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AN EVALUATION OF HIGHER-ORDER MODAL METHODS FOR CALCULATING TRANSIENT STRUCTURAL RESPONSE

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SUMMARY

The present study evaluates a higher-order modal method proposed by Leung for transient structural analysis entitled the force-derivative method. This method repeatedly integrates by parts with respect to time the convolution-integral form of the structural response to produce successively better approximations to the contribution of the higher modes which are neglected in the modal summation. Comparisons are made of the force-derivative, the mode-displacement, and the mode-acceleration methods for several numerical example problems for various times, levels of damping, and forcing functions. The example problems include a tip-loaded cantilevered beam and a simply-supported multispan beam. The force-derivative method is shown to converge to an accurate solution in fewer modes than either the mode-displacement or the mode-acceleration methods. In addition, for problems in which there are a large number of closely-spaced frequencies whose mode shapes have a negligible contribution to the response, the force derivative method is very effective in representing the effect of the important, but otherwise neglected, higher modes.

SYMBOLS

A	cross-sectional area
$[C]$	damping matrix
E	modulus of elasticity
e	error norm (see eq. (17))
$F_i(t)$	i^{th} modal forcing function
I	moment of inertia
$[K]$	stiffness matrix
L	beam length
$[M]$	mass matrix
M	bending moment
m	number of modes used in truncated modal summation
n	total number of degrees of freedom
$\{Q\}$	applied non-dimensional force vector ($Q=qL^2/EI$ or $Q=qL^3/EI$)
q	applied load
$\bar{q}$	distributed load
r_i	frequency ratio $\frac{\gamma}{\omega_i}$
S	shear force
t	time
T	normalized time $T = \omega_0 t$
$\{U\}$	structural displacement response vector
x,y,z	cartesian coordinates
$Y_i(t)$	i^{th} modal displacement function

Greek:

γ	forcing frequency
$\{\delta\}$	vector of difference between converged response and approximated response using m modes
ζ_i	i^{th} modal viscous damping factor
$\mu(t)$	unit step function
ρ	mass density
τ	dummy variable of integration
$\{\phi\}_i$	i^{th} natural vibration mode
$[\Phi]$	modal matrix
$[\Omega]$	diag $[\omega_1,\omega_2 \ldots \omega_n]$ diagonal matrix of circular natural frequencies
ω_{d_i}	i^{th} circular frequency of the damped free vibration
ω_i	i^{th} circular natural frequency
ω_0	normalizing frequency $\omega_0 = \sqrt{\frac{EI}{\rho A L^4}}$

Subscripts:

c	converged
m	response using a truncated set of modes (1 to m)
o	initial

Superscripts:

$(\cdot)$	differentiation with respect to time
(n)	n^{th} derivative with respect to time

INTRODUCTION

Dynamic analysis of complicated structures which are modeled as discrete multidegree-of-freedom systems often requires the solution of very large systems of equations. Reducing the order of such systems is highly desirable from the standpoint of increased computational efficiency. Some of the many methods for reducing the order of discrete multidegree-of-freedom dynamic systems include mass condensation methods (refs. 1-2) and reduced basis methods (refs. 3-7). The reduced basis methods use the expansion theorem and a truncated set of basis vectors (e.g. undamped free-vibration modes (modal methods) or Ritz-vectors) to approximate the dynamic response. Some problems such as large space structures require a large number of basis vectors to accurately represent the dynamic response and in some cases, such as when singularities occur in the loading, convergence of a solution is not guaranteed. Also, for problems which require the derivatives of the response with respect to a design parameter (e.g. system identification or optimization problems) not only is the size of the problem increased but the convergence of the derivative equation is not generally guaranteed.

When a reduced basis method uses the natural vibration modes of the structure the method is referred to as a modal method. Two of the most widely-used modal methods are the mode-displacement method (MDM) and the mode-acceleration method (MAM). Comparisons of the MDM and MAM methods (ref. 6) indicate that the MAM converges to an accurate solution with fewer modes than the MDM. One reason for this improved convergence is that the MAM incorporates a pseudo-static response (the inverse of the stiffness matrix multiplied by the applied forcing function) which approximates, to some degree, the flexibility of the higher modes which are neglected in the modal summation. The work of reference 7 reveals that the MAM method can be derived by integrating-by-parts with respect to time the convolution-integral form of the original MDM method. It is also shown in reference 7 that higher-order modal methods may be obtained by further integration-by-parts. Thus, integrating the convolution integral two more times produces a higher-order modal method (ref. 7) than the MAM which is called here the force-derivative method (FDM). It is called the force-derivative method because, analogous to the MAM, the FDM produces a term which is a function of the forcing function and additional terms which are functions of the time-derivatives of the forcing function. These additional terms produce successively higher-order approximations to the higher, neglected modes. It is shown that for the case of zero damping the MAM is a higher-order method than the MDM, and can be derived by integrating the convolution integral by parts two times. The purpose of the present study is to evaluate the use of higher-order modal methods which have the ability to significantly reduce the number of degrees-of-freedom (modes) necessary to accurately represent the transient structural response. The present study extends the work of reference 7 to include modal damping and clarifies the relation of the MDM and MAM to the FDM.

The rate of convergence or accuracy of each of the modal summation methods (MDM, MAM, and FDM) is investigated with respect to the differentiability of the forcing function, the level of damping, and the time at which the response is calculated. A relative, spatial error norm is used to measure accuracy and convergence of the transient response. The forcing functions were chosen to illustrate the effect of a continuous forcing function (with respect to time) which has continuous higher derivatives and a discontinuous forcing function (a unit step function) on the solution. A quintic function of time was selected to illustrate what happens when the force or one of its derivatives vanish at some point in time. The convergence for a sinusoidal forcing function is also studied. A series of numerical examples has been selected to illustrate the adequacy and/or inadequacy of each method: (1) a uniform cantilevered beam subject to a tip loading condition of (a) a unit step load input, and (b) a quintic function of time; and (2) a simply-supported multispan beam (10 equal length spans) subjected to (a) a uniformly distributed load varying as a quintic function of time and b) two concentrated loads located about the center of the first span and varying as a quintic function of time.

UNIFIED DERIVATION OF MODAL METHODS

The equations of motion, in matrix form, of an n-degree of freedom system, together with the initial conditions, are given by

$$[M]\{\ddot{U}(t)\} + [C]\{\dot{U}(t)\} + [K]\{U(t)\} = \{Q(t)\} \tag{1}$$

$$\{U(o)\} = \{U_o\}, \; \{\dot{U}(o)\} = \{\dot{U}_o\}$$

where [M], [C], and [K] are the mass, damping, and stiffness matrices of the structure; U(t) and Q(t) are the displacement and force vectors, respectively, and a dot represents differentiation with respect to time. For certain forms of [C], equation (1) can be transformed into a set of uncoupled equations of motion using the undamped normal modes, $\{\phi\}_i$, of the structure and the expansion theorem, namely, by

$$\{U(t)\} = \sum_{i=1}^{n} \{\phi\}_i Y_i(t) \tag{2}$$

where the natural vibration modes $\{\phi\}_i$ are obtained from the solution of the following eigenvalue problem

$$[K]\{\phi\}_i = \omega_i^2 [M]\{\phi\}_i \tag{3}$$

The modes $\{\phi\}_i$ may be normalized such that

$$\{\phi\}_i^T [M]\{\phi\}_i = 1. \quad \text{and then} \quad \{\phi\}_i^T [K]\{\phi\}_i = \omega_i^2 \tag{4a}$$

Using the modal matrix $[\Phi]$ which contains all n modes equations 4(a) can be written as

$$[\Phi]^T [M][\Phi] = [I] \quad \text{and} \quad [\Phi]^T [K][\Phi] = [\Omega^2] \tag{4b}$$

In modal coordinates, equation (1) can be expressed

$$\ddot{Y}_i(t) + 2\zeta_i\omega_i\dot{Y}_i(t) + \omega_i^2 Y_i(t) = F_i(t) \qquad i=1,2,3...n \tag{5}$$

$$Y_{o_i} = \{\phi\}_i^T[M]\{U_o\}, \quad \dot{Y}_{o_i} = \{\phi\}_i^T[M]\{\dot{U}_o\}$$

where the i^{th} modal force $F_i(t)$, is defined by

$$F_i(t) = \{\phi\}_i^T \{Q(t)\} \tag{6}$$

and ζ_i is the i^{th} modal viscous damping factor.

The mode-displacement method (MDM) calculates m-values of $Y_i(t)$ by solving the first m-equations (eq. 5) and substituting this into a truncated form of equation (2).

$$\{U(t)\} \approx \sum_{i=1}^{m} \{\phi\}_i Y_i(t) \qquad \text{where } m \leq n \tag{7}$$

The solution of equation (5) can be written by using a convolution integral as follows

$$Y_i(t) = e^{-\zeta_i\omega_i t}\left[\cos(\omega_{d_i}t)\, Y_{o_i} + \frac{1}{\omega_{d_i}}(\dot{Y}_{o_i} + \zeta_i\omega_i Y_{o_i})\sin(\omega_{d_i}t)\right] \tag{8}$$

$$+ \frac{1}{\omega_{d_i}} \int_0^t e^{-\zeta_i\omega_i(t-\tau)} \sin(\omega_{d_i}(t-\tau))\{F_i(\tau)\}\, d\tau$$

where

$$\omega_{d_i} = \omega_i\sqrt{1 - \zeta_i^2}$$

Substituting equation (8) into equation (7) gives

$$\{U(t)\} = \sum_{i=1}^{m} \{\{\phi\}_i[e^{-\zeta_i\omega_i t}(\cos(\omega_{d_i}t)\, Y_{o_i} + \frac{1}{\omega_{d_i}}(\dot{Y}_{o_i} + \zeta_i\omega_i Y_{o_i})\sin(\omega_{d_i}t))] \tag{9}$$

$$+ \{\phi\}_i[\frac{1}{\omega_{d_i}} \int_0^t e^{-\zeta_i\omega_i(t-\tau)} \sin(\omega_{d_i}(t-\tau))\, F_i(\tau)\, d\tau]\}$$

Equation (9) is an equivalent representation of the MDM. Integrating the convolution integral of equation (9) by parts twice with respect to time and assuming the forcing function and its first derivative are continuous over the interval gives

$$\{U(t)\} = \sum_{i=1}^{m}\Big\{ \{\phi\}_i[e^{-\zeta_i\omega_i t}(\cos(\omega_{d_i}t)\, Y_{o_i} + \frac{1}{\omega_{d_i}}(\dot{Y}_{o_i} + \zeta_i\omega_i Y_{o_i})\sin(\omega_{d_i}t))]$$

$$+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{-1}{\omega_i^2}(\frac{\zeta_i\omega_i}{\omega_{d_i}}\sin(\omega_{d_i}t) + \cos(\omega_{d_i}t))]\{\phi\}_i^T\{Q_o\} \tag{10}$$

$$+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{1}{\omega_i^4}((\frac{(\zeta_i\omega_i)^2-\omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}t) + 2\zeta_i\omega_i\cos(\omega_{d_i}t))]\{\phi\}_i^T\{\dot{Q}_o\}$$

$$+ \{\phi\}_i[\frac{1}{\omega_i^4}\int_0^t e^{-\zeta_i\omega_i(t-\tau)}\{(\frac{(\zeta_i^2\omega_i) - \omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}(t-\tau))$$

$$+ 2\zeta_i\omega_i\cos(\omega_{d_i}(t-\tau))\}\{\phi\}_i^T\{\ddot{Q}_i(\tau)\}d\tau]$$

$$+ \{\phi\}_i[\frac{1}{\omega_i^2}]\{\phi\}_i^T\{Q(t)\} - \{\phi\}_i[\frac{2\zeta_i\omega_i}{\omega_i^4}]\{\phi\}_i^T\{\dot{Q}(t)\}\Big\}$$

If the forcing function or its first derivative were discontinuous at some point in time the appropriate jump conditions would have to be included in equation (10).

If all the modes are used in calculating the last two terms of equation (10) these terms can be represented as

$$\sum_{i=1}^{n}\{\phi\}_i(\frac{1}{\omega_i^2})\{\phi\}_i^T\{Q(t)\} = [\Phi][\Omega^{-2}][\Phi]^T\{Q(t)\} = [\Phi][[\Phi]^T[K][\Phi]]^{-1}[\Phi]^T = [K]^{-1}\{Q(t)\} \tag{11}$$

and similarly

$$\sum_{i=1}^{n}\{\phi\}_i(\frac{2\zeta_i\omega_i}{\omega_i^4})\{\phi_i\}^T\{\dot{Q}(t)\} = [\Phi][\frac{2\zeta_i\omega_i}{\omega_i^4}][\Phi]^T\{\dot{Q}(t)\} = [K]^{-1}[C][K]^{-1}\{\dot{Q}(t)\}$$

where $[\Phi]$ is the complete (nxn) matrix of eigenvectors and $[\Omega^{-2}]$ is the diagonal matrix $[\frac{1}{\omega_1^2}, \frac{1}{\omega_2^2}, \ldots \frac{1}{\omega_n^2}]$ and $[\frac{2\zeta_i\omega_i}{\omega_i^4}]$ is the diagonal matrix

$$[\ \frac{2\zeta_1\omega_1}{\omega_1^4}\ , \frac{2\zeta_2\omega_2}{\omega_2^4}\ , \ldots \frac{2\zeta_n\omega_n}{\omega_n^4}\ ].$$

The term, $[K]^{-1}\{Q(t)\}$, is called the pseudo-static response in reference 6. Hence, if all the modes are used in the last term of equation (10) it is expressed as

$$\begin{aligned}\{U(t)\} = \sum_{i=1}^{m} \{ & \{\phi\}_i[e^{-\zeta_i\omega_i t}(\cos(\omega_{d_i}t)\, Y_{o_i} + \frac{1}{\omega_{d_i}}(\dot{Y}_{o_i} + \zeta_i\omega_i Y_{o_i})\sin(\omega_{d_i}t))] \\ & + \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{-1}{\omega_i^2}(\frac{\zeta_i\omega_i}{\omega_{d_i}}\sin(\omega_{d_i}t) + \cos(\omega_{d_i}t))]\{\phi\}_i^T\{Q_o\} \qquad (12) \\ & + \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{1}{\omega_i^4}((\frac{(\zeta_i\omega_i)^2 - \omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}t) + 2\zeta_i\omega_i\cos(\omega_{d_i}t))]\{\phi\}_i^T\{\dot{Q}_o\} \\ & + \{\phi\}_i[\frac{1}{\omega_i^4}\int_0^t e^{-\zeta_i\omega_i(t-\tau)}\{(\frac{(\zeta_i^2\omega_i) - \omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}(t-\tau)) \\ & \qquad + 2\zeta_i\omega_i\cos(\omega_{d_i}(t-\tau))\}\{\phi\}_i^T\{\ddot{Q}_i(\tau)\}d\tau]\ \} \\ & + [K]^{-1}\{Q(t)\} - [K]^{-1}[C][K]^{-1}\{\dot{Q}(t)\}\end{aligned}$$

For the undamped case, equation (12) can be easily shown to be equivalent to the MAM given as

$$\{U(t)\} \approx [K]^{-1}\{Q(t)\} - \sum_{i=1}^{m}\{\phi\}_i[(\frac{1}{\omega_i^2})\ddot{Y}_i(t)] \qquad (13)$$

As shown by equation (13) the MAM uses the modal expansion to approximate the deviation from a pseudo steady-state response. It can be expected to perform better than the MDM for low frequency excitation, where the response can track the load well. The pseudo-static term, by making use of the inverse of the stiffness matrix of the structure, is a first-order approximation to the flexibility of the higher, neglected modes.

