Topological Susceptibility with Three Flavors of Staggered Quarks

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Abstract

As one test of the validity of the staggered-fermion fourth-root determinant trick, we examine the suppression of the topological susceptibility of the QCD vacuum in the limit of small quark mass. The suppression is sensitive to the number of light sea quark flavors. Our study is done in the presence of 2+1 flavors of dynamical quarks in the improved staggered fermion formulation. Variance-reduction techniques provide better control of statistical errors. New results from staggered chiral perturbation theory account for taste-breaking effects in the low-quark mass behavior of the susceptibility, thereby reducing scaling violations from this source. Measurements over a range of quark masses at two lattice spacings permit a rough extrapolation to zero lattice spacing to remove the remaining lattice artifacts. The results are consistent with chiral perturbation theory.

Objectives

1. Test the improved (Asqtad) staggered fermion formalism

2. Demonstrate the expected suppression of topological fluctuations at small quark mass.

What is New Here?

1. Staggered Chiral Perturbation Theory corrects for some lattice artifacts.

2. New variance reduction technique improves precision.

3. New small-quark-mass $40^3 \times 96$ lattices at $a \approx 0.09$ fm.

Simulation sample improved fermion (Asqtad) configurations

Coarse lattices $a \approx 0.12$ fm.

$am_{u,d}$ / am_s	$10/g^2$	L	lats.
quenched	8.00	20	408
0.05 / 0.05	6.85	20	425
0.04 / 0.05	6.83	20	351
0.03 / 0.05	6.81	20	564
0.02 / 0.05	6.79	20	484
0.01 / 0.05	6.76	20	658
0.007 / 0.05	6.76	20	493

Fine lattices $a \approx 0.09$ fm.

$am_{u,d}$ / am_s	$10/g^2$	L	lats.
quenched	8.40	28	396
0.031 / 0.031	7.18	28	496
0.0124 / 0.031	7.11	28	527
0.0062 / 0.031	7.09	28	592
0.0031 / 0.031	7.08	40	80

Staggered Fermion pion taste multiplet

pion taste	$M_\pi r_1$ (coarse)	$M_\pi r_1$ (fine)
γ_5	0.8251(45)	0.7659(7)
$\gamma_0\gamma_5$	0.9386(19)	0.8127(11)
$\gamma_i\gamma_5$	0.9426(16)	0.8116(26)
$\gamma_i\gamma_j$	1.0033(34)	0.8372(41)
$\gamma_i\gamma_0$	1.0044(29)	0.8383(26)
γ_i	1.0555(53)	0.8576(56)
γ_0	1.0558(32)	0.8602(37)
I	1.1029(80)	0.8899(93)

Coarse ensemble $m_{ud}/m_s = 0.02/0.05$.
Fine ensemble $m_{ud}/m_s = 0.0124/0.031$

Staggered Chiral Lagrangian

The Lagrangian is written in terms of N-flavor 4-taste pseudoscalar meson fields, parameterized by an $U(4N)$ matrix.

$$U = \exp(iT_{ia}\phi_{ia}/f).$$

with generators

$$T_{ia} = \Lambda_i t_a,$$

where $a \in \{1 \ldots 16\}$ is the taste index and $i \in \{1 \ldots N^2\}$ is the flavor index, the first N of which are taken to be the flavor neutral (diagonal) generators.

Then

$$\phi_{0a} = \sum_{i=1}^{N} \phi_{ia}/\sqrt{N}$$

is a flavor singlet field.

Staggered Chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\mu U\right) - \frac{\mu f^2}{4}\mathrm{Tr}[\mathcal{M}(U^\dagger + U)] + \frac{m_0^2}{2}\phi_{0I}^2 + a^2\mathcal{V}(U)$$

$\mathcal{M}$ is the taste-degenerate quark mass matrix

$$\mathcal{M}_{ia,jb} = \delta_{ij} m_i \delta_{ab}$$

and the dimension-six taste-breaking potential is

$$-\mathcal{V}(U) = \sum C_i \mathcal{O}_i.$$

The taste-breaking four-fermion terms $\mathcal{O}_i$ were described by Lee and Sharpe and Aubin and Bernard and are not displayed here.

