Beams-doc-1066-v2
March 15, 2004

Pattern of Phases From the TeV BPM Using the Echotek Board

Rob Kutschke

Abstract

This note explains the pattern of phases observed when the TeV BPM signals are processed with the modified recycler Echotek board. It also suggests a possible explanation for the observed bistable values of Proton-Pbar relative phase, which was discussed in Beams-doc-1059. There were several mistakes in v1 of this document so make sure you have the latest version.

1 Introduction

When the Echotek boards are used to measure an approximately constant beam, the phase of the measurement on a cable is not approximately constant from one measurement to the next. Instead the phase varies randomly among 5 possible values, separated by much more than the statistical error on the measurement. To illustrate the effect, I have used some data taken on March 5, 2004, using the modified recycler Echotek board on BPM HA15. Figure 1 shows the phase of the signal on each of the 4 cables. The phase is defined as the argument of the complex number (I,Q), where I and Q were read from the data logger. The data are discussed in more detail in Beams-doc-1062.

The critical observation is that, on each cable, the phase has 5 allowed values, separated from its neighbors by $1/7$ of 2π; remember that the cut line at phases of $\pm\pi$ is an artifact so the points at the bottom of the plots are separated from those on the top by $1/7$ of 2π. The pattern of which phase follows which phase in time is not important — it is an artifact of the timing of the 15 Hz data logger. The 5 allowed values differ from cable to cable, which represents true phase differences among the signals

2 A Toy Model

The first job is to explain why there are 5 stable values. Figure 2 has some plots from a toy model which will help explain this. In this toy model, times and frequencies are expressed in units in which the RF frequency $f_{RF} = 1$. Figure 2a) shows 10 periods of a sine wave with the RF frequency. The next plot shows a histogram of the waveform which results from digitizing the sine wave at a sampling frequency of $f_s = 7/5\ f_{RF}$. In this model, the sampling occurs at the leading edge of each period of the sampling clock. The sampled waveform repeats after seven measurements, which corresponds to 5 RF periods.

If one were to FFT this digitized waveform, power would appear not only a the RF frequency but also at image frequencies of $mf_s \pm f_{RF}$, where m is a positive integer. These image frequencies are artifacts of the digitization process and they contain the same information as is present at the RF frequency. That is, one may down convert the digitized waveform at either the RF frequency, or at any of its images, and the same information is available.

The Echotek boards downconvert at the lowest image frequency, $f_i = 2/5\ f_{RF}$. I am not 100% certain why this frequency was chosen. It probably has something to do with the following: if the downconversion process is driven by the $7/5\ f_{RF}$ clock then only the $2/5\ f_{RF}$ image frequency has a Nyquist frequency less than $7/5\ f_{RF}$. So choosing one of the other frequencies would either require a different clock or would result in loss of information.[1]

The math of the downconversion is:

$$(I,Q) \quad = \quad \sum_n e^{-i\omega_i t_n} f(t_n), \tag{1}$$

where (I,Q) is a complex number, $t_n = t_0 + n(5\,T_{RF}/7)$ is the time at the n^{th} tick of the sampling clock, $n = 0, 1, 2\ldots$, $T_{RF} = 1/f_{RF}$, $f(t_n)$ is value of the digitized waveform at t_n, and where $\omega_i = 2\pi f_i$. The constant t_0, depends on the relative phase of the $7/5\ f_{RF}$ clock at the time that a trigger is received. This expression can be simplified a little,

$$(I,Q) \quad = \quad e^{-i\delta} \sum_n e^{-i2\pi 2n/7} f(t_n), \tag{2}$$

where $\delta = 2\pi\omega_i t_0$. Because phase shifts of integer multiples of 2π are not observable, the important part of $\omega_i t_n$ is $\mathrm{mod}(2n, 7)$, the remainder of $2n$ with respect to 7. The third plot on the page shows how $\mathrm{mod}(2n, 7)$ varies as a function of t_n. After 7 ticks of the sampling clock, corresponding to 5 RF cycles, the phase of the exponential is back in phase with the RF frequency. Equation 2 also shows that a time offset at the start of a measurement appears as a phase shift in the final result.

In short, the down conversion process does a bin by bin multiplication of the second histogram by an exponential with the phase of the third histogram.

3 Explaining the Pattern

On the Echotek board, the sampling clock is derived from the Tevatron RF frequency and runs free. One can think of the sampling clock as counting

[1] Would it also introduce additional frequency artifacts?

