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# A Possible Design of the Linac-to-LEB Injection Girder for 1133 MeV $H^-$ Ions

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## Abstract

We present a design of the girder to accommodate injection of 1133 MeV kinetic energy $H^-$ ions from the linac [1] into the LEB. This design is based on Colton and Thiessen's design [2] for 600 MeV ions, and does not require a modification of the current [3] LEB lattice.

## 1 Introduction

The CDR [4] specifies the injection momentum in the LEB to be $p = 1.219$ GeV/$c$ and describes a lattice with a 250 m circumference. Because of a desire to decrease the dispersion in that design, a new lattice has been proposed [3] with a circumference of 343 m. For the nominal value of $1 \times 10^{10}$ particles per bunch this increase in circumference implies a substantial space-charge tune shift of $-0.21$ at injection, which means that many resonances are unavoidably crossed. Some of these may cause emittance dilution due to phase-space distortion, with the corresponding potential for degradation of the luminosity in the SSC. Although tracking studies [3] show that this is not a serious problem if the lattice is properly tuned, it is natural to consider the option of injecting at higher energy. This would provide a safety margin and would also allow the possibility of operating the SSC at higher current (say three times nominal) or at lower emittance (say half of nominal). Since the space-charge tune shift varies roughly like $\beta^{-1}\gamma^{-2}$, a modest increase in energy has a large payoff in reducing it. At a recent meeting [5] a linac design was considered that would upgrade the kinetic energy from 600 to 1133 MeV with a corresponding increase in momentum from 1.219 to 1.847 GeV/$c$, and a decrease of the space-charge tune shift by more than a factor of 2.

In this note we present a design of the injection girder for such an upgrade. We base our design on Colton and Thiessen's design [2] for 600 MeV kinetic energy. Although our design has the virtue of fitting in the 6.23 m long drift space of the present LEB lattice design, it does not have the simplicity of Colton and Thiessen's because it requires the orbit "bumps" to have different magnetic fields. In fact, it requires the second bump to have a substantial field of 0.66 T.

Because of the desire to achieve controllable and possibly large currents in the LEB, the CDR calls for multi-turn injection, in which the buckets are filled gradually over many turns. This allows the possibility of using a linac with low peak current and low emittance, but requires using $H^-$ ions instead of protons. The ions are coalesced with the existing protons in the bucket and are immediately stripped of the two electrons with a stripping foil. Since the extra electron in the $H^-$ ion is very loosely bound, requiring only 0.755 eV to strip, it is important to pay attention to the magnetic fields in the trajectory of the ions. If the magnetic field or momentum are large enough, the Lorentz force will strip the ions prematurely. The magnets in our design are sufficiently weak that this not a problem.

In Section 2 we present: (1) Colton and Thiessen's girder design for 600 MeV ions; (2) a modification of this design, also for 600 MeV ions, based on a shallower injection angle; and (3), our design for 1133 MeV ions. In Section 3 we present relevant facts about the $H^-$ ion and hydrogen atom lifetimes in a magnetic field based on experimental measurements

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and theoretical calculations. Section 4 contains a discussion of our results.

## 2 Girder Designs

The girder basically consists of a septum magnet, two orbit "bumps," and a stripping foil.

Consider first Fig. 1, in which we show the trajectories of a charged particle through a bend and through a bump. The bump consists of two juxtaposed bending magnets of equal and opposite magnetic fields, so that the trajectory is parallel-displaced. It is straightforward to show that

$$\frac{h}{L} = \tan\frac{\alpha}{2}, \quad \frac{L}{\rho} = \sin\alpha \quad \text{(bend)} \tag{2.1a}$$

$$\frac{h}{L} = \tan\frac{\alpha}{2}, \quad \frac{L}{\rho} = 2\sin\alpha \quad \text{(bump)} \tag{2.1b}$$

from which one obtains the radii of curvature

$$\rho = \frac{L^2+h^2}{2h} \qquad \text{(bend)} \tag{2.2a}$$

$$\rho = \frac{L^2+h^2}{4h} \qquad \text{(bump)} \tag{2.2b}$$

For a given momentum the magnetic field is obtained from

$$p = \kappa\rho B \tag{2.3}$$

where $\kappa \simeq 0.2997925$ (GeV/$c$)/T-m.

