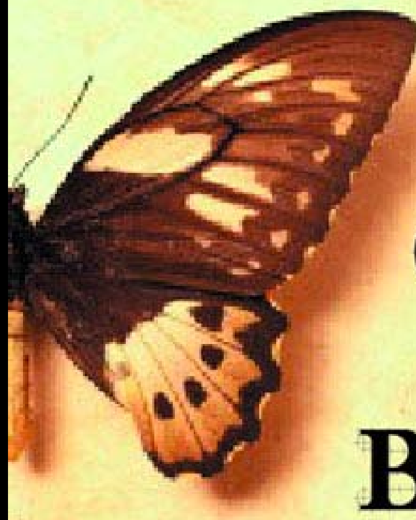
The Equation That Couldn't Be Solved



How Mathematical Genius Discovered the Language of

SYMMETRY

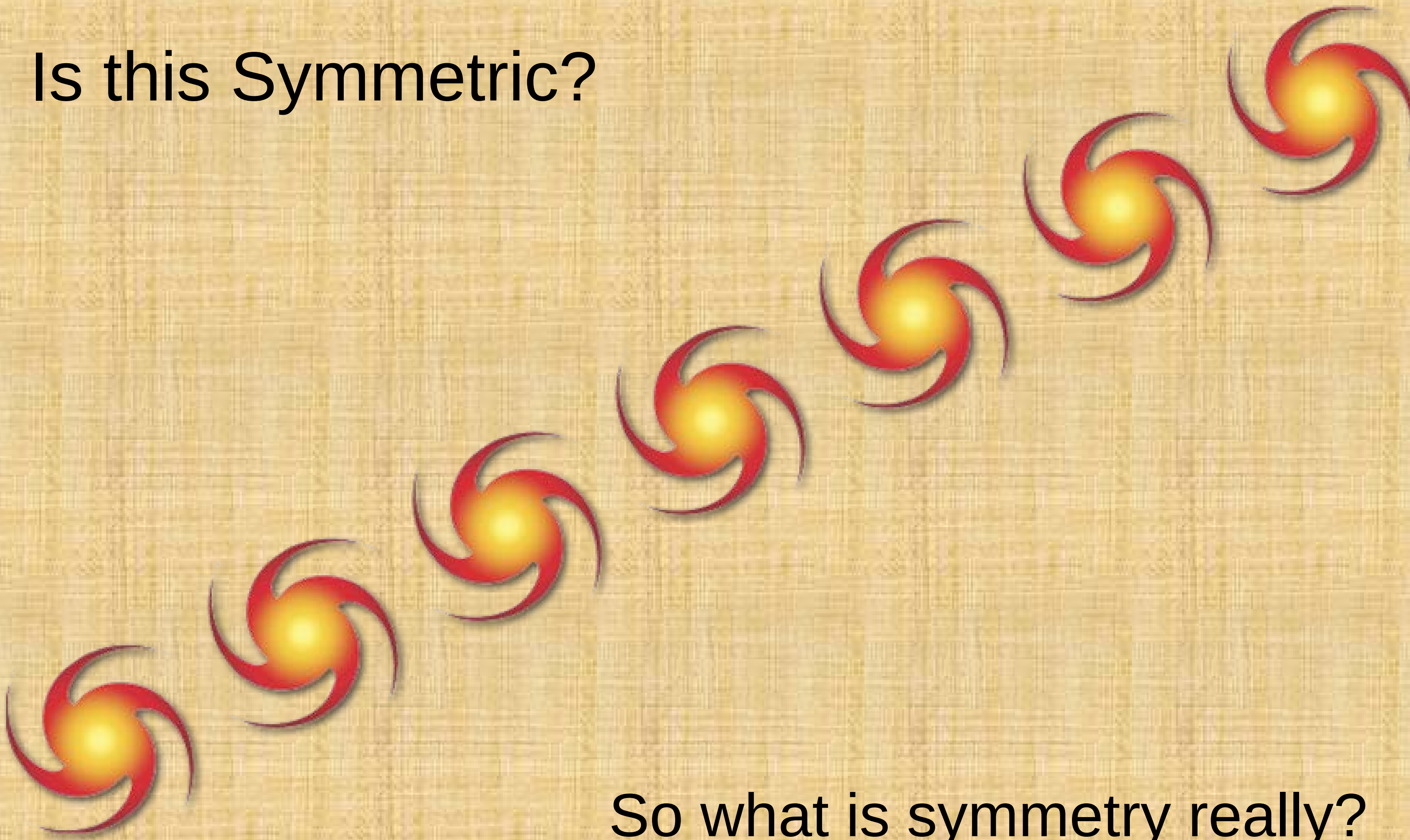
MARIO LIVIO

Author of THE GOLDEN RATIO

An Inkblot

If you fold the paper before the ink dries

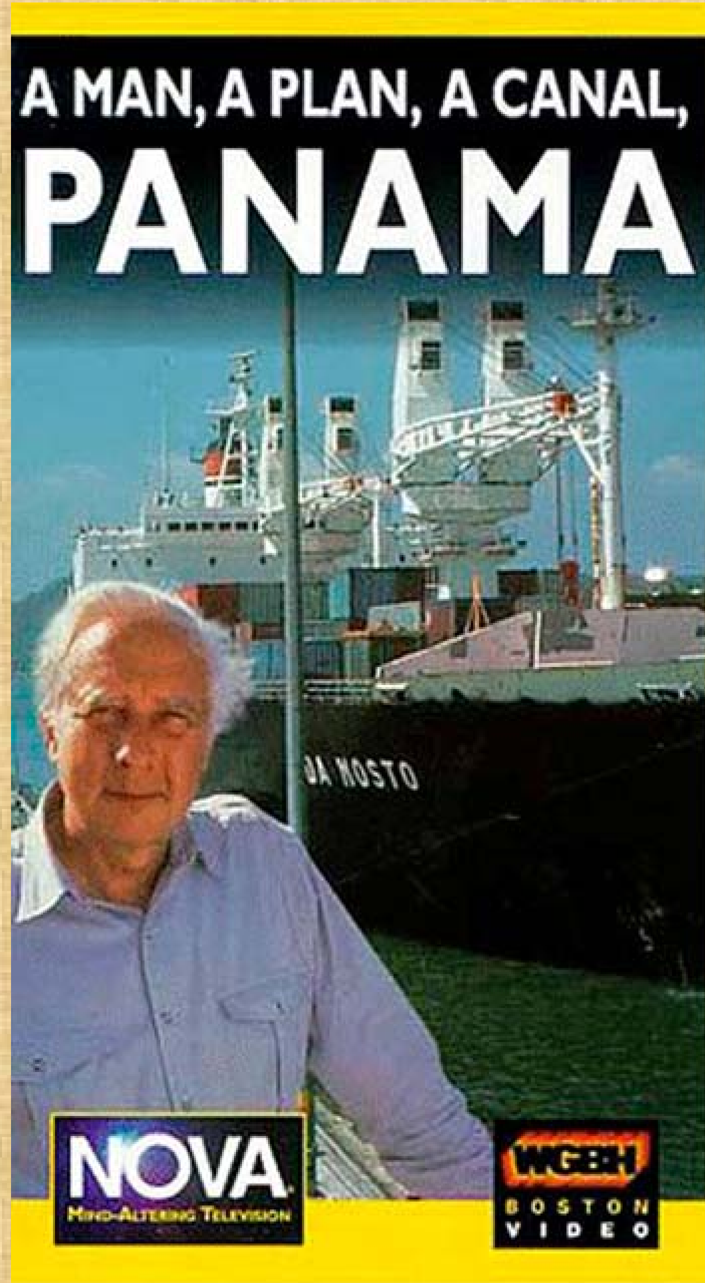
Is this Symmetric?



So what is symmetry really?

