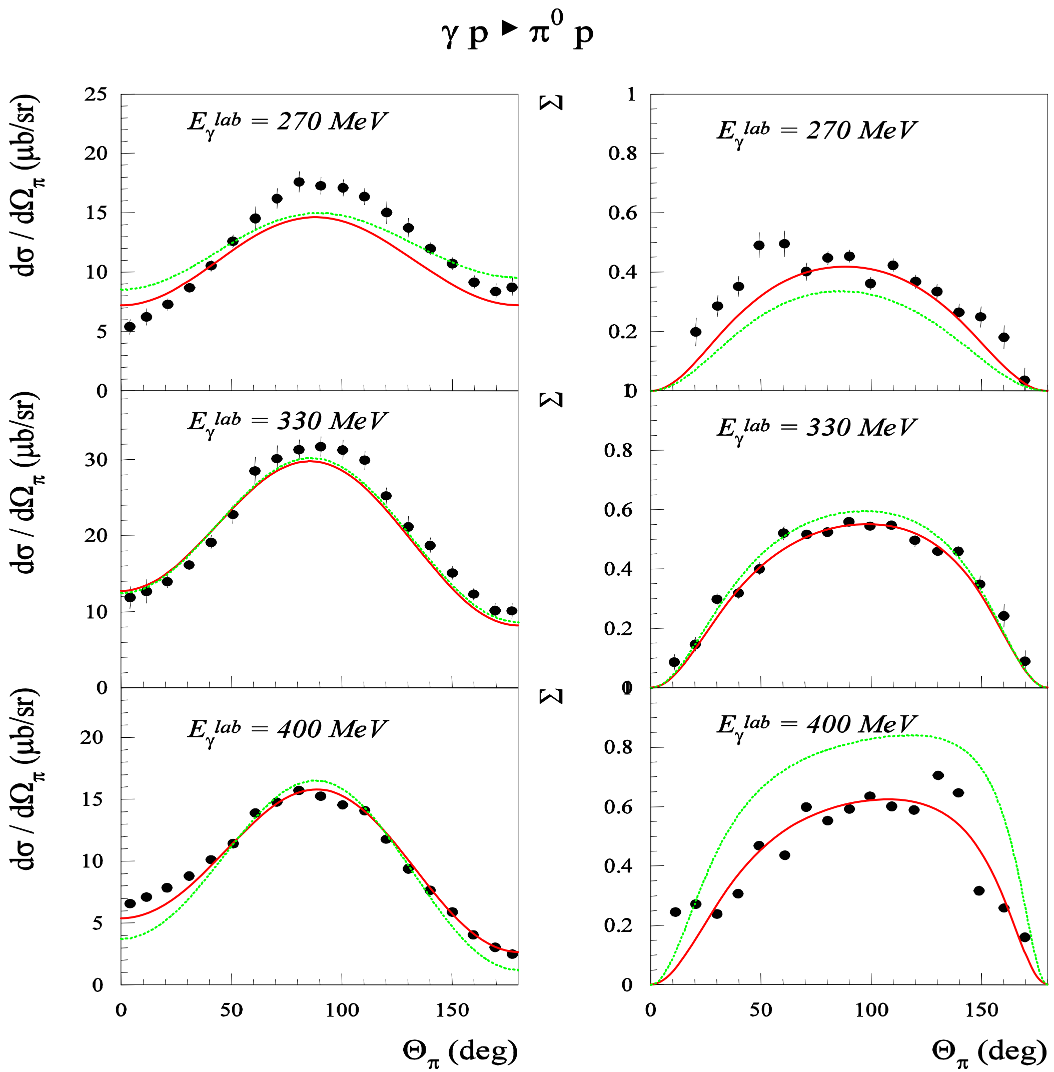
2 free parameters to NLO



G^*_M = 2.95

G^*_E = 0.07 (E2/M1 = -2.4 %)

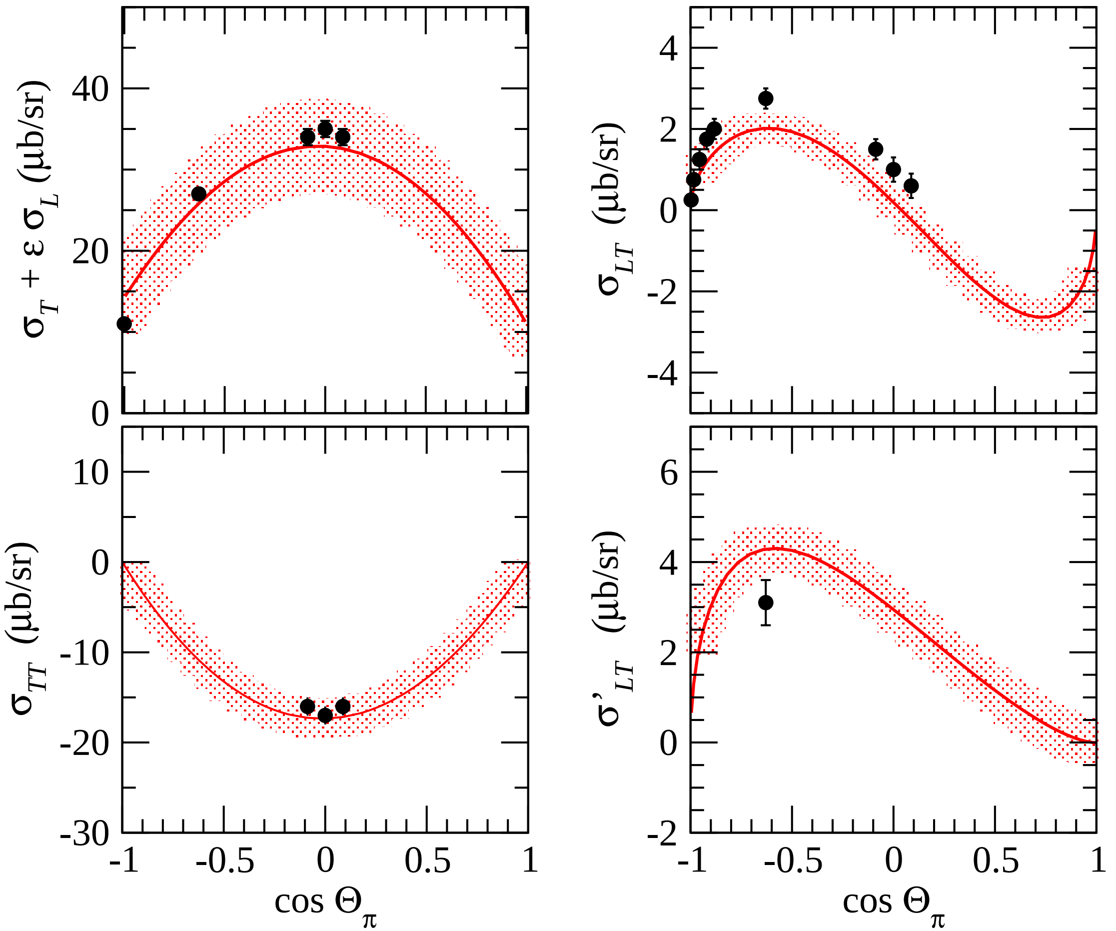
Photoproduction: magnetic (M1) & electric (E2) N – Δ transition (observables)



NLO calculation

DMT model

ables



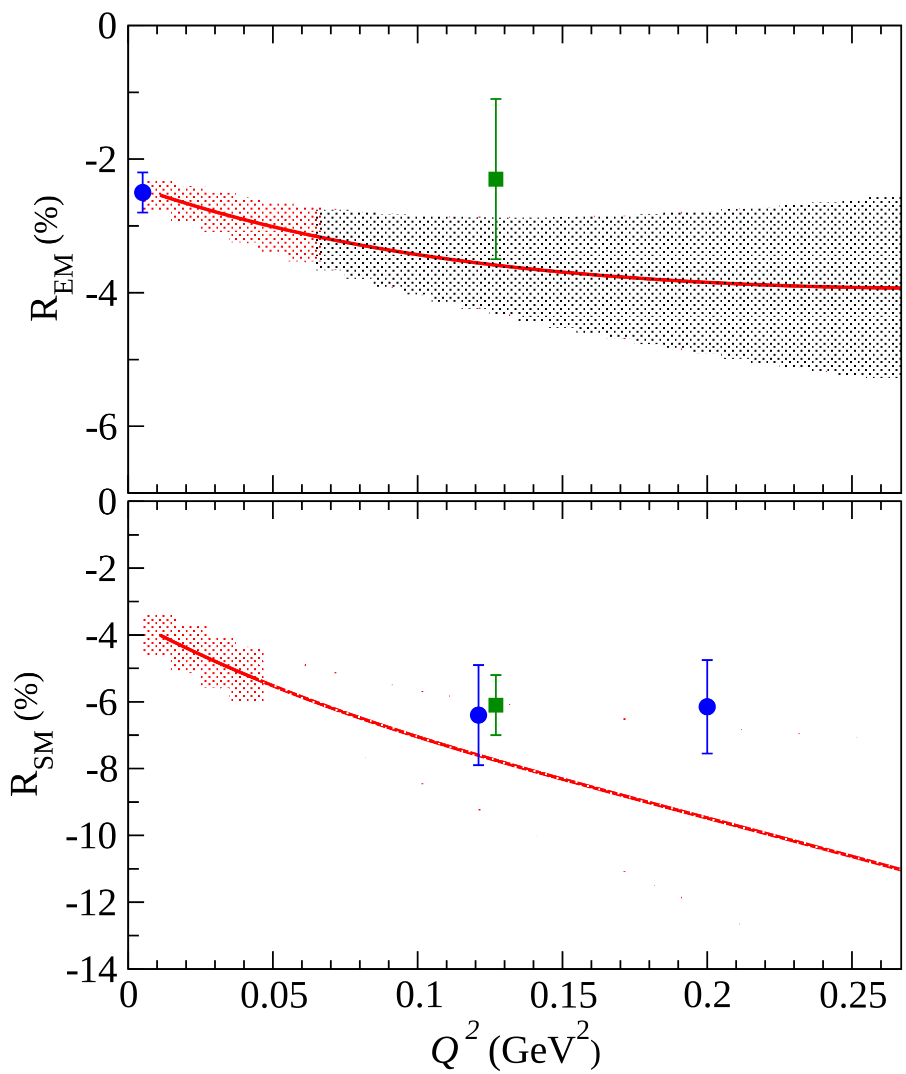
EFT calculation

error bands due to

NNLO, estimated as:

$\Delta\sigma \sim 2|\sigma|\ \delta^2$

I1 ratios



data points :

MIT-Bates

(Sparveris et al., 2005)

MAMI :