Partition function for fixed winding number

The partition function for winding number ν is

$$Z_\nu = \int dU d\theta \, \mathrm{exp}(-i\theta\nu) \, \mathrm{exp}\left[-\mathcal{L}(\phi_{0I} - \theta) dV\right].$$

We work to quadratic order in the fields and in the mean-field approximation and follow Leutwyler and Smilga to obtain the susceptibility.

$$\chi = \left\langle \nu^2 \right\rangle / V = \frac{f^2/16}{\sum_{i=1}^{N} 1/m_{iI}^2 + N/m_0^2}$$

where the bare taste-shifted flavor-neutral pseudoscalar masses are

$$m_{iI}^2 = 2\mu m_i + a^2 \Delta_I.$$

Formula for Susceptibility

The susceptibility with a degeneracy of n_i flavors of four tastes each is then

$$\chi = \left\langle \nu^2 \right\rangle / V = \frac{f^2/16}{\sum_{i=1} n_i (1/m_{iI}^2 + 1/m_0^2)}$$

Replica trick: to count two degenerate u and d quarks and one s quark we put $n_{ud} = 1/2$ and $n_s = 1/4$. Finally

$$\chi = \frac{f^2 m_{\pi,I}^2/8}{1 + m_{\pi,I}^2/2m_{ss,I}^2 + 3m_{\pi,I}^2/2m_0^2}$$

This formula interpolates between the perturbative and quenched regimes and reproduces the Witten-Veneziano formula.

Variance Reduction Method

With large lattice volumes we gain statistically by recognizing that the susceptibility is really a local, intensive observable, rather than a global observable:

$$\left\langle \nu^2 \right\rangle / V = \sum_r C_{\text{meas}}(r),$$

where the topological charge density correlation function, is

$$C_{\text{meas}}(r) = \langle \rho(0) \rho(r) \rangle$$

and ρ is the topological charge density operator.

We define ρ through the HYP smearing of Knechtli and Hasenfratz and the topological density operator of DeGrand, Hasenfratz, and Kovacs as described in `hep-lat/0308019`. The $J^{PC} = 0^{-+}$ correlator has the asymptotic form (Shuryak and Verbaarschot)

$$C_{\text{fit}}(r) = b_\eta D(m_\eta, r) + b_{\eta'} D(m_{\eta'}, r) + \ldots$$

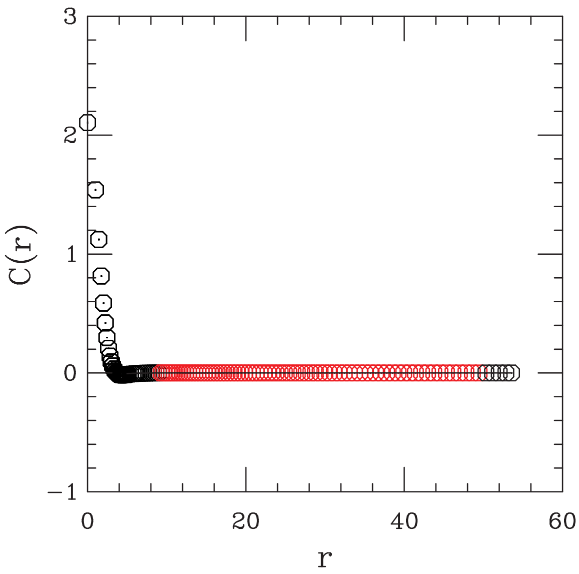
where $D(m, r)$ is the Euclidean scalar propagator. Note that $b_\eta \leq 0$ and $b_{\eta'} \leq 0$ because $F\tilde{F}$ is time-reflection negative.

Variance Reduction Method

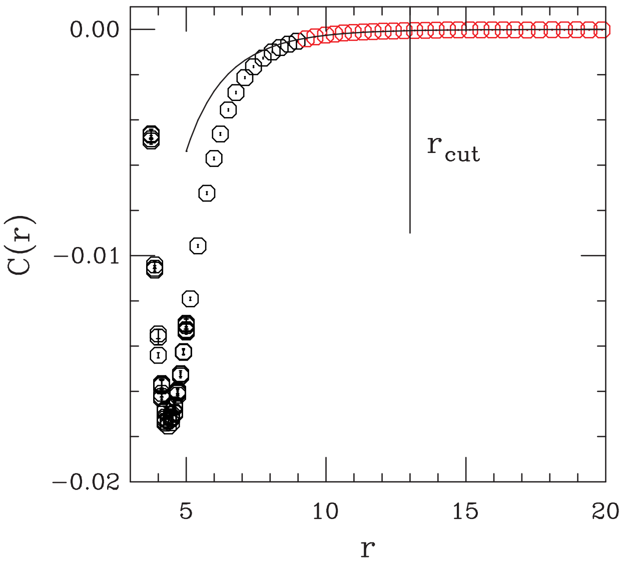
We fit the correlator to this form for large $|r|$ and use it to model the large distance behavior, thereby reducing the variance due to contributions at large $|r|$:

$$\left\langle \nu^2 \right\rangle / V = \sum_{|r| \leq |r_{\text{cut}}|} C_{\text{meas}}(r) + \sum_{|r| > |r_{\text{cut}}|} C_{\text{fit}}(r).$$

A typical fit is shown below for m_{ud}/m_s = 0.0062/0.031 at $a \approx 0.09$ fm. (Red points are included in the fit.)