IMMUNITY
TO A
POSSIBLE
CHANGE

Madam I’m Adam



Palindromes

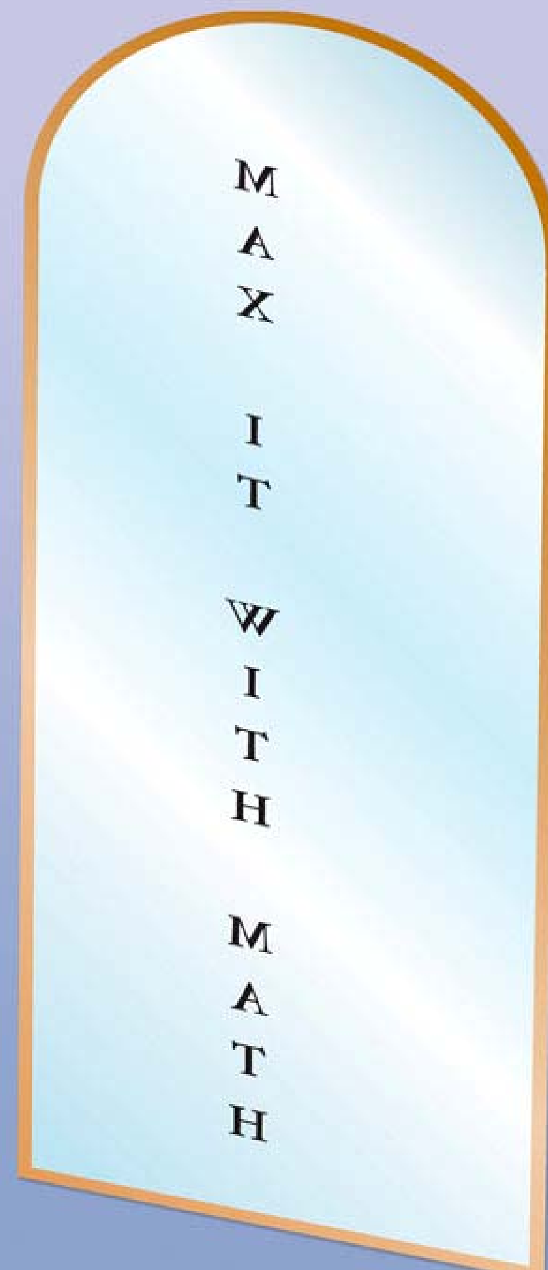
Reflections and bilateral Symmetry

MAX

IT

WITH

MATH



Rotations

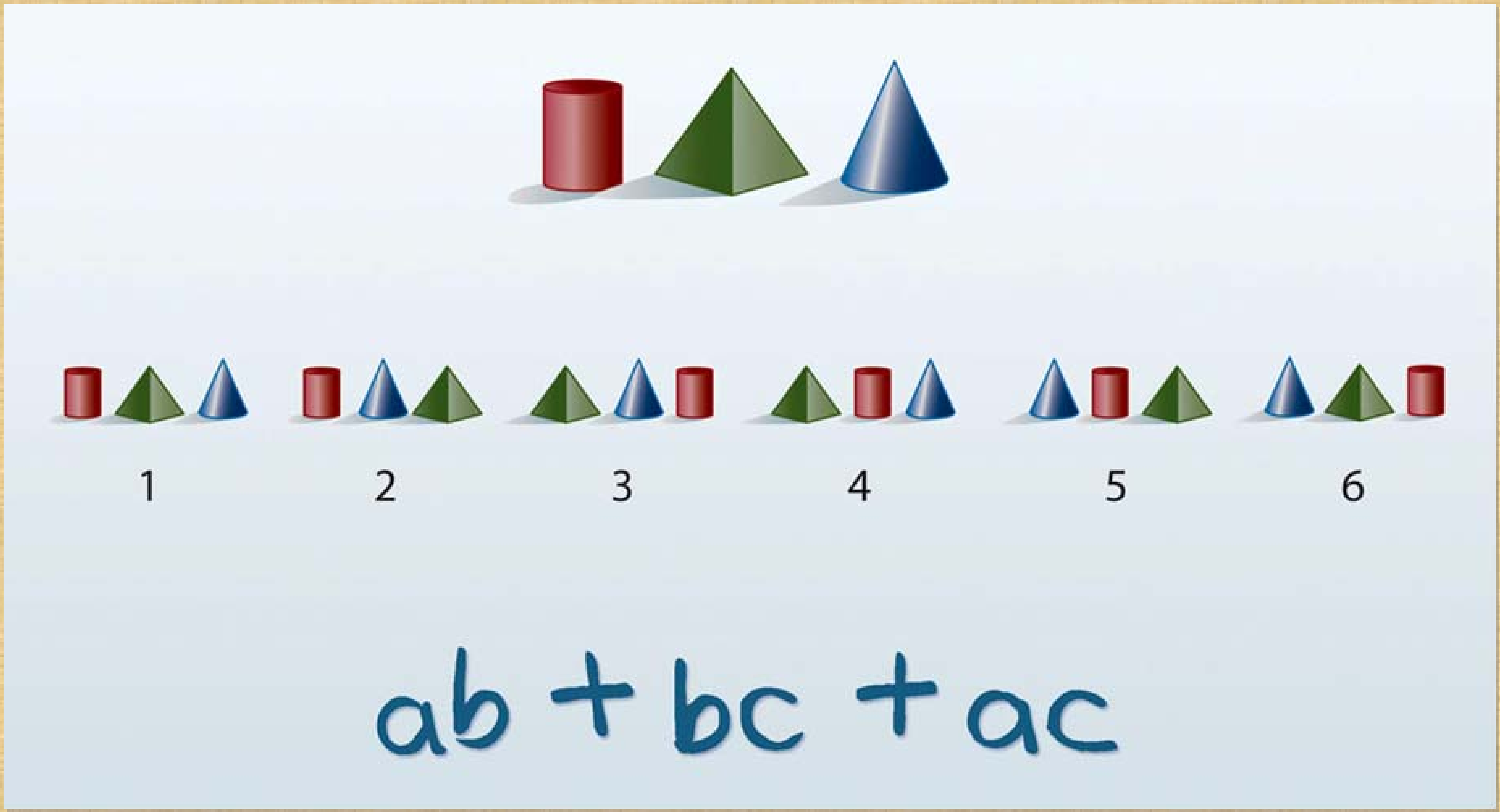
By 60°, 120°, 180°, 240°, 300°, 360°

Glide reflections

Symmetry In Music