The force derivative method makes use of the fact that successive integrations by parts of the convolution integral produces terms which are functions of the natural frequency to successively higher negative powers and terms which, if all the modes are used in the summation, can be represented by the system mass, stiffness, and damping matrices and the forcing function and its derivatives. Hence, higher-order methods can be developed by successively integrating the convolution integral by parts.

Integrating the convolution integral of equation (10) two more times, one obtains

$$\{U(t)\} = \sum_{i=1}^{m}\Big\{ \{\phi\}_i[e^{-\zeta_i\omega_i t}\{\cos(\omega_{d_i}t) + \frac{1}{\omega_{d_i}}(\dot{Y}_{o_i} + \zeta_i\omega_i Y_{o_i})\sin(\omega_{d_i}t)\}]$$

$$+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{-1}{\omega_i^2}(\frac{\zeta_i\omega_i}{\omega_{d_i}}\sin(\omega_{d_i}t) + \cos(\omega_{d_i}t))\{\phi\}_i^T\{Q_o\}] \qquad (14)$$

$$+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{1}{\omega_i^4}((\frac{(\omega_i\zeta_i)^2-\omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}t) + 2\zeta_i\omega_i\cos(\omega_{d_i}t))]\{\phi\}_i^T\{\dot{Q}_o\}$$

$$+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\{\frac{1}{\omega_i^4}((\frac{\zeta_i\omega_i(3-4\zeta_i^2)}{\omega_{d_i}})\sin(\omega_{d_i}t) + (1-4\zeta_i^2)\cos(\omega_{d_i}t))\}]\{\phi\}_i^T\{\ddot{Q}_o\}$$

$$+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\{\frac{1}{\omega_i^4}((\frac{1-8\zeta_i^2+8\zeta_i^4}{\omega_{d_i}})\sin(\omega_{d_i}t) + \frac{(8\zeta_i^3-4\zeta_i)}{\omega_i}\cos(\omega_{d_i}t))\}]\{\phi\}_i^T\{Q_o\}^{(3)}$$

$$+ \{\phi\}_i[\int_0^t \frac{e^{-\zeta_i\omega_i(t-\tau)}}{\omega_i^4}[\frac{(1-8\zeta_i^2+8\zeta_i^4)}{\omega_{d_i}}\sin(\omega_{d_i}(t-\tau))$$

$$+ \frac{(8\zeta_i^2-4\zeta_i)}{\omega_i}\cos(\omega_{d_i}(t-\tau))][\phi\}_i^T\{Q(\tau)\}^{(4)}d\tau]$$

$$+ \{\phi\}_i[\frac{1}{\omega_i^2}]\{\phi\}_i^T\{Q(t)\} - \{\phi\}_i[\frac{2\zeta_i\omega_i}{\omega_i^4}]\{\phi\}_i^T\{\dot{Q}(t)\} - \{\phi\}_i[\frac{(1-4\zeta_i^2)}{\omega_i^4}]\{\phi\}_i^T\{\ddot{Q}(t)\}$$

$$- \{\phi\}_i[\frac{(8\zeta_i^3-4\zeta_i)}{\omega_i^5}]\{\phi\}_i^T\{Q(t)\}^{(3)}\Big\}$$

where

$$\{Q(o)\} = \{Q_o\},\ \{\dot{Q}(o)\} = \{\dot{Q}_o\},\ \{\ddot{Q}(o)\} = \{\ddot{Q}_o\},\ \text{and}\ \{Q(o)\}^{(3)} = \{Q_o\}^{(3)}$$

if all the modes are used in the modal summation of the last four terms they can be represented as

$$\sum_{i=1}^{n} \{\phi\}_i[\frac{1}{\omega_i^2}]\{\phi\}_i^T = [K]^{-1}$$

$$\sum_{i=1}^{n} \{\phi\}_i[\frac{2\zeta_i\omega_i}{\omega_i^4}]\{\phi\}_i^T = [K]^{-1}[C][K]^{-1} \tag{15}$$

$$\sum_{i=1}^{n} \{\phi\}_i[\frac{(1-4\zeta_i^2)}{\omega_i^4}]\{\phi\}_i^T = [K]^{-1}[M][K]^{-1} - [K]^{-1}[C][K]^{-1}[C][K]^{-1}$$

and

$$\{\phi\}_i[\frac{(8\zeta_i^3-4\zeta_i)}{\omega_i^5}]\{\phi\}_i^T = [K]^{-1}[C][K]^{-1}[C][K]^{-1}[C][K]^{-1} - [K]^{-1}[C][K]^{-1}[M][K]^{-1}$$

Using equation (15), equation(14) becomes

$$\begin{aligned}\{U(t)\} = \sum_{i=1}^{m}\{ &\{\phi\}_i[e^{-\zeta_i\omega_i t}\{\cos(\omega_{d_i}t) + \frac{1}{\omega_{d_i}}(\dot{Y}_{o_i} + \zeta_i\omega_i Y_{o_i})\sin(\omega_{d_i}t)\}] \\
&+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{-1}{\omega_i^2}\{\frac{\zeta_i\omega_i}{\omega_{d_i}}\sin(\omega_{d_i}t) + \cos(\omega_{d_i}t)\}\{\phi\}_i^T\{Q_o\}] \\
&+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{1}{\omega_i^4}\{(\frac{(\omega_i\zeta_i)^2-\omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}t) + 2\zeta_i\omega_i\cos(\omega_{d_i}t)\}]\{\phi\}_i^T\{\dot{Q}_o\} \\
&+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\{\frac{1}{\omega_i^4}((\frac{\zeta_i\omega_i(3-4\zeta_i^2)}{\omega_{d_i}})\sin(\omega_{d_i}t) + (1-4\zeta_i^2)\cos(\omega_{d_i}t))\}]\{\phi\}_i^T\{\ddot{Q}_o\} \\
&+ \{\phi\}_i[e^{-\zeta_i\omega_i t}\{\frac{1}{\omega_i^4}((\frac{1-8\zeta_i^2+8\zeta_i^4}{\omega_{d_i}})\sin(\omega_{d_i}t) + \frac{(8\zeta_i^3-4\zeta_i)}{\omega_i}\cos(\omega_{d_i}t))\}]\{\phi\}_i^T\{Q_o\}^{(3)}\end{aligned} \tag{16}$$

$$+ \{\phi\}_i [\int_0^t \frac{e^{-\zeta_i \omega_i (t-\tau)}}{\omega_i^4} [\frac{(1-8\zeta_i^2+8\zeta_i^4)}{\omega_{d_i}} \sin(\omega_{d_i}(t-\tau))$$

$$+ \frac{(8\zeta_i^2-4\zeta_i)}{\omega_i} \cos(\omega_{d_i}(t-\tau))] \{\phi\}_i^T \{Q(\tau)\}^{(4)} d\tau] \} \quad + [K]^{-1}\{Q(t)\}$$

$$- [K]^{-1}[C][K]^{-1}\{\dot{Q}(t)\} - [[K]^{-1}[M][K]^{-1} - [K]^{-1}[C][K]^{-1}[C][K]^{-1}]\{\ddot{Q}(t)\}$$

$$- [[K]^{-1}[C][K]^{-1}[C][K]^{-1}[C][K]^{-1} - [K]^{-1}[C][K]^{-1}[M][K]^{-1}]\{Q(t)\}^{(3)}$$

Hence, analogous to the MAM for zero damping which retains a first order approximation to the effect of the neglected modes by the $[K]^{-1}$ Q(t) or $[\Phi][\Omega^{-2}][\Phi]^T\{Q(t)\}$ term, the FDM offers a second-order approximation to the neglected modes by including the $[K]^{-1}[M][K]^{-1}\{\ddot{Q}(t)\}$ or $[\Phi][\Omega^{-4}][\Phi]^T\{\ddot{Q}(t)\}$ term. Results of reference 7 indicate that this higher-order-approximation can significantly reduce the number of modes required for convergence, depending on the nature of the forcing function. Higher-order approximations than those presented for the FDM (using four integrations by parts) can be developed by succesively integrating the convolution integral by parts several more times.

IMPLEMENTATION

The above equations were programmed for beam examples using analytical expressions for the mode shapes $\{\phi\}_i$ and modal displacements and their derivatives. Modal vectors assume 51 equally-spaced points along the length of beam. Moment and shear forces were calculated from $M(t)=EI\partial^2U(t)/\partial x^2$ and $S(t)=EI\partial^3U(t)/\partial x^3$. For most cases a converged solution was assumed to occur using 30 modes and, hence, a total of 30 modes were used to approximate expressions such as

$$[K]^{-1} \approx \sum_{i=1}^{30} \{\phi\}_i (\frac{1}{\omega_i^2}) \{\phi\}_i^T, \quad [K]^{-1}[M][K]^{-1} \approx \sum_{i=1}^{30} \{\phi\}_i (\frac{1}{\omega_i^4}) \{\phi\}_i^T \text{ , etc.}$$

Whenever necessary a total of 50 modes is used in the above expressions to represent a converged solution.

The following section investigates the effect of various forcing functions, load distributions, and damping levels on the transient response of a uniform cross-section cantilevered beam, and a uniform simply-supported multispan beam.

Cantilevered Beam With Tip Loading

The first problem studied is a uniform cantilevered beam under tip loading. The first 30 natural frequencies of the beam are listed in table 1. The beam is subjected to various levels of modal damping (same for all modes) and a variety of loading conditions which are described below.

Case 1: $Q(T)=1000(T^4-T^5)$.- This problem was presented in reference 7 for the case of zero damping ($\zeta_i=0$). The function or one of its derivatives vanishes at various times: $Q(T)=0$ when T=1.0, $\dot{Q}(T)=0$ when T=0.8, $\ddot{Q}(T)=0$ when T=0.6 $Q^{(3)}(T)=0$ when T=0.4, and $Q^{(4)}(T)=0$ when T=0.2 and this fact affects the convergence of the method as will be shown subsequently. As shown in figure 2 for $\zeta_i=0.05$ and T=1.2, the FDM offers an improvement in accuracy of several orders of magnitude in the error norm over either the MDM or the MAM methods. The advantage in using higher-order modal methods (either the MAM or the FDM) lies in the ability of those methods to approximate the flexibility of the higher, but neglected, modes with terms which are functions of the stiffness, mass, and damping matrices and the forcing function and, in the case of FDM, its derivatives (see for example eqns. 14 and 16). The FDM offers a higher-order approximation to the neglected modes by the use of additional terms in addition to the pseudo-static response ($[K]^{-1}\{Q(t)\}$). Figure 3 is a plot of the moment error norm (using 5 modes) as a function of time; at T=1.0, the moment error norm associated with the MDM is equivalent to the MAM value because at time T=1.0, Q(T)=0 and there is no difference between MDM and MAM (see eq. 12). A similar dip in the MAM error occurs at time T=0.8 which corresponds to a time when $\dot{Q}(T)=0$. These narrow regions where there is a sharp increase in solution accuracy can be anticipated a priori from the zeroes of the forcing function and its derivatives.

A comparison of displacement, moment, and shear errors for $\zeta_i=0.$ and times T=0.6 and 1.0 are shown in figures 4a and 4b, respectively. The error associated with the displacements is lowest, the moment errors are greater and the shear errors are the largest. This occurs because the moments and shears are functions of successively higher spatial derivatives of the displacements (ref. 6). When T=0.6 and $\zeta_i=0.$, the MAM and FDM methods coincide (fig. 4a) because $\ddot{Q}(T)=0$. When T=1.0 the MDM and MAM methods coincide (fig. 4b). From figure 4b, when T=1. and $\zeta_i=0$ the only difference between the MDM and MAM methods and the FDM lies in the term $\{\phi\}_i[\frac{1}{\omega_i^4}]\{\phi\}_i^T\{\ddot{Q}(T)\}$ (eq. 16).

The difference in errors between the methods is less at T=1.0 (Q(T)=0) than at T=0.6 (fig 4a) where the difference between the methods lies in the

$\{\phi\}_i[\frac{1}{\omega_i^2}]\{\phi\}_i^T\{Q(T)\}$ term. The higher-order terms are functions of the frequencies to successively higher negative exponents and, hence, should have a negligible effect as higher modes are used providing the time-function term does not grow proportionally.

RESULTS AND DISCUSSION

The convergence of each method (number of modes versus accuracy of the transient response) is measured by using a relative, spatial error norm. The error norm, e, of an approximation to the displacement vector based on the first m modes $\{U\}_m$ is given as

$$e = \sqrt{\frac{\{\delta\}^T\{\delta\}}{\{U\}_c^T\{U\}_c}} \tag{17}$$

where

$$\{\delta\} = \{U\}_c - \{U\}_m$$

$\{U\}_m$ and $\{U\}_c$ represents a converged solution (for most of this study the first 30 modes are assumed to provide a converged solution). Similarly, moment and shear error norms are calculated by substituting the moment or shear response vectors in equation 17.