$0, 1, 2, 3, 4, 5, 6, 0, \ldots$. The sampling clock returns to 0 every 7 ticks, which corresponds to exactly 5 RF cycles. The period of the Tevatron is 1113 buckets, which is not evenly divisible by 5. Finally, Echotek measurements are triggered by the turn marker, which is fixed in phase relative to the RF frequency. In this toy model it means that the triggers may arrive only at integer values of the time.

Consider the case that, at the marker for one turn, the sampling clock is in phase with the RF frequency, as is drawn in Figure 2c). At the marker for the second turn, the remainder $\mathrm{mod}(1113, 5) = 3$ and, at $t = 3$, Figure 2c) shows that the sampling clock will be part way through its state[2] 4. At the start of the third turn, the remainder $\mathrm{mod}(2 \times 1113, 5) = 1$, and the sampling clock will be part way though its state 1. At the start of the fourth turn, the remainder $\mathrm{mod}(3 \times 1113, 5) = 4$, and the sampling clock will is part way through its state 5. At the start of the fifth turn, $\mathrm{mod}(4 \times 1113, 5) = 2$ and the sampling clock is part way through its state 2. Finally, at the start of the sixth turn, the sampling clock is back in phase with the RF frequency. The cycle then repeats indefinitely.

In summary, although the sampling clock has 7 possible states, only 5 of these may occur at the time of a turn marker. These states are: 0, 1, 2, 4 and 5. On the Echotek boards, the command to arm for a measurement is controlled by the data logger, which is not synchronized with the beam. Therefore any group of measurements should sample all of the 5 allowed states.

I am uncertain about one detail of the Echotek downconversion and I see that one of two scenarios is possible. Fortunately both give the same answer. The options are: either the sum in the down conversion starts with the data which is current at the time of the trigger or it starts on the next state transition of the sampling clock. I will describe the first option now.

Suppose that a measurment is triggered when the sampling clock is in its state 0. Suppose that another measurement, on an identical beam, is triggered when the sampling clock is in its state 1. Inspection of equation 2 and Figure 2c), shows that these two measurements will differ by an overall phase of $2/7$ of 2π. Similarly, if a measurement is triggered when the sampling clock is in its state 2, the measurement will differ from the first by an overall phase of $4/7$ of 2π. When the other possibilities are considered, this gives rise to 5 possible phases for the (I, Q) measurement. Relative to the phase of the first measurement they are, $0/7$, $1/7$, $2/7$, $3/7$, and $4/7$ of 2π. This pattern of 5 allowed states, separated by $1/7$ of 2π, is observed in Figure 1.

The alternate scenario is that the Echotek board starts its sum on the next transition of the sampling clock. Inspection of Figure 2c), shows that at the transitions following times (0,1,2,3,4) the clock states are (0,2,3,5,6), which have relative phases of (0,4,6,3,5) in units of $2\pi/7$, respectively. This is also consistent with the pattern in Figure 1.

So long as all 4 cables have their clocks synchronized, they will have the same overall phase shift and the BPMs will work properly.

[2] Counting the states as $0, 1, \ldots 6$.

4 A Candidate Explanation for the Bad Data

In the data taken before March 5, 2004, the Pbar channels had an additional delay, relative to the proton channels, of 3 ticks of the clock. This data had a bistable pattern for the difference between the phase of the Pbar signals and the phase of the proton signals. When the 3 tick delay was removed, the phase difference between the Pbar and proton signals became stable at a single value. In this section I will discuss an idea for how a delay can create a bistable phase difference.

The bad data is illustrated in Figure 3. Part a) shows the familiar pattern of 5 stable phases for the Pbar A cable from data taken on Feb 27, 2004. Part b) shows the phase of Pbar A relative to the phase of proton A+B; this is the bistable pattern mentioned above. Part c) shows that the two values in part b) do not occur randomly — one value is associated with 3 of the phases from a) and the other value is associated with the remaining two phases from a). Finally part d) shows that the two values from b) yield very different measured Pbar intensities, after correction for proton contamination on the Pbar cables. There was no beam in the machine at this measurement so it is clear that the relative phase near π radians is the correct value. If I add 2/7 of $2\pi/7$ to the points with a phase difference near 1.1 radians then the Pbar intensity for these points becomes consistent with zero.