Opening measures from Mozart’s *Symphony No. 40* in G minor

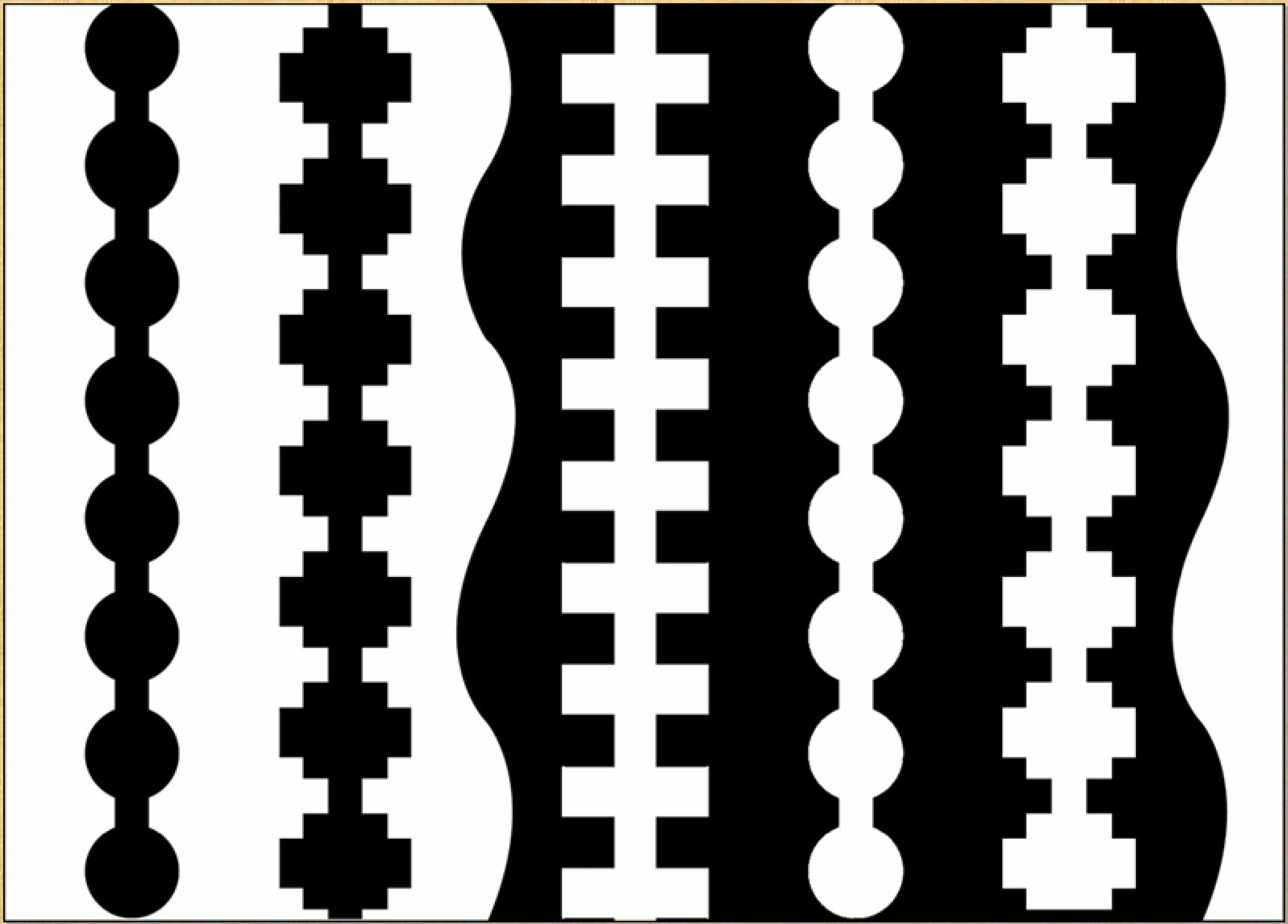
Symmetry Under Permutations

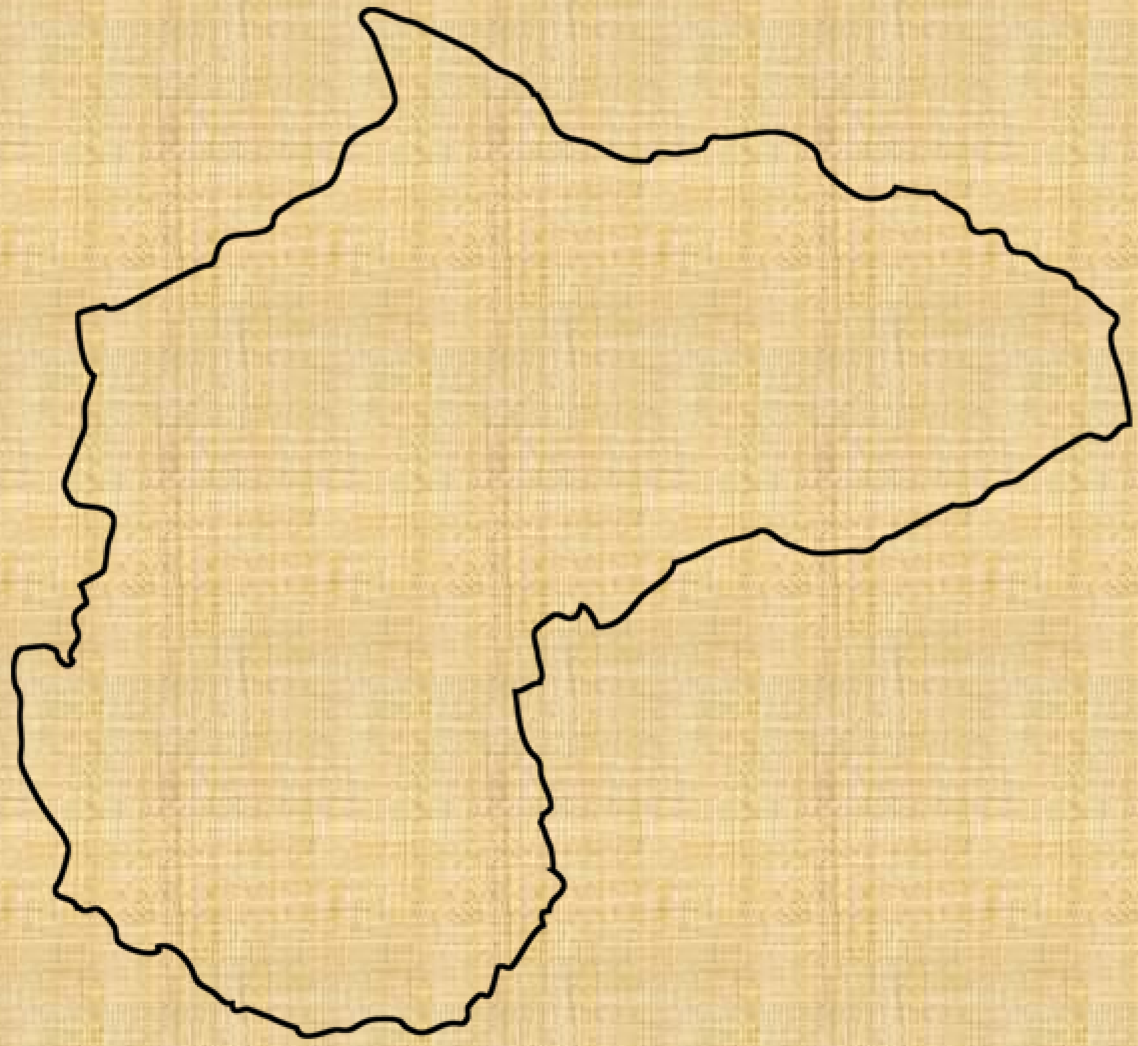


Perception

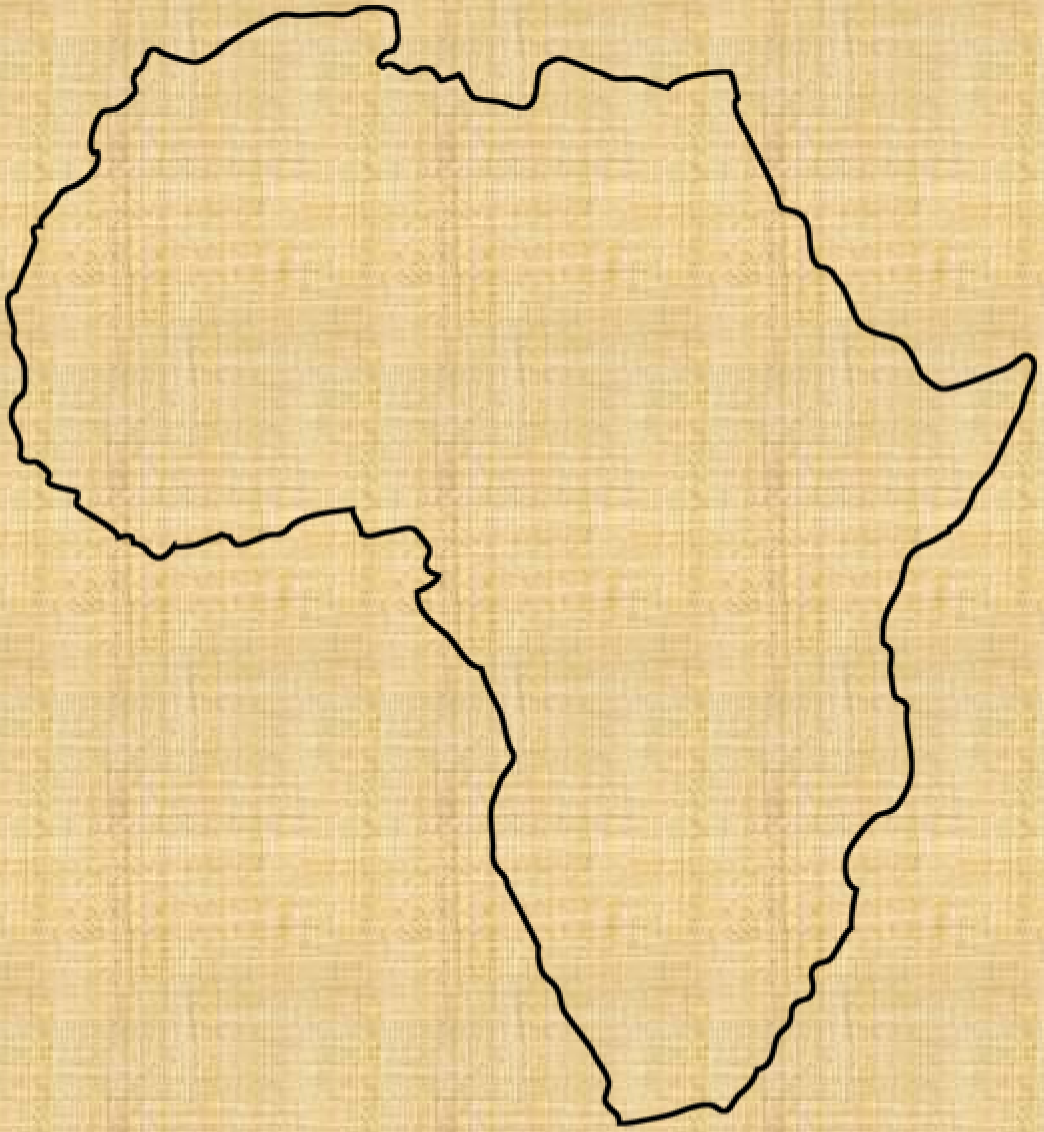
QuickTime™ and a
Sorenson Video 3 decompressor
are needed to see this picture.

