BFKL $\gamma^*\gamma^*$ cross-sections at LC and CLIC

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Abstract. The BFKL dynamics can be successfully tested at the future high energy e^+e^- linear colliders. The total $\gamma^*\gamma^*$ cross-section is calculated in the Leading Log QCD dipole picture of BFKL dynamics, and compared with the one from 2-gluon exchange. The rapidity dependence of the cross-section remains a powerful tool to increase the ratio between the BFKL and the 2-gluon cross-sections and is more sensitive to BFKL effects, even in the presence of higher order corrections.

I DIFFERENTIAL CROSS-SECTIONS

We want to calculate the total $\gamma^*\gamma^*$ cross-section derived in the Leading Log QCD dipole picture of BFKL dynamics [1,2]. This could be a good test of the BFKL equation which can be performed at e$^+$-e$^-$ colliders (LEP or linear collider). The advantage of this process is that it is a process. which does not involve non perturbative couplings.

In this study, we compare the 2-gluon and the BFKL cross-sections. Defining y_1 (resp. y_2), and Q_1^2 (resp. Q_2^2) to be the rapidities and the squared transfered energies for both virtual photons, one gets [4,6]

$$d\sigma_{e^+e^-}(Q_1^2, Q_2^2; y_1, y_2) = \frac{4}{9}\left(\frac{\alpha_{e.m}^2}{16}\right)^2 \alpha_s^2\,\pi^2\sqrt{\pi}\,\frac{dQ_1^2}{Q_1^2}\frac{dQ_2^2}{Q_2^2}\frac{dy_1}{y_1}\frac{dy_2}{y_2}\frac{1}{Q_1 Q_2}$$
$$\frac{e^{\frac{4\alpha_s N_c}{\pi} Y \ln 2}}{\sqrt{\frac{14\alpha_s N_c}{\pi} Y\zeta(3)}} \times e^{-\frac{\ln^2 \frac{Q_1^2}{Q_2^2}}{\frac{56\alpha_s N_c}{\pi} Y\zeta(3)}} \left[2l_1 + 9t_1\right]\left[2l_2 + 9t_2\right], \tag{1}$$

for the BFKL-LO cross-section, where $t_1 = \frac{1+(1-y_1)^2}{2}$, $\quad l_1 = 1-y_1$, and an analogous definition for t_2 and l_2, and $Y = \ln \frac{s y_1 y_2}{\sqrt{Q_1^2 Q_2^2}}$.

The 2-gluon cross-section has been calculated exactly within the high energy approximation (NNNLO calculation) and reads

$$
\begin{aligned}
d\sigma_{e^+e^-}(Q_1^2, Q_2^2; y_1, y_2) = &\frac{dQ_1^2}{Q_1^2}\frac{dQ_2^2}{Q_2^2}\frac{dy_1}{y_1}\frac{dy_2}{y_2}\frac{64(\alpha_{e.m}^2\alpha_s)^2}{243\pi^3}\frac{1}{Q_1^2} \\
&\left[t_1 t_2 \ln^3\frac{Q_1^2}{Q_2^2} + (7t_1t_2 + 3t_1l_2 + 3t_2l_1)\ln^2\frac{Q_1^2}{Q_2^2}\right. \\
&+ \left((\frac{119}{2} - 2\pi^2)t_1t_2 + 5(t_1l_2 + t_2l_1) + 6l_1l_2\right)\ln\frac{Q_1^2}{Q_2^2} \\
&+ \left.\left(\frac{1063}{9} - \frac{14}{3}\pi^2\right)t_1t_2 + (46 - 2\pi^2)(t_1l_2 + t_2l_1) - 4l_1l_2\right]. \qquad (2)
\end{aligned}
$$

In Ref. [6], it is shown that the 2-gluon cross-section is almost always dominating the DGLAP one in the Double Leading Log approximation. and that the difference between the BFKL and 2-gluon cross-sections increase with Y.

The comparison between the DGLAP-DLL and the 2-gluon cross-section in the LO approximation shows that both cross-sections are similar when Q_1 and Q_2 are not too different, so precisely in the kinematical domain where the BFKL cross-section is expected to dominate. However, when Q_1^2/Q_2^2 is further away from one, the LO 2-gluon cross-section is lower than the DGLAP one, especially at large Y. This suggests that the 2-gluon cross-section could be a good approximation of the DGLAP one if both are calculated at NNNLO and restricted to the region where Q_1^2/Q_2^2 is close to one. In this paper we will use the exact NNNLO 2-gluon cross-section to evaluate the effect of the non-BFKL background, since the 2-gluon term appears to constitute the dominant part of the DGLAP cross-section in the region $0.5 < Q_1^2/Q_2^2 < 2$.