The girder design is specified by the magnets lengths, positions and magnetic fields. Consider Fig. 2, which is a reproduction of Colton and Thiessen's design (Fig. 2 of Ref. 2). We assume that the septum $S$ is such that the $\mathrm{H}^-$ ions emerge from it parallel to the closed orbit at a height $h_o$, that the coalesced trajectories emerge out of the bump $\mathrm{O}_1$ at a height $h_o/2$ also parallel to the closed orbit, and that bump $\mathrm{O}_2$ displaces the trajectory back to the closed orbit. The $\mathrm{H}^-$ ions and the protons have the same momenta and we neglect their mass difference. Then the geometry implies the equations

$$\frac{H-h_i}{d} = \tan\alpha\,, \qquad \frac{h_i-h_o}{L_S} = \tan\frac{\alpha}{2} \tag{2.4}$$

For specified initial height $H$, distance $d$, septum length $L_S$ and output height $h_o$, these equations determine the entrance height $h_i$ and angle $\alpha$. Then Eq. (2.3) determines the magnetic field for a given momentum $p$. In the small-angle approximation ($\alpha \ll 1$, or $L \ll \rho$), which is a good approximation in the cases considered here, these equations readily yield

$$h_i = \frac{HL_S + 2h_o d}{L_S + 2d} \tag{2.5}$$

and then Eq. (2.1) determines the radii of curvature in the septum and bumps,

$$\rho_S = \frac{L_S^2 + (h_i-h_o)^2}{2(h_i-h_o)}, \quad \rho_n = \frac{4L_n^2 + h_o^2}{8h_o} \tag{2.6}$$

where $n = 1,\, 2$ refers to bumps $\mathrm{O}_1$ and $\mathrm{O}_2$.

Colton and Thiessen's design on Fig. 2 has the following values for the parameters: $H = 34.7$ cm, $h_i = 17.9$ cm, $h_o = 5.35$ cm, $L_S = 1.5$ m, $d = L_1 = L_2 = 1$ m and $\alpha = 0.16667$ rad $= 9.55°$. The separation between the bumps is 50 cm, and the stripping foil is half-way in between. For 600 MeV kinetic energy ($p = 1.219$ GeV/$c$), the magnetic fields and radii of curvature are $B_S = B_1 = B_2 = 0.45$ T and $\rho_S = \rho_1 = \rho_2 = 9.03$ m.

Before attempting a design for a kinetic energy of 1133 MeV, we take an intermediate step by modifying the above design for 600 MeV. We choose as shallow an injection angle as possible, by having the transfer line from the linac grazing the quadrupole magnet Q. Since the magnet is 6″ high, we choose $H = 7'' = 17.8$ cm, and leave the rest of the design as before. In particular, the septum length and position, and the height $h_o$ remain unchanged. This is shown in Fig. 3. From the above equations we obtain $h_i = 10.7$ cm, $\alpha = 0.0711$ rad $= 4.07°$. For a momentum of 1.219 GeV/$c$ we obtain $B_S = 0.19$ T and $\rho_S = 21.4$ m. The fields and orbits in the bumps remain unchanged.

Assume now that the ions have a kinetic energy of 1133 MeV, corresponding to a momentum $p = 1.847$ GeV/$c$. The Lorentz force is larger than before so there is the danger of premature stripping in the septum and in the first bump. Therefore the magnetic fields must be smaller, which implies longer magnets in order to achieve the necessary bending. Our design is presented in Fig. 4. As we show in the next Section, the magnetic fields are small enough that the premature stripping is not a problem. The main difference with the previous design is the length of bump $\mathrm{O}_1$, which we now choose to be $L_1 = 1.5$ m. The heights $H$, $h_i$ and $h_o$ are the same as before, as is the length $L_2$ of the second bump. Eq. (2.6) yields $\rho_S = 21.1$ m, $\rho_1 = 21.0$ m and $\rho_2 = 9.35$ m, so that, for $p = 1.847$ GeV/$c$, we get $B_S = 0.292$ T, $B_1 = 0.293$ T, $B_2 = 0.658$ T. The large magnetic