The effectiveness of the error norm, e, as a measure of accuracy and convergence in quantifying the global error associated with each of the modal methods is demonstrated for a cantilever beam problem. For this problem $\omega_0 = \sqrt{EI/\rho AL^4}$ and time t is normalized such that $T = \omega_0 t$. Figures 1a and 1b demonstrate the use of the error norm in describing the accuracy of displacement and moment distributions for a tip load $(Q(T) = 1000(T^4-T^5))$ at a normalized time T = 0.4 and for $\zeta_i = 0.05$. In figure 1a only one mode is used, and for the MDM e = 0.045 and displacement errors are noticeable, and MAM and FDM results having error norms of e = 0.00627 and e = 0.0000317, respectively, are indistinguishable from the converged solution (MDM(30)). Similarly, for the normalized moment distribution (fig. 1b) the MDM (using the first two modes) has an error e = .128, the MAM and FDM (both using only the first mode) have errors of e = .0379 and e = .00019 respectively. There is a qualitative improvement in the solution (response distribution) as the error norm e decreases in magnitude.

The rate of convergence of each of the methods (MDM, MAM, and FDM) is expected to depend on the nature of the forcing function, the level of damping and the time at which the response is calculated. The forcing fuctions were selected to investigate the effects of continuous forcing functions with vanishing higher derivatives at various times ($Q(T) = 1000(T^4-T^5)$), the effect of a discontinuous forcing function, a unit step at t = 0 $(Q(T) = \mu(T))$, and the effects of a sinusoidal forcing function $(Q(T)=\sin \gamma T)$. It is assumed that for all forcing functions Q(T) = 0 for T<0. Several example problems, described below, were selected to evaluate the accuracy of each method.

The effect of damping on the accuracy of the response is shown in figures 5a and 5b. Increasing the modal damping ζ_i does not always increase the accuracy of the MAM as suggested in reference 6. For the case studied in reference 6, a uniformly loaded cantilevered beam subjected to a step loading, the accuracy of the MAM is enhanced in the presence of damping as can be seen from equation (16). Since all the derivatives of the forcing function vanish, the only terms remaining are the pseudo-static response and a term which is a function of $e^{-\zeta_i \omega_i t}$. Hence, as ζ_i increases, the relative importance of this term as compared to the pseudo-static term ($[K]^{-1}\{Q(t)\}$) decreases and therefore the accuracy of the MAM increases.

For other forcing functions, terms such as $\{\phi\}_i[\frac{2\zeta_i\omega_i}{\omega_i^4}]\{\phi\}_i^T\{\dot{Q}(t)\}$ increase in importance as ζ_i increases and so the effect of ζ_i on accuracy is more complex. For example, the case of a linearly varying forcing function, $Q(t) = t$, the damped system response can be represented, assuming zero initial conditions, as (see eq. (16)):

$$\{U(t)\} \approx \sum_{i=1}^{m}\{\phi\}_i[e^{-\zeta_i\omega_i t}\frac{1}{\omega_i^4}[(\frac{(\omega_i\zeta_i)^2 - \omega_{d_i}^2}{\omega_{d_i}})\sin(\omega_{d_i}t) + 2\zeta_i\omega_i\cos(\omega_{d_i}t)]\{\phi\}_i^T\{\dot{Q}_o\}]$$

$$+ [\Phi][\Omega^{-2}][\Phi]^T\{Q(t)\} - [\Phi][\frac{2\zeta_i\omega_i}{\omega_i^4}][\Phi]^T\{\dot{Q}(t)\}$$

From this equation, the MAM assumes all the modes are used in the second to last term and it is represented as the pseudo-static response $[K]^{-1}\{Q(t)\}$. The remaining terms are represented by a truncated modal summation. The first term decreases exponentially as damping (ζ_i) increases but the last term increases proportionally to ζ_i. If the magnitude of this last term does not decrease with respect to the pseudo-static term the increase in damping will not necessarily result in an increase in accuracy as in the unit step function case.

<u>Case 2: $Q(T) = \mu(T)$</u>.- This forcing function is discontinuous at time T=0 and hence the integration by parts of the convolution integral must begin at a time $T=0^+$. The MAM and the FDM produce the same results for a step forcing function because for T>0, Q(T) is constant and all its derivatives vanish (see for example eq. (16)). As shown in figure 6, the FDM and MAM methods are more accurate than the MDM and require several fewer modes than the MDM for the same degree of accuracy. Figure 7 is a plot of moment error, using the first 25 modes, as a function of time. Over the time range considered (T=0.001 to 0.01) the moment errors of FDM and MAM methods are about one-half the magnitude of the error using the MDM method. Also, as time increases the error associated with each method for a given number of modes used tends to decrease.

For a discontinuous forcing function, such as a unit step, $\mu(T)$ at T=0, the modal method exactly predicts a zero response at time T=0. The MAM and FDM methods, however, require a summation of all the modes to exactly predict a zero response. Therefore, the MDM will produce qualitatively better results for times near T=0 or close to discontinuities. To calculate the transient response at very small times a large number of modes is necessary. The MDM using two modes, qualitatively approximates the displacement distribution with the same accuracy as the MAM and FDM methods using 17 modes for a unit step loading at T=.0002 and ζ_i = .05 (see fig. 8). Figure 9 shows that the error norm associated with the MAM and FDM methods is exceedingly large when fewer than about five modes are used to approximate the response. If a sufficient number of modes are used to predict the displacement distribution accurately ($m > 20$) the FDM and MAM methods appear to give better results.

For cases where the forcing function or its derivatives are discontinuous at some point other than T=0, the appropriate jump conditions must be included to use the FDM correctly. The jump conditions are necessary because the integration by parts requires the functions and their derivatives to be continuous. This complicates the implementation of the FDM (integrating-by-parts four times) for cases where the forcing function is not continuous at least up to its third derivative. The FDM can be applied up to the point of discontinuity and then re-started using the proper initial conditions.

Simply-Supported Multispan Beam (10 Spans)

The second problem studied is a uniform multispan beam (10 equal length spans) subject to two loading distributions and one forcing function. The beam has a nominal frequency $\omega_0 = \sqrt{EI/\rho AL^4}$ and time, t, is normalized such that $T = \omega_0 t$. The first 30 natural frequencies of the beam are normalized and listed in table 2. An analytical solution for the mode shapes and frequencies of multispan beams was obtained using equations from reference 8. This problem was selected because the frequencies are closely spaced (in groups equal to the number of spans (10 in this example)) and the chances of a neglected higher mode having a considerable effect on the response is increased.

Case 1: Uniformly distributed load with $Q(T)=1000(T^4-T^5)$.- Figure 10 is a plot of the moment distribution, normalized to the maximum value of M, of the multispan beam at T=1.2 and ζ_i=0.05. The FDM converges using only one mode whereas the MDM and MAM methods require 30 and 10 modes, respectively for $e<.01$. The moment error norm as a function of the number of modes for T=1.2 is shown in figure 11. The first nine modes are nearly orthogonal to the uniform load distribution, hence, the modal load $\{\phi\}_i^T\{Q(t)\}$ is negligible and has a negligible effect on the response (fig. 11). The 10th and 30th modes, however, have an effect on the solution as shown in the figure. The effect of these higher modes, however, is taken into account to some degree, by the pseudo-static response (note the MAM curve for $m<10$) and to a greater degree by the higher-order approximation of these neglected modes used in the FDM. Hence, the FDM using only one mode gives a more accurate response than the MDM using 49 modes or the MAM using 9 modes. The moment error norm as a function of time for each method (using 10 modes) is shown in figure 12. The accuracy of the FDM is at least two orders of magnitude greater than the MDM and one order of magnitude greater than the MAM. The MAM and MDM methods are identical at T=1.0 as expected because Q=0. At T=1.0, the error associated with the MDM decreases an order of magnitude and that associated with the MAM increases an order of magnitude as shown.

Case 2: Two concentrated loads equally spaced about the center of the first span with $Q(T)=1000(T^4-T^5)$.- For this loading distribution, the solution does not converge in a step like manner as in figure 11 but does so gradually as shown in figure 13. This occurs because the loading distribution is not nearly orthogonal to many mode shapes and hence the associated modal load $(\{\phi\}_i^T\{Q(T)\})$ is not negligible as in the previous case and these modes contribute to the response. Once again, the FDM converges more rapidly than either of the other methods and is at least an order of magnitude more accurate. The accuracy of the FDM tends to increase as T increases (fig. 14).

Comparison of Convergence for a Sinusoidal Forcing Function

In this section it is shown that, for the case of a sinusoidal forcing function, the higher-order modal methods such as FDM converge faster than the lower-order methods provided that the modal summation encompasses the forcing function frequency. For a sinusoidal forcing function $(\{Q_0\}\sin\gamma t)$ with zero initial conditions and no damping, the response using the mode-displacement method (MDM) is:

$$\{U(t)\} = \sum_{i=1}^{m} \left(\frac{\{\phi\}_i(\frac{1}{\omega_i^2})\{\phi\}_i^T\{Q_0\}\sin\gamma t}{(1 - r_i^2)} - \frac{\{\phi\}_i(\frac{\gamma}{\omega_i^3})\{\phi\}_i^T\{Q_0\}\sin\omega_i t}{(1 - r_i^2)} \right)$$

where $r_i = \gamma/\omega_i$. The mode-acceleration method (MAM) form of the response can be written as:

$$\{U(t)\} = [K]^{-1}\{Q_0\}\sin\gamma t + \sum_{i=1}^{m} \left(\frac{\{\phi\}_i(\frac{r_i^2}{\omega_i^2})\{\phi\}_i^T\{Q_0\}\sin\gamma t}{(1 - r_i^2)} - \frac{\{\phi\}_i(\frac{\gamma}{\omega_i^3})\{\phi\}_i^T\{Q_0\}\sin\omega_i t}{(1 - r_i^2)} \right)$$

Comparing the two equations, the last term in the modal summation is exactly the same, the first term in the modal summation is different and will converge faster (proportional to r_i^2) using the MAM provided that $r_i<1$ or $\omega_i>\gamma$. This is not unreasonable since the frequencies used in the modal summation must usually encompass the forcing frequency to provide an accurate transient response. The FDM method, using four integration by parts (eq. 16), results in the following expression:

$$\{U(t)\} = [K]^{-1}\{Q_0\}\sin\gamma t + [K]^{-1}[M][K]^{-1}\gamma^2\{Q_0\}\sin\gamma t + \sum_{i=1}^{m} \left(\frac{\{\phi\}_i(\frac{r_i^4}{\omega_i^2})\{\phi\}_i^T\{Q_0\}\sin\gamma t}{(1 - r_i^2)} - \frac{\{\phi\}_i(\frac{\gamma}{\omega_i^3})\{\phi\}_i^T\{Q_0\}\sin\omega_i t}{(1 - r_i^2)} \right)$$

The first term in the modal expansion of the FDM converges proportional to r_i^4 or $(\gamma/\omega_i)^4$ and so for $r_i<1$ this term converges much faster than the MDM or MAM methods. The last term, however, in the modal expansion remains the same for all of the methods.

CONCLUSIONS

The present study evaluates an improved modal method proposed by Leung for transient structural analysis and generalizes it to damped systems. The improved method, entitled the force-derivative method, is a higher-order method relative to the mode-displacement or mode-acceleration methods and is based on repeated integrations by parts of the convolution-integral form of the response. The repeated integration-by-parts produces terms which give successively better approximations to the contribution of the higher modes which are neglected in the modal summation. It is shown that the procedure used for deriving force-derivative method can be used to derive even higher-order methods and to explain the lower-order mehtods such as the mode-displacement and mode-acceleration methods. Comparisons are made of the force-derivative, mode-acceleration and mode-displacement methods for several numerical example problems for various times, levels of damping, and forcing functions. The forcing functions were chosen to study the effects of both continuous and discontinuous forcing functions (with respect to time) and to study what happens when the force or one of its derivatives vanishes at some point in time. The example problems include a tip-loaded cantilevered beam and a simply-supported multispan beam (10 equal-length spans).

In general, the force-derivative method was found to be more accurate than either the mode-displacement or mode-acceleration methods and results in a converged solution in fewer modes. In addition, for problems in which there are a large number of closely-spaced frequencies (e.g., large truss-type space structures and multispan beams) the force derivative method is very effective in representing the effect of the important, but otherwise neglected, higher modes. Results of a sinusoidal forcing function indicate that the higher-order modal methods will generally converge faster (proportional to the ratio of the forcing frequency to the natural frequency raised to some power) providing the natural frequencies are higher than the forcing frequency. This is to be expected since accuracy suggests that the modal summation include frequencies which encompass the dominant forcing function frequency components. Convergence of the moments and shears was found to be slower than convergence of displacements, as expected, since the former quantities are functions of the spatial derivatives of the displacements. At response times close to discontinuities in the forcing function and/or its derivatives, the mode-displacement method qualitatively gives better results using fewer modes than the mode-acceleration or force-derivative methods. Implementation of the force-derivative method at the discontinuity requires the inclusion of the necessary jump conditions. It was also found that increasing modal damping levels does not always increase the convergence rate of the mode-acceleration method. The relative importance of damping on the convergence rate of the modal solution is shown to be complex and dependent upon several factors including the existence of the derivatives of the forcing function.

REFERENCES

1. Guyan, R. J.: Reduction of Stiffness and Mass Matrices. AIAA Journal, Vol. 3, No. 2, 1965, p. 380.

2. Leung, Andrew Yee-Tak: An Accurate Method of Dynamic Condensation in Structural Analysis. International Journal for Numerical Methods in Engineering, Vol. 12, 1978, pp. 1705-1715.

3. Wilson, Edward L.; Yuan, Ming-Wu; and Dickens, John M.: Dynamic Analysis by Direct Superposition of Ritz Vectors. Earthquake Engineering and Structural Dynamics, Vol. 10, 1982, pp. 813-821.

4. de Veubeke, Fraeijs, B. M.; Geradin, M.; Huck, A.; and Hogge, M. A.: Structural Dynamics -- Heat Conduction. International Center for Mechanical Sciences -- Courses and Lectures, no. 126. Wien-New York, 1972.

