Gauge Theories & Particle Physics

Physics 539 Spring Semester 1997 Chris Quigg

February 27, 1997

Recommended Reading:

Gauge Theories, Chapter 5.
M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Massachusetts, 1995), §11.1 and §20.1.
Okun, Chapter 5.
Aitchison and Hey, Chapter 13.

Cultural Reading:

Alan Lightman, *Einstein's Dreams* (Pantheon, New York, 1993).
On the analogy between the Ginzburg-Landau picture of the superconducting phase transition and the Higgs mechanism: Robert E. Marshak, *Conceptual Foundations of Modern Particle Physics* (World Scientific, Singapore, 1993), §4.4.
S. Weinberg, *The Quantum Theory of Fields,* vol. 2, "Modern Applications" (Cambridge University Press, Cambridge, 1996), §19.1, 19.2, 21.1.

Problems (due March 4, 1997)

17. The two-dimensional Ising model is an instructive cartoon model of a ferromagnet that displays a second-order phase transition—spontaneous magnetization—at a critical (inverse) temperature $\beta_c = 0.4407$. It is easily studied by Monte Carlo simulation. For this exercise, take advantage of Kenji Harada's JAVA applet at http://fndsys.kuamp.kyoto-u.ac.jp/~harada/Monte-eg.html or the Ising simulation XISING in Mike Creutz's collection of XTOYS at http://penguin.phy.bnl.gov/www/xtoys/xtoys.html.
(a) Heat the system above the critical temperature and observe the disordered state.
(b) Cool the system just below the critical temperature to see the formation of domains.
(c) Cool the system further and watch the evolution of the spontaneous magnetization.
(d) Beginning from a zero-temperature (ordered) configuration, heat the system and observe the formation of small bubbles of flipped spins. When the temperature exceeds the critical temperature, the bubbles expand and merge, and up-down symmetry is restored.
Repeat the experiments enough times to develop some intuition about the occurrence of configurations far from the average. [The original paper is E. Ising, *Z. Phys.* **31**, 253 (1925). For more than you want to know, see B. M. McCoy and T. T. Wu, *The Two-Dimensional Ising Model* (Harvard University Press, Cambridge, 1973).]

18. Analyze the spontaneous breakdown of a global $SU(2)$ symmetry. Consider the case of three real scalar fields ϕ_1, ϕ_2, and ϕ_3, which constitute an $SU(2)$ triplet, denoted

$$\boldsymbol{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}.$$

The Lagrangian density is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \boldsymbol{\phi}) \cdot (\partial^\mu \boldsymbol{\phi}) - V(\boldsymbol{\phi} \cdot \boldsymbol{\phi}),$$

where as usual

$$V(\boldsymbol{\phi} \cdot \boldsymbol{\phi}) = \tfrac{1}{2}\mu^2 \boldsymbol{\phi} \cdot \boldsymbol{\phi} + \tfrac{1}{4}\,|\lambda|\,(\boldsymbol{\phi} \cdot \boldsymbol{\phi})^2.$$

Assume that for $\mu^2 < 0$ the potential has a minimum at

$$\langle \boldsymbol{\phi} \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}.$$

Then show that (a) the vacuum remains invariant under the action of the generator T_3, but not under T_1 or T_2; (b) the particles associated with T_1 and T_2 become massless (Goldstone) particles; (c) the particle associated with T_3 acquires a mass $= \sqrt{-2\mu^2}$.

19. Generalize the preceding example to a Lagrangian that describes the interactions of n scalar fields and is invariant under global transformations under the group $O(n)$. After spontaneous symmetry breaking and the choice of a vacuum state, show that the vacuum is invariant under the group $O(n-1)$. Verify that the number of Goldstone bosons corresponds to the number of broken generators of the original symmetry group—*i.e.*, to the difference between the number of generators of $O(n)$ and $O(n-1)$.

20. The Ginzburg–Landau theory of superconductivity provides a phenomenological understanding of the Meissner effect: the observation that an external magnetic field does not penetrate the superconductor. Ginzburg and Landau introduce an "order parameter" ψ, such that $|\psi|^2$ is related to the density of superconducting electrons. In the absence of an impressed magnetic field, expand the free energy of the superconductor as

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4\,,$$

where α and β are phenomenological parameters.
(a) Minimize $G_{\text{super}}(0)$ with respect to the order parameter and discuss the circumstances under which spontaneous symmetry breaking occurs. Compute $\langle|\psi|^2\rangle_0$, the value at which $G_{\text{super}}(0)$ is minimized.
(b) In the presence of an external magnetic field $\mathbf{B}$, a gauge-invariant expression for the free energy is

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{2} + \frac{1}{2m^*}\psi^*(-i\nabla - e^*\mathbf{A})^2\psi.$$

(The effective charge e^* turns out to be $2e$, because $|\psi|^2$ represents the density of Cooper pairs.) Derive the field equations that follow from minimizing $G_{\text{super}}(\mathbf{B})$ with respect to ψ and $\mathbf{A}$. Show that in the weak-field approximation ($\nabla\psi \approx 0$, $\psi \approx \langle\psi\rangle_0$) the photon acquires a mass within the superconductor. [Reference: V. L. Ginzburg and L. D. Landau, *Zh. Eksp. Teor. Fiz.* **20,** 1064 (1950); English translation: see *Men of Physics: Landau,* Vol. II, edited by D. ter Haar, Pergamon, New York, 1965. For further information, see §21.6 of Weinberg.]

* * * * *

Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by `EFGHIJ-MA-89`):

$$\langle \boldsymbol{\phi} \rangle_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (5.2.4)$$

$$\boldsymbol{\phi} = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad (5.3.2)$$

* * * * *

Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to http://lutece.fnal.gov/Princeton/.