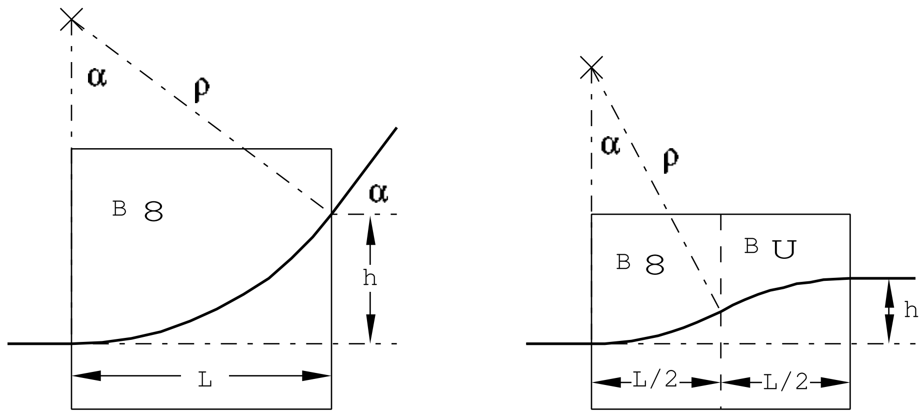


Figure 1: The trajectory of a charged particle through a bending magnet and through an orbit bump. The bump is a juxtaposition of two magnets of equal and opposite magnetic fields such that the trajectory is parallel-displaced. The magnets have magnetic field $B$ and length $L$, and the particle's trajectory is a sector of circle with radius $\rho$. The quantities $B$, $\alpha$, $h$, $\rho$ and $L$ are not necessarily equal in the bend and in the bump.

field of $O_2$ is not of concern for the problem of stripping because the ions are stripped by the foil anyway. However, it may present operational problems that we discuss in the last Section.

# 3 $H^-$ Ion and Hydrogen Atom Lifetimes

## 3.1 $H^-$ Ion Lifetime

When the $H^-$ ion moves in a magnetic field $B$ it experiences a Lorentz force that bends its trajectory but also tends to break it up since the proton and the electrons experience the force in opposite directions. Since the binding energy of the extra electron is only 0.755 eV, this breakup can occur readily. The breakup is a probabilistic process since it is essentially quantum-mechanical in nature. In the reference frame where the ion is at rest, the stripping force is effected by the electric field $\mathcal{E}$ that is the Lorentz-transform of the magnetic field $B$ in the lab (of course the ion also experiences a magnetic field in its own rest frame). The electric field is given by

$$\mathcal{E} = \kappa' \beta\gamma B \tag{3.1}$$

where $\kappa'$ has the same numerical value as $\kappa$, but has dimensions of GV/T-m. Note that $\mathcal{E}$ is proportional to $\beta\gamma$, which is in turn proportional to the momentum. For the $H^-$ ion (or the hydrogen atom) this equation is conveniently rewritten as

$$\mathcal{E}\,[\mathrm{MV/cm}] = 3.197\, p\,[\mathrm{GeV}/c]\, B\,[\mathrm{T}] \tag{3.2}$$

The lifetime of the ion in an electric field can be calculated by applying the WKB approximation to the tunneling probability [6, 8] It has also been measured in several experiments [9, 12] whose results, for the ion's lifetime $\tau$ *in its own rest frame* is very well parametrized by the formula

$$\tau = \frac{A}{\mathcal{E}} \exp\left(\frac{C}{\mathcal{E}}\right) \tag{3.3}$$

whose validity has recently been justified from first principles [13].

In the region of values of $\mathcal{E}$ where they overlap, the measurements in Refs. 9, 11 and 12 are fairly consistent with each other within errors, but are not consistent with Ref. 10. Ref. 11, which covers the range $\mathcal{E} = 1.87 - 2.14$ MV/cm, has $A = 7.96 \times 10^{-14}$ MV-s/cm and $C = 42.56$ MV/cm, while Ref. 12, which covers $\mathcal{E} = 1.87 - 7.02$ MV/cm, has $A = (2.47 \pm 0.09) \times 10^{-14}$ MV-s/cm and $C = 44.94 \pm 0.10$ MV/cm. These two fits yield fairly similar results in the region of values of $\mathcal{E}$ of interest to us. Since the data in Ref. 12 span a wider range of values of $\mathcal{E}$ and is very well fitted by (3.3), we assume that this fit is more robust, so we adopt it for our purposes, which require a slight extrapolation to lower electric field values.