II INTEGRATED CROSS-SECTIONS

Results based on the calculations developed above will be given for a future Linear Collider (500 GeV center of mass energy for LC, and either 3 or 5 TeV CLIC). $\gamma^*\gamma^*$ interactions are selected at e^+e^- colliders by detecting the scattered electrons, which leave the beampipe, in forward calorimeters. For the LC it has been argued [4] that angles as low as 20 mrad should be reached. Presently angles down to 40 mrad are foreseen to be instrumented for a generic detector at the LC.

Let us first specify the region of validity for the parameters controlling the basic assumptions made in the previous chapter. The main constraints are required by the validity of the perturbative calculations. The "perturbative" constraints are imposed by considering only photon virtualities Q_1^2, Q_2^2 high enough so that the scale μ^2 in α_S is greater than 3 GeV^2. μ^2 is defined using the Brodsky Lepage Mackenzie (BLM) scheme [5], $\mu^2 = \exp(-\frac{5}{3})\sqrt{Q_1^2 Q_2^2}$. In this case α_S remains always small enough such that the perturbative calculation is valid. In order that gluon contributions dominates the QED one, Y is required to stay larger than $\ln(\kappa)$ with $\kappa = 100$. (see Ref. [5] for discussion). Furthermore, in order to suppress DGLAP evolution, while maintaining BFKL evolution will constrain $0.5 < Q_1^2/Q_2^2 < 2$

for all nominal calculations. In this paper we will use the exact NNNLO 2-gluon cross-section in the following to evaluate the effect of the non-BFKL background, since the 2-gluon term appears to constitute the dominant part of the DGLAP cross-section in the region $0.5 < Q_1^2/Q_2^2 < 2$.

We will not discuss here all the phenomenological results, and some detail can be found in [6], as well as the detailed calculations. An important issue on the BFKL cross-section is the importance of the NLO corrections and we adopt a phenomenological approach to estimate the effects of higher orders. First, at Leading Order, the rapidity Y is not uniquely defined, and we can add an additive constant to Y. A second effect of NLO corrections is to lower the value of the so called Lipatov exponent in formula 1. We applied this idea phenomenologically for the $\gamma^*\gamma^*$ cross-section as it was done in Ref. [3,6] and in Ref [7]. For this purpose, we modify the scale in α_S so that the effective value of α_S for $Q_1^2 = Q_2^2 = 25$ GeV^2 is about 0.136. Finally, the results of the BFKL and 2-gluon cross-sections are given in Table 1 if we assume this effect and if tagging of scattered leptons can be done down to 40 mrad and to respectively 50 and 100 GeV at LC and CLIC. The ratio between BFKL-HO and 2-gluon cross-sections is lower at CLIC than at LC due to the phase space. With the value of the cross section we obtain, we can expect to get about 97 (28) events for a 3 (5) TeV CLIC.

In figure 1 is plotted the BFKL-HO and the 2-gluon cross sections, as well as their ratio, for a 3 TeV CLIC for different values of the tagging angle. As we already noticed in Ref. [6], it is important to decrease the tagging angle to obtain a better ratio and large values of the cross-sections. The conclusions are similar for a 5 TeV machine.

In figure 2, we study the dependence on the tagging energy. The ratio between BFKL-HO and 2-gluon cross-sections remain almost constant, but lowering the tagging energy has an important effect on the cross-section value, and on the number of events expected per year.

Another idea to establish the BFKL effects in data is to study the energy or Y dependence of the cross-sections, rather than the comparison with total cross-sections itself. To illustrate this point, we calculated the BFKL-HO and the 2-gluon cross-sections, as well as their ratio, for given cuts on rapidity Y (see table 2). We note that we can reach up to a factor 6 difference keeping a cross-section measurable at CLIC if one can tag leptons down to 25 mrad. If this is not possible, a cut on Y at 8 would probably be enough to see BFKL effects.

III CONCLUSION

We first discussed the difference between the 2-gluon and BFKL $\gamma^*\gamma^*$ cross-sections at al linear collider (LC or CLIC). The LO BFKL cross-section is much larger than the 2-gluon cross-section. Unfortunately, the higher order corrections of the BFKL equation (which we estimated phenomenologically using the recent measurement of the L3 collaboration) are large, and the 2-gluon and BFKL-NLO

	BFKL$_{HO}$	2-gluon	ratio
CLIC (3 TeV)	9.7E-2	3.7E-2	2.6
CLIC (5 TeV)	2.8E-2	1.1E-3	2.5
LC (500 GeV)	8.7	2.6	3.3

TABLE 1. Final cross-sections (fb), for selections described in the text.