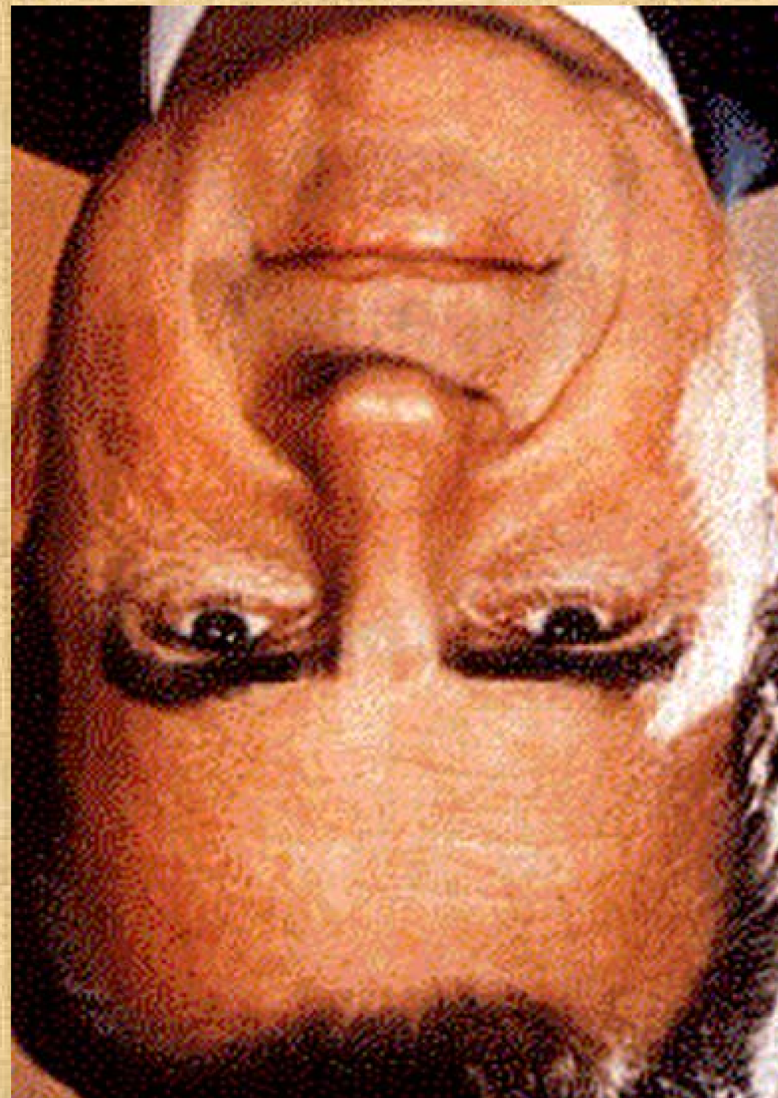
Symmetry can help perception

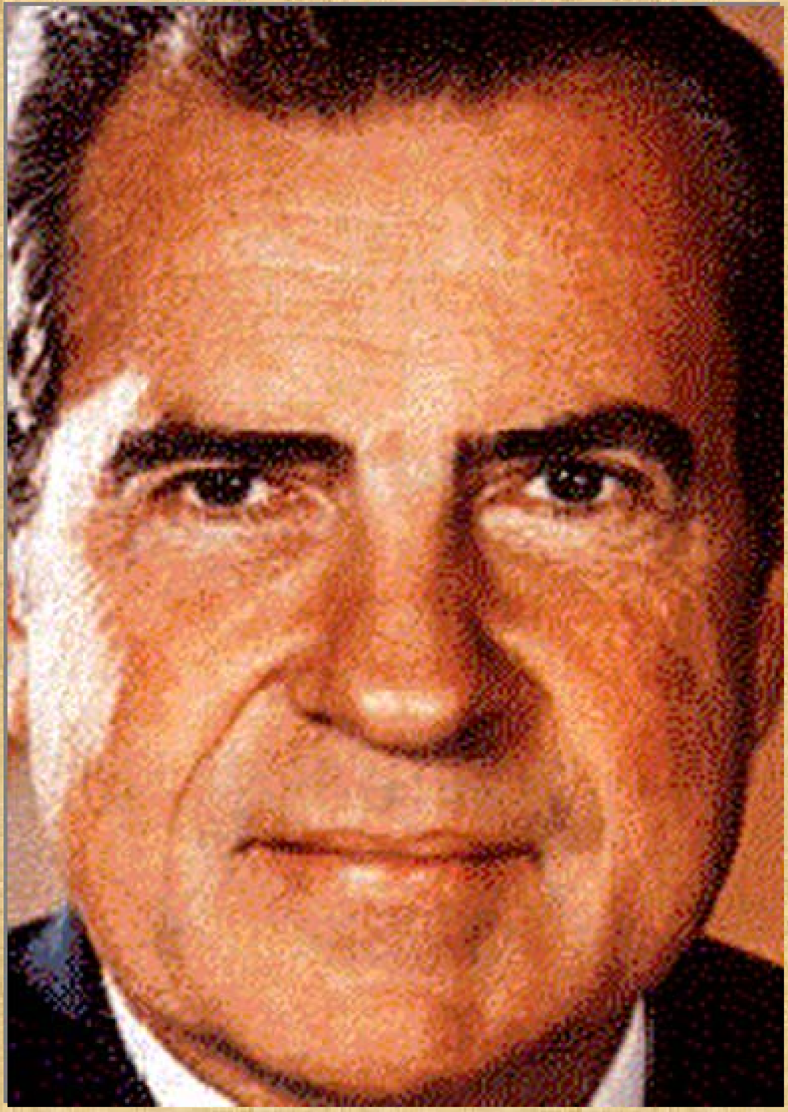




Orientation



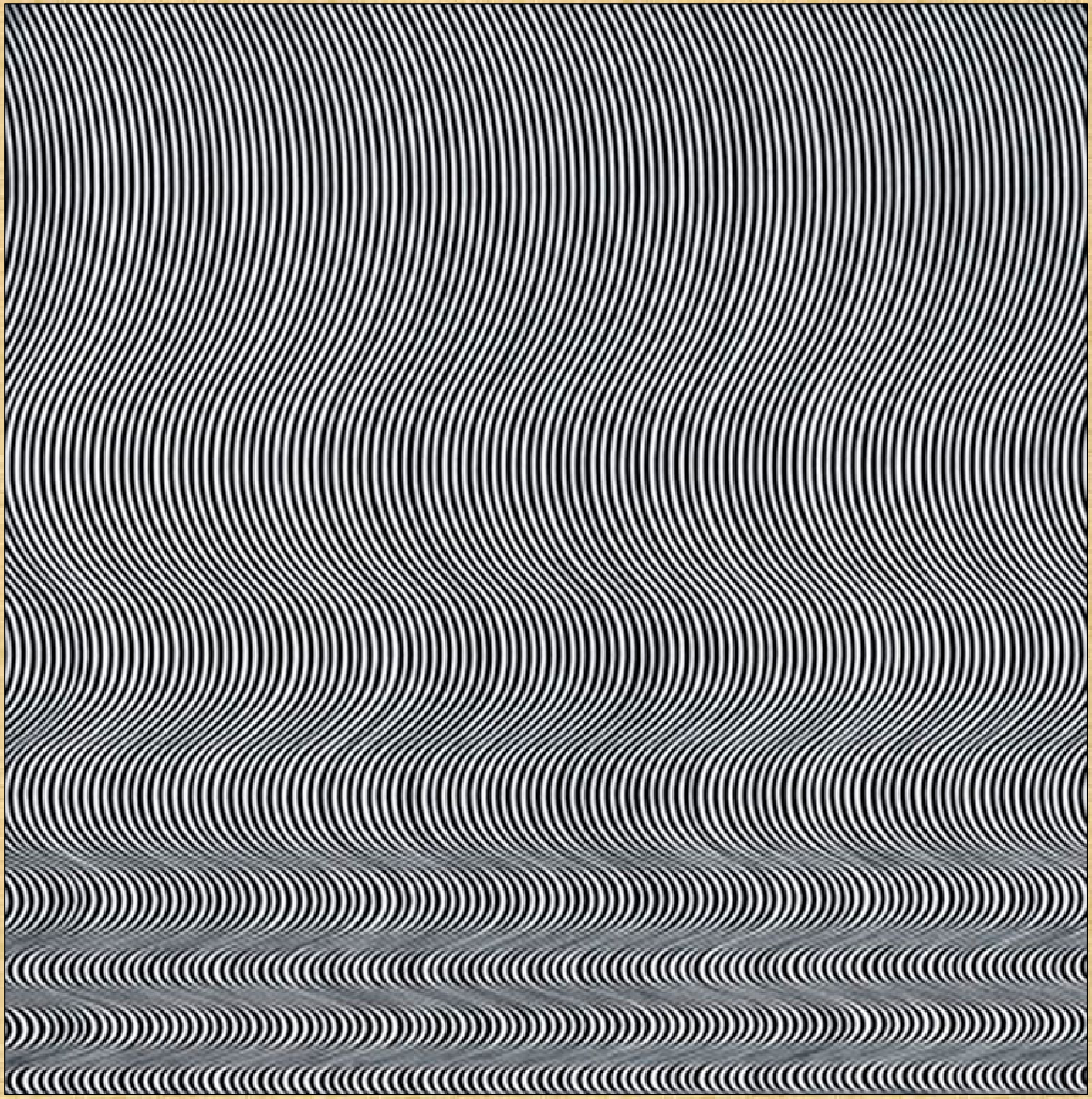




Perception

Sometimes symmetry can mislead

Or give a sense of motion



Language

For evaluating companies

Language is arithmetic

For Abstract Art

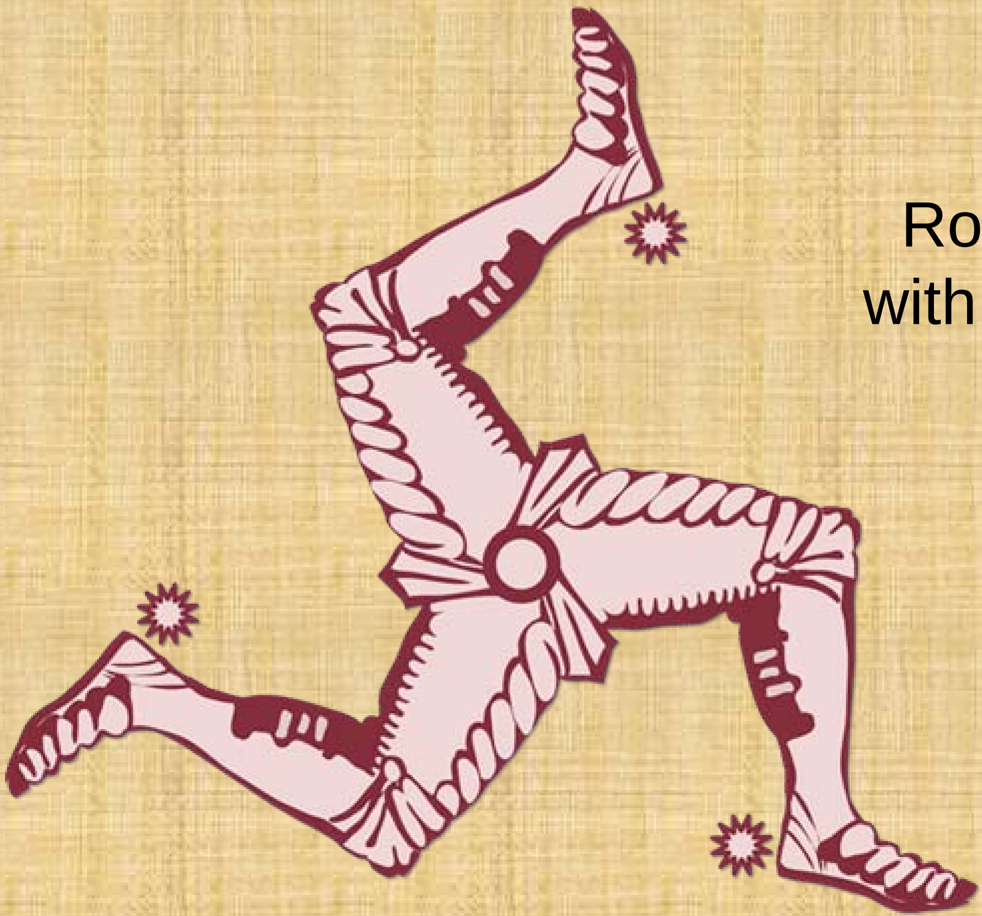
Language is color

For symmetries

Language is Group Theory

A *group* is a set and an operation that obey four rules:

Closure   *Associativity*   *Identity*   *Inverse*



Rotation by: 120°, 240°, 360°
with the operation "followed by."

Integers:
… -5, -4, -3, -2, -1,
0, 1, 2, 3, 4, 5, …
with the simple
operation of addition.

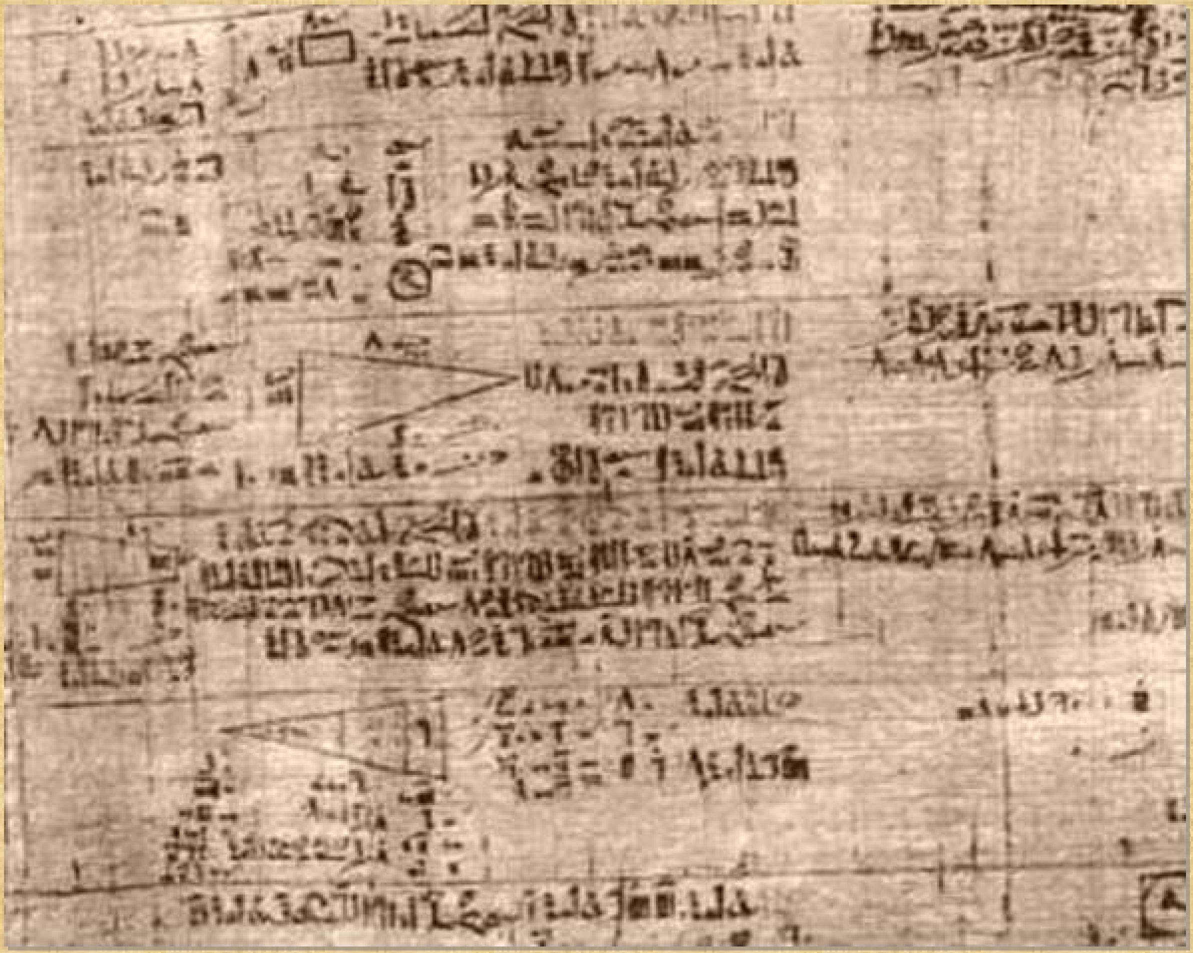
The symmetry transformations of any system form a GROUP

How Was This Language Invented?

Linear equations

$ax + b = c$

$2x + 5 = 7$



The Ahmes Papyrus (Egyptian, Middle Kingdom, ~2000-1800 BCE)

Quadratic Equations

$$ax^2 + bx + c = 0$$

$$x^2 + 3x - 4 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Plimpton 322 cuneiform tablet