In order to calculate the mean decay length in the lab $\lambda$, we multiply $\tau$ by the Lorentz dilatation factor $\gamma$ and by the velocity of the ion,

$$\lambda = \beta\gamma c\tau \tag{3.4}$$

Thus for a given momentum $p$ we compute $\beta$ and $\gamma$, then obtain $\mathcal{E}$ from (3.1) or (3.2), and $\lambda$ from the above equation. Table 1 below shows $\lambda$ and $\mathcal{E}$ for an 1133 MeV kinetic energy $H^-$ ion ($p = 1.847$ GeV/$c$) in various magnetic fields.

Table 1: $\lambda$ and $\mathcal{E}$ for various values of $B$.

| $B$ [T] | $\mathcal{E}$ [MV/cm] | $\lambda$ [m] |
|---|---|---|
| 0.2 | 1.18 | $4.28 \times 10^{11}$ |
| 0.3 | 1.77 | $8.76 \times 10^{5}$ |
| 0.4 | 2.36 | $1.15 \times 10^{3}$ |
| 0.5 | 2.95 | 20.4 |
| 0.6 | 3.54 | 1.34 |
| 0.7 | 4.13 | 0.188 |
| 0.8 | 4.72 | 0.042 |

For $p = 1.219$ GeV/$c$ and $B = 0.45$, as appropriate for Colton and Thiessen's design, the electric field is $\mathcal{E} = 1.75$ MV/cm. For $p = 1.847$ GeV/$c$ and $B = 0.293$ T, as appropriate for our design, the field is almost the same, $\mathcal{E} = 1.73$ MV/cm. This was the criterion we had in mind in our redesign of the girder, namely that the ions should be as stable as in Colton and Thiessen's case. A mean decay length of 876 km for $B = 0.3$ T implies that fewer than 4 out of every $10^6$ ions are stripped by the magnets before they reach the foil, and therefore this problem is not significant. However, because of the rapid variation of the mean decay length with magnetic field, one should be cautious about this issue, especially because of the possible effects of fringe fields.

### 3.2 Hydrogen Ionization

The binding energy of the hydrogen atom in its ground state is sufficiently large that it is extremely difficult to ionize it by a magnetic field in all practical cases, so we present the expression for its lifetime for the sake of completeness only.

Consider a hydrogen atom in its ground state, at rest in an electric field $\mathcal{E}$. The probability for ionization per unit time, $\tau^{-1}$, can be expressed in terms of the Bohr radius $a_0 = \hbar^2/me^2 = 0.529 \times 10^{-10}$ m and the ground state binding energy $E_0 = me^4/2\hbar^2 = 13.61$ eV. The result of applying the WKB approximation to the tunneling probability yields for the lifetime [8]

$$\tau = \frac{\tau_0 \mathcal{E}}{16\mathcal{E}_0} \exp\left(\frac{4\mathcal{E}_0}{3\mathcal{E}}\right) \tag{3.5}$$

where

$$\mathcal{E}_0 = \frac{E_0}{ea_0} = \frac{e}{2a_0^2} = 2.57 \text{ GV/cm} \tag{3.6a}$$

$$\tau_0 = \frac{\hbar}{E_0} = 0.484 \times 10^{-16} \text{ s} \tag{3.6b}$$

Note that the exponential factor, which is characteristic of the WKB approximation, has the same form as for the $H^-$ ion, Eq. (3.3), but that the prefactor has an inverted dependence on $\mathcal{E}$. We don't know whether this difference is significant since the potential seen by the extra electron in the $H^-$ is not Coulombic. We attempted to fit the $H^-$ data with a formula of the form (3.5), but the fit is not nearly as good as with (3.3) (this discrepancy is of no concern for our present purposes because we require only a small extrapolation from the experimental data for the $H^-$ lifetime). The constant $\mathcal{E}_0$ is much larger than the corresponding constant $C$ in the $H^-$ case on account of the larger binding energy and smaller Bohr radius. As a result, the mean decay length obtained from Eq. (3.5) for an 1133 MeV kinetic energy hydrogen atom in a 10 T magnetic field yields $\lambda \simeq 0.07$ light years, so the ionization of hydrogen is not an issue.

## 4 Discussion

The design of the girder we have presented here for 1133 MeV kinetic energy ions fits in the 6.23 m long drift in the present design of the LEB, and is satisfactory from the point of view of premature ion stripping. However, the second bump has a substantial magnetic field, which is also different from the field in the first bump and in the septum. This implies that $O_2$ has to be on a different power supply circuit as the other two magnets. Thus the simplicity of Colton and Thiessen's design, in which all three magnets have the same field, is lost.