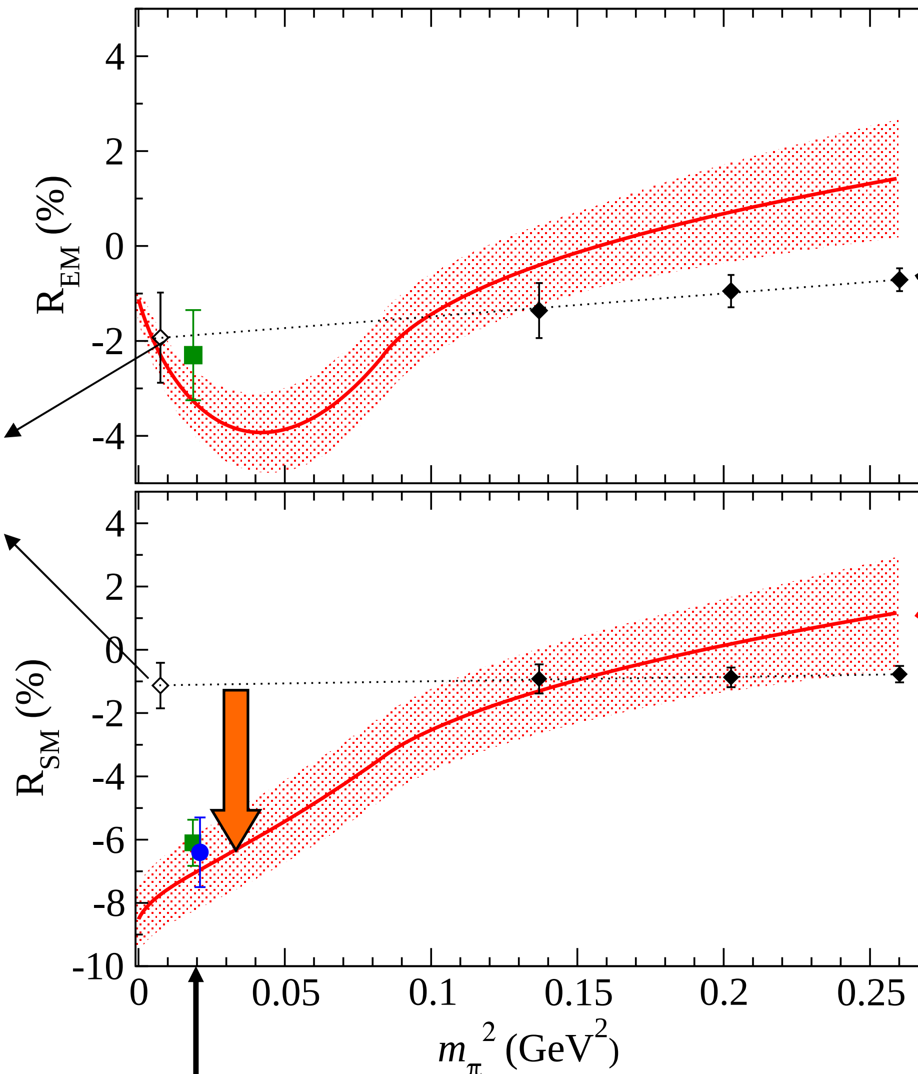
eck et al., 2000), R_{SM} (Pospischil
: al., 2001; Elsner et al., 2005)

EFT calculation

error bands due to NNLO

n

linear extrapolatio
in $m_q \sim m_\pi^2$



ıched lattice
QCD results :

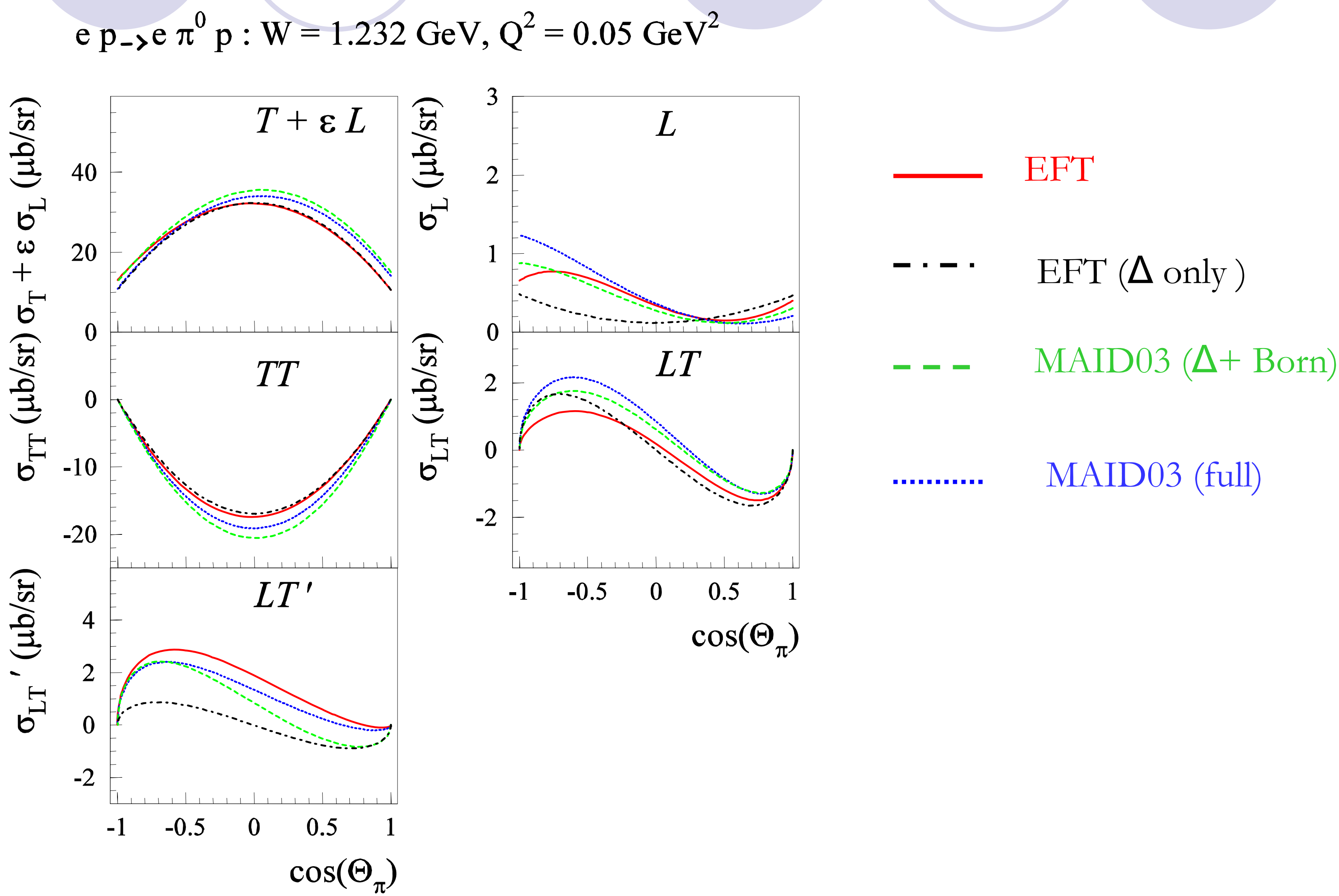
: 0.37, 0.45, 0.51
GeV

ia – MIT group
xandrou et al., PRL
94 (2005)

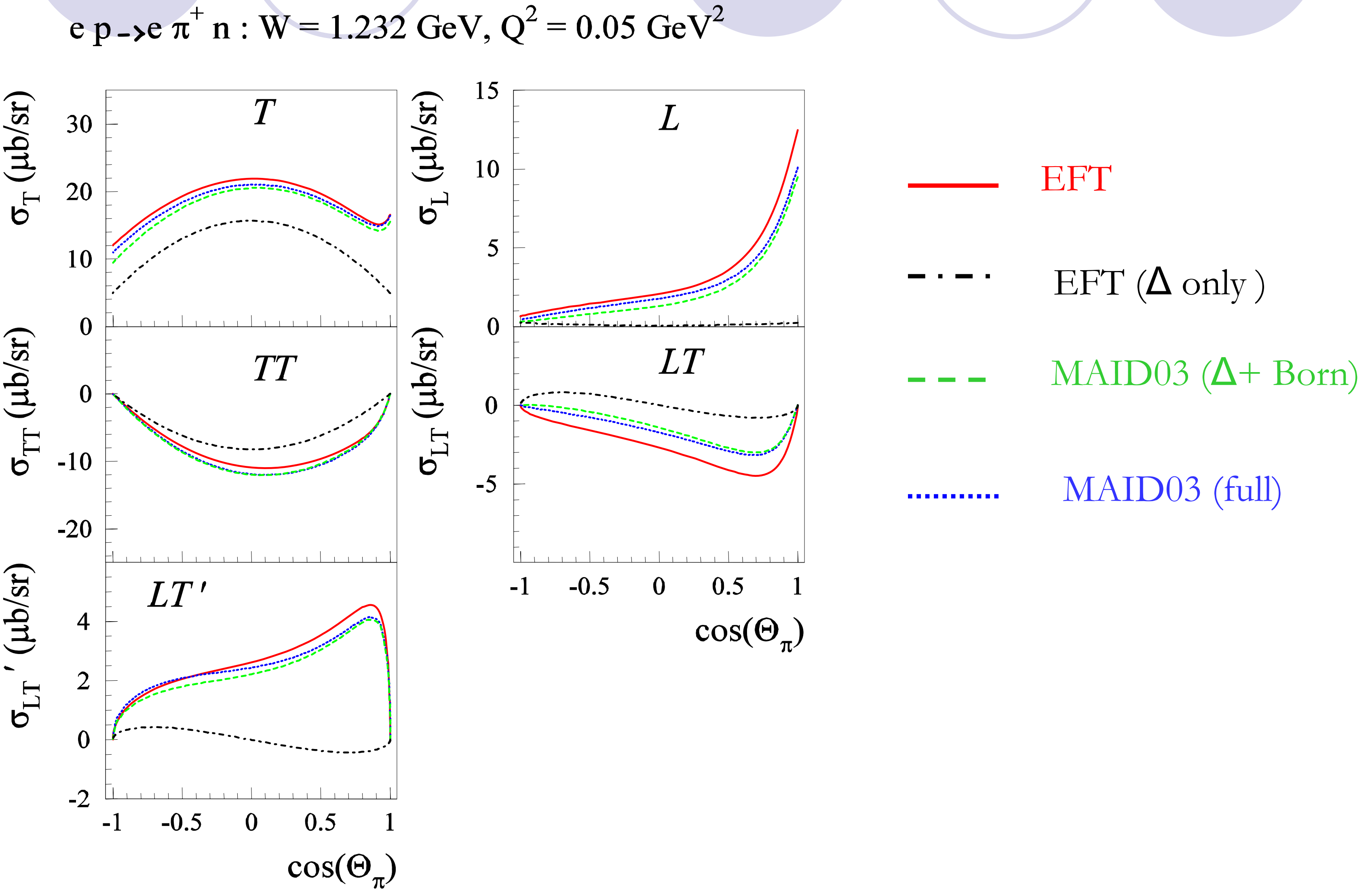
alculation

discrepancy with
lattice explain
by chiral loop
(pion cloud)