Peter Prieto tells me that the meaning of "3 ticks" is 3 ticks of the 7/5 f_{RF} clock. Figure 2d) shows the same histogram as part c) but delayed by 3 ticks of the 7/5 f_{RF} clock. Comparison of parts c) and d) shows that at, at any given time, the value in histogram d) is the value in histogram c) plus $2\pi/7$. Therefore, if the Pbar signals are delayed by 3 ticks, they will have a phase shift relative to the proton cables but that phase shift will not depend on the phase of the 7/5 f_{RF} clock at the start of the measurement. Therefore this sort of delay cannot cause the observed pattern.

I have discovered what sort of delay can cause the bistable pattern: if the Pbar signals are delayed by a fraction of a period of the 7/5 f_{RF}, then the relative phase of the Proton and Pbar signals will depend on the phase of the 7/5 f_{RF} at the start of the measurement.

How could such a delay arise? The only clock input received by the Echotek board is the 7/5 f_{RF} clock signal, from which it must construct any other clocks which it needs. I don't really know what other clocks are needed but let's guess that it requires a clock at 2 times the input clock, that is at 14/5 f_{RF}.

Figure 2d) shows the same histogram as part 3 but shifted by 3 ticks of 14/5 f_{RF}. At the times $t = 0, 1, 2, 3, 4$, the phase differences between plots e) and c) are, respectively, $(+4, +4, +2, +4, +2)$, all in units of $2\pi/7$. This reproduces the observed effects:

- The 5 allowed values of the Pbar phase split into a group of 3 and a group of 2.
- The two groups are distinguished by their phase with resect to the phase of the signal on the proton cable.

- Adding 2/7 of 2π to the group of 2 matches their relative phase with that of the group of 3.

I did a similar study with other sorts of fractional delays and they produce similar patterns. In general the 5 phases fall into two groups. In each case the groups two groups are distinguished by their phase difference with respect to the signal on the proton cables. The two phase differences differ by 2/7 of 2π. The number of phases in each group change in the pattern: (0,5), (1,4), (2,3), (3,2) or (4,1) as the length of the delay increases. Each of these patterns persists for a delay of 1/5 of the period of the 7/5 f_{RF} clock.

5 Summary and Conclusions

The pattern of 5 stable phases on each cable arises because the 7/5 f_{RF} clock does not evenly divide the 1113 RF buckets in one turn. Every 5 turns there is an integer number of periods of the 7/5 f_{RF} clock.

The bad data which exhibited bi-stable phase differences between the Pbar and proton cables can be explained if there is a relative delay between the proton and Pbar cables which is a fraction of a period of the 7/5 f_{RF} clock. It is not yet understood how such a delay can be generated by the programmable delays on the Echotek board.