The present designs of the LEB [3] and linac [1] call for a 26-turn injection. Since the revolution period of the LEB at injection is 1.44 $\mu$s, this means that the magnets in the girder have to be simultaneously powered for 37.44 $\mu$s out of the LEB's cycle time of 0.1 s. Although this means a duty cycle of only $\sim$ 0.04%, the substantial magnetic field of 0.66 T in $O_2$ may be a problem. However, it is easy to show, in the small angle approximation, $L \ll \rho$, that the magnetic field and magnet length of either bump satisfy

$$BL^2 = 2h_o \frac{p}{\kappa} \tag{4.1}$$

Thus for a given momentum $p$ and height $h_o$, the magnetic field decreases as $L^{-2}$, so this possible problem may be avoided by a modest increase in magnet length.

Although we have not optimized our design, we have shown that a girder for 1133 MeV ions is possible within the present LEB design. The girder would fit snugly, and its operation may be not as simple as for lower injection energy.

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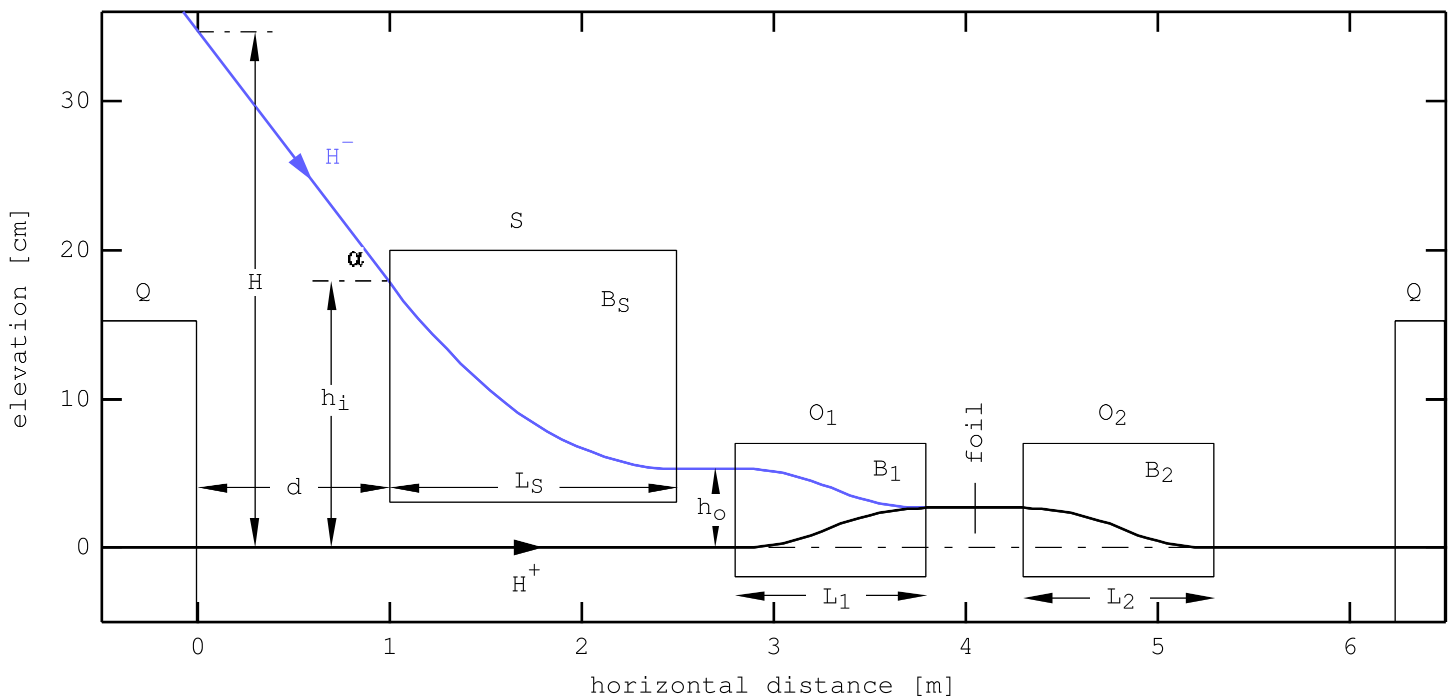