(Old Babylonian, ~1900-1600 BCE)

Page from al-Khwarizmi's Kitab al-Jabr w'al-Muqabala, the oldest Arabic work on algebra (Baghdad, 9th century CE)

The Dramatic History of the Cubic and Quartic Equations

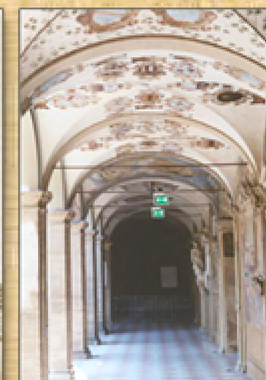
$$ax^3 + bx^2 + cx + d = 0$$
$$2x^3 + x^2 + x - 4 = 0$$

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$
$$x^4 + 2x^3 + x^2 + x - 5 = 0$$

Inscribed stone marking the birthplace of Scipione Dal Ferro

Nicolo Tartaglia (1499–1557)

Girolamo Cardano (1501–1576)



Corridor in the old building of the University of Bologna


HIERONYMI CAR
DANI, PRÆSTANTISSIMI MATHE
MATICI, PHILOSOPHI, AC MEDICI,
ARTIS MAGNÆ,
SIVE DE REGVLIS ALGEBRAICIS,
OPVS PERFECTVM

Cardano's Ars Magna (1545)

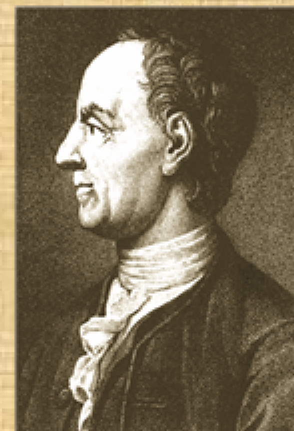
The Story of the Quintic Equation

$$ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$$

James Gregory
(1638–1675)

Ehrenfried Tschirnhaus
(1651–1708)

Leonhard Euler
(1707–1783)



Erland Samuel Bring
(1736–1798)

Edward Waring
(1736-1798)

Joseph Lagrange
(1736–1813)

Carl Friedrich Gauss
(1777–1855)