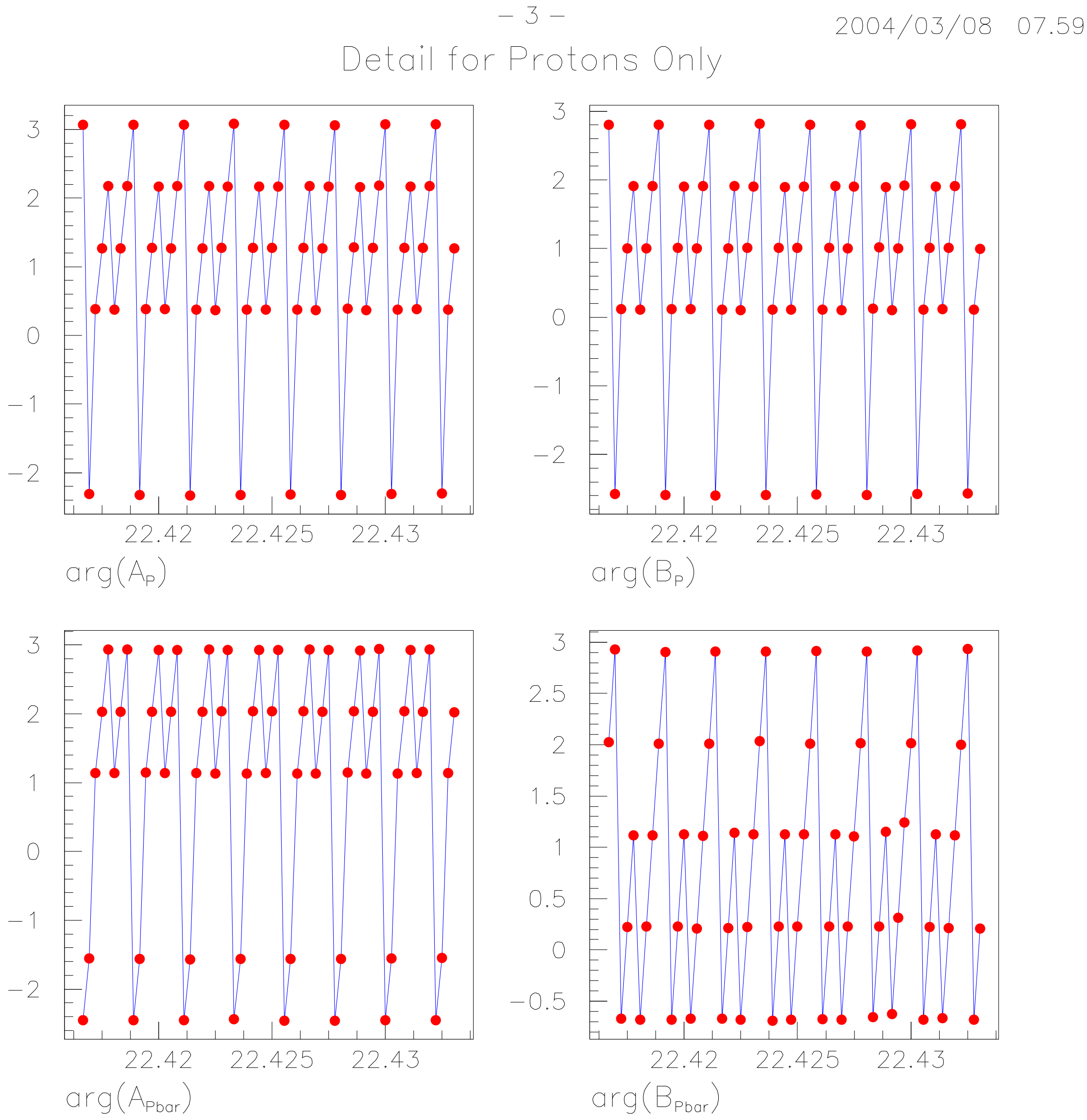

Figure 1: Plots for HA15 for the data of Mar 5, 2004. The plots show the phases of the signals on the 4 cables as a function of time. The units on the horizontal axis are hours and the units on the vertical axis are radians. For each cable, the phase has 5 possible allowed values.

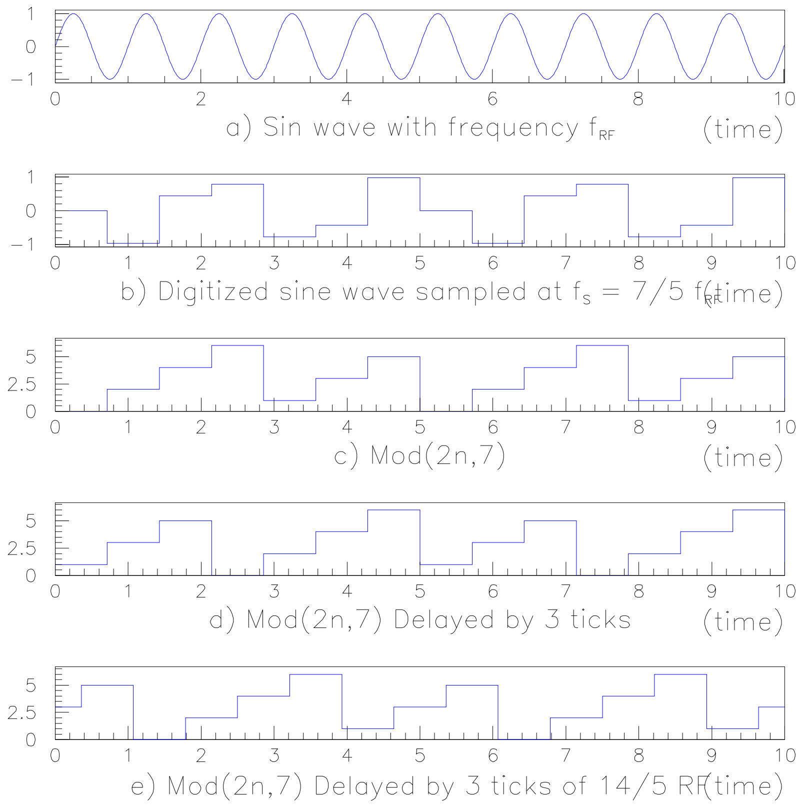

Figure 2: Plots a) to c) are used to illustrate the pattern of 5 allowed phases. Plots d) and e) are used to illustrate the effect of the extra delay which was present on the Pbar cables before March 5, 2004. In all plots the horizontal axis is time in units where the RF frequency is 1. In the last three captions, n counts the number of ticks of the sampling clock. The plots are discussed in detail in the text.