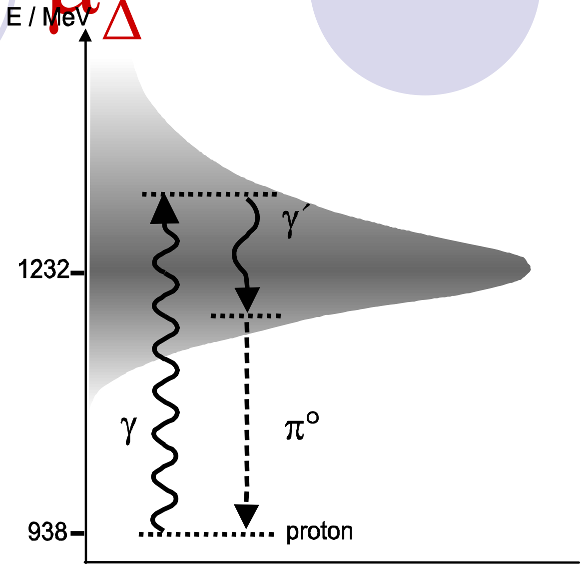
e p -> e p π^0 in $\Delta(1232)$ region : observables



e p -> e n π^+ in $\Delta(1232)$ region : observables



Motivation for μ_{Δ}



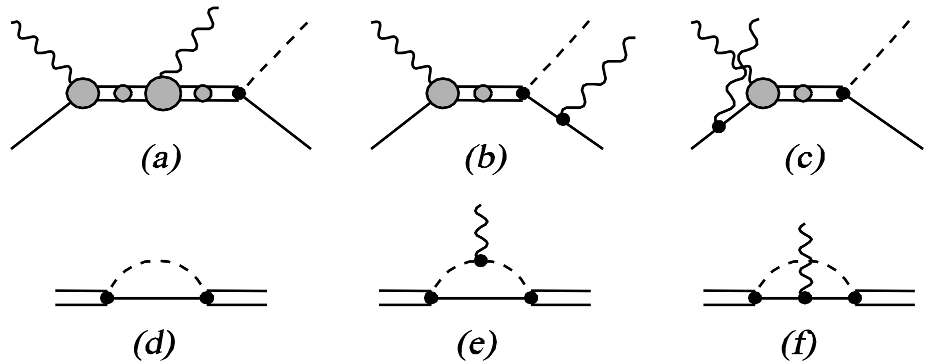
- Observation of the magnetic moment of a strongly unstable particle.
- Δ-resonance is the best-studied example of such a particle.
- Excitation energy of the Δ-resonance, M_{Δ}-M_{N}´ Δ ' 300 MeV is relatively low, which allows to treat it as a low-energy scale in an EFT expansion, the Δ can be incorporated in ChPT.

$\gamma p \longrightarrow \gamma\, \pi^0\, p$: ChEFT calculation

Power counting: $\delta=(M_\Delta - M_N)/\Lambda_{\chi\,PT}$, $m_\pi/\Lambda_{\chi\,PT}$ » δ^2

To next-to-leading order in the resonance region

(counting ω'» m_π , ω» M_Δ-M_N)



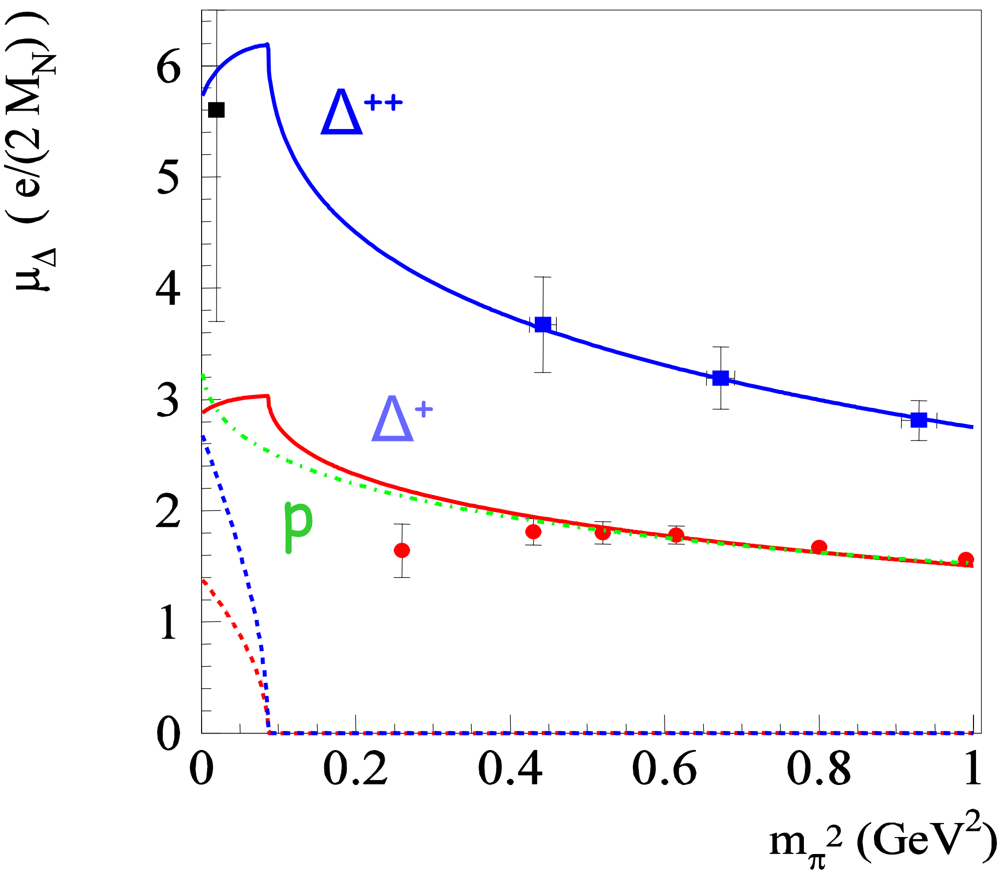
$$S_{\mu\nu}(p) = \frac{-\mathcal{P}^{(3/2)}_{\mu\nu}(p)}{(p\cdot\gamma - M_\Delta)[1 - i\mathrm{Im}\,\Sigma'(M_\Delta)] - i\mathrm{Im}\,\Sigma(M_\Delta)},$$

$$\bar{u}_\alpha(p')\,\Gamma^{\mu\alpha\beta}_{\gamma\Delta\Delta}(p',p)\,u_\beta(p)\,\epsilon_\mu$$
$$= e\,\bar{u}_\alpha(p')\left[\epsilon\cdot\gamma\,F(q^2) + \frac{(p'+p)\cdot\epsilon}{2M_\Delta}\,G(q^2)\right]u^\alpha(p)\,,\quad \mu_\Delta = F(0),\ Z_\Delta = 1 - \Sigma'(M_\Delta)$$

The Ward-Takahashi identity,

$$q_\mu\Gamma^{\mu\alpha\beta}(p',p) = e\left[(S^{-1})^{\alpha\beta}(p') - (S^{-1})^{\alpha\beta}(p)\right],\ \text{demands}\ F(0)+G(0) = 1-\Sigma'(M_\Delta).$$

Chiral behavior of the Δ^{++} and Δ^{+} magnetic moments



Lattice data points from

[1] D.B. Leinweber, Phys. Rev. D (1992); I.C. Cloet, D.B. Leinweber and A.W.Thomas, Phys. Lett. B563 (2003).
[2] F.X. Lee *et.al.*, hep-lat/0410037