5. Williams, D.: Displacements of a Linear Elastic System Under a Given Transient Load - Part I. The Aeronautical Quarterly, Vol. 1, August 1949, pp. 123-136.

6. Cornwell, R. E.; Craig Jr., R. R.; and Johnson, C. P.: On the Application of the Mode-Acceleration Method to Structural Engineering Problems. Earthquake Engineering and Structural dynamics, Vol. 11, pp. 679-688, 1983.

7. Leung, Y. T.: Fast Response Method for Undamped Structures. Engineering Structures, Vol. 5, April 1983.

8. Darnley, E. R.: The Transverse Vibrations of Beams and the Whirling of Shafts Supported at Intermediate Points. Philosophical Magazine, Vol. 41, 1982, p. 81.

TABLE 1 - NATURAL FREQUENCIES OF A CANTILEVERED BEAM

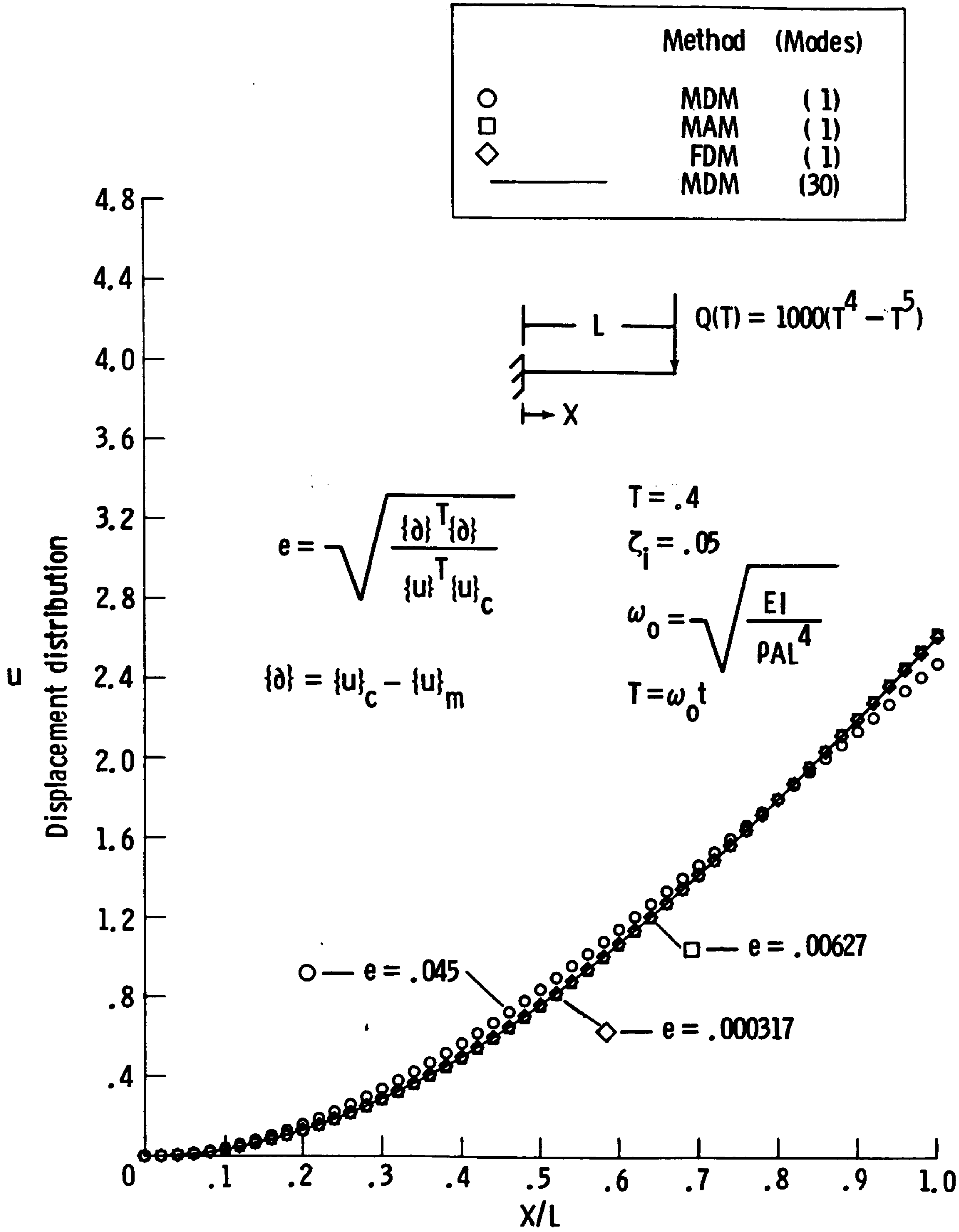
$$\left(\omega_o = \sqrt{\frac{EI}{\rho A L^4}}\right)$$

MODE NUMBER	NORMALIZED NATURAL FREQ. ω/ω_o
1	3.51601
2	22.0345
3	61.6972
4	120.902
5	199.860
6	298.556
7	416.991
8	555.165
9	713.079
10	890.732
11	1088.12
12	1305.26
13	1542.13
14	1798.74
15	2075.08
16	2371.17
17	2687.00
18	3022.57
19	3377.87
20	3752.92
21	4147.70
22	4562.22
23	4996.49
24	5450.49
25	5924.23
26	6417.71
27	6930.93
28	7463.89
29	8016.59
30	8589.02

TABLE 2 - NATURAL FREQUENCIES OF A SIMPLY-SUPPORTED MULTISPAN BEAM (10 SPANS)

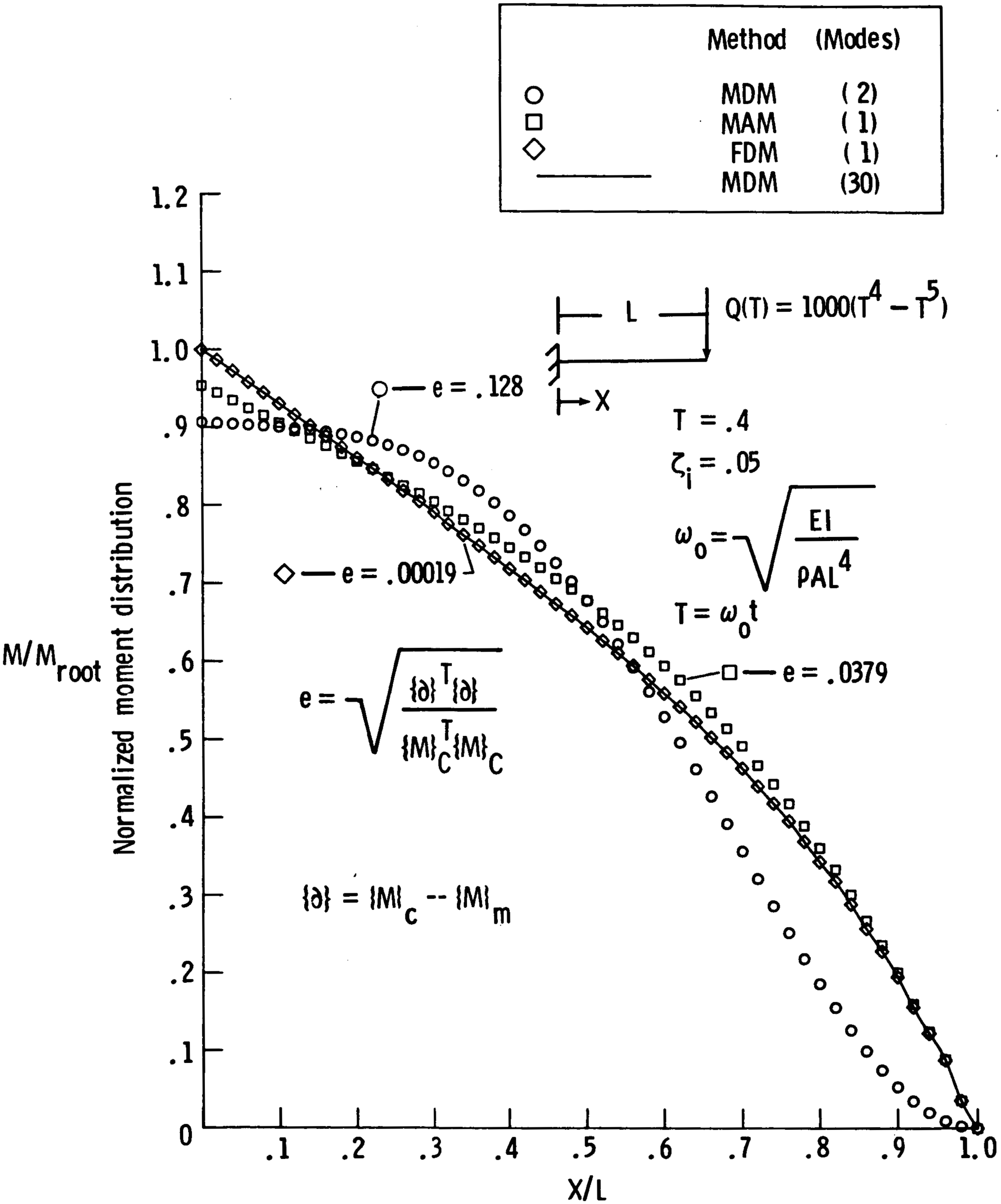
$$(\omega_0 = \sqrt{\frac{EI}{\rho A L^4}})$$

MODE NUMBER	NORMALIZED NATURAL FREQ. ω/ω_0
1	9.86960
2	10.1479
3	10.9435
4	12.1691
5	13.6959
6	15.4212
7	17.2490
8	19.0691
9	20.7508
10	21.9115
11	39.4784
12	40.0728
13	41.7202
14	44.1072
15	46.9043
16	49.9649
17	53.1222
18	56.2347
19	58.9525
20	60.9470
21	88.8264
22	89.7765
23	92.2339
24	95.7363
25	99.8682
26	104.248
27	108.879
28	113.289
29	117.133
30	119.972



a) Displacement distribution

Figure 1.- Representation of displacement and moment errors in a cantilevered beam with tip loading using error norm e ($Q(T) = 1000(T^4-T^5)$, $T = 0.4$ and $\zeta_i = 0.05$).



b) Moment distribution

Figure 1.- Concluded.

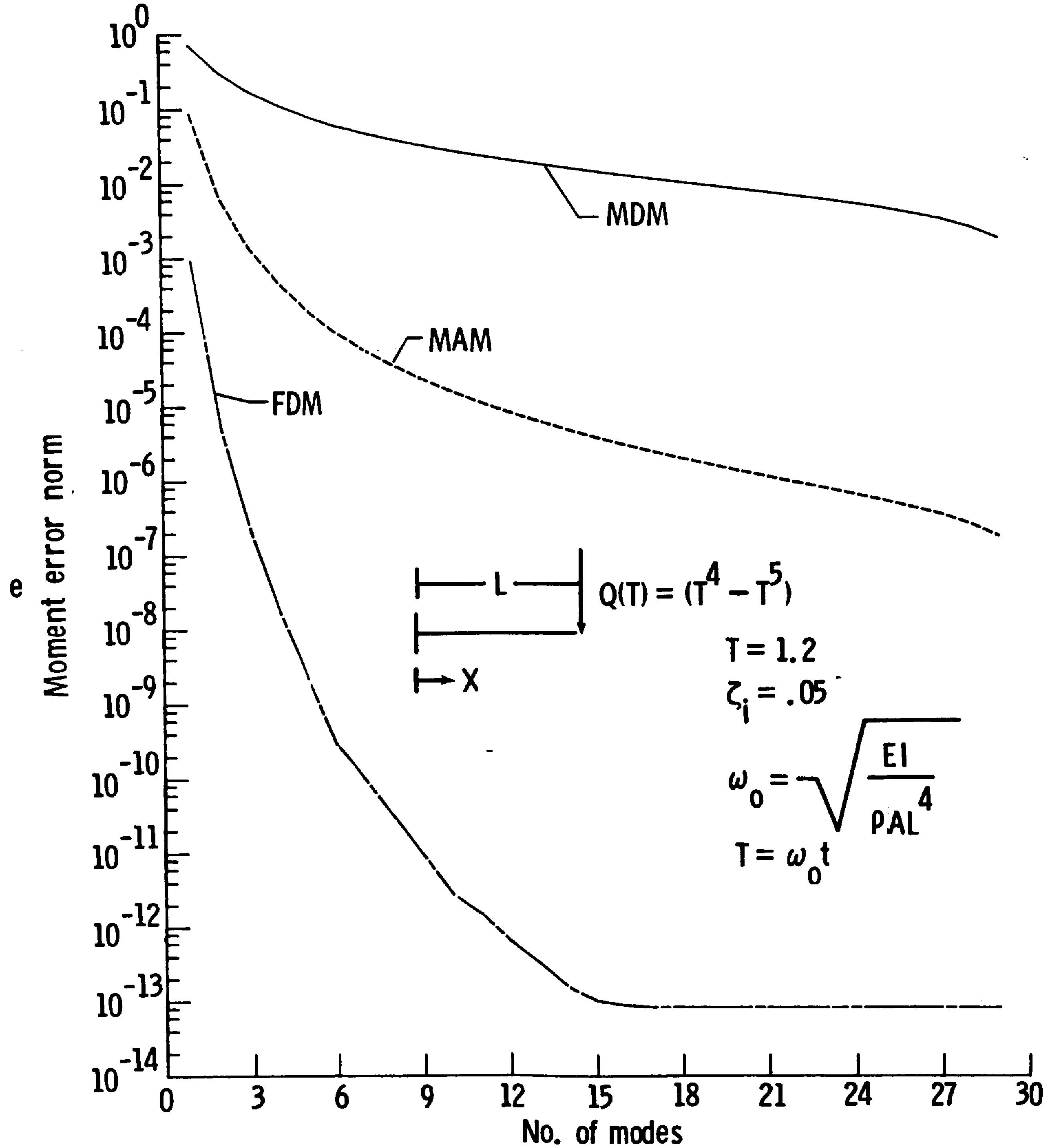


Figure 2.- Comparison of moment errors of a cantilevered beam subject to a tip load $Q(T) = 1000(T^4 - T^5)$ ($T=1.2$ and $\zeta_i = 0.05$) using three different modal methods.