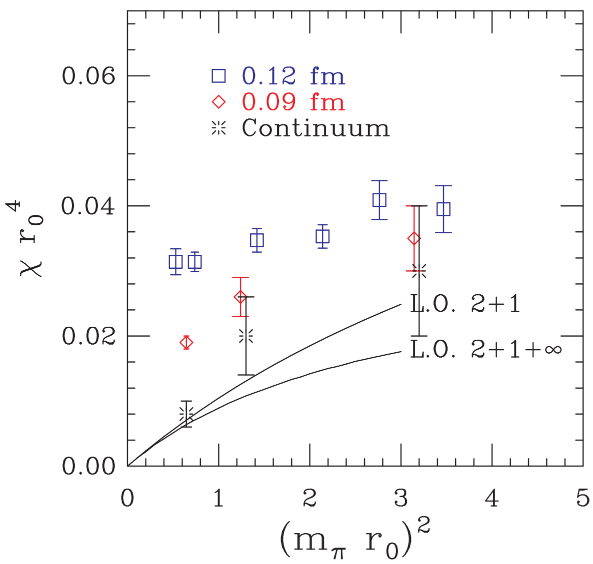


Full



Detail

Result



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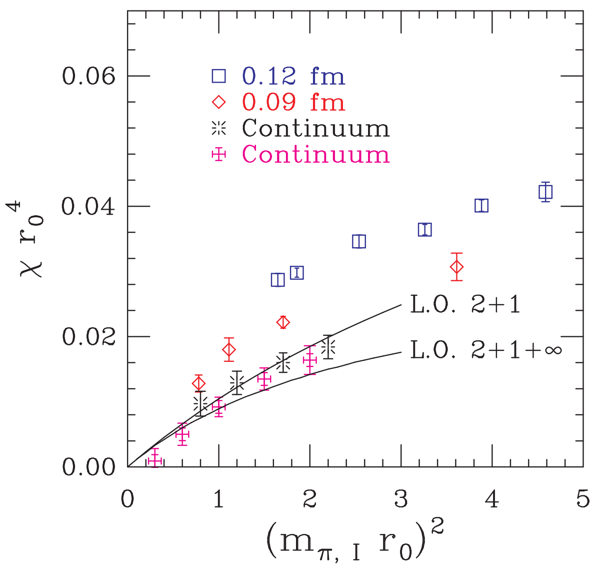

New

Residual lattice artifacts come from the lattice operator definition of topological charge density. It is plausible that they scale as $\mathcal{O}(a^2)$. The continuum extrapolation is based on that assumption. Black points are extrapolated at fixed abscissa. Magenta points are less justifiable extrapolations at fixed Goldstone pion mass, because the extrapolation mixes $\mathcal{O}(a^2)$ and $\mathcal{O}(\alpha_s^2 a^2)$ effects.

Summary and Conclusion

We have measured the topological susceptibility for the improved staggered fermion (Asqtad) action and compared our results with the predictions of staggered chiral perturbation theory. A variance reduction method, based on the expected asymptotic form of the density-density correlator reduces statistical errors in the measurement of the susceptibility. The remaining lattice artifacts can be attributed to the topological charge density operator itself. An extrapolation to the continuum limit gives a result consistent with chiral perturbation theory.

References

- W. J. Lee and S. R. Sharpe, Phys. Rev. D **60**, 114503 (1999) [arXiv:hep-lat/9905023].

- C. Aubin and C. Bernard, Phys. Rev. D **68**, 074011 (2003) [arXiv:hep-lat/0306026].

- H. Leutwyler and A. Smilga, Phys. Rev. D **46**, 5607 (1992).

- Brian Billeter, C. DeTar, and J. Osborn, [arXiv:hep-lat/0406032].

- S. Dürr, Nucl. Phys. B **611**, 281 (2001)

- C. Bernard *et al.*, Phys. Rev. D **68**, 114501 (2003) [arXiv:hep-lat/0308019].

- E. V. Shuryak and J. J. M. Verbaarschot, Phys. Rev. D **52**, 295 (1995) [arXiv:hep-lat/9409020].

- F. Knechtli and A. Hasenfratz, Phys. Rev. D **63**, 114502 (2001).

- T. DeGrand, A. Hasenfratz and T. G. Kovacs, Nucl. Phys. B **505**, 417 (1997)

- E. Follana, A. Hart, and C.T.H. Davies, [arXiv:hep-lat/0406010].

- S. Dürr, C. Hoelbling, and U. Wenger, [arXiv:hep-lat/0406027].

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