Real parts

Imag. parts

$\gamma p \longrightarrow \gamma \pi^0 p$: observables

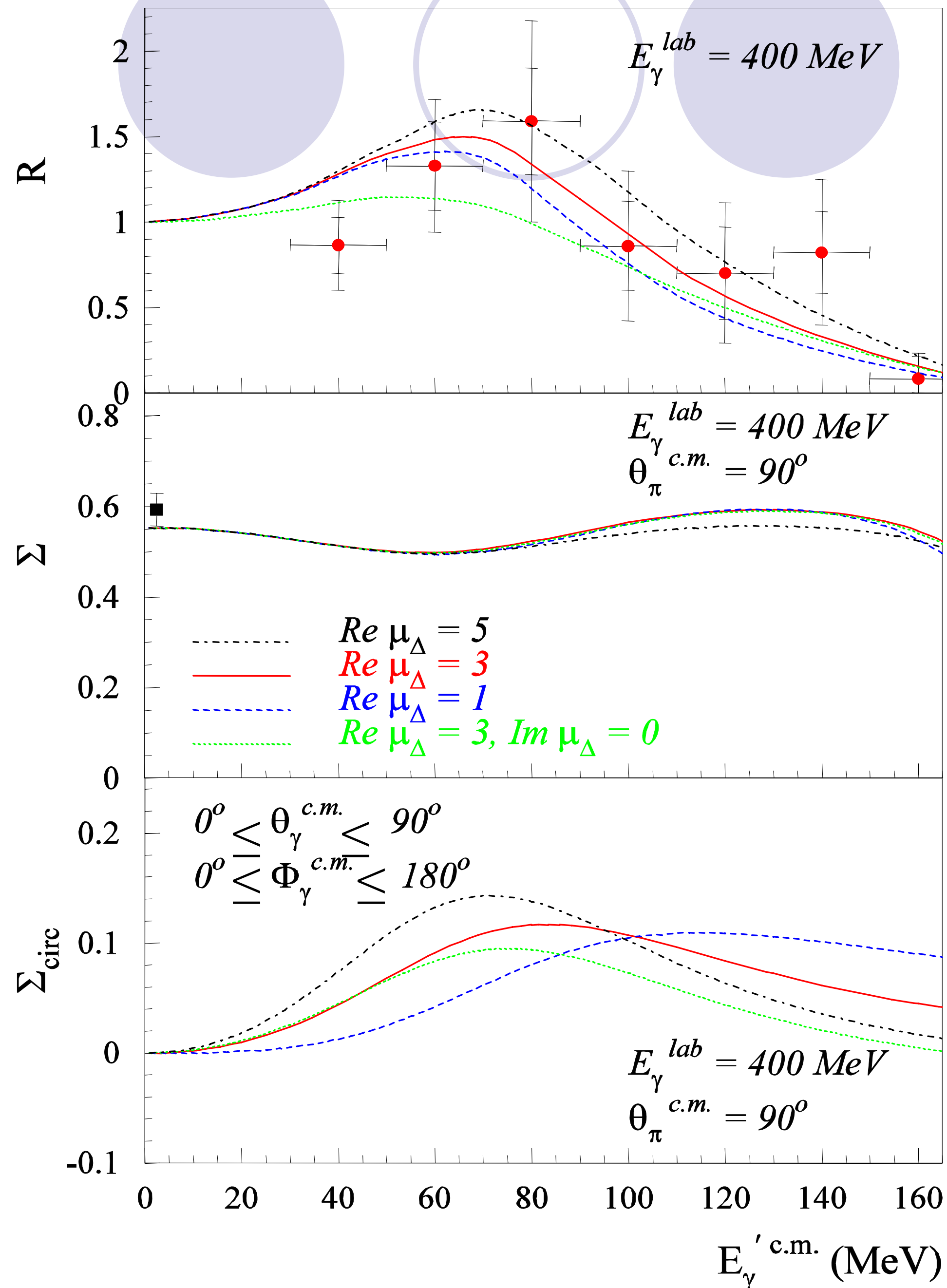
Exp. data points from TAPS@MAMI 2002

1. Ratio of the angle-integrated cross-section to the soft-photon limit:

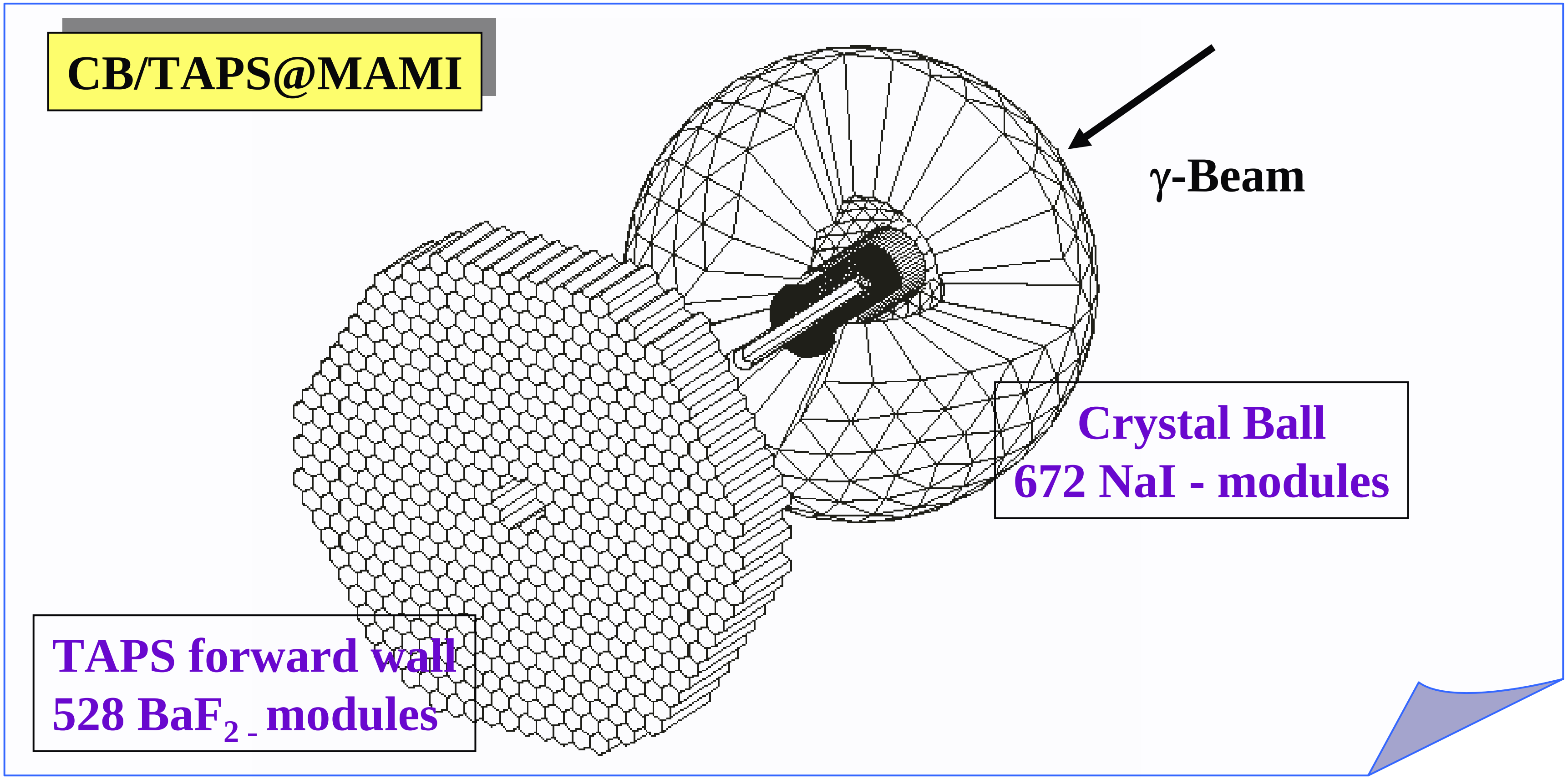
$$R \equiv \frac{1}{\sigma_\pi} \cdot E'_\gamma \frac{d\sigma}{dE'_\gamma}$$

2. Σ – linear-pol. photon beam asymmetry.

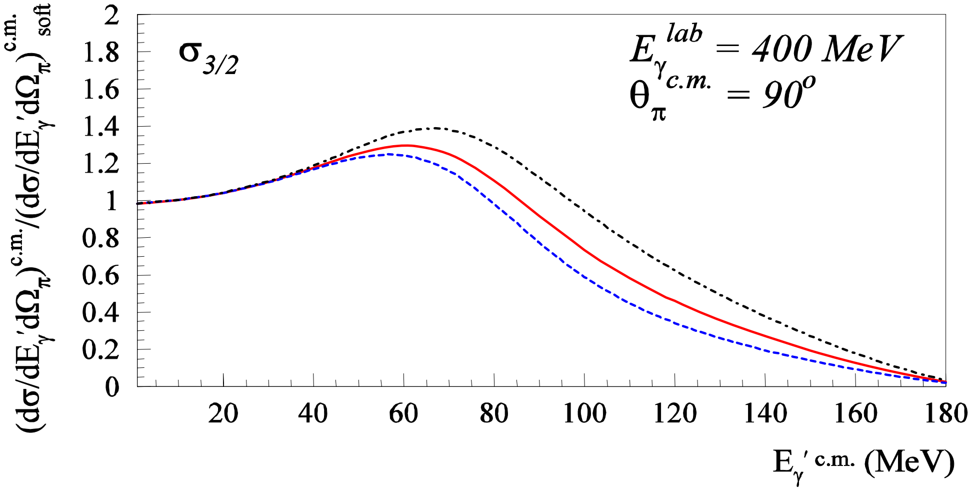
3. Σ_{circ} – circular-pol. photon beam asymmetry.

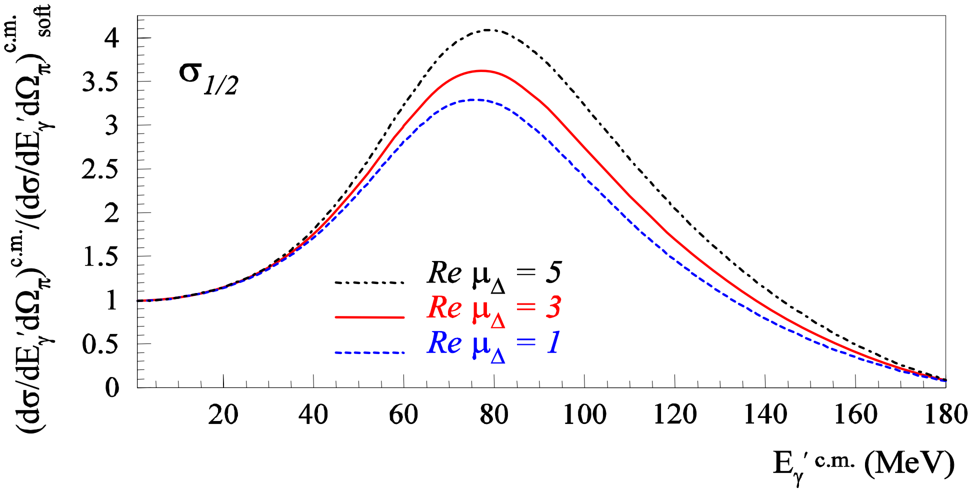


2005: New Crystal Ball Collaboration dedicated expt to improve statistics by two orders of magnitude.



$\gamma\, p \longrightarrow \gamma\, \pi^0\, p$: doubly polarized cross sections

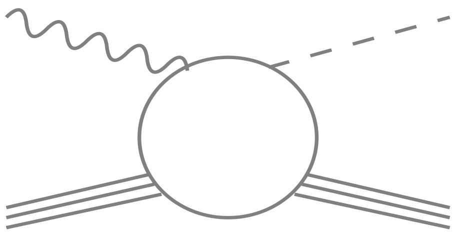




Ohio-Utrecht dynamical model

Relation to the πN scattering: Watson's theorem

Pion production on nucleons by (real and virtual) photons



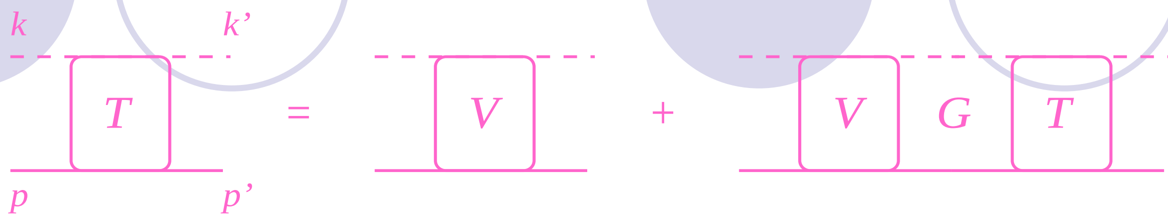
⇨ At lower energies the phase of this amplitude is given by the πN phase shift:

$$T_{\pi\gamma} = |T_{\pi\gamma}|\, e^{i\delta_{\pi N}}$$ [K. Watson (1954)]

In the Δ-resonance region this condition is fulfilled exactly to lowest order in *e*.