Figure 2: Girder design for 600 MeV kinetic energy $H^-$ ions by Colton and Thiessen. The girder consists of a septum S, two orbit bumps $O_1$ and $O_2$, and a stripping foil. The Q's are the quadrupoles at the ends of the 6.23 m drift. The injection angle is $\alpha = 0.16667$ rad=9.552°, and the initial elevation is $H = 34.7$ cm. For $d = 1$ m, $L_S = 1.5$ m and $h_o = 5.35$ cm one obtains $h_i = 17.9$ cm and a field $B_S = 0.45$ T. The bumps are 50 cm apart and have lenghts $L_1 = L_2 = 1$ m and fields $B_1 = B_2 = 0.45$ T. The radii of curvature are the same in the septum and the bumps, $R = 9.0$ m. The heavy lines are the actual beam trajectories for these fields and geometry. The coalesced beams emerge out of $O_1$ at a height $h_o/2$.

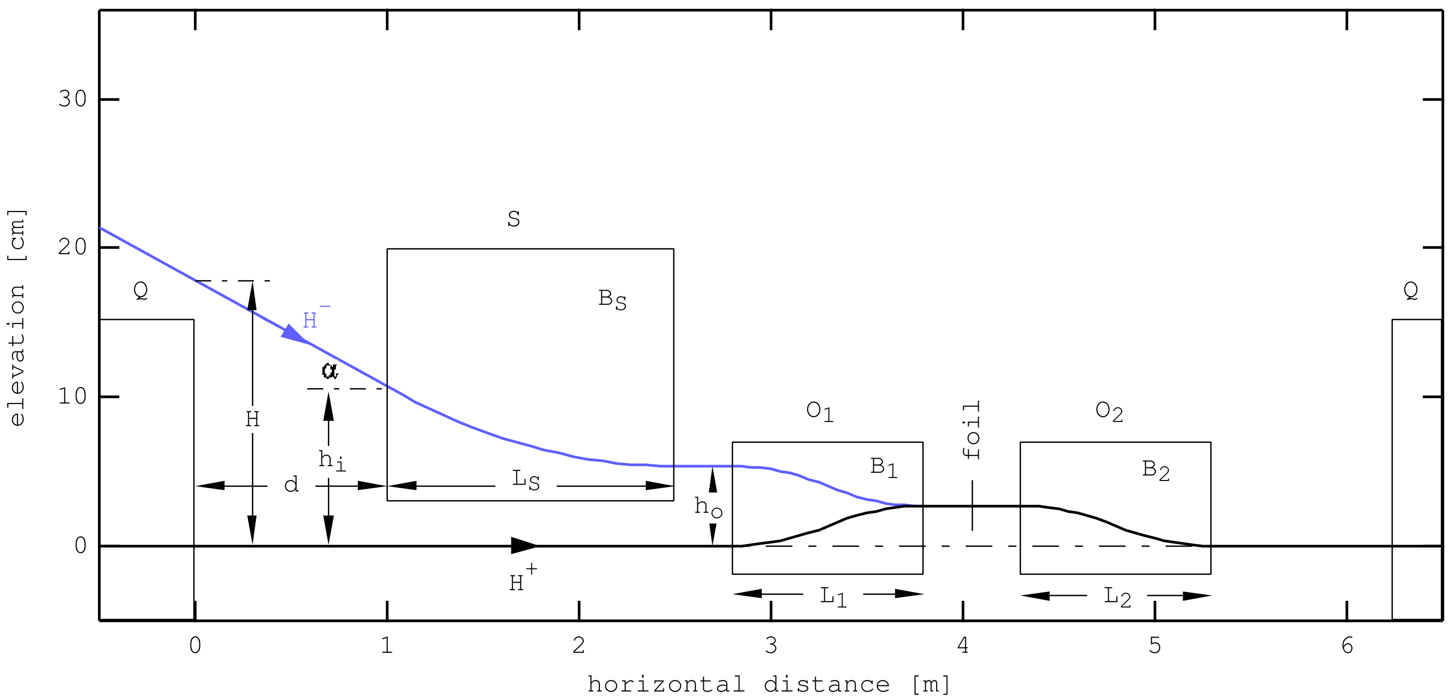


Figure 3: Our proposed girder design for 600 MeV kinetic energy $H^-$ ions is almost identical to Colton and Thiessen's, the difference arising from a choice of a lower initial elevation $H = 7'' = 17.8$ cm. This results in $\alpha = 0.0711$ rad $= 4.07°$, $h_i = 10.7$ cm, $B_S = 0.19$ T, and a radius of curvature $R = 21$ m in the septum. The rest of the parameters are as specified in Fig. 2.