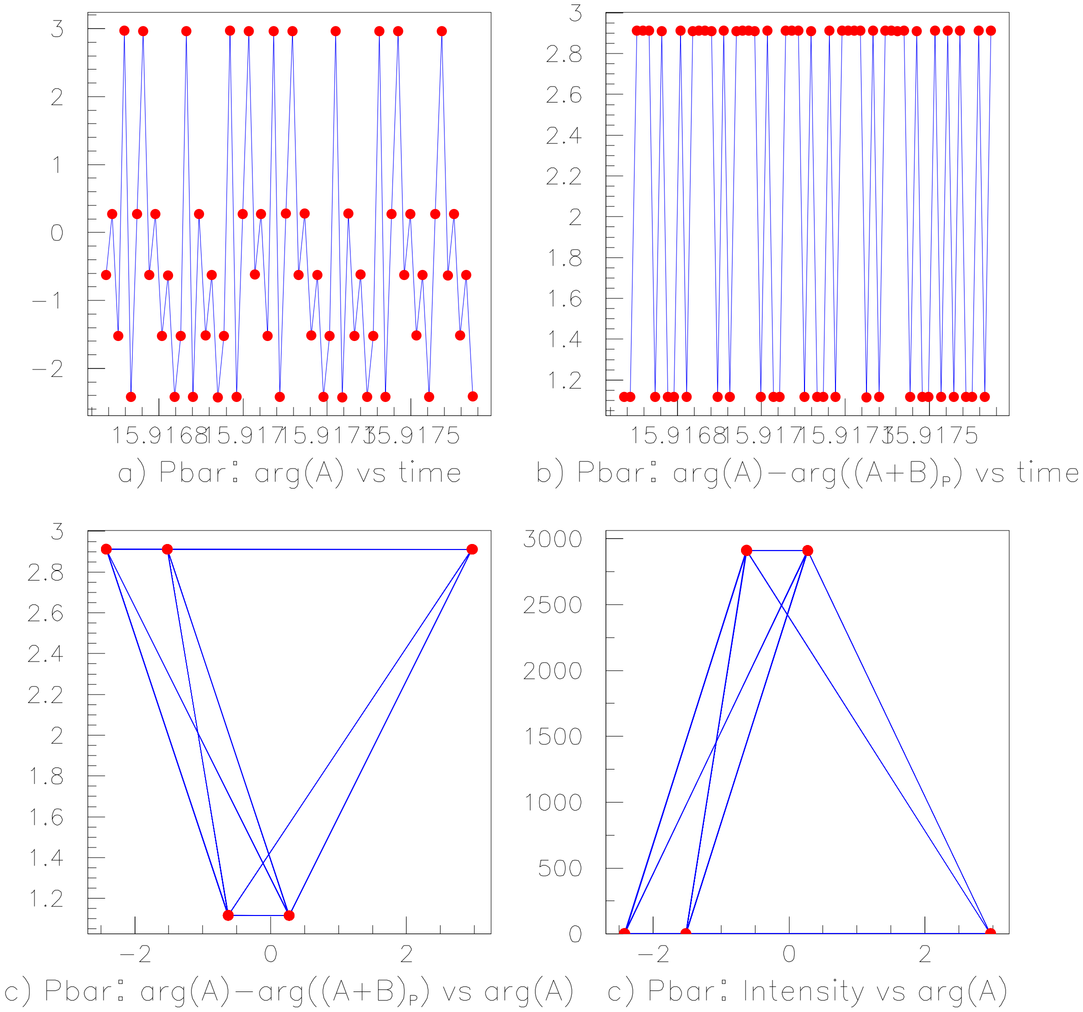

Figure 3: Details of the bistable phase problem. Part a) shows the phase of the Pbar A cable, as a function of time for 4 seconds on Feb. 27 2004, during a time that there were protons only in the machine. There were no orbit or intensity changes during this time. Part b) shows this phase relative to the phase of Proton A+B for the same time interval. Part c) shows a scatter plot of the phase differences in b) vs the phases in a). Part c) shows the Pbar A+B, after the cancelation of proton contamination, plotted against the phase from part a).