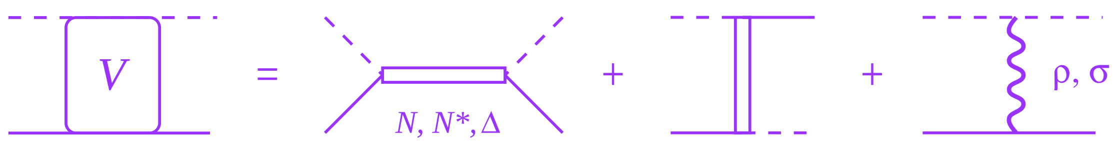
⇨ Other general constraints: e.m. gauge invariance, chiral symmetry lead to low-energy theorems [de Baenst (1970), Vanshtein & Zakharov (1970) Bernard, Kaiser, Gasser & Meissner (1991)]

πN equation



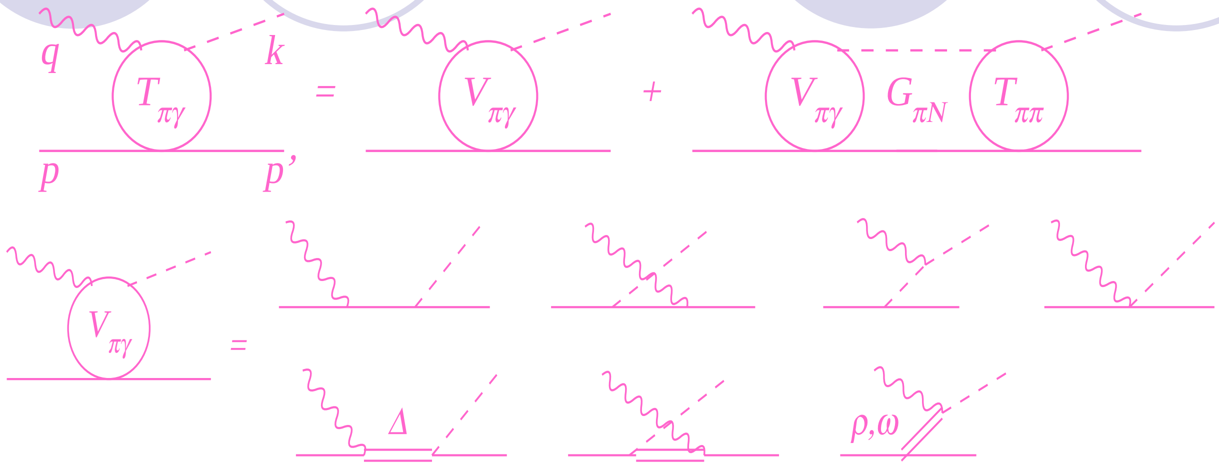
Here covariant 3D reduction
of the Bethe-Salpeter equation – Salpeter equation. Other approaches
[e.g., Pierce &Jennings NPA (1991) – "BbS, Thomson", Gross & Surya PRC (1993) – "Spe
S.N. Yang *et al*. JPG (1994) – Cooper-Jennings , Lahiff & Afnan PRC (1999) – 4D BS].

Solved by Pade approximants for the following one-hadron-exchange potential:



Resulting solution fitted to KA86 and SM95 phase-shift analyses

Pion photo/electro-production



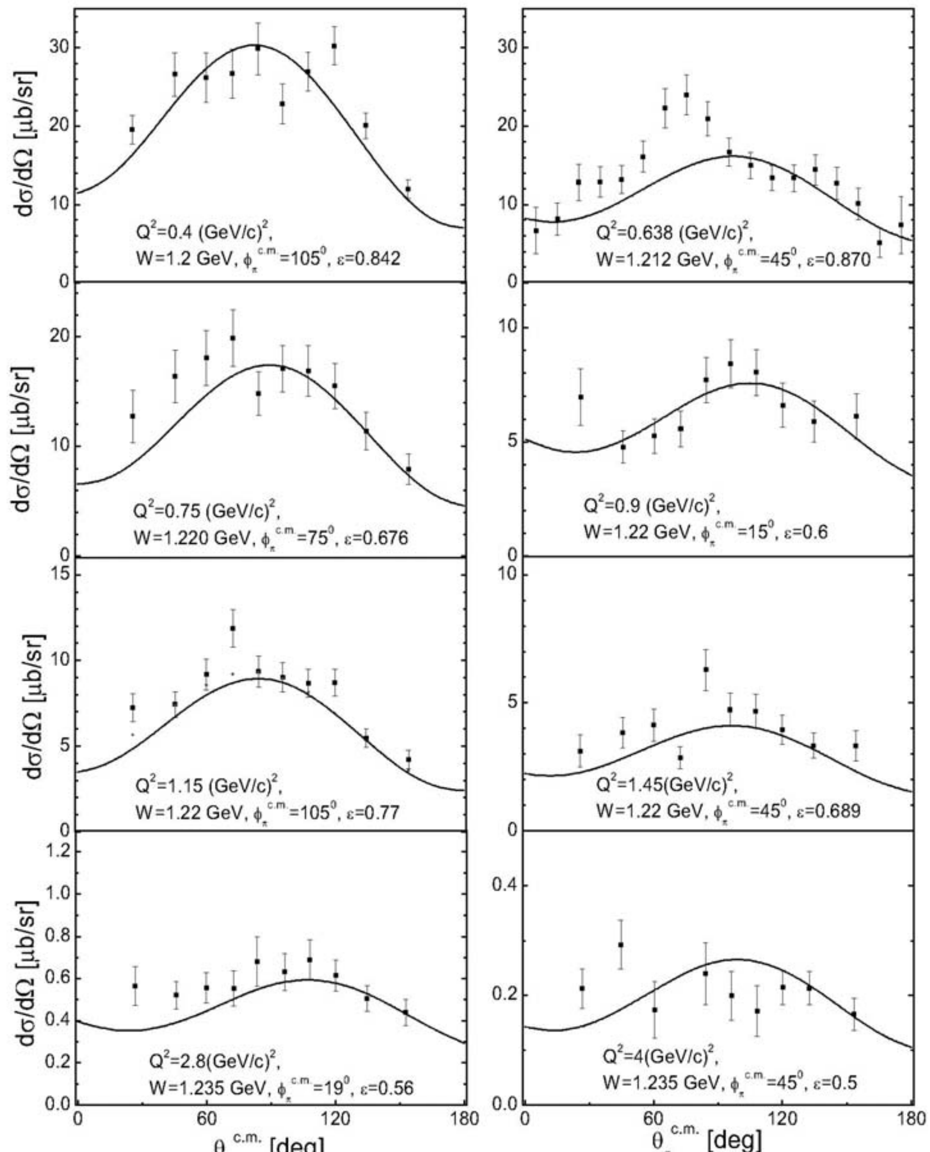
Electromagnetic form factors are introduced via

$$J^\mu \to J'^\mu(Q^2) = J^\mu + [F(Q^2) - 1]\, O^{\mu\nu} J_\nu\,, \qquad O^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$$

This allows to use realistic form factors without loss of e.m. gauge invariance. E.g.

$$\begin{aligned}
\Gamma^\mu_{NN\gamma} &= e\,\gamma^\mu + e\,[F_1(Q^2) - 1]\, O^{\mu\nu}\gamma_\nu + \frac{e\kappa_N}{2m_N}\, F_2(Q^2)\, i\sigma^{\mu\nu} q_\nu \\
\Gamma^\mu_{\pi\pi\gamma} &= e\,(k + k')^\mu + e\,[F_\pi(Q^2) - 1]\, O^{\mu\nu}(k + k')_\nu \\
J^\mu_{KR} &= \frac{e g_{\pi N}}{2m_N}\, \{\gamma^\mu + [F_A(Q^2) - 1]\, O^{\mu\nu}\gamma_\nu\}\, \gamma_5 .
\end{aligned}$$

Neutral pion electroproduction at JLab, diff. cross sections:



Chiral EFT vs. Dynamical models

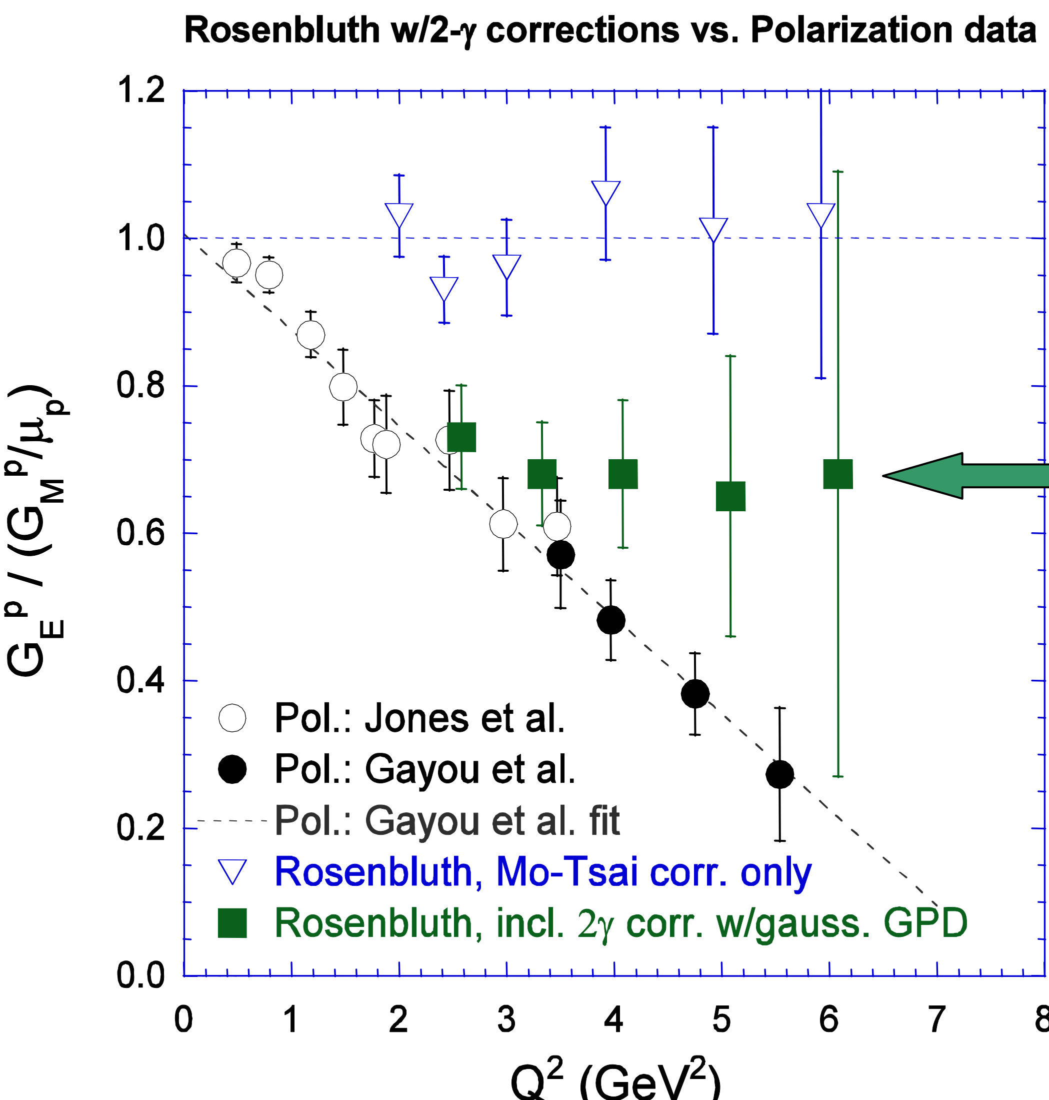
Chiral EFT

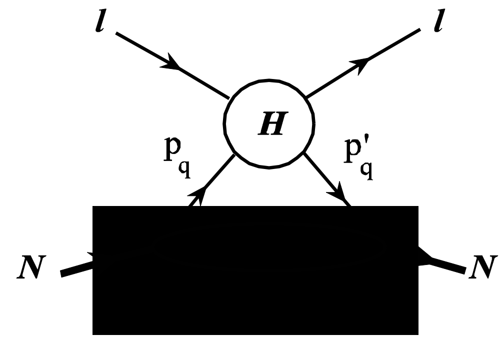
- Consistent with principles of QCD, an EFT of QCD.
- Systematic expansion in powers of soft scales and momenta – perturbation theory
- Quantum field theory: symmetries of the Lagrangian are satisfied at the level of amplitudes and observables (crossing, gauge-, chiral, relativistic-, unitarity).
- Phenomenologically viable, provides connection of lattice QCD to observables
- Very limited range of applicability

Dynamical models

- Unitarity, relativity are good
- Possibilty to study dynamical resonances, pion cloud effects
- Local symmetries are a problem
- Systematic improvement ?
- Control of model dependence ?
- Useful phenomenology: physics motivated (QCD-inspired) parametrization and extrapolation of experimental results in a wide range of kinematics

Two-photon exchange effect in nucleon form factors



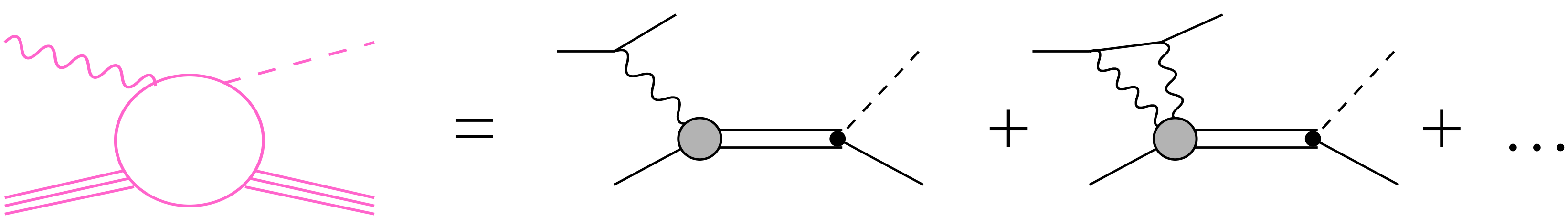


Partonic calculation

[Chen, Afanasev, Brodsky, Carlson, Vanderhaeghen, PRL (2004)]

Electromagnetic Nucleon to Delta transition

- γ¤NΔ is specified by three form-factors: G_M¤ (Q^2) [M1], G_E¤ (Q^2) [E2], G_C¤ (Q^2) [C2]
- studied in *pion electroproduction on the nucleon* at the resonance kinematics, $s=M_\Delta^2$



EM nucleon to Delta transition: general formalism

- In general (any # of exchanged photons), the e N! e Δ transition can be described by 16 form factors, or, 16 independent helicity amplitudes: $T^{h',h}_{\lambda_\Delta,\lambda_N} \equiv \langle k', h'; p_\Delta, \lambda_\Delta | T | k, h; p, \lambda_N \rangle$.
- m_e=0, cuts this number down to 8: $(\varepsilon_\pm = \sqrt{1 \pm \varepsilon})$

$$T_1 \equiv T_{+3/2,+1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-} - 1\right) \frac{(M_N + M_\Delta)\, Q_-}{4 M_N\, Q} \,(G_M^* + G_E^*) \xrightarrow{\text{PQCD}} \frac{1}{Q^6}$$

$$T_2 \equiv T_{+3/2,-1/2} = 0$$

$$T_3 \equiv T_{+1/2,+1/2} = -\frac{\sqrt{2\varepsilon}}{\varepsilon_-} \frac{(M_N + M_\Delta)\, Q_-}{2 M_N M_\Delta} G_C^* \xrightarrow{\text{PQCD}} \frac{1}{Q^5}$$

$$T_4 \equiv T_{+1/2,-1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-} - 1\right) \frac{(M_N + M_\Delta)\, Q_-}{4 M_N\, Q} \,(G_M^* - 3G_E^*)$$

$$T_5 \equiv T_{-1/2,+1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-} + 1\right) \frac{(M_N + M_\Delta)\, Q_-}{4 M_N\, Q} \,(G_M^* - 3G_E^*) \longrightarrow \frac{1}{Q^4}$$

$$T_6 \equiv T_{-1/2,-1/2} = \frac{\sqrt{2\varepsilon}}{\varepsilon_-} \frac{(M_N + M_\Delta)\, Q_-}{2 M_N M_\Delta} G_C^*$$

$$T_7 \equiv T_{-3/2,+1/2} = 0$$

$$T_8 \equiv T_{-3/2,-1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-} + 1\right) \frac{(M_N + M_\Delta)\, Q_-}{4 M_N\, Q} \,(G_M^* + G_E^*)$$

$$R_{EM} = \frac{E2}{M1} = -\frac{G_E^*}{G_M^*} \xrightarrow{\text{PQCD}} 1$$

$$R_{SM} = \frac{C2}{M1} = -\frac{Q_+ Q_-}{4 M_\Delta^2} \frac{G_C^*}{G_M^*} \longrightarrow \text{const}$$

[Carlson, PRD (1986)]

one-photon exchange approximation

EM Nucleon to Delta transition: general formalism

❖ The unpolarized pion electroproduction cross-section is, *in general*, written as:

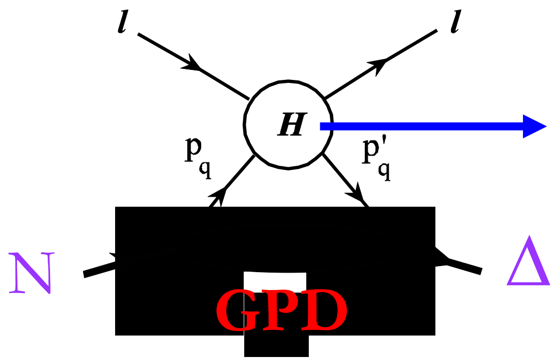
$$\frac{d\sigma}{(dE'_e\, d\Omega'_e)^{lab}\, d\Omega_\pi} \equiv \Gamma_v \frac{d\sigma}{d\Omega_\pi}, \qquad \text{Flux: } \Gamma_v = \frac{e^2}{(2\pi)^3} \left(\frac{E'_e}{E_e}\right)^{lab} \frac{(s_{\pi N} - M_N^2)/(2M_N)}{Q^2\,(1-\varepsilon)}.$$

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\, \cos(2\Phi)\, \frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1+\varepsilon)}\, \cos\Phi\, \frac{d\sigma_{LT}}{d\Omega_\pi} + \varepsilon\, \sin(2\Phi)\, \frac{d\sigma_{TTi}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1-\varepsilon)}\, \sin\Phi\, \frac{d\sigma_{LTi}}{d\Omega_\pi}.$$

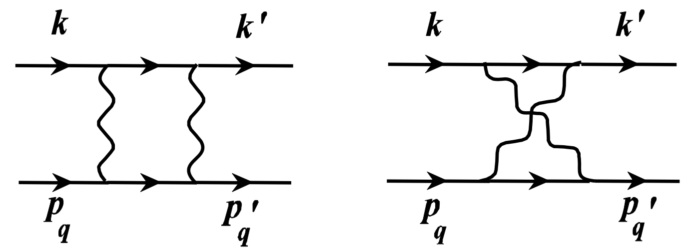
❖ At the Δ-resonance, these cross-sections are expressed in terms of e N! e Δ helicity amplitudes

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\pi} &= \frac{1}{\pi}\, \frac{9\,Q^2\,(1-\varepsilon)}{16\,M_\Delta\,(M_\Delta^2 - M_N^2)\,\Gamma_\Delta} \\
&\times \left\{ \frac{1}{2}\sin^2\theta_\pi\, \left[|T_1|^2 + |T_2|^2 + |T_7|^2 + |T_8|^2 \right] + \frac{1}{6}(1 + 3\,\cos^2\theta_\pi)\, \left[|T_3|^2 + |T_4|^2 + |T_5|^2 + |T_6|^2 \right] \right. \\
&+ \cos\Phi\, \frac{1}{\sqrt{3}}\, \sin(2\theta_\pi)\, \Re\left[T_1\,T_3^* + T_2\,T_4^* - T_7\,T_5^* - T_8\,T_6^* \right] - \cos(2\Phi)\, \frac{1}{\sqrt{3}}\, \sin^2\theta_\pi\, \Re\left[T_1\,T_5^* + T_2\,T_6^* + T_7\,T_3^* + T_8\,T_4^* \right] \\
&\left. + \sin\Phi\, \frac{1}{\sqrt{3}}\, \sin(2\theta_\pi)\, \Im\left[T_1\,T_3^* + T_2\,T_4^* + T_7\,T_5^* + T_8\,T_6^* \right] - \sin(2\Phi)\, \frac{1}{\sqrt{3}}\, \sin^2\theta_\pi\, \Im\left[T_1\,T_5^* + T_2\,T_6^* - T_7\,T_3^* - T_8\,T_4^* \right] \right\}.
\end{aligned}$$

Two-photon exchange : partonic calculation



Hard scattering amplitude



$$A^* = \int_{-1}^{1} \frac{dx}{x} \left(\frac{\hat{s} - \hat{u}}{Q^2} g_M^{hard} + g_A^{(2\gamma)} \right) \sqrt{\frac{2}{3}} \frac{1}{6} H_M^{(3)}$$ “magnetic” GPD

$$C^* = \int_{-1}^{1} \frac{dx}{x} \left(\frac{\hat{s} - \hat{u}}{Q^2} g_A^{(2\gamma)} + g_M^{hard} \right) \mathrm{sgm}(x) \frac{1}{6} C_1^{(3)}$$ “axial” GPD