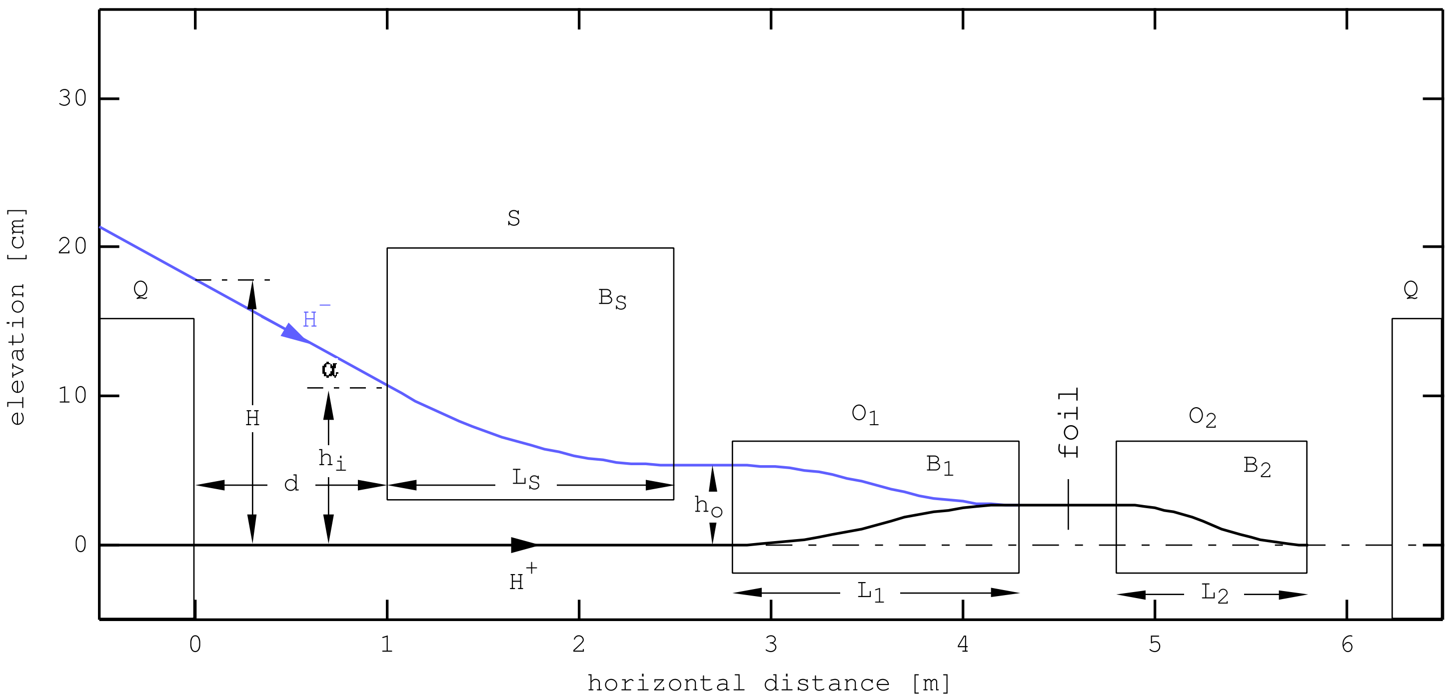


Figure 4: Our proposed girder design for 1133 MeV kinetic energy ions assumes the same beam elevations as in Fig. 3, namely $H = 17.8$ cm and $h_o = 5.35$ cm. This implies $\alpha = 0.0711$ rad $= 4.07°$ and $h_i = 10.7$ cm. Bump $O_1$ is longer than before, with $L_1 = 1.5$ m. The fields of the magnet are $B_S = B_1 = 0.29$ T and $B_2 = 0.66$ T, and the radii of curvature are $R_S = R_1 = 21$ m and $R_2 = 9.4$ m.