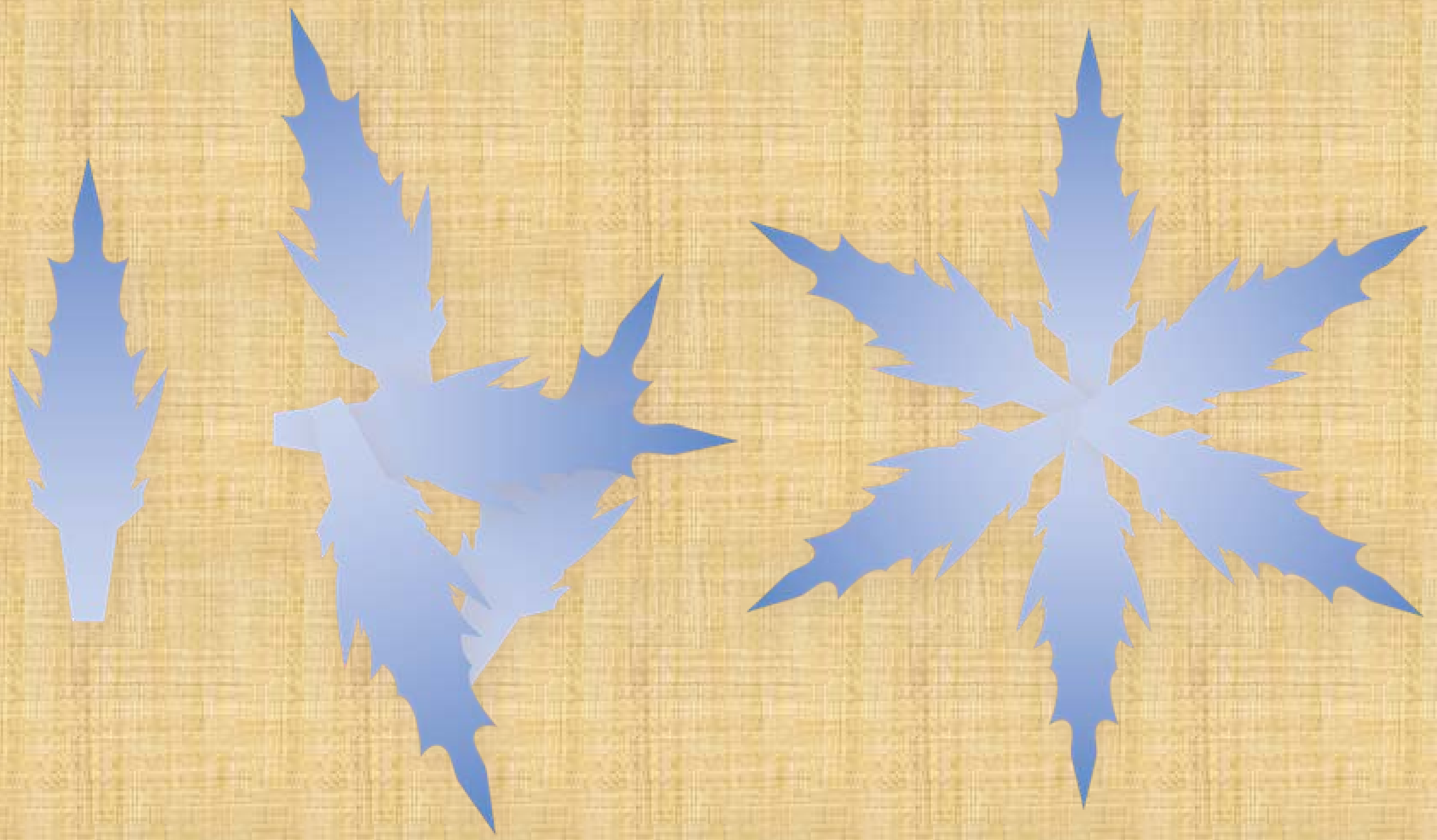
Paolo Ruffini
(1765–1822)

Two young men who *proved* that the quintic equation *cannot be solved by a formula!*

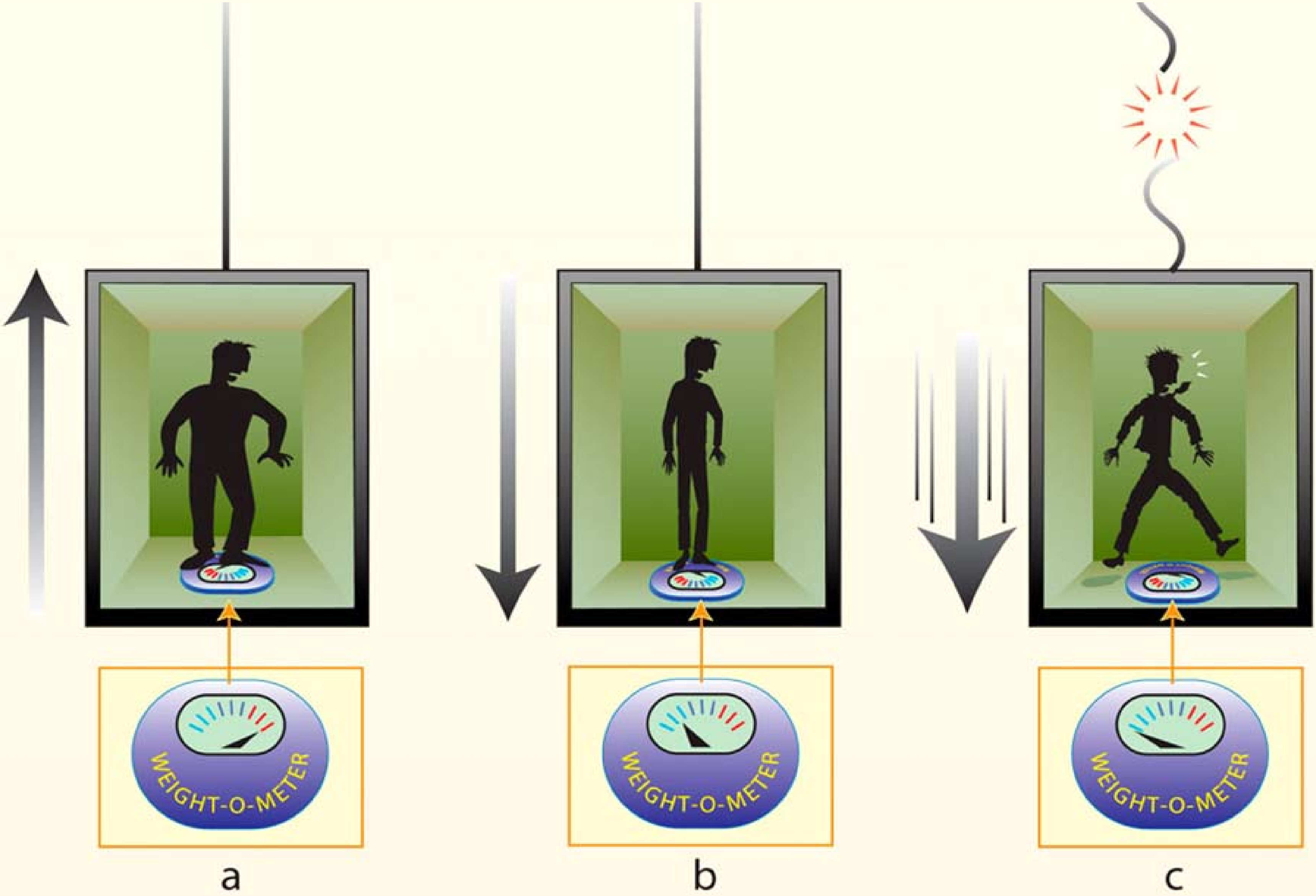
Niels Abel
1802-1829, Norway

Evariste Galois
1811-1832, France

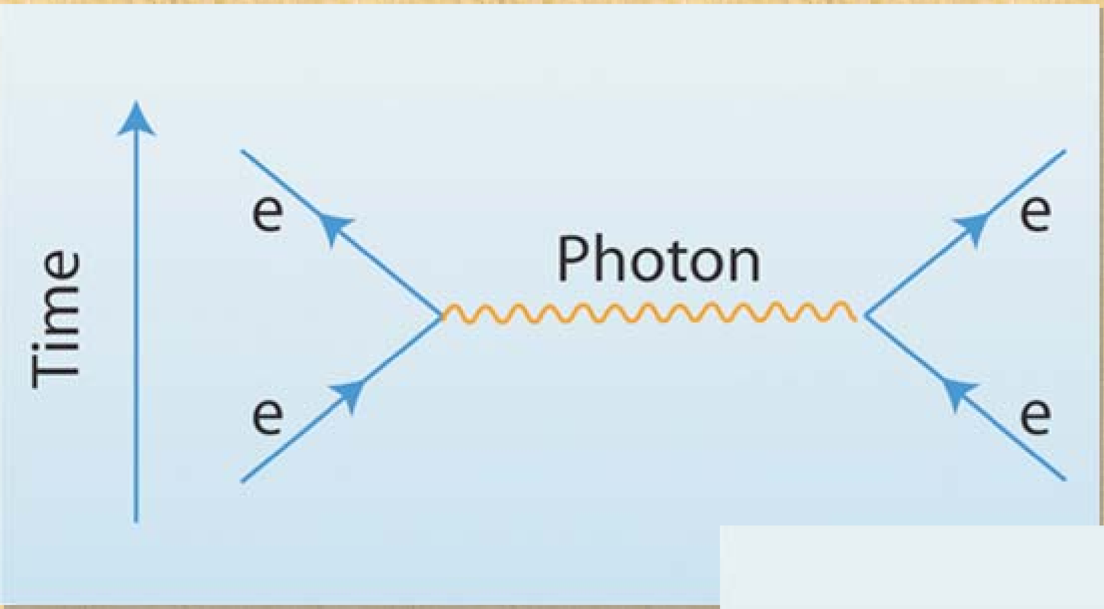
Why is symmetry so crucial in theories of the universe?