Large N_C

$$H_M^{(3)}(x, 0, Q^2) = 2 \frac{G_M^*(0)}{\kappa_V} \left[E^u(x, 0, Q^2) - E^d(x, 0, Q^2) \right]$$

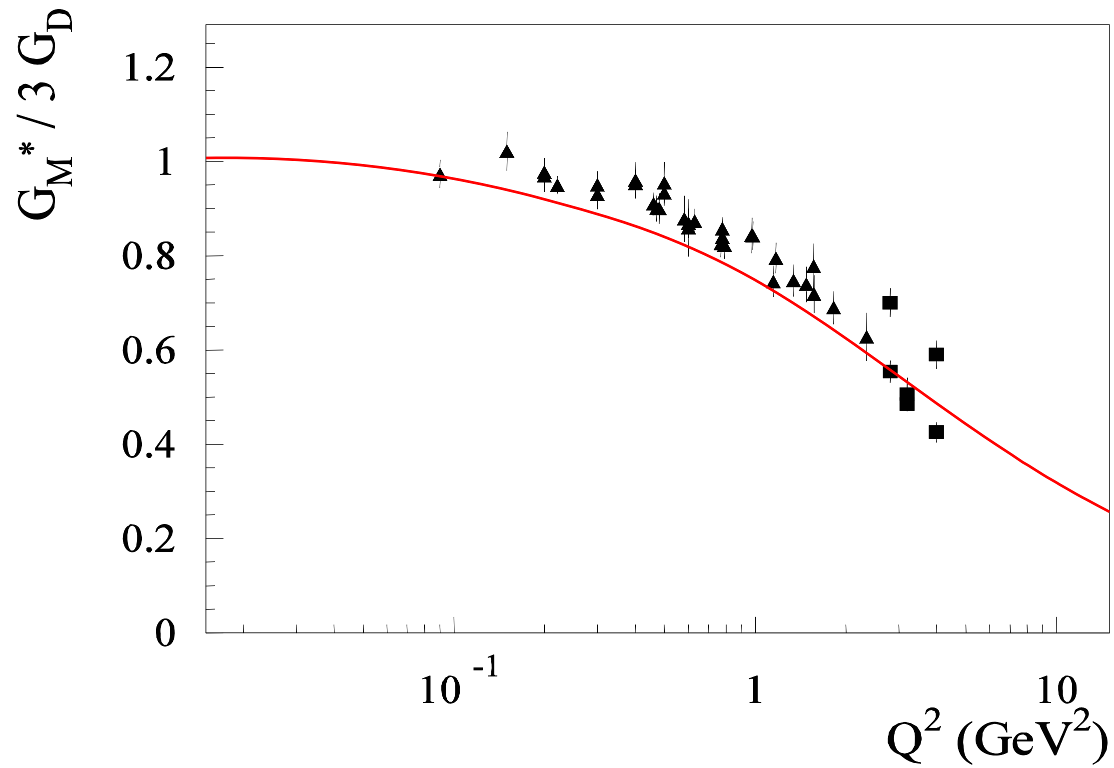
$$C_1^{(3)}(x, 0, Q^2) = \sqrt{3} \left[\tilde{H}^u(x, 0, Q^2) - \tilde{H}^d(x, 0, Q^2) \right]$$

Nucleon GPD’s

N -> Δ transition form factors from GPDs

large N_c

$$G_M^*(t) = \frac{G_M^*(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x,\xi,t) - E^d(x,\xi,t) \right\} = \frac{G_M^*(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$



``modified Regge model”

One + two-photon-exchange in observables

- The unpolarized pion electroproduction cross-section:

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\cos(2\Phi)\frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\Phi\frac{d\sigma_{LT}}{d\Omega_\pi} + \varepsilon\sin(2\Phi)\frac{d\sigma_{TTi}}{d\Omega_\pi} + \sqrt{2\varepsilon(1-\varepsilon)}\sin\Phi\frac{d\sigma_{LTi}}{d\Omega_\pi}$$

$$= \frac{1}{\pi}\frac{9\,Q^2(1-\varepsilon)}{16\,M_\Delta(M_\Delta^2-M_N^2)\Gamma_\Delta} \times \Big\{ \frac{1}{2}\sin^2\theta_\pi\,\left[|T_1|^2+|T_2|^2+|T_7|^2+|T_8|^2\right] + \frac{1}{6}(1+3\cos^2\theta_\pi)\,\left[|T_3|^2+|T_4|^2+|T_5|^2+|T_6|^2\right]$$

$$+\cos\Phi\,\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Re\left[T_1T_3^*+T_2T_4^*-T_7T_5^*-T_8T_6^*\right] - \cos(2\Phi)\,\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Re\left[T_1T_5^*+T_2T_6^*+T_7T_3^*+T_8T_4^*\right]$$

$$+\sin\Phi\,\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Im\left[T_1T_3^*+T_2T_4^*+T_7T_5^*+T_8T_6^*\right] - \sin(2\Phi)\,\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Im\left[T_1T_5^*+T_2T_6^*-T_7T_3^*-T_8T_4^*\right]\Big\}.$$

$$T_1 \equiv T_{+3/2,+1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-}-1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*+G_E^*)$$

$$T_2 \equiv T_{+3/2,-1/2} = 0$$

$$T_3 \equiv T_{+1/2,+1/2} = -\frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N+M_\Delta)\,Q_-}{2M_NM_\Delta}\,G_C^*$$

$$T_4 \equiv T_{+1/2,-1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-}-1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*-3G_E^*)$$

$$T_5 \equiv T_{-1/2,+1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-}+1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*-3G_E^*)$$

$$T_6 \equiv T_{-1/2,-1/2} = \frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N+M_\Delta)\,Q_-}{2M_NM_\Delta}\,G_C^*$$

$$T_7 \equiv T_{-3/2,+1/2} = 0$$

$$T_8 \equiv T_{-3/2,-1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-}+1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*+G_E^*)$$

$$T_1^{2\gamma} = \frac{e^2}{2}\left\{A^*\frac{3(M_\Delta+M_N)Q}{2\sqrt{2}M_NQ_+} - C^*\frac{\sqrt{2}Q}{Q_-}\right\},$$

$$T_2^{2\gamma} = 0,$$

$$T_3^{2\gamma} = -\frac{e^2}{2}C^*\sqrt{\frac{2}{3}}\frac{M_\Delta^2-M_N^2-Q^2}{M_\Delta Q_-}$$

$$T_4^{2\gamma} = \sqrt{\frac{1}{3}}T_1^{2\gamma},$$

$$T_5^{2\gamma} = \sqrt{\frac{1}{3}}T_8^{2\gamma},$$

$$T_6^{2\gamma} = T_3^{2\gamma},$$

$$T_7^{2\gamma} = 0,$$

$$T_8^{2\gamma} = \frac{e^2}{2}\left\{A^*\frac{3(M_\Delta+M_N)Q}{2\sqrt{2}M_NQ_+} + C^*\frac{\sqrt{2}Q}{Q_-}\right\}.$$

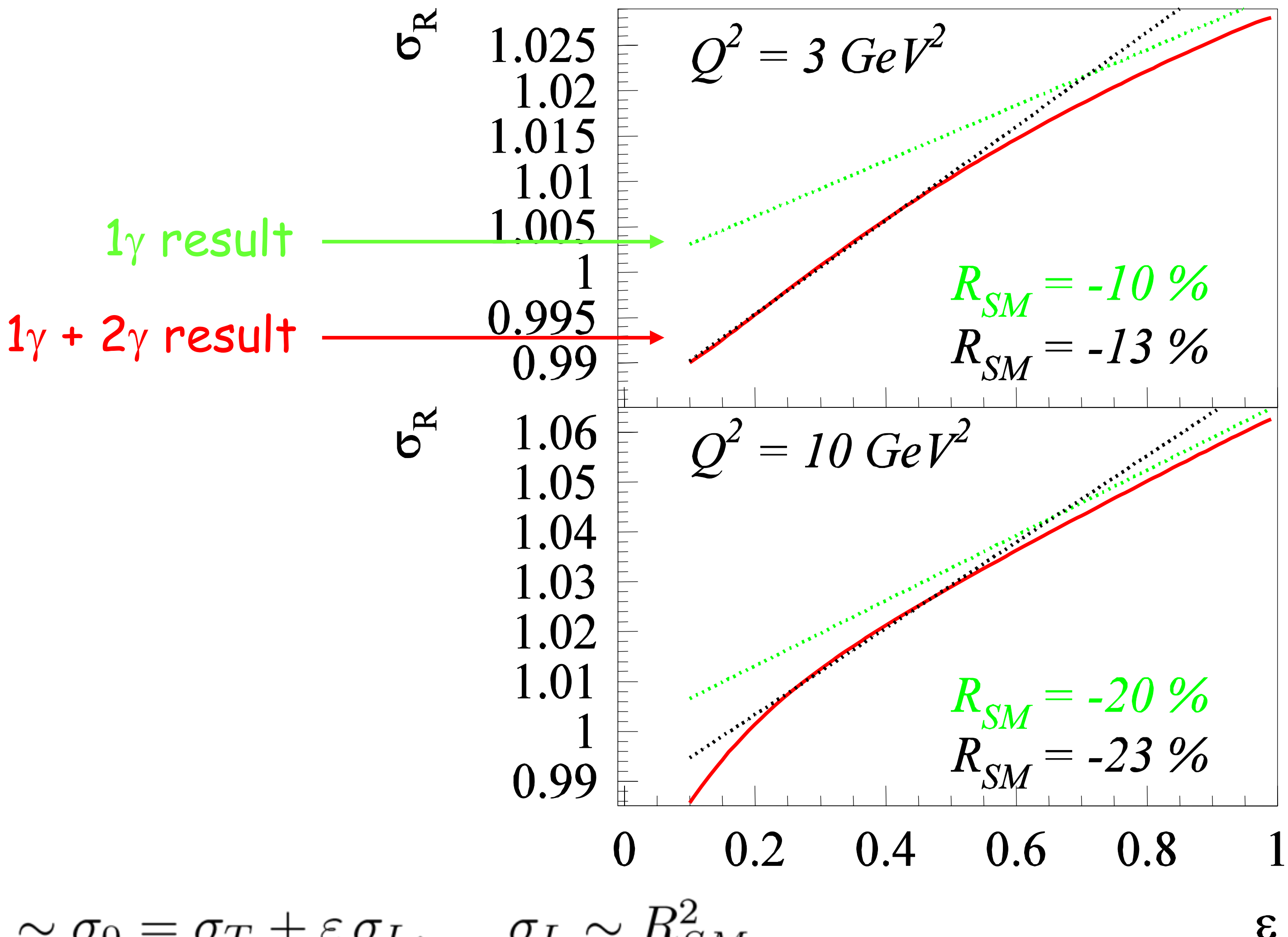
Two-photon effects on observables

$$\sigma_R = 1 + 3\,(R^{1\gamma}_{EM})^2 + \varepsilon\,\frac{16\,M_\Delta^2\,Q^2}{Q_+^2\,Q_-^2}\,(R^{1\gamma}_{SM})^2$$
$$+ \frac{1}{G_M^*}\sqrt{\frac{2}{3}}\left[\frac{1}{2}\,A^*\,\frac{Q^2}{Q_+\,Q_-}\,\varepsilon_+\,\varepsilon_- + 2\,C^*\,\frac{Q^2}{Q_-^2}\,\varepsilon_-^2\,\frac{M_N}{M_N+M_\Delta}\right].$$