Y cut	BFKL$_{NLO}$ 3 TeV	2-gluon 3 TeV	ratio 3 TeV	BFKL$_{NLO}$ 5 TeV	2-gluon 5 TeV	ratio 5 TeV
no cut	9.7E-2	3.7E-2	2.6	2.8E-2	1.1E-3	2.5
$Y \geq 8.$	7.8E-3	1.6E-3	4.8	2.8E-3	6.2E-4	4.5
$Y \geq 9.$	1.9E-3	3.1E-4	6.2	9.3E-4	1.6E-4	5.7
$Y \geq 9.$ $\theta \geq 25$ mrad	3.3E-2	4.4E-3	7.5	1.1E-2	1.8E-3	6.2

TABLE 2. Final cross-sections (fb), for selections described in the text, after different cuts on Y

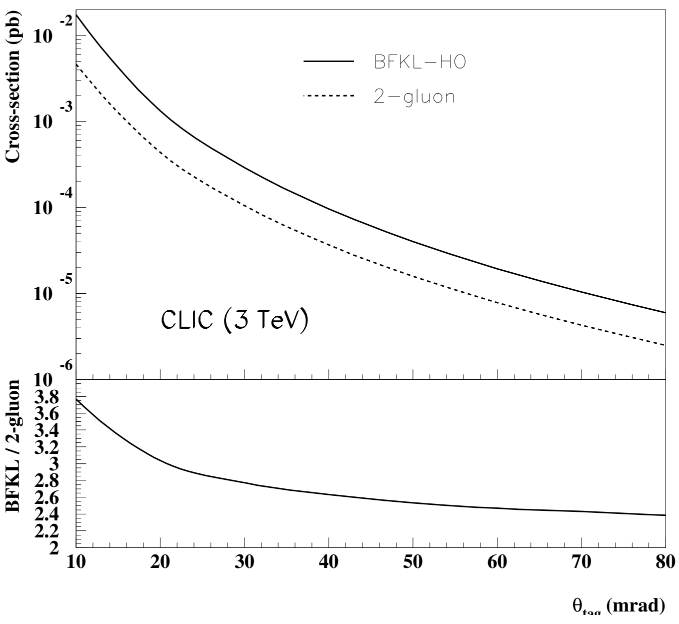

FIGURE 1. BFKL-HO, 2-gluon cross sections and their ratio as a function of the tagging angle for a 3 TeV CLIC. Leptons are tagged beyond 100 GeV.

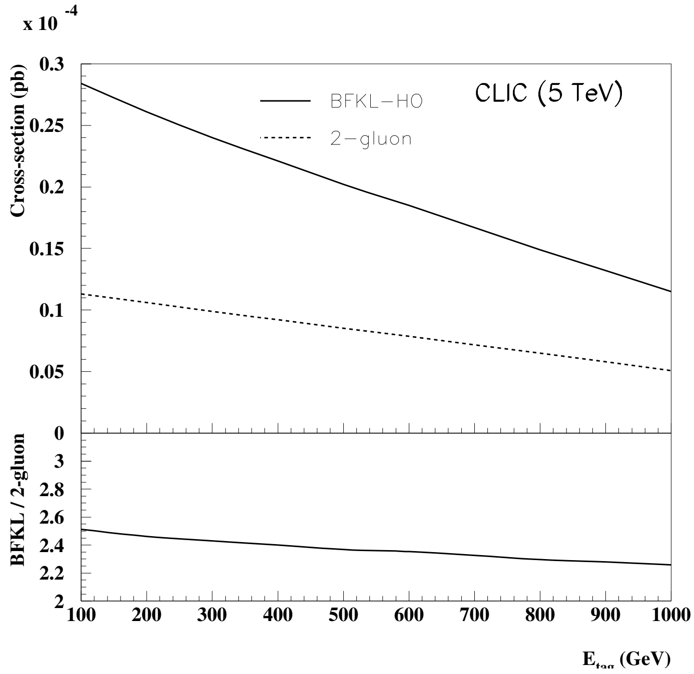

FIGURE 2. BFKL-HO, 2-gluon cross sections and their ratio as a functio of the tagging energy for a 5 TeV CLIC. Leptons are tagged beyon 40 mrad.

cross-sections ratios are reduced to a factor two to four. The Y dependence of the cross-section remains a powerful tool to increase this ratio and is more sensitive to BFKL effects, even in the presence of large higher order corrections. The uncertainty on the BFKL cross-section after higher order corrections is still quite large. We thus think that the measurement which will be performed at LC/CLIC should be compared to the precise calculation of the 2-gluon cross-section after the kinematical cuts described in this paper, and the difference can be interpreted as BFKL effects. A fit of these cross-sections will then be a way to determine the BFKL pomeron intercept after higher order corrections as we did for LEP or HERA data [8]. A possible meaurement at LC/CLIC would then be of great importance provided it is possible to tag electrons at low scattering angles.

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