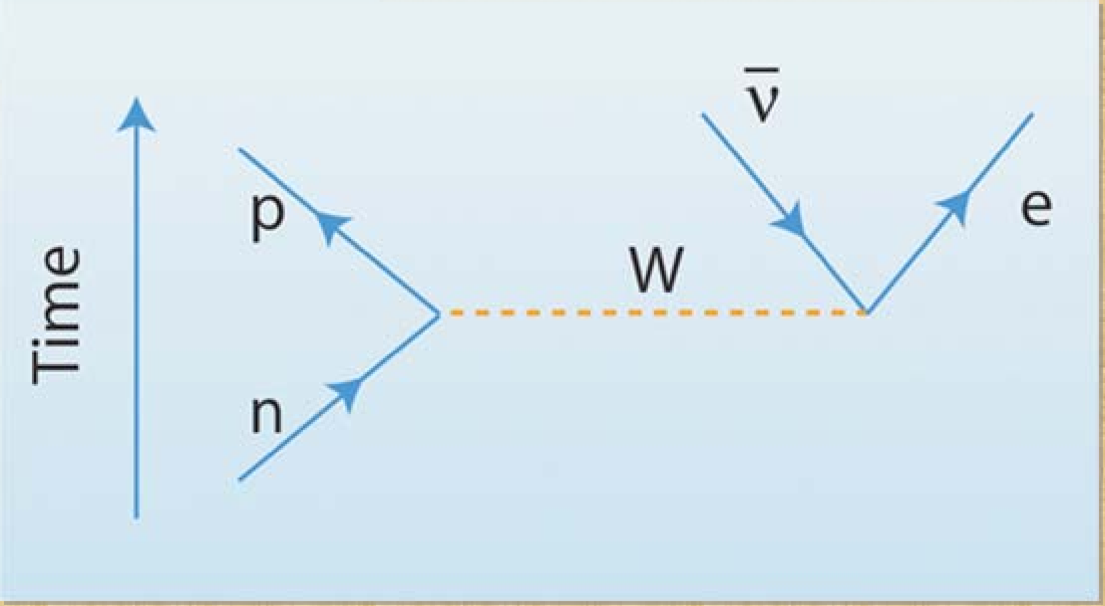


Einstein's Equivalence Principle



The Electroweak Theory

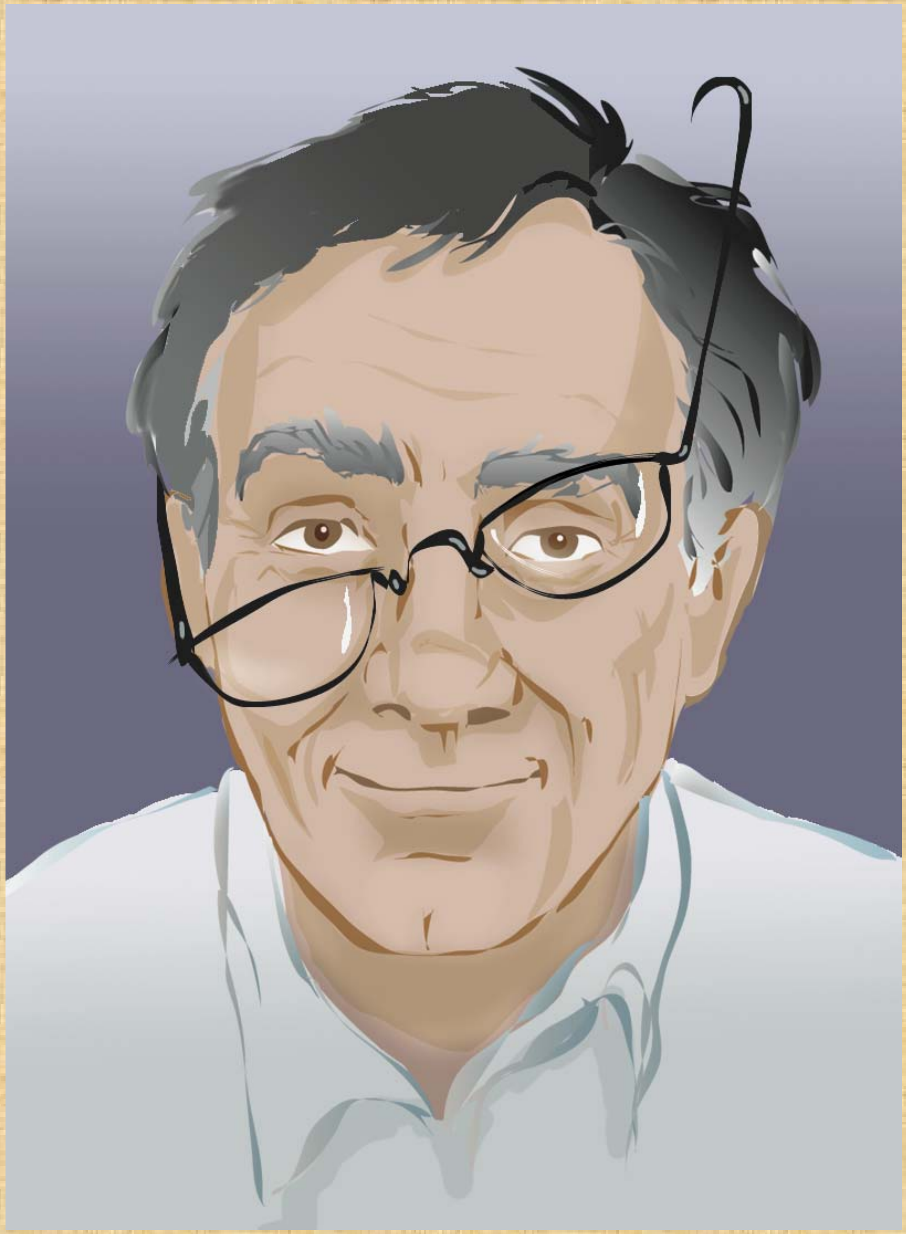




Is symmetry truly fundamental in the universe?

Or is the human mind fine-tuned to latch onto only the symmetric aspects?

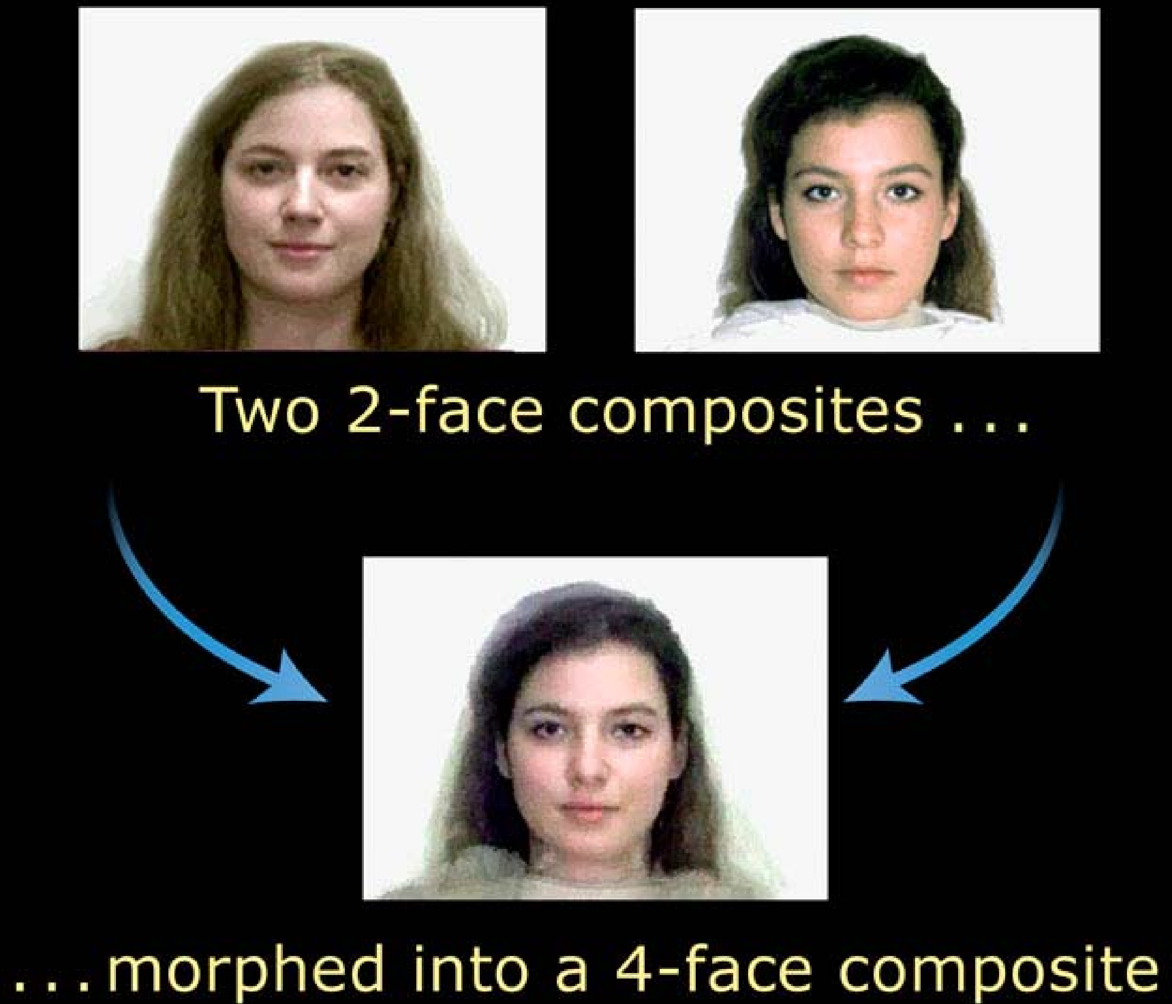
QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.