$$R_{EM} = R^{1\gamma}_{EM} + \frac{1}{8}\sqrt{\frac{3}{2}}\frac{Q^2}{Q_+\,Q_-}\,\frac{\varepsilon_-^3\,\varepsilon_+}{\varepsilon}\,\frac{1}{G_M^*}\,A^* - \frac{1}{4}\sqrt{\frac{2}{3}}\,\frac{Q^2}{Q_-^2}\,\frac{\varepsilon_-^2\,\varepsilon_+^2}{\varepsilon}\,\frac{M_N}{(M_N+M_\Delta)}\,\frac{1}{G_M^*}\,C^*$$

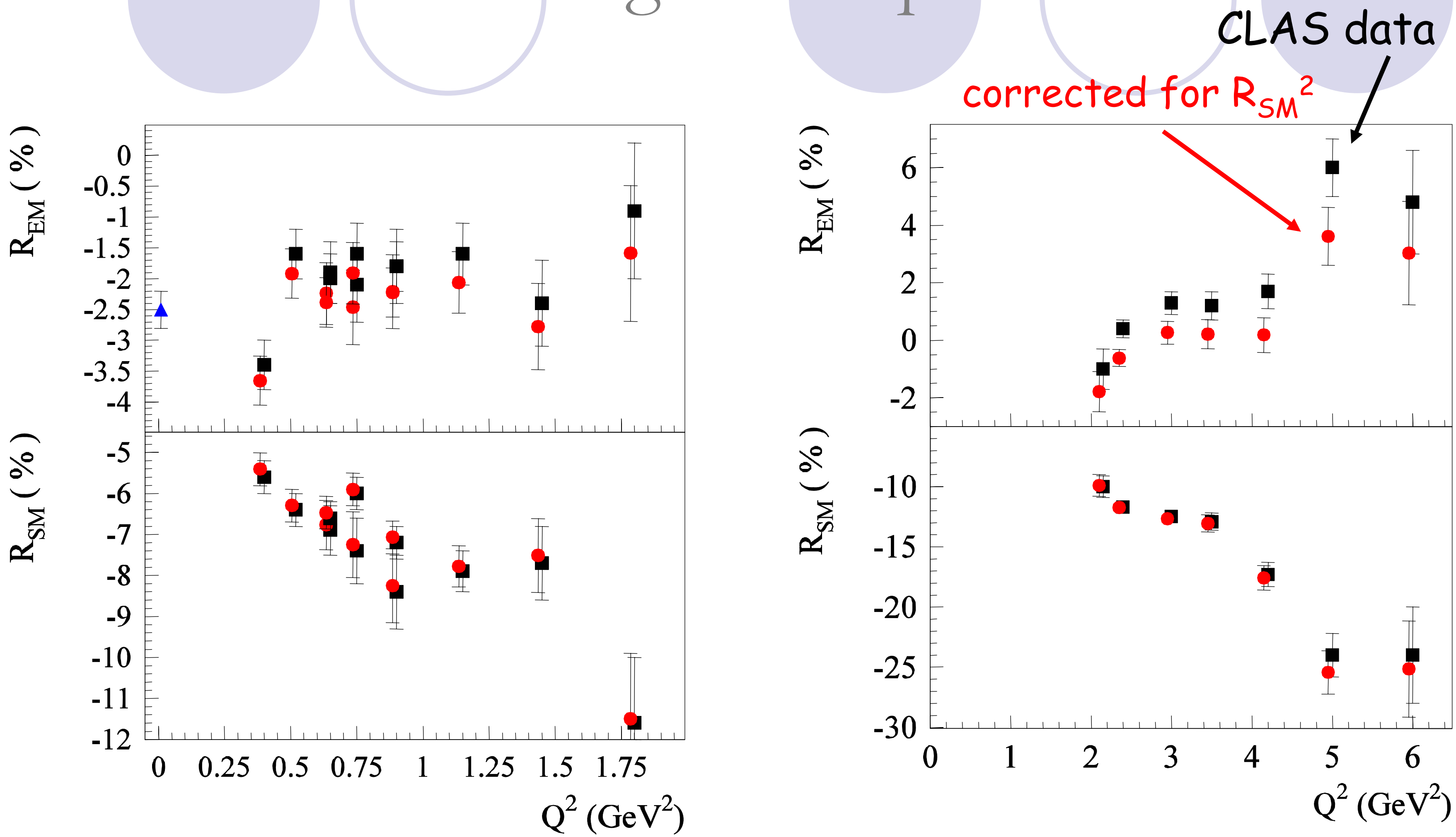
$$R_{SM} = R^{1\gamma}_{SM} - \sqrt{\frac{2}{3}}\,\frac{(Q^2 - M_\Delta^2 + M_N^2)}{4\,M_\Delta^2}\,\frac{Q_+}{Q_-}\,\frac{1}{\sqrt{2\,\varepsilon}}\,\frac{\varepsilon_-^2}{\varepsilon_+}\,\frac{M_N}{(M_N+M_\Delta)}\,\frac{1}{G_M^*}\,C^*$$

Two-photon effects on cross sections

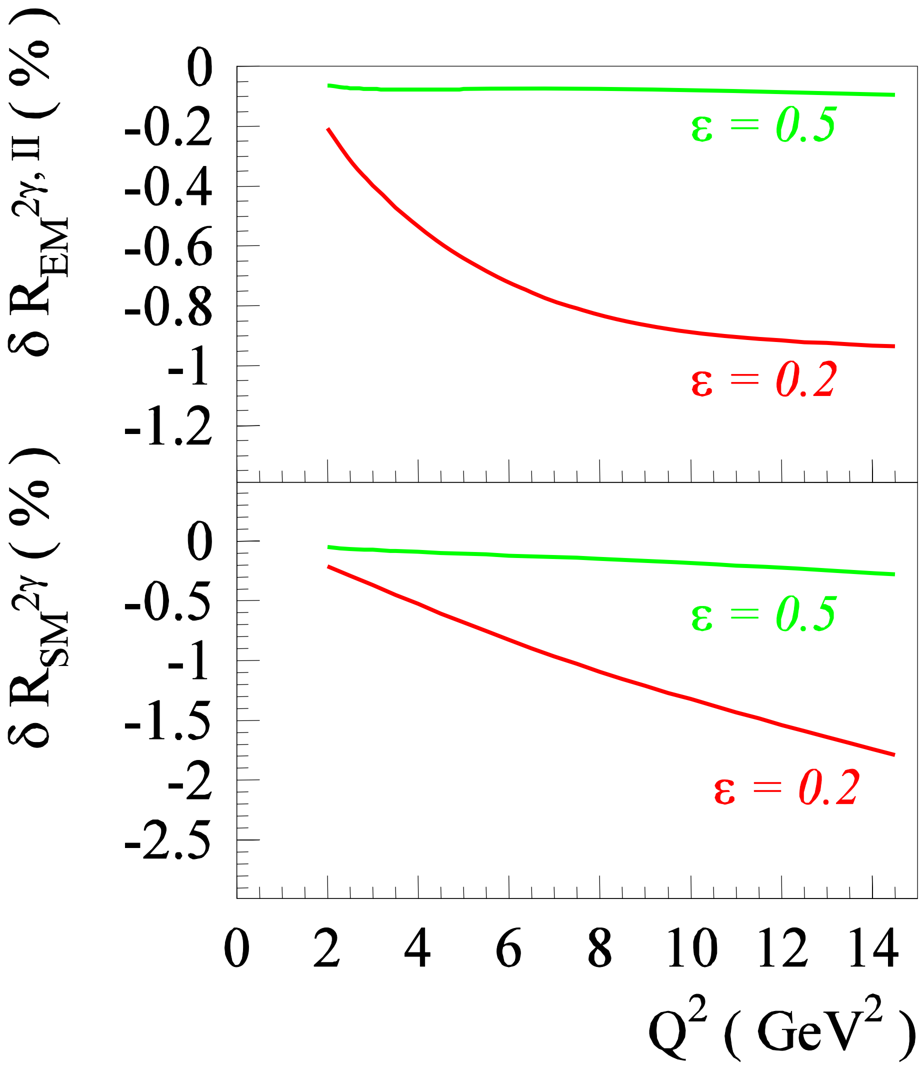


$$\sigma_R \sim \sigma_0 = \sigma_T + \varepsilon\, \sigma_L, \qquad \sigma_L \sim R_{SM}^2$$

Effect due to neglect of quadratic terms



Two-photon effects on the ratios (Q^2 dependence)



Conclusions on 2-photon exchange

✓ General formalism for the unpolarized pion electroproduction cross-sections in terms of N!Δ helicity amplitudes.

✓New responses σ_{TTi} and σ_{LTi}.

✓Parton-model calculation of two-photon exchange N!Δ helicity amplitudes.

✓A first glance at two-photon effects on inelastic observables.

❖ Corrections on R_{EM} are small, analogous to corrections in the nucleon polarization-transfer observables.

❖ Substantial (up to 10%) corrections to the L responses (σ_L and σ_{LT}). Therefore, mainly R_{SM} is affected.

❖Effects increasing with increasing $1/\varepsilon$ and Q^2

Summary

- Relativistic spin-3/2 field:
- Power counting for the Δ resonance in chiral EFT (ChEFT):
- Chiral extrapolations in relativistic baryon ChPT
- Pion electroproduction and γ N Δ form factors in ChEFT:
- Radiative pion photoproduction and Δ's magnetic moment in ChEFT:
- OU dynamical model for pion photo- and electroproduction on the nucleon:
- 2-photon effects in γ N -> Δ transition

Major theoretical tools in hadronic physics