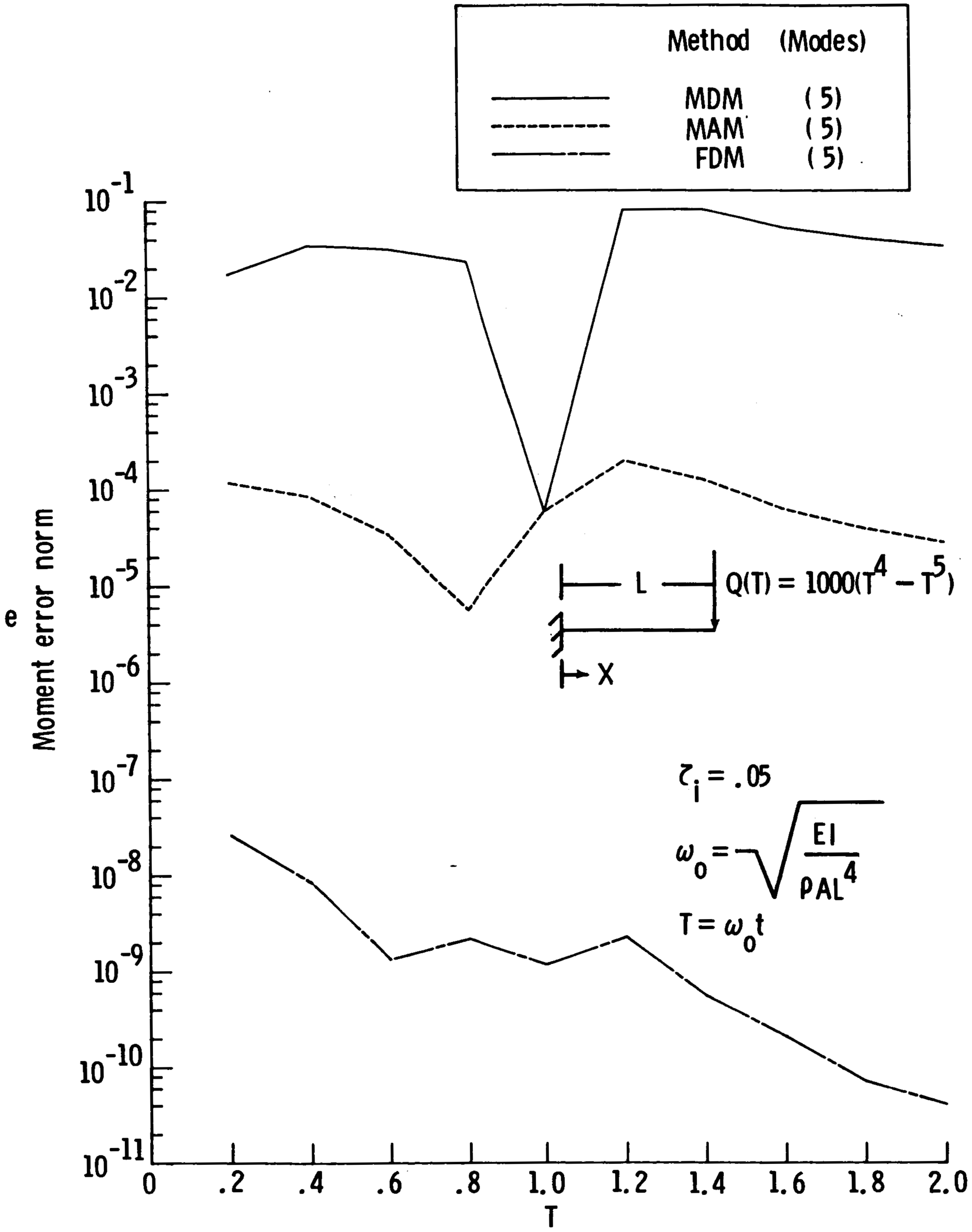
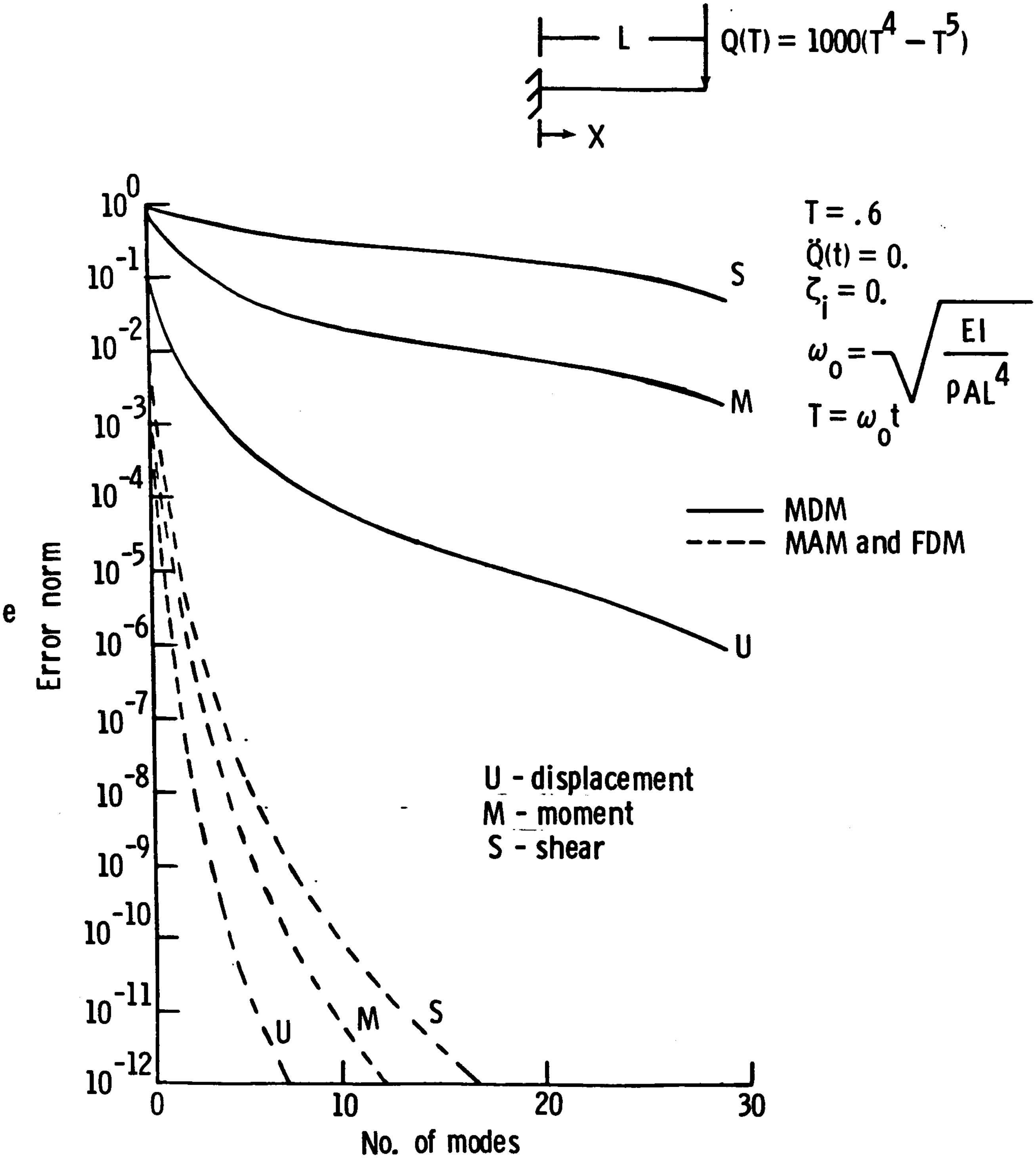
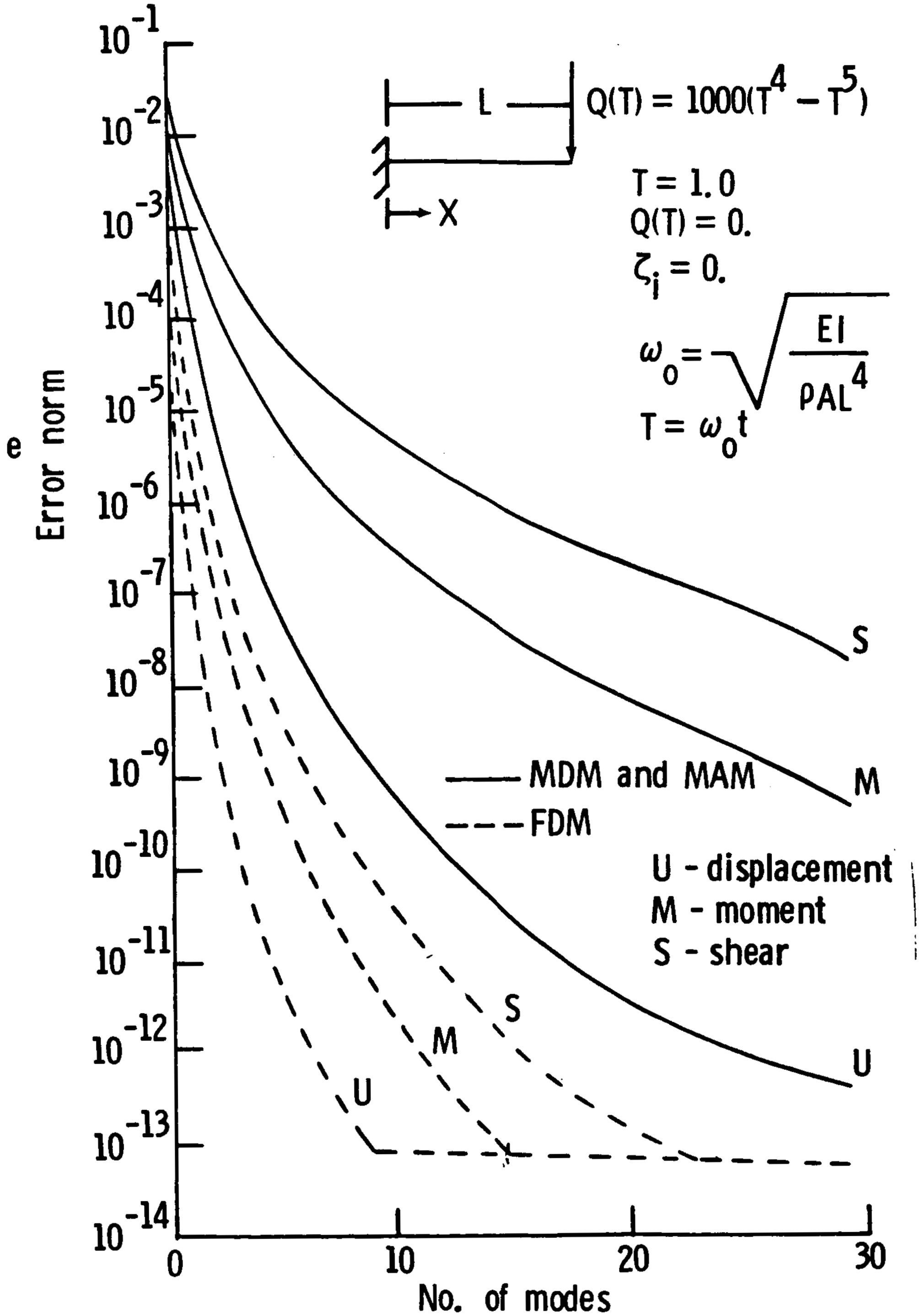


Figure 3.- Variation of moment errors as a function of time, using 5 modes in the modal summation. Cantilevered beam with a tip load $Q(T) = 1000(T^4 - T^5)$ ($\zeta_i = 0.05$).



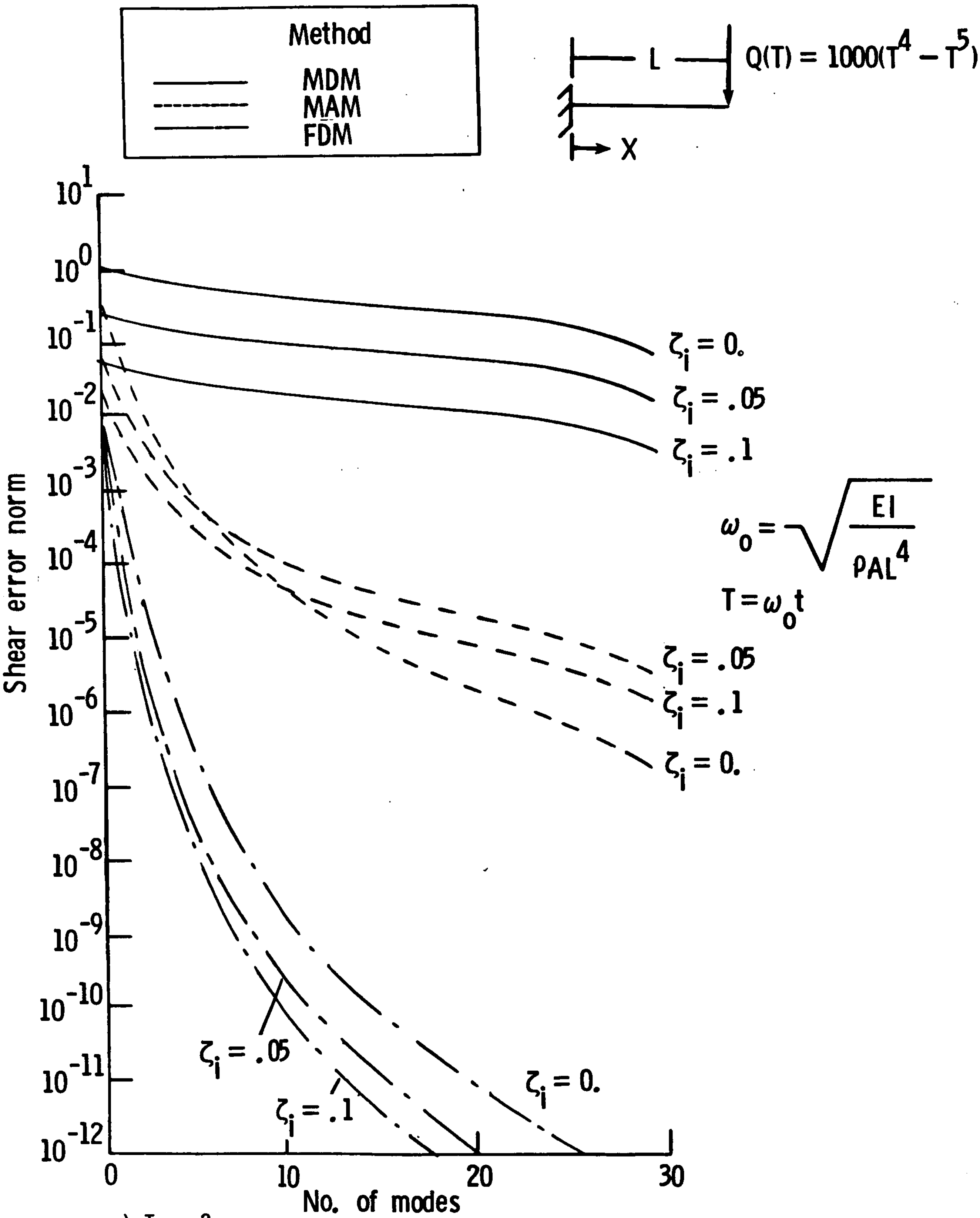
a) T = .6

Figure 4.- Comparison of displacement, moment, and shear errors of a cantilevered beam using three different modal methods (ζ_i = 0. and tip load $Q(T) = 1000(T^4 - T^5)$).



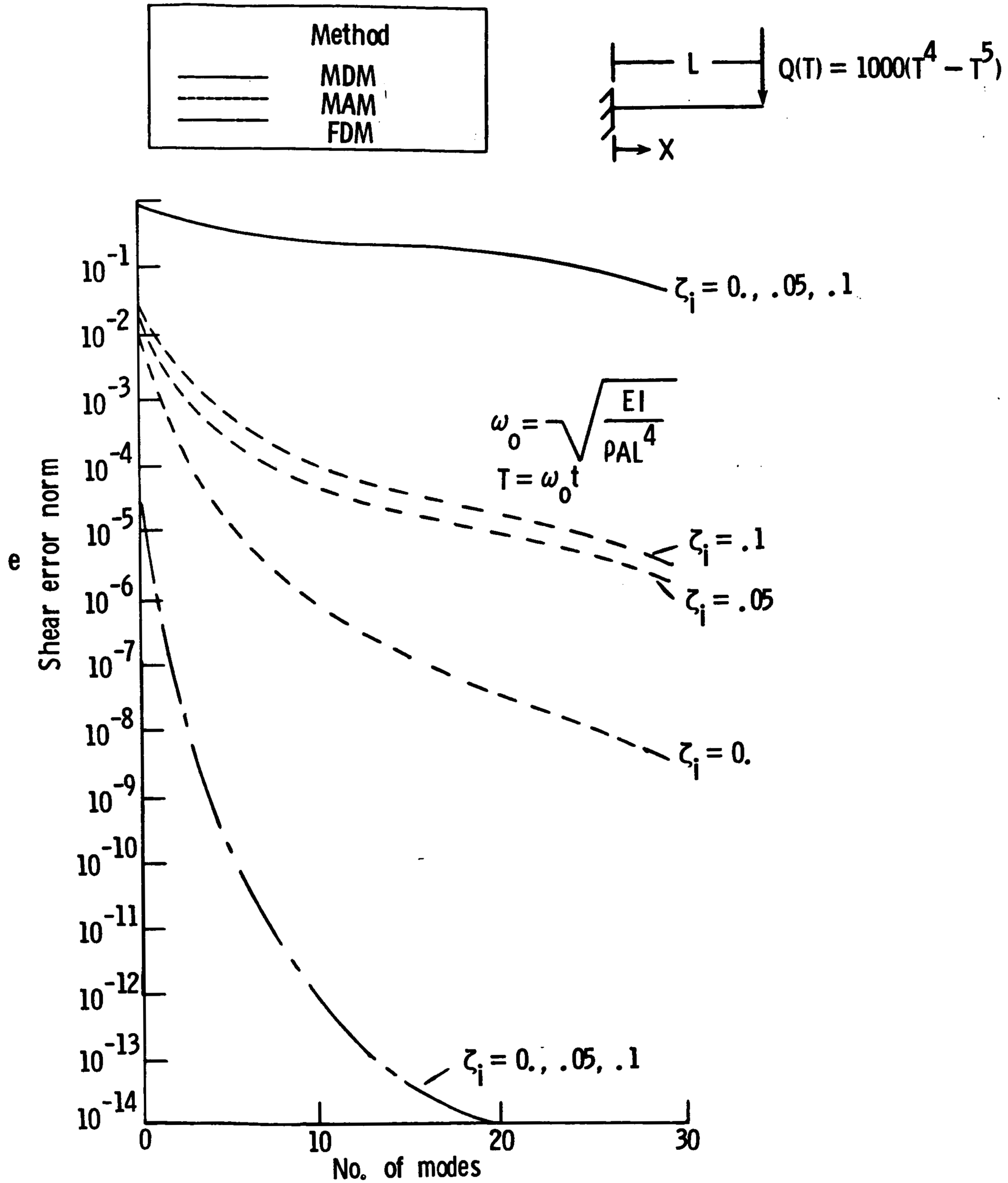
b) T = 1.0

Figure 4.- Concluded.



a) T = .2

Figure 5.- Comparison of shear errors of a cantilevered beam for various damping levels (tip load $Q(T) = 1000\ (T^4 - T^5)$).



b) $T = 2$

Figure 5.- Concluded.

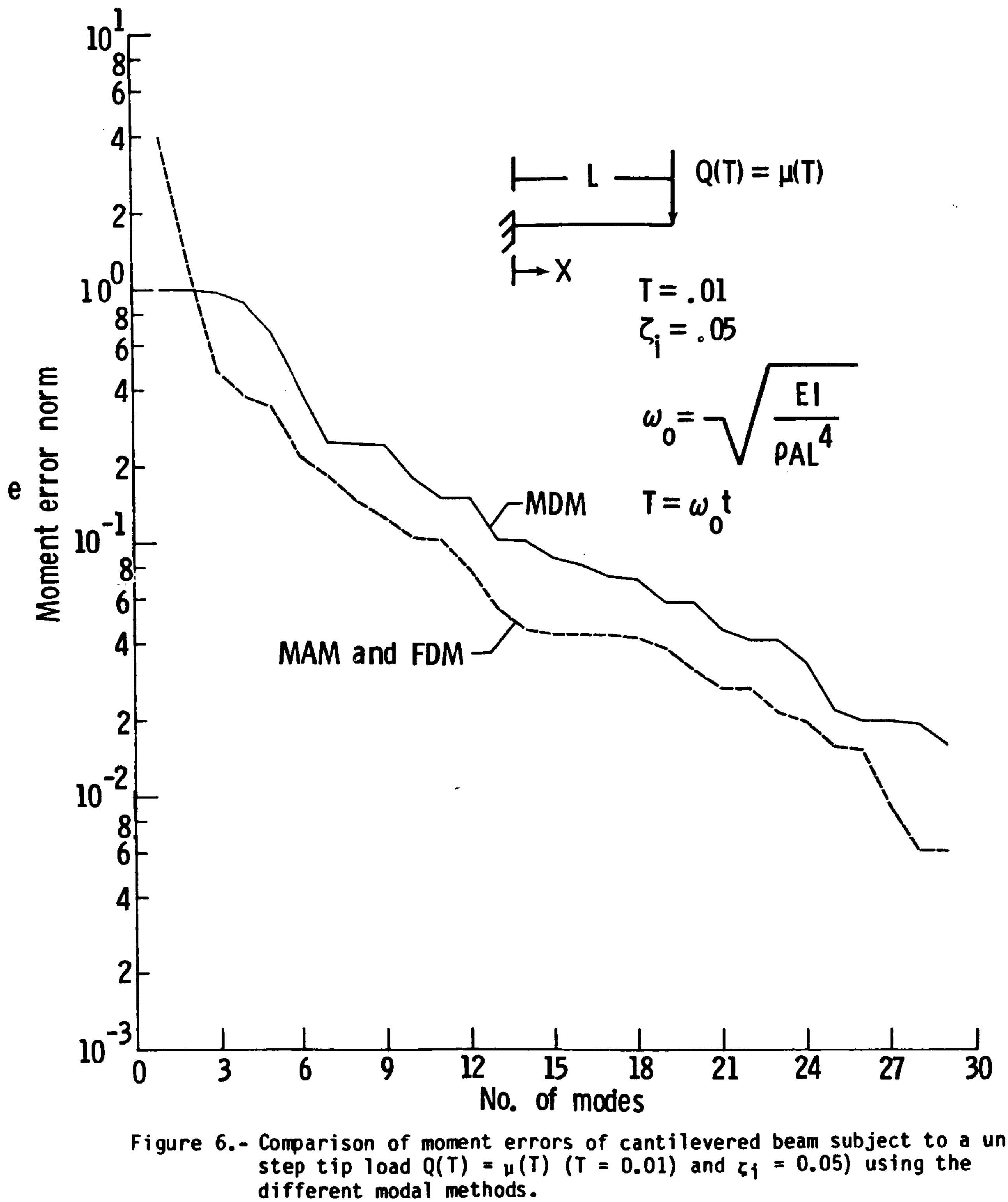


Figure 6.- Comparison of moment errors of cantilevered beam subject to a unit step tip load $Q(T) = \mu(T)$ $(T = 0.01)$ and $\zeta_i = 0.05$) using the different modal methods.

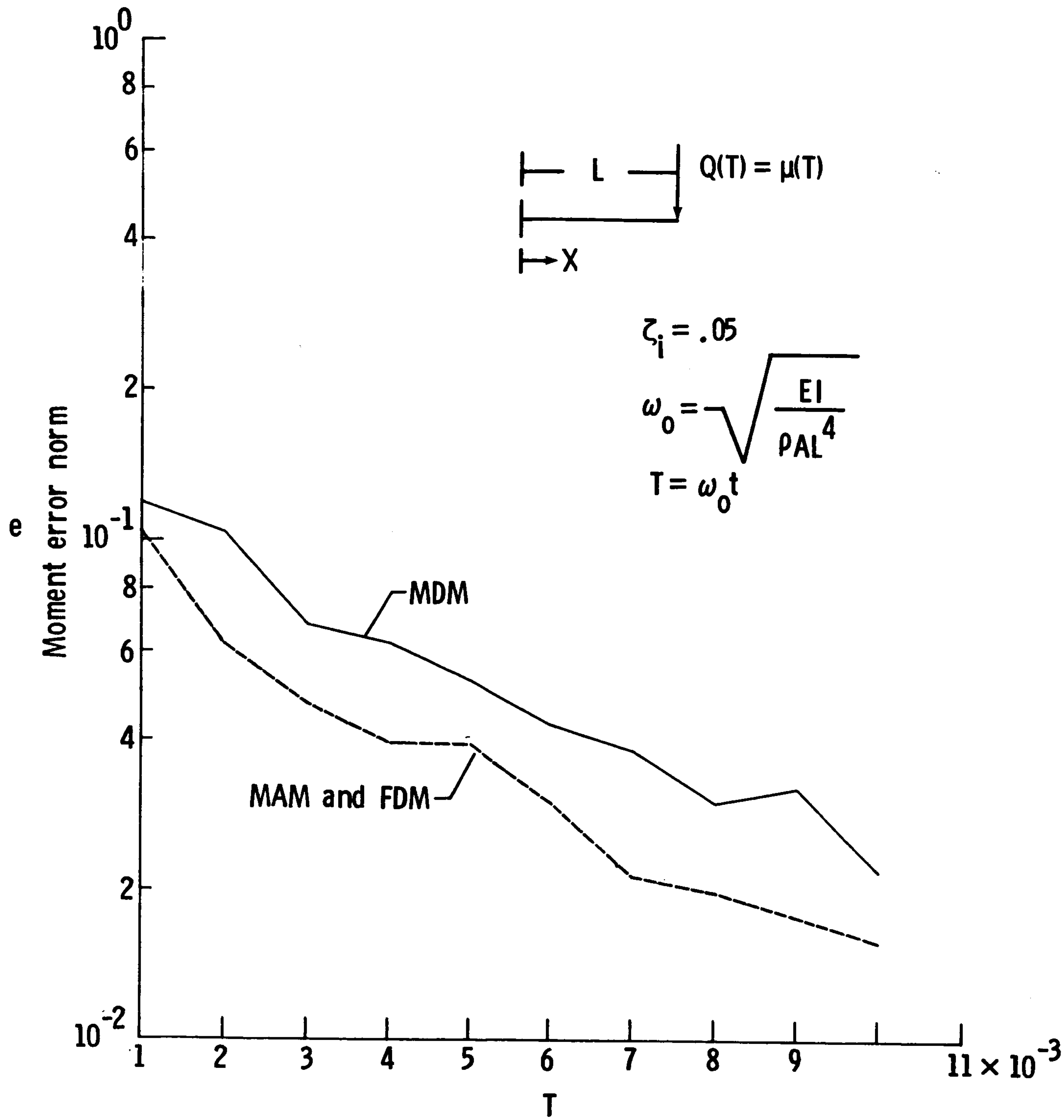


Figure 7.- Variation of moment errors as a function of time , assuming 25 modes are used in the modal summation,for a cantilevered beam with a unit step tip load $Q(T) = \mu(T)$ ($\zeta_i = 0.05$).

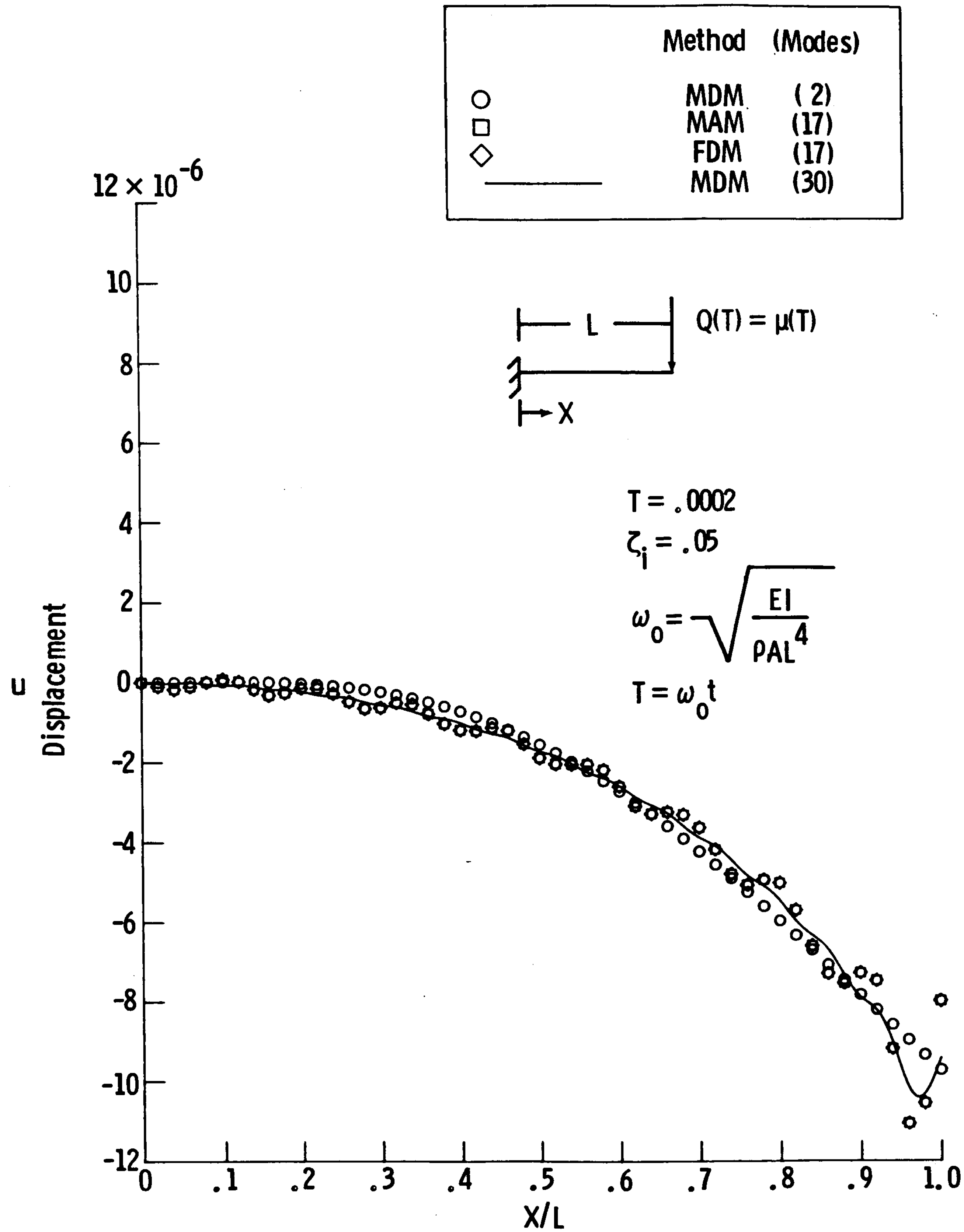


Figure 8.- Displacement distribution in a uniform cantilevered beam with a unit step tip loading at time T = 0.0002 ($Q(T) = \mu(T)$ and $\zeta_i = 0.05$).

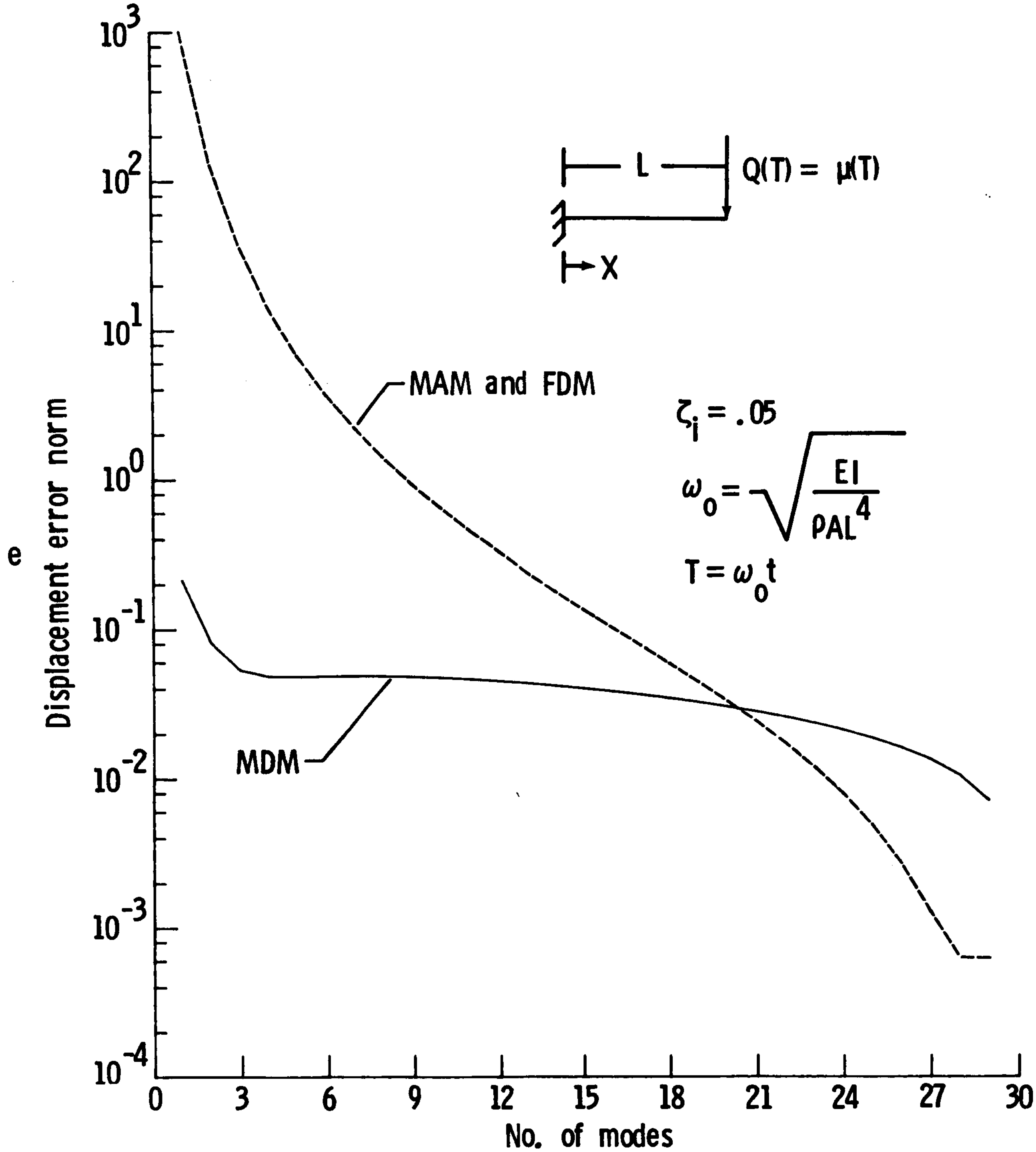


Figure 9.- Comparison of displacement errors of uniform cantilevered beam subject to a unit step tip load at time T = 0.0002 ($Q(T) = \mu(T)$ and $\zeta_i = 0.05$).

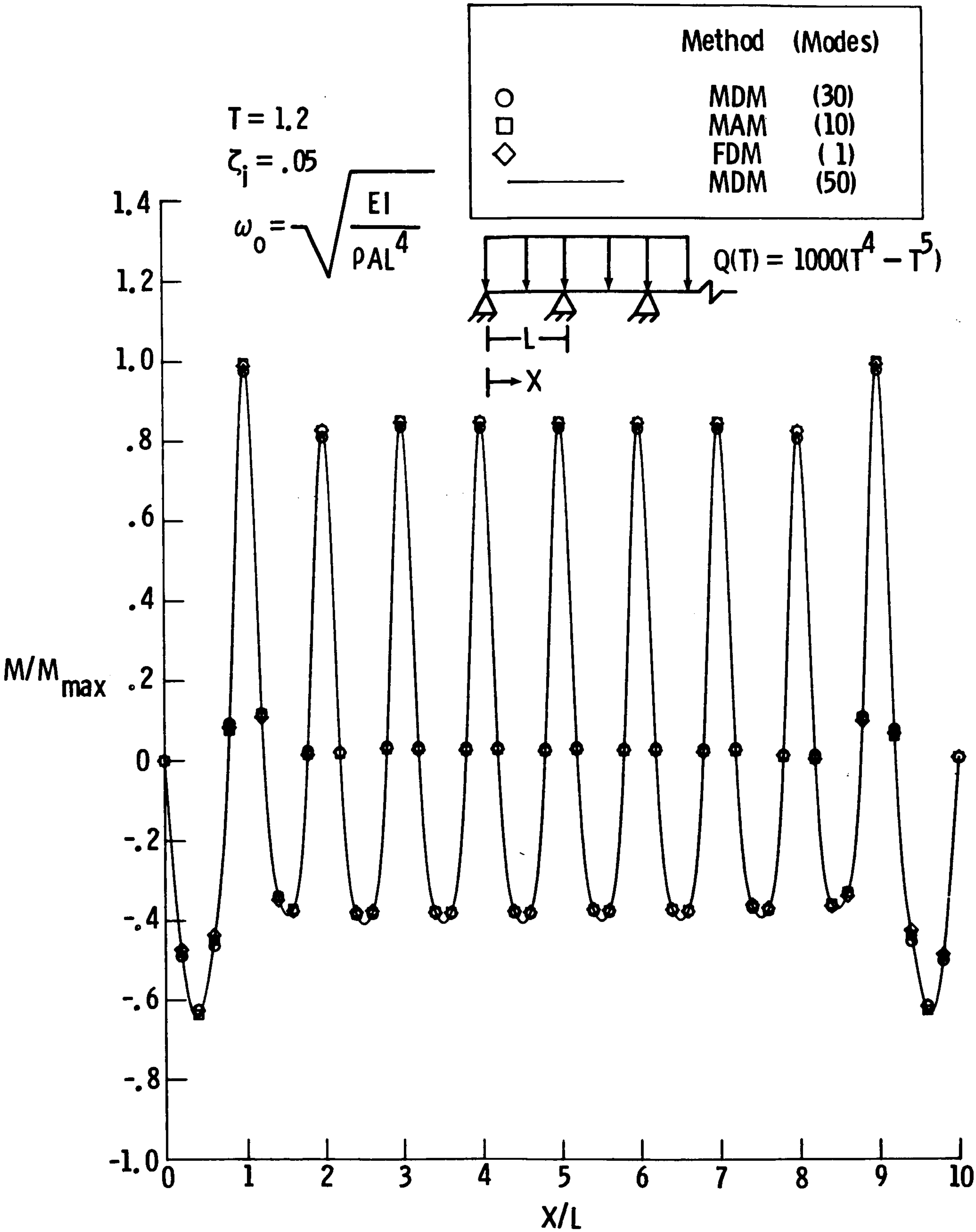


Figure 10.- Comparison of normalized moment distributions along a simply-supported multispan beam (ten equally-spaced spans) and a uniformly distributed load $Q(T) = 1000\ (T^4 - T^5)$ ($T = 1.2$ and ζ_i = 0.05).

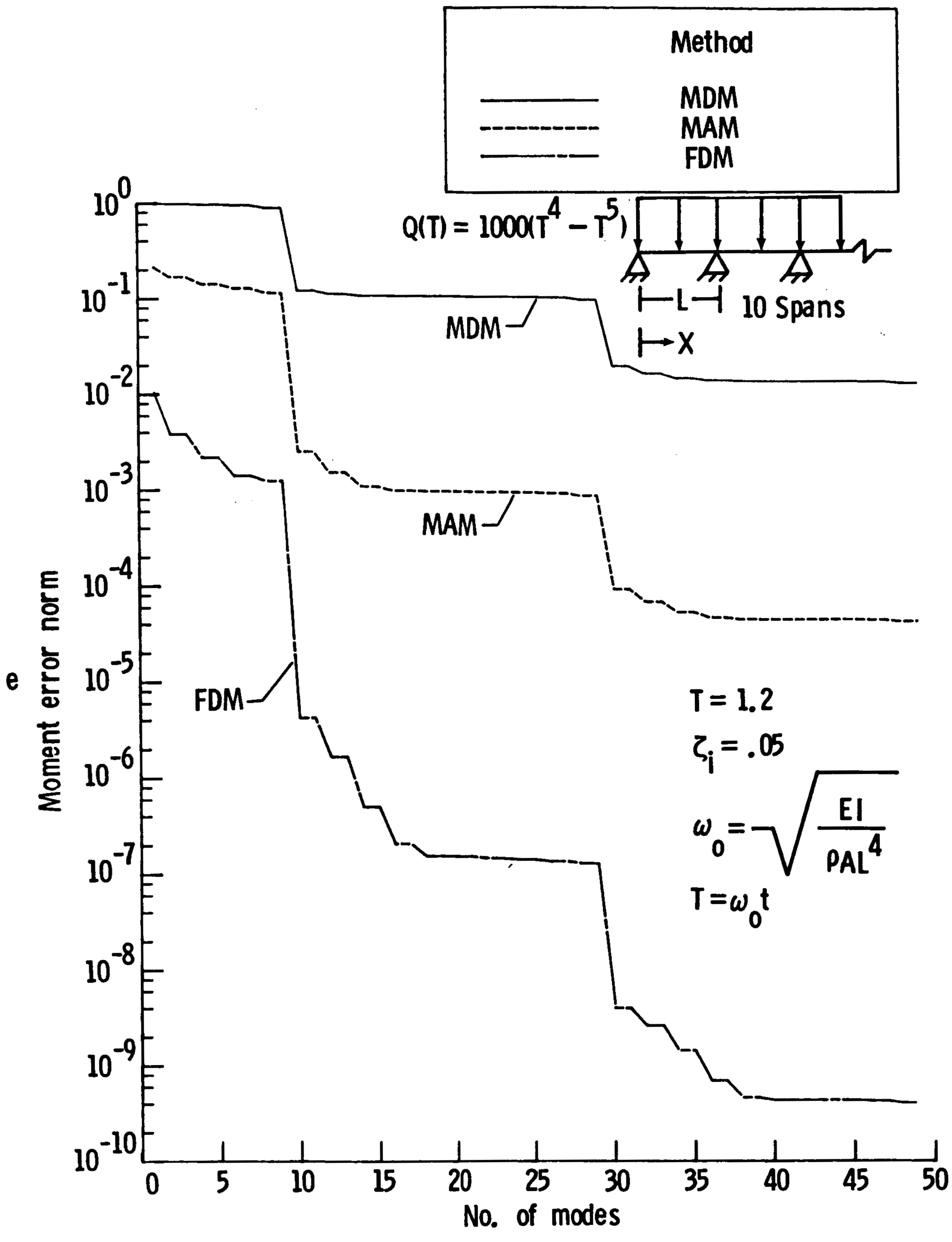


Figure 11.- Comparison of moment errors of a simply-supported multispan beam (ten equally-spaced spans) subject to a uniformly distributed load $Q(T) = 1000\ (T^4 - T^5)$ ($T = 1.2$ and $\zeta_i = 0.05$).

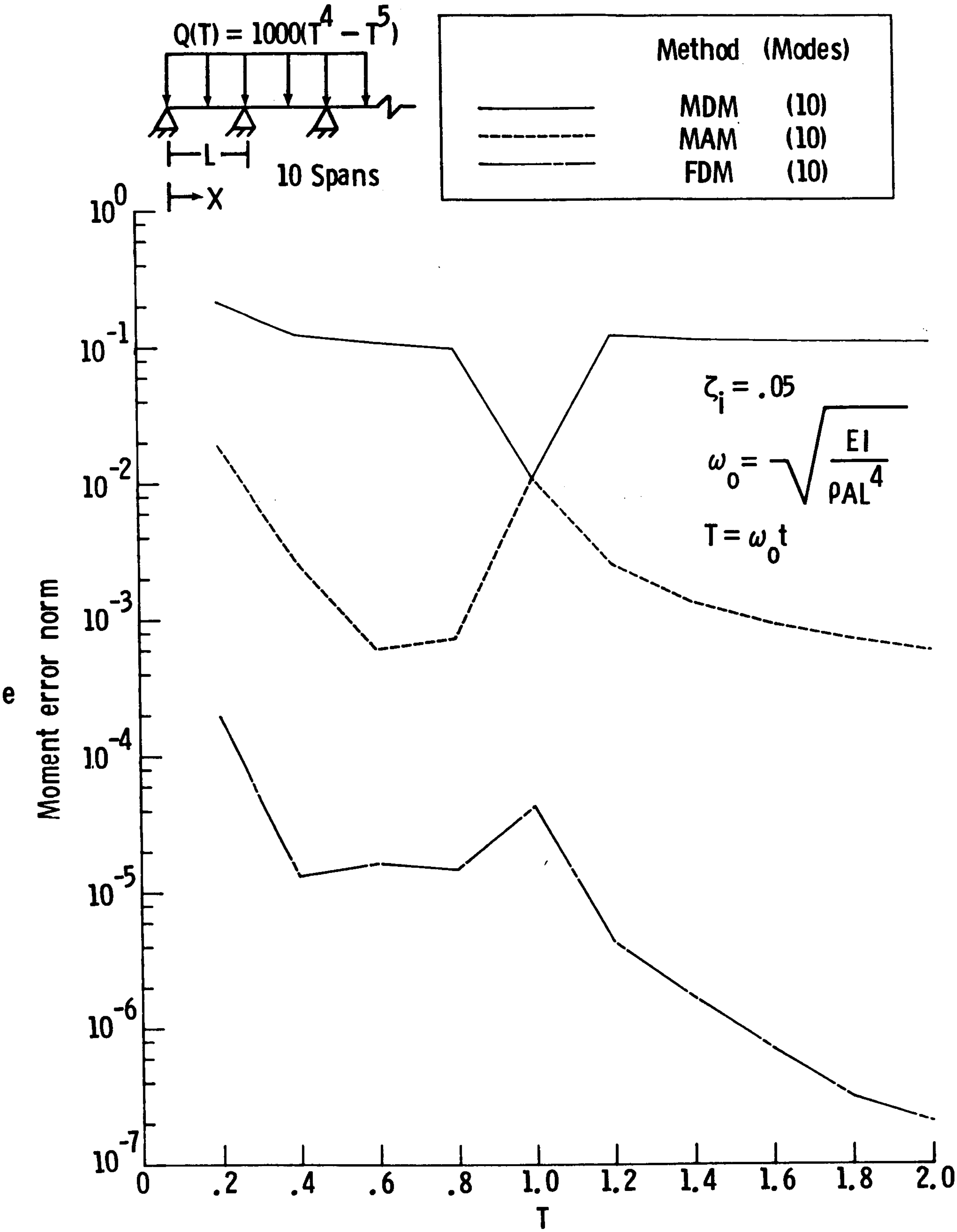


Figure 12.- Variations of moment errors as a function of time, using ten modes in the modal summation, for a simply-supported multispan beam (ten equally-spaced spans) subject to a uniformly-distributed load $Q(T) = 1000(T^4 - T^5)$ ($\zeta_i = 0.05$).

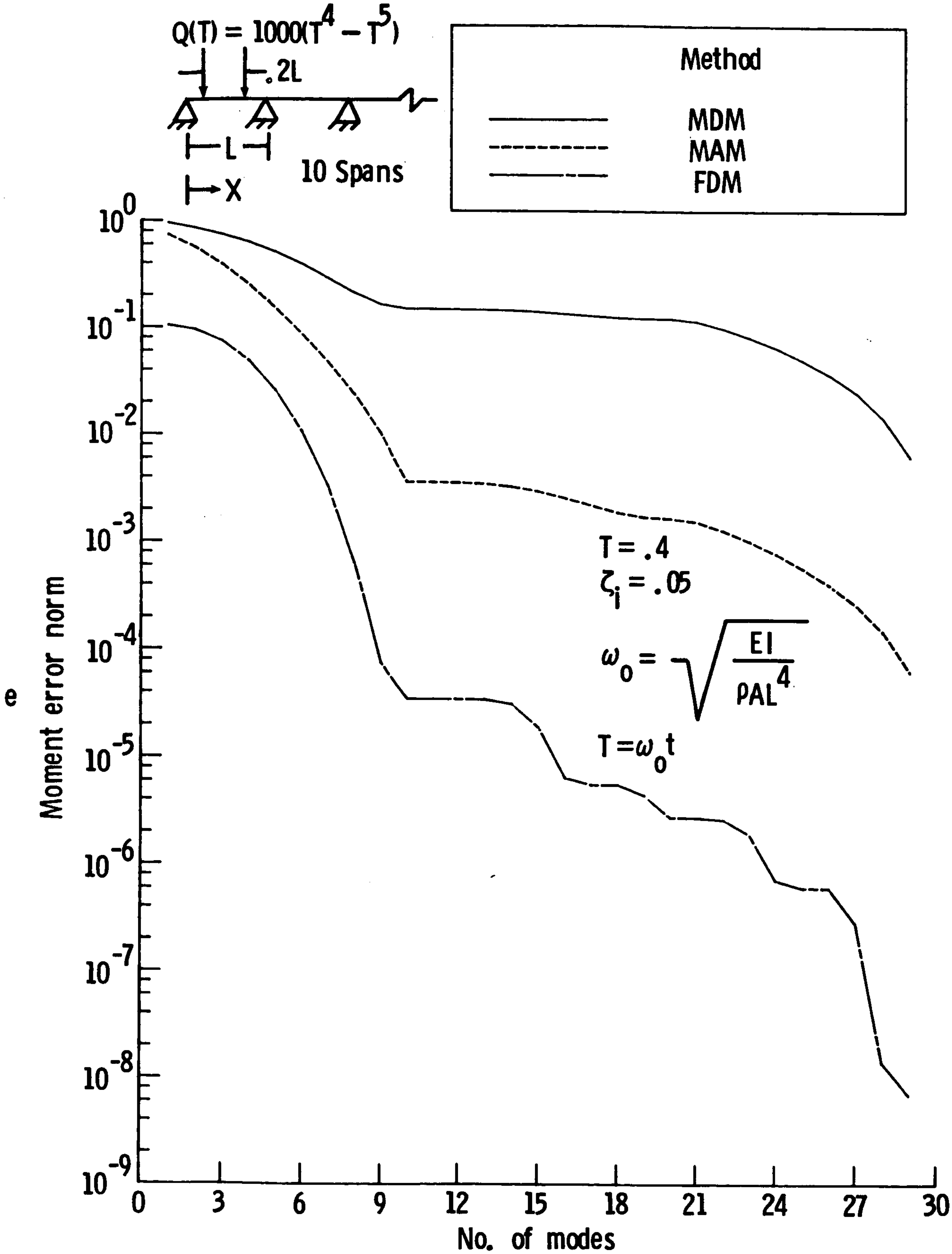


Figure 13.- Comparison of moment errors of a simply-supported multispan beam (ten equally-spaced spans) subject to two concentrated loads spaced about the center of the first span and varying as $Q(T) = 1000(T^4 - T^5)$ ($T = 0.4$ and $\zeta_i = 0.05$).

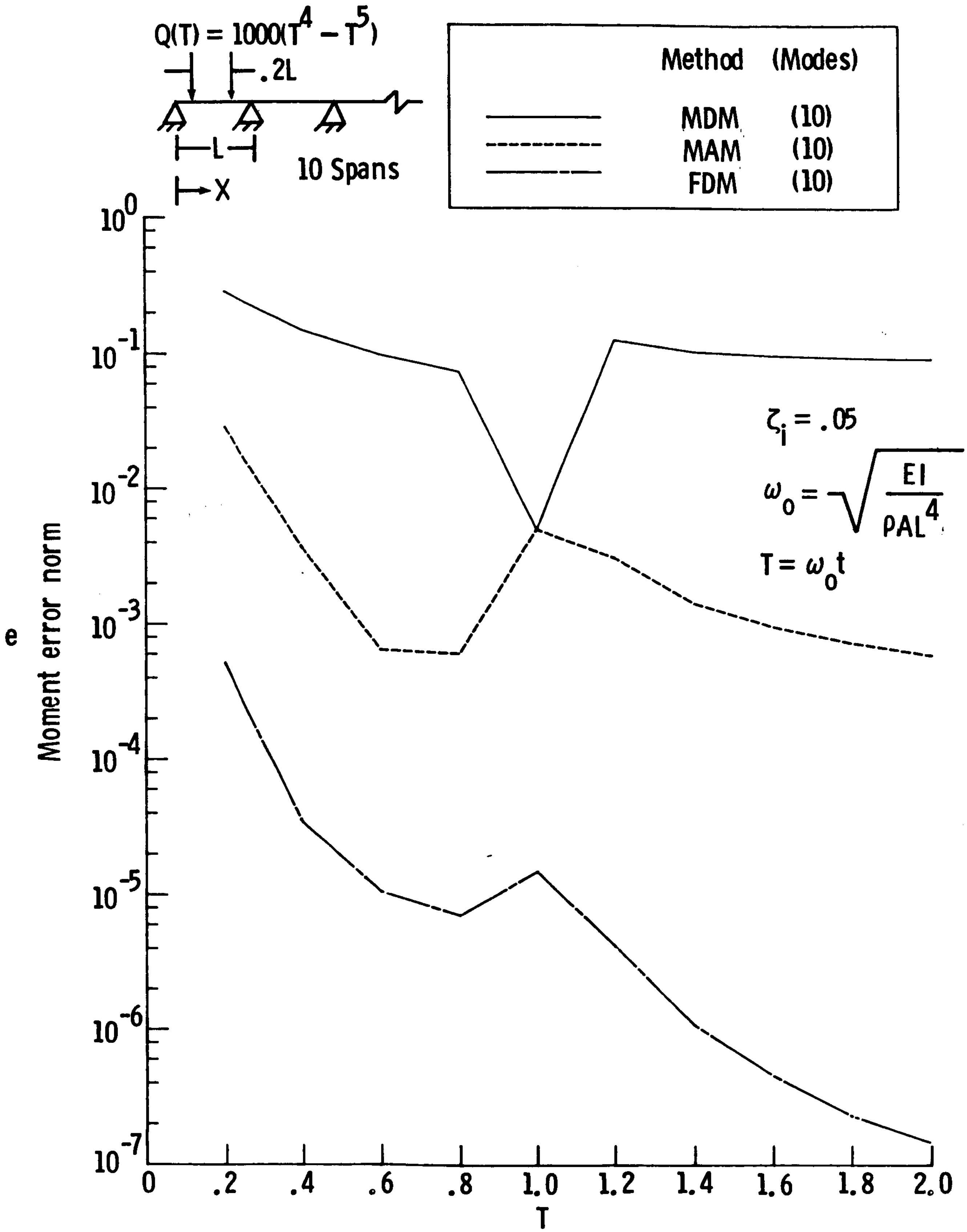


Figure 14.- Variation of moment errors as a function of time, using 10 modes in the modal summation, for simply-supported multispan beam (ten equally-spaced spans) subject to two concentrated loads spaced about the center of the first span and varying as $Q(T) = 1000(T^4 - T^5)$ ($\zeta_i = 0.05$).

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16. Abstract

The present study evaluates a higher-order modal method proposed by Leung for transient structural analysis entitled the force-derivative method. This method repeatedly integrates by parts with respect to time the convolution-integral form of the structural response to produce successively better approximations to the contribution of the higher modes which are neglected in the modal summation. Comparisons are made of the force-derivative, the mode-displacement, and the mode-acceleration methods for several numerical example problems for various times, levels of damping, and forcing functions. The example problems include a tip-loaded cantilevered beam and a simply-supported multispan beam. The force-derivative method is shown to converge to an accurate solution in fewer modes than either the mode-displacement or the mode-acceleration methods. In addition, for problems in which there are a large number of closely-spaced frequencies whose mode shapes have a negligible contribution to the response, the force derivative method is very effective in representing the effect of the important, but otherwise neglected, higher modes.

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