What do we find attractive?

Two individual (unmorphed) faces . . .

. . . morphed into a 2-face composite


Two 2-face composites ...
...morphed into a 4-face composite

Two 4-face composites ...

...morphed into an 8-face composite

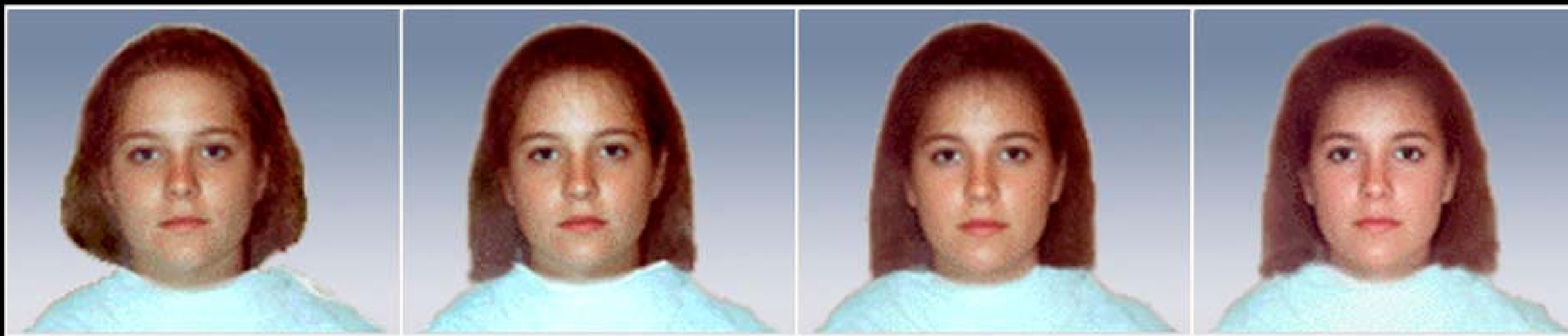
Two 8-face composites . . .

. . . morphed into a 16-face composite

Two 16-face composites ...

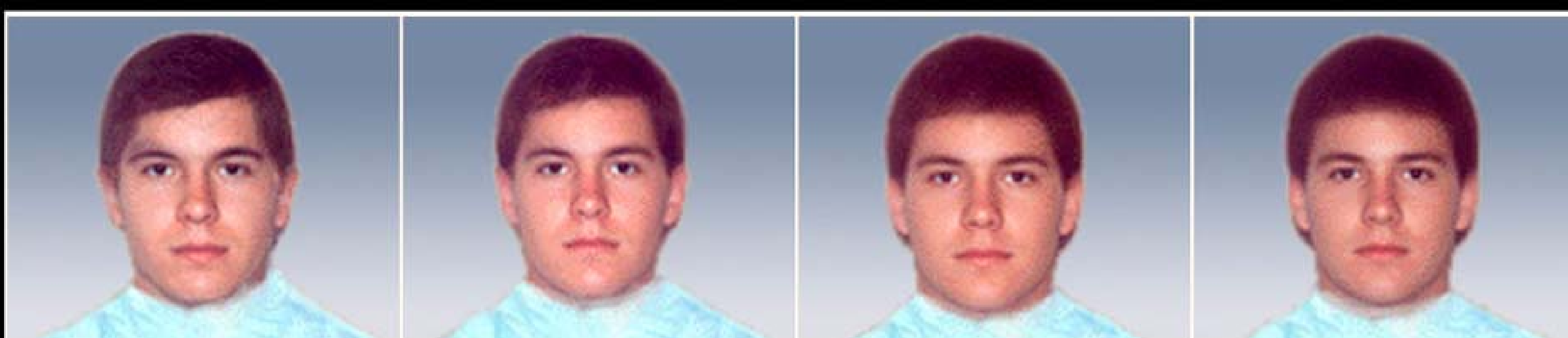
...morphed into a 32-face composite

Mathematically averaged Caucasian female faces



4-face composite 8-face composite 16-face composite 32-face composite

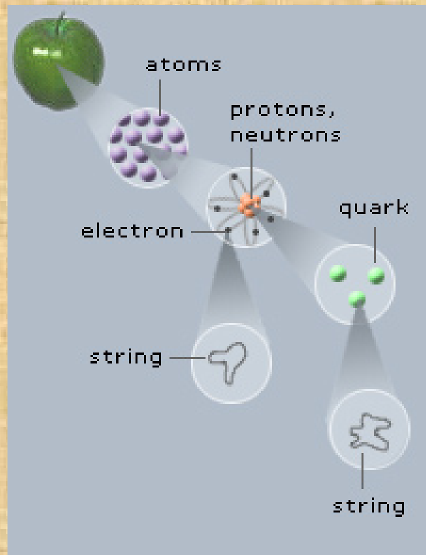
Mathematically averaged Caucasian male faces

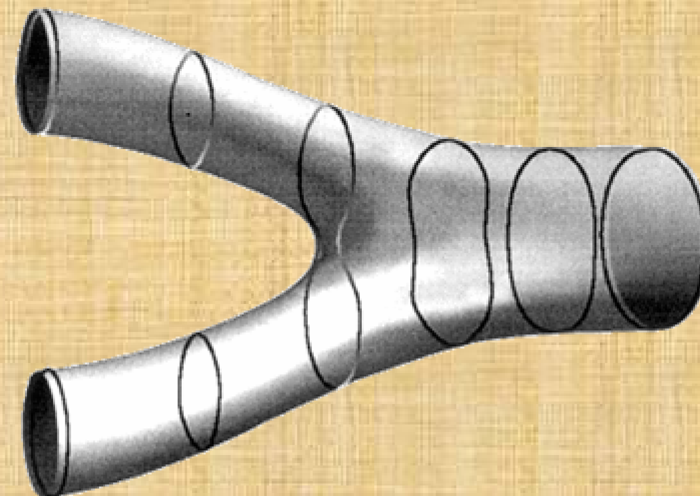


4-face composite 8-face composite 16-face composite 32-face composite

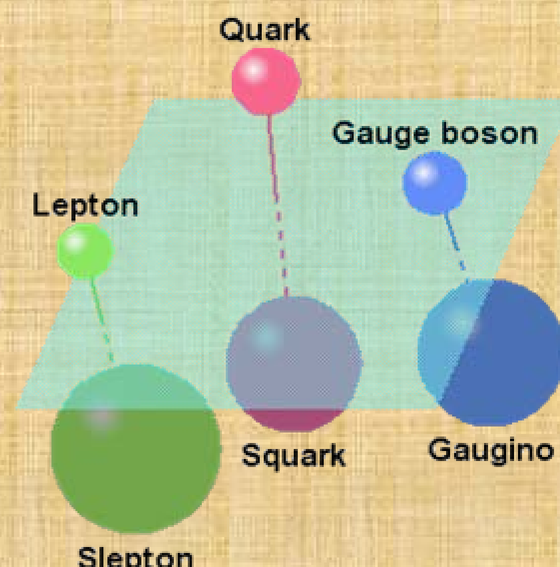
Our minds are tuned to prefer symmetry.

String Theory





Supersymmetry is an Output



Not clear yet what the fundamental principle is.

"So our problem is to explain where symmetry comes from. Why is nature so nearly symmetrical? No one has any idea why. The only thing we might suggest is something like this: There is a gate in Japan, a gate in Neiko, which is sometimes called by the Japanese the most beautiful gate in all Japan; it was built in a time when there was great influence from Chinese art. This gate is very elaborate, with lots of gables and beautiful carving and lots of columns and dragon heads and princes carved into the pillars, and so on. But when one looks closely he sees that in the elaborate and complex design along one of the pillars, one of the small design elements is carved upside down; otherwise the thing is completely symmetrical. If one asks why this is, the story is that it was carved upside down so that the gods will not be jealous of the perfection of man. So they purposely put an error in there, so that the gods would not be jealous and get angry with human beings. We might like to turn the idea around and think that the true explanation of the near symmetry of nature is this: that God made the laws only nearly symmetrical so that we should not be jealous of His perfection!"

Richard Feynman

QuickTime™ and a
TIFF (Uncompressed) decompress
are needed to